Introduction To Adaptive Signal Processing

Prof. Mrityunjoy Chakraborty

Department of Electronics and Electrical Communication Engineering

Indian Institute of Technology, Kharagpur

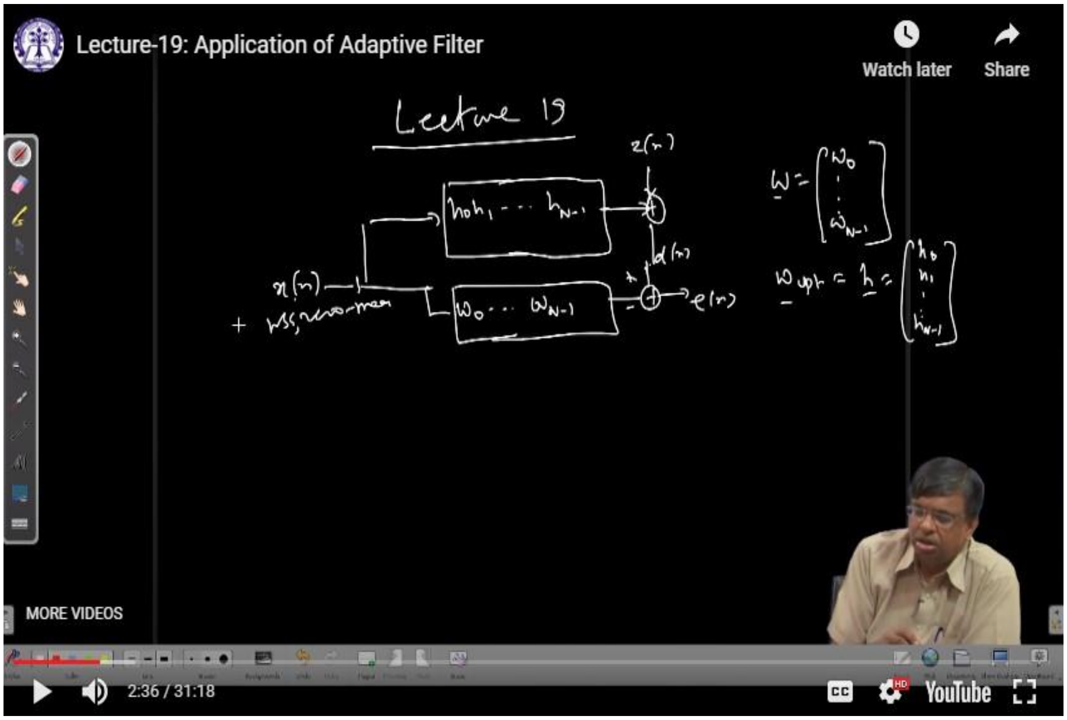
Lecture No # 19

Application of Adaptive Filter

Okay, so in the last class we are considering applications. So, one category was the forward modeling and that time what I did I showed that if there is an unknown system with some impulse response coefficients if I have system if I have model for the system, I do not know maybe I took up to capital N, so okay. So, if you give an WSS zero-mean random process at the input and the noise which gets added in various parts of the system the entire noise effect you can add at the end call it Zn and this noise is zero-mean and statistically independent statistically independent with Xn then if I we have shown that if I have an optimal filter W0 dot dot dot Wn minus 1 and this I take as dn. So, essentially this error is to be minimized is variance is to be minimized this is dn okay. Then we showed that optimal filter there is W is W0 dot dot dot Wn minus 1. So, W opt will become W opt is W opt actual optimal filter is nothing, but the filter coefficient vector H. So, if I can compute W opt from the study autocorrelation matrix of Xn and cross correlation between dn and Xn okay. There is R inverse P in that case from W opt W opt gives us this unknown system parameters which is fine.

$$\underline{w} = \begin{bmatrix} w_0 \\ \vdots \\ w_{N-1} \end{bmatrix}$$

$$\underline{w}_{opt} = \underline{h} = \begin{pmatrix} h_0 \\ h_1 \\ \vdots \\ h_{N-1} \end{pmatrix}$$



But as it is you know I mean we often do not know the statistical behavior or statistical properties of the input two inputs are here one is the Xn input another is the desired response input I mean I need the statistical properties like the autocorrelation on the input and the cross correlation between the two ok dn and Xn or all samples of Xn. Now these things may not be static in time we change with time that is why we cannot have a you know optimal filter computed once for all ok. That is why you have to make it adaptive. For instance, suppose this filter coefficients suppose Xn is such that the input autocorrelation matrix is fixed it does not change input statistics is not changing. But still this if the system coefficients are changing from time to time maybe very slowly, but changing from time to time then when we have one set of coefficients then I know what is dn, dn is Xn transpose H, Xn is the data vector we have done all these in the last class input data vector at the current time.

