Semiconductor Device Modelling and Simulation
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Lecture-49
Hydrodynamic Model

Hello, welcome to lecture number 49. The topic is hydrodynamic model. In previous classes, we have discussed about the drift diffusion model. And in this lecture, we will consider extension of drift diffusion model and we will try to incorporate the changes.

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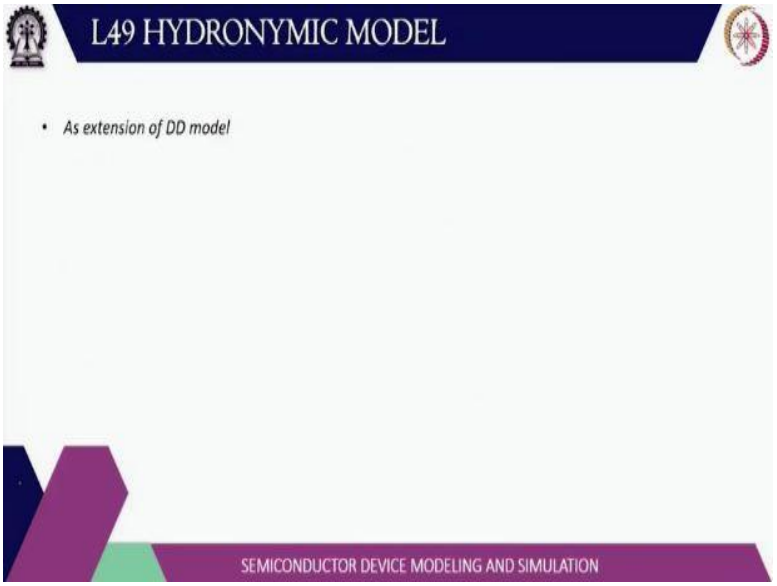

So, that the validity of the drift diffusion model can be extended.

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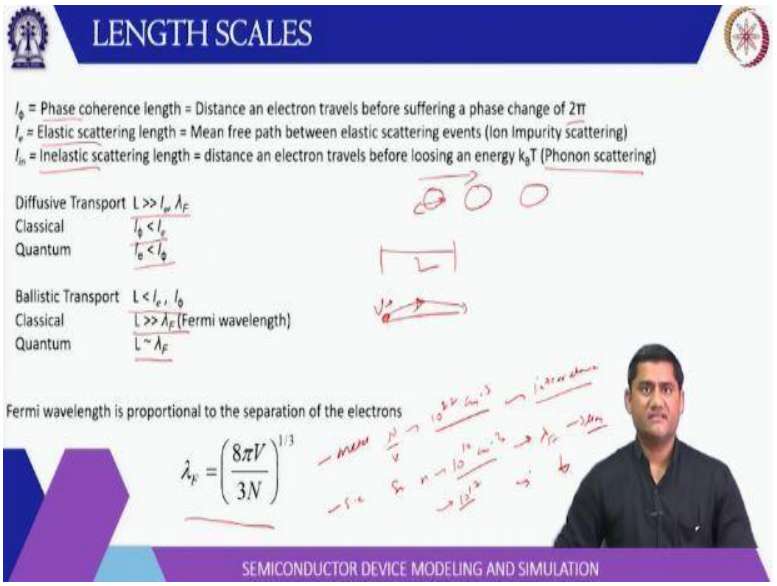

So, before getting to the topic, let us consider the different length scales that are encountered by electron traveling in the lattice. So, one is called phase coherence length. So, phase coherence length is basically a length, which electron will travel before suffering scattering or any event which will change its phase by 2 pi. So, typically the elastic scattering they give rise to certain phase change and inelastic they are randomized basically the phase.

And there are some events, such as spin flip and all. There the phase basically, without being introducing another type of scattering. So, this is the phase coherence length. Then, there is elastic scattering length and this is a length corresponding to the scattering events. So, it is basically the length scale or the mean free path between consecutive scattering events. And elastic scattering events are those where there is no change in energy.