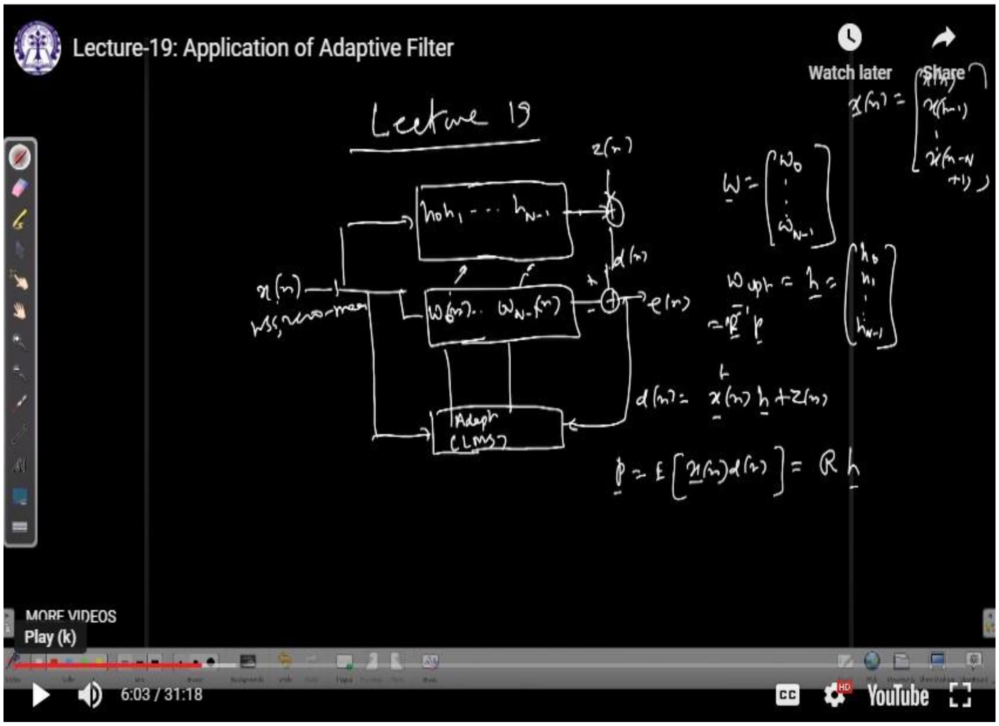
So, this is a convolution output X transpose H here plus Zn and we know what is p vector because W of t is R inverse p, R is fixed input statistics is not changing suppose. But if the system coefficients change that will affect this output and therefore, dn and if dn is affected p will change that is what I am saying I am I am showing. So, p is we know E of Xn dn. So, as we have seen in the previous class if I replace dn by this E of Xn with Zn will be 0 because just realistically independent each have been 0 means it will be E of Xn into E of Zn each is 0 mean gone. So, we will left with E of Xn X transpose n H.

$$d(n) = \underline{x}^t(n)\underline{h} + z(n)$$

So, it will be R H. Now suppose right now I have some H vector ok. So, for that a particular p I calculate R inverse p that gives me optimal filter from that I find out H, but if H changes from time to time ok then what is down p will not be p in future ok. So, current R inverse p current W of t will not be equal to your H vector in practice. In fact, this filter coefficients may be continuously time varying, but in a slow manner. So, in such cases I cannot go on computing R inverse p like this right at all points of time.

$$\underline{p} = E\left[\underline{x}(n)d(n)\right] = R\underline{h}$$

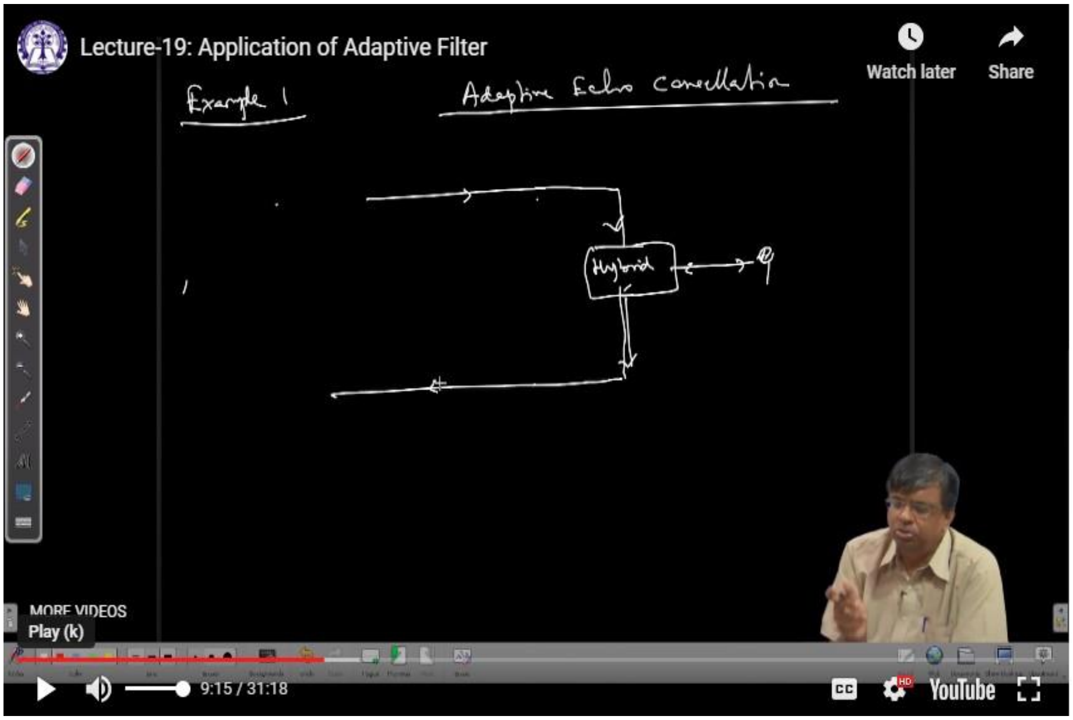
So, I have to make it adaptive. So, therefore, if you want to make it adaptive it will be like you know there will be function of n function of n these are adaptation part maybe a adaptation example LMS. So, you take this E n and use the LMS rule to change these coefficients. These are schematic we have shown for adaptive filters right. So, this forward modeling of adaptive field forward modeling by adaptive filtering forward modeling of an unknown time varying system by an adaptive filter these are general scheme. Now I will take up some special cases of this special applications of this ok.



Example 1 adaptive echo cancellation what happens is it is considered a telephone line this is line actually it is not a single line along with this there is a ground line, but ground line I am not showing this common for all presence. So, through this signal comes speech signal from a distant speaker comes to this end and there is something called a hybrid device. What it does is this at this end I got a speaker is a microphone a speaker. So, what the hybrid does is this this outgoing signal it channelizes to this side and this incoming signal it channelizes to this speaker side. So, he is ideally the speaker yeah this speaker here will be hearing this incoming signal, but his own speech will not come back to him will go in this path ok this is the ideal hybrid.

But problem is these ideal hybrids do not exist in practice often they are found to be leaky, leaky in the sense part of this signal this signal ideally should have gone entirely to the speaker side none should have leaked out. But in practice what happens the part of the signal may be after passing through this filter in the another forms some kind of distorted form goes through the hybrid in this direction. So, it gets mixed with the incoming speech from this speaker which goes of course, in this direction and this leakage component also goes in this direction. So, this leakage component also reaches the distant speaker he hears it as an echo and this echo is often very disturbing especially if the distance is large like what you have been satellite phones. If the distance is large then obviously, there is a gap between you know what he told and what is hearing because what he told it was some time ago they need to call the time to reach here and coming back twice the journey twice the distance by that time his own speech has finished.

But even if this guy does not speak even if he has his own speech has finished that now part of it now comes back as an echo. So, he distinctly hears the echo which is very disturbing ok and this is not only in the case of telephone line even I mean one faces the same problem in data network. In data network this will be carrying data incoming data that will go to a server at this end and servers it will come through this path. So, if the hybrid is leaking part of incoming data will get mixed with the server's data. So, whatever goes in this direction will be erroneous ok it will be like you know added with some kind of noise coming from this side.