So, for example ionized impurity scattering, you can have defect the scattering or you can have other examples like alloy scattering, surface scattering. So, these are examples of elastic scattering. So, where there is no moving particle is there as such. Then there are inelastic scatterings and corresponding to inelastic scattering, there is inelastic scattering length.

That is again the distance stable between two inelastic scattering events or distance table before losing an energy equaling to some thermal energy; it is also called phonon scattering. Now, it can be acoustic phonon or optical phonon, different types of phonon scattering. Phonon basically involves some kind of movement. So, there are moving particles basically. So, this lattice vibration as a function of lattice temperature; this gave rise to some change in the potential and that deformation potential basically scatters these carriers.

If you consider a crystal at zero Kelvin, then also this electron will see a variation in the crystal potential, but we do not call that as scattering because the effect of this periodic potential is already taken into account in the band structure, but as a temperature increases, these atoms start to vibrate. So, this vibration basically leads to a potential profile which is different from the original potential profile.

So, there is some you can take the difference and say this is the difference potential that they will experience and that gives to elastic scattering. Now, based on the length of the device, for example, channel length, we can say whether the transport is diffusive transport or it is a

ballistic transport so, when length of the device is larger than this elastic scattering length or the Fermi wavelength then, we say it is a diffusive transport.

And of course, if the phase coherence length is less than the elastic scattering length, then we say that is classical. So, that means classical laws are applicable there. And if the elastic is getting is length less than the phase coherence length, then we say this is a quantum transport. So, then you have to basically consider the quantization effects into the transport mechanism. Then of course if the length is less than the elastic scattering of the phase coherence length, then we set is ballistic transport.

And of course, in the ballistic transport, basically the length is not enough that electron will encounter scattering, so because the device length is less than the scattering length. So, the number of scattering events is negligible. So, we can say this: the current will basically depend on how this electron what initial velocity this electron goes and cross over this region basically or this length.

So, it basically depends on the initial velocity of this carrier, it is like a ballistic missile or like bullet it just goes, basically. And again, in the ballistic regime also if the length is more than the Fermi wavelength then we can apply the classical laws and if length is comparable or smaller than the Fermi wavelength then we apply the quantum mechanical loss. Now, Fermi wavelength is basically a parameter, which is related to the number of carriers. So, for example, if you consider metals, number of carriers is same as number of atoms.

So, for example, typically these are 10 is power 22 per cubic centimeter. So, that means this is one over N; this will be N by v**;** this 10 is to 22 per cubic centimeter. So, if you take 1 by 3 root of this thing, it will basically correspond to the interatomic spacing. And that is possible because in metals all the atoms give one electron at least. So, the number of electron concentration is quite high.

And therefore, this Fermi wavelength is small. Now, in case of semiconductors, for example, silicon n is around 10 is to power 10 per cubic centimeter, thus entry security concentration. So, in this case, this Fermi length can be actually quite large; it may be water of micrometer. Of course, if you dope it, then the carry concentration becomes 10 raised to power 17 or 18 then you basically reduce the Fermi wavelength. So, these are the length scales that guide us

with mechanism we should consider, whether diffusive or ballistic or classical or quantum. So, this can be determined by these length scales.

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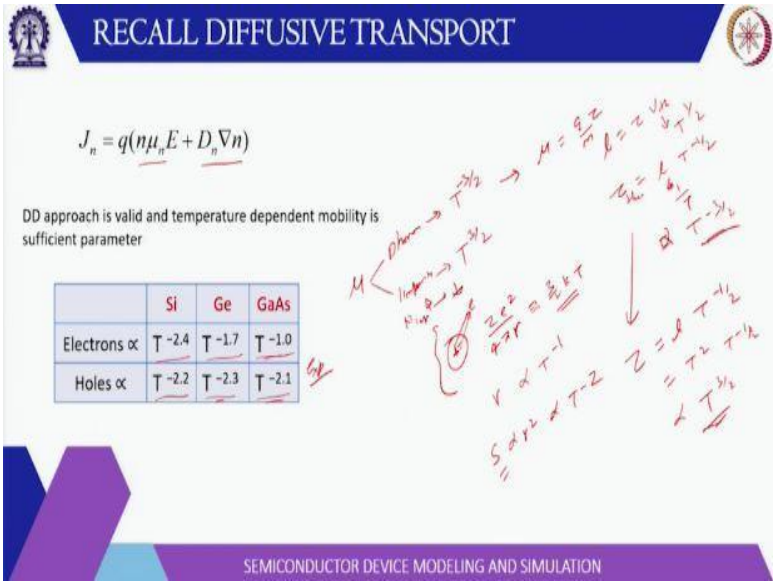