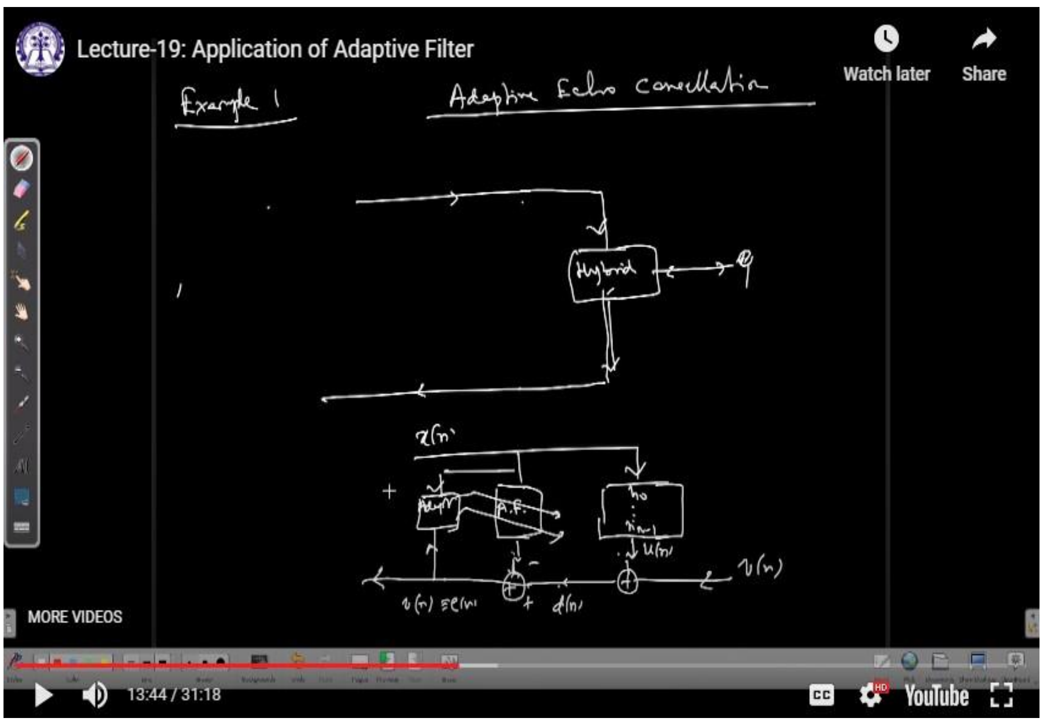
So, we have to cancel this. So, what we do the hybrid we model as an unknown system we have already shown that unknown system in the previous slide previous page unknown system and model it by an FIR filter all right model it by an FIR filter. So, this actually is like this. So, the leakage generating mechanism that part not the entire hybrid the leakage generation mechanism which takes this as input and gives a leakage component here that we take as I mean that that process is my unknown process unknown system ok that leaking phenomena and the leakage part of this hybrid ok. So, that is an unknown system.

So, we take it to be linear and model it by an FIR filter maybe a 0 dot dot dot a is n minus 1. So, if xn is the input you know this xn convolved with them this coefficient filter coefficients that will be the output this is the leakage component and that gets mixed with this fellow speech. So, this fellow speech is coming here like this maybe Vn. So, what goes out is this component ok let this component be un. So, this is the leakage disturbance un plus vn goes out to this direction.

So, what I do is like an unknown I am I am going to see it in a system identification framework where this is an unknown system it is driven by an input and this is the output and this is like a noise in that context in the previous system identification context there was an additive component which was noise there. It will be something like this because v of n and x of n they are statistically independent because this is from this speaker this is from this speaker there is no connection between them and they are you can take them to be 0 means. So, they are 0 means statistically independent ok. Therefore, the previous theory the theory of the previous thing you know that system identification thing will work. If I could have an optimal filter here ok this output and if I take this as dn because remember in that unknown system identification problem system output plus that additive component which was noise there in this case it is just speech signal. So, it is not a noise, but then it is statistically independent with the xn and 0 mean. So, you can apply the same theory here.

So, this system output plus this component they get added if I use this as dn and this as xn and this is an optimal filter if I have filter then we know that optimal filter will be nothing, but this fellows h0 to hn minus 1 which means un and this will be same. So, this will also produce un if I could make an optimal filter it will be un. So, if I subtract un from here. So, this un will go. So, what will go here is vn.

So, the echo part ideally is cancelled, but optimal filter you know we cannot do because system parameter may change input statistics may change from time to time. So, I cannot have because the speaker may change. So, the statistics of input can change optimal filter can change ok. So, I cannot put an optimal filter here I have to make it just adaptive. So, adaptive filter AF adaptive filter.



So, what it does filter output dn minus the filter output this and this xn. These are used in adaptation adapt the filter output dn dn minus this is the error error is that is equivalent to error. So, en is given to the error adaptive filter xn is given and it changes the coefficients like this. Remember as when it is adaptive filter I have seen adaptive filters converge only mean not absolutely. So, in the previous slide also I forgot to mention for a minute that adaptive filter coefficients if they are I mean under optimal filtering they would be ideally equal to this unknown system coefficients, but here adaptive filter means the coefficients will be each coefficient will be fluctuating around the corresponding optimal value ok, but mean that is expected value of that fluctuation will be equal to the v true system value this parameter. So, we make sure that the fluctuation ranges are small by adjusting mu.

In fact, it is known that in mu the step size in the LMS is made very small fluctuation range will be small and with that we have we are happy. So, technically it is not an actual optimal filter. So, it will be it will not be exactly get the un there will be some error you know coming because of the fact that adaptive filters do not converge absolutely they converge only mean. So, anytime the coefficients here will not be exactly equal to h 0 it will be fluctuating around I mean particular h say you can say the h 0 corresponding coefficient here because it is adaptive it will be fluctuating around h 0. So, that mean of the fluctuation is h 0.

So, it will not be h 0 always. So, since it is fluctuating and same for all the coefficients the filter output will not be exactly equal to un there will be some error, but the error will be less because by adjusting mu we make sure the fluctuation ranges is very small ok. So, these are very simple example of adaptive eco cancellation and in this topic lots of research have gone more and many interesting structures have come up and all tons of research have gone into it, but that is all old now 17, 18, 19, but still a very important topic.

Next example. This is the example I gave in the very beginning of this course that is maybe in the digital domain you are given a signal x n which has a discrete time cosine I mean sinusoid function like a cosine sin a cosine some particular omega cc for carrier omega c a n plus phi phi can be a random phase.

So, it is a random signal you can say every time you observe you get I mean same cosine waveform, but time shifted because of this phase phase takes values from 0 to 2 pi randomly. So, it is a random thing plus noise. So, this is a sinusoid signal which is mixed with some noise of high reasonable I mean some high power stationary is not high. Then I said that if you want to remove noise as much as possible you should design a band pass filter where you know very short very narrow band center frequency omega c and here also very narrow band pi minus pi.

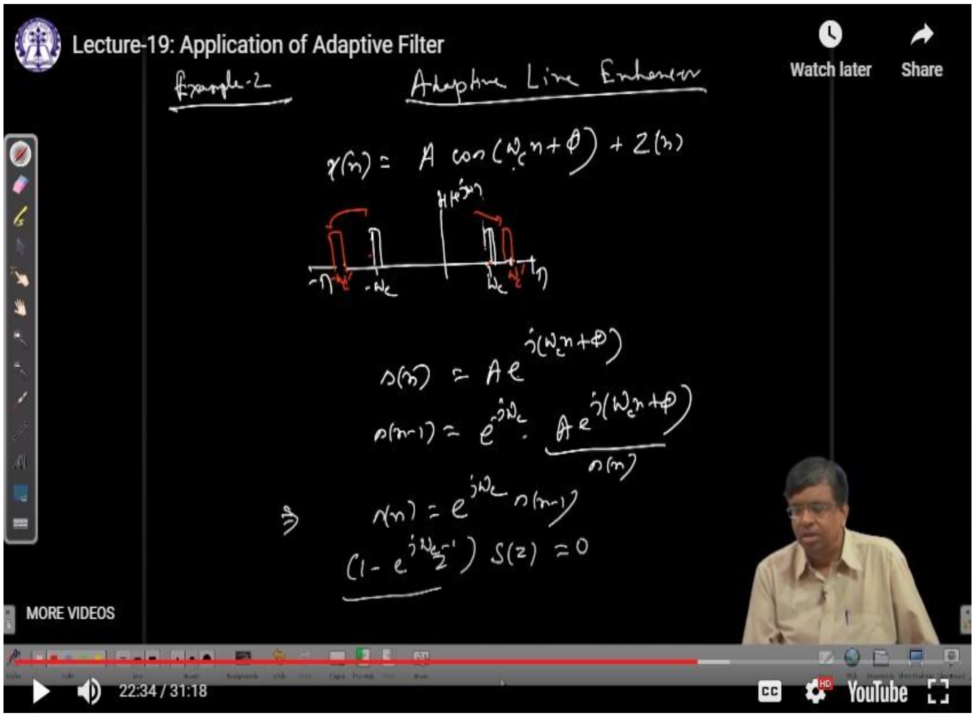
This is your ok. If you can design like this then what will happen this noise is white band maybe white noise. So, all the noise outside the pass band will go. So, most of the noise almost 99 percent may say you know will be gone because the filter is very narrow in all other places 0, but it will capture this signal because it is center frequency omega c. So, in impulse response will be I mean if you take a discrete time Fourier transform that will be an impulse at omega c and that minus omega c. So, they will pass and therefore, you will be able to get by this signal maximally, but problem is in this approach is there is after some time the frequency changes.

Then this filter will remove this input also because now omega c has changed to some frequency say here some frequency may be here and here. So, my pass band should have been here now ok, but there is no other case there is my pass band should have been here and here. So, it is now omega c prime minus omega c prime omega c has changed to omega c prime, but there is not the case I have designed the filter once for all. So, filter remains as it is centered at omega c and minus omega c. Therefore, in this frequency changes from omega c to omega c prime after some time the frequency changes it falls outside the pass band of this filter ok and therefore, signal also is gone.

So, this is not the way to do ok. So, how to still recover this adaptively that is what this adaptive line is. Line because I mean if you take this as a random process its autocorrelation function again will be cosine because it will like you know I mean a cosine this and a cosine omega c within bracket n minus k in k is the gap plus phi. So, a square and cos into cos will give rise to cos effectively it will be a cosine function of k. You can work out take it as an exercise. So, if you take discrete time Fourier transform of that you will get the power spectral density power spectral density if it is cosine it will be impulses, impulses at omega c and minus omega c that here and here impulses are like line. So, line spectra power spectral density coming from here and Zn is a white noise in general.

So, its presence will be everywhere. So, our purpose is to enhance the line that is removing the noise as much as possible in a and therefore, enhancing the impulses ok. That is why it is called line enhancer it is done adaptively. So, that even if the input frequency changes our previously redesigned filter adjust itself adapt itself to the new filter a new filter with this pass band and so on and so forth ok. That is what I will do again we will be using the concept of the forward modeling, but before that you need to know something some fact which maybe you do not know. So, see one thing suppose I give you Sn as some a e to the power j omega cn plus phi instead of cosine I am taking complex sinusoidal fine cos you can always write as e to the power j theta plus e to the power minus j theta like that e to the power j this plus e to the power minus j this by 2 ok.

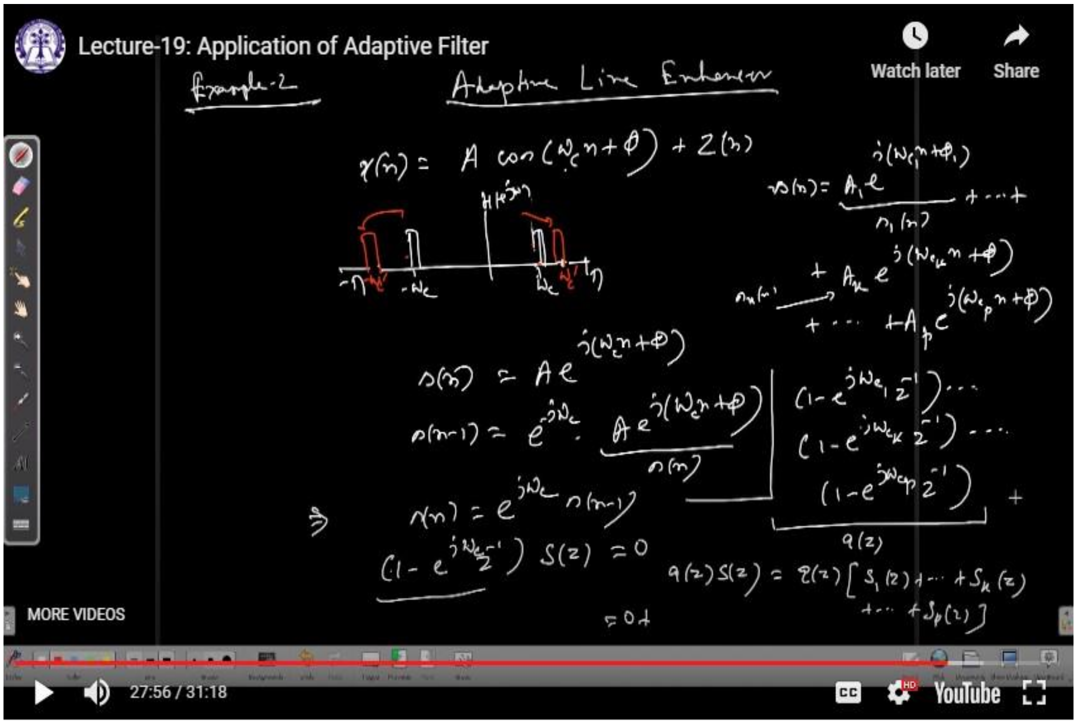
So, Sn what is Sn minus 1 if you put n minus 1 here. So, one component e to the power minus j omega c ok that will go out we put n minus 1 here. So, we get into minus 1 that part e to the power minus j omega c goes out and remaining remains same e to the power minus j omega cn plus phi which is Sn which means Sn is e to the power plus j omega c is n minus 1. So, if I do the Z transform of both sides ok it will be capital SZ and here it will be this times Z inverse capital SZ if you take everything on one side it will be 1 minus. So, this is the first order polynomial in the Z inverse it is such a polynomial it is called annihilator, because it annihilates SZ if you multiply this SZ by this you get 0 all right.



Now suppose you got not just 1 you have got many hm suppose Sn has some a1 e to the power j omega c1 n plus phi 1, which I call signal S1n with another component a2 or maybe dot dot dot dot ak take a general equation e to the power j omega ck n plus phi dot dot dot dot dot maybe you have got p fellow e to the power j omega c p n plus phi ok. This is my Skn, this fellow is Skn, like that ok and if I had only this then I know if I construct a polynomial like it is 1 minus e to the power j omega c1. So, omega c1 Z inverse that into S1Z 0, similarly here from here if I call it Skn, and if I construct a polynomial 1 minus e to the power j omega ck Sk omega ck Z inverse that into capital Skz equal to 0 and likewise. So, suppose I form a product polynomial that is 1 minus e to the power j omega c1 for this first guy Z inverse. So, polynomial for the first dot dot dot dot I am multiplying them now 1 minus e to the power j omega ck Z inverse dot dot dot dot dot 1 minus e to the power j omega c p Z inverse.

If I have this product ok and this product let me call not a let me call may be qz then question is what happens to qz into Sz? Sz is summation of S1z plus S2z plus dot dot dot. So, this is nothing, but qz into S1z dot dot dot S kz dot dot dot S pz. Now qz into S1z ok qz is this much into S1z, but this is a product of the first order factors these into the. So, I can always bring this factor in the first position is not it I am multiplying the factors say a plus b into c plus d is same as c plus d into a plus b. So, I can bring this in the first and then if I multiply S1z by that. So, this first polynomial this will first work on S1z, but that will be 0 because of this theory.

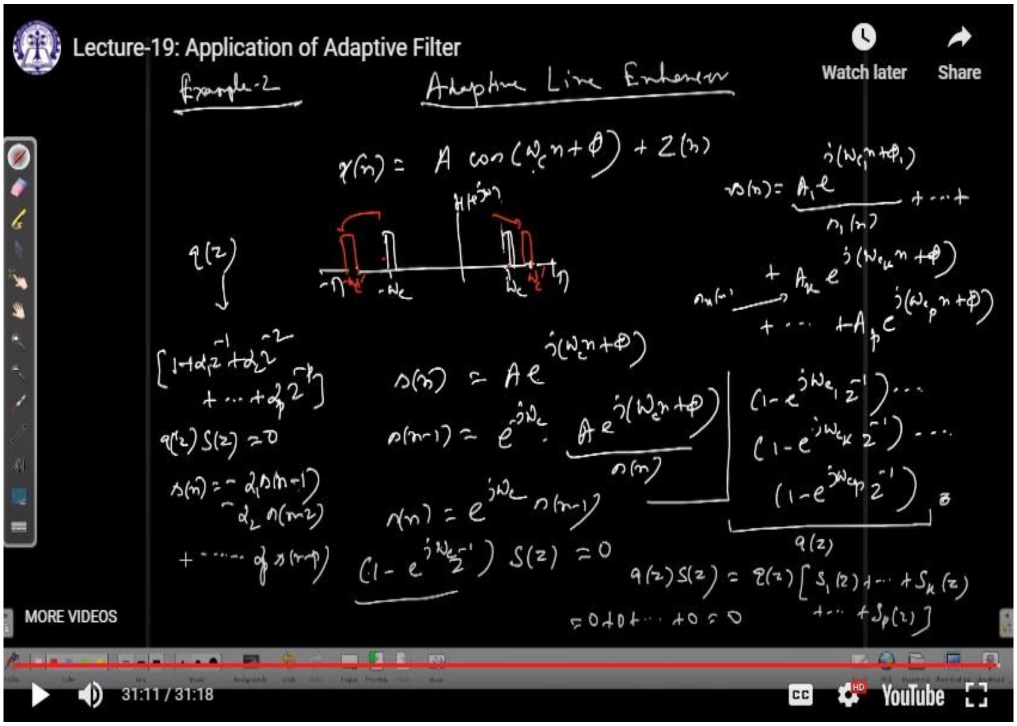
So, if other polynomials then multiply still they will multiply 0 anything multiplying 0 is 0. So, that will become 0 I hope you understand this S1z and qz. Qz has this factor S1 has e to the power j omega c1 ok and e to the power j omega c1 z inverse 1 minus this this first factor in this chain the product, but product of these factors I can always shuffle I can bring the first guy in the very beginning. So, that into S1z because of this theory is 0 and remaining factors multiplying 0 is 0. So, qz into S1z would be 0 plus dot dot dot say you consider Skz this guy when I am multiplying Skz by this qz qz times Skz I will pick up this polynomial 1 minus e to the power j omega c k z inverse. I will bring it to the front and so, that will first work on that polynomial will first work on Skz and that is 0.



So, whole product is 0 understand when the same qz I am using to multiply Skz that time I will pick up not the first polynomial, but the kth 1 minus e to the power j omega c k z inverse. So, if I bring it to the front. So, first that will multiply Skz, but because of this theory that is 0. So, I will get 0. So, that way 0 plus 0 plus dot dot dot dot 0 which is 0.

So, this means qz into Sz, this is my qz right, this is my qz, qz into Sz is 0, but now if I concentrate the product 1 into 1 into 1. So, first term will be 1 and there are p terms ok. So, it will be all together highest power will be z to the power minus p and coefficients can be complex I do not mind. So, it will be a polynomial like some 1 plus just a minute it will be a polynomial like you know 1 plus maybe alpha 1 they are inverse alpha 2, they are inverse 2 alpha 1 alpha 2 are coefficients you multiply and carry out the whole product these are the coefficients they can be complex, but this is a polynomial up to order up to power z to the power minus p because there are p factors. So, this is my qz when you multiply this is my qz.

So, qz into Sz equal to 0 that means, in time domain it will be what Sn, Sz into 1, Sz if you keep that on one side Sn and bring the others on the left hand side. So, minus alpha 1 z inverse Sz. So, minus alpha 1 z inverse Sz S is the summation of all. So, alpha 1 Sn minus 1, similarly minus alpha 2, alpha 2 Sn minus 2 dot dot dot dot minus alpha k, but no need to have alpha k you can directly go to the last term alpha p Sn minus p. That means, in this case current sample can be given exactly as a linear combination of past p samples that is why it is purely predictable. Any current sample of Sn can be given obtained by a linear combination of past p samples as though if I have a filter that takes Sn minus also as the input and I have this coefficient as the filter coefficients minus alpha 1 minus alpha 2 minus alpha p then filter output will be same as Sn ok.



So, this fact will be used in that adaptive line in answer at that I will show in the next class ok. Thank you very much.