Now, let us recall the drift diffusion model. So, in that model we wrote the expression for the current density in terms of two terms: One is the drift current mu E, other is a diffusion current which is proportional to the derivative of the carrier concentration. And of course, this drift diffusion is valid and the temperature dependent mobility is basically sufficient to model the effects basically.

So, for example, we mentioned that mu in semiconductor is proportional to is a function of temperature. And two main scattering mechanisms: One is the phonon scattering, other is the ionized impurity scattering. So, the temperature dependence as seen by these carriers for example electrons, holes in silicon, it is order of 10 raised to -2.4, T to the power -2.2; for germanium, for electron is T raised to power -1.7.

For holes it is T to the power -2.3; for gallium arsenide, is T to the power -1 and for holes in gallium arsenide is T to the power - 2.1. So, these are the experimentally measured characteristic, but we can get some idea. The phonon scattering generally has a dependence T to the power -3 by 2. So, that means the mobility decreases as T to the power -3 by 2. So, as the temperature increases, the mobility decreases. In case of impurity scattering is T to the power +3 by 2. So, as the temperature increases the mobility increases. That means the scattering is reduced.

Although if you increase the impurity concentration then that will reduce the mobility. The way these scattering takes place, for example in phonon. We know that mu is q tau by m, where tau is the some kind of relaxation time. And we can say that now the length you can say is tau times the thermal velocity. And thermal velocity, we can say it is half m v square is 3 by 2 k T.

So, you can set is proportional T to the power half. So, if you find tau here. So, tau can be written as L into T to the power minus half and where L is basically the length scale of the scattering. If you assume that it is inversely purpose to do the thermal energy. So, then for tau thermal will be basically 1 by T, the length corresponding will be 1 by T. So, that becomes T to the power -3 by 2.

So, that is how you get this dependence here due to the phonon scattering. So, in silicon we see it is close to 10 is to power -2.4 and T to the power -1.7 T to the power -2.2. So, that means you know phonon scattering has a major role here. If you consider ionized impurity scattering here, so, let us say this is your ion. So, it will have certain potential energy. So, if you say that z is the size of the ion, so z e square by 4 pi epsilon r**.**

So, that will be the potential energy due to this ion and the electron and if you compare this to let us say 3 by 2 k T, so, you can say that the effective radius because this is the energy of the electron, the thermal energy so, which is basically half m v square. This is a potential energy due to this charge. So, if they are equal first certain so that will give you some r. So, this r will be proportional to T to the power minus r.

And then, if you consider the cross section of this impurity the scattering cross section because, it will have certain area in that area it will basically have a greater influence. So, the cross section area can be proportional to r square and that because T to the power -2. And then again, you apply the same thing from here. So, the tau will be L into T to the power minus half.

So, L will be now T to the power 2 because, this is universally proportional to this cross section area and then this is T to the power minus half. So, that tells you that tau is around T to the power 3 by 2. So, that you can roughly see these things, that how the phonon is scattering

and the impurity is scattering affect the dependence. So, these two basically are the mechanism, which recombine and give rise to these different temperature dependence.

Apart from this, we have the effective masses. So, effective mass in gallium arsenide is quite a small. So, therefore, they have much higher mobility in case of gallium arsenide and even in germanium compared to silicon.

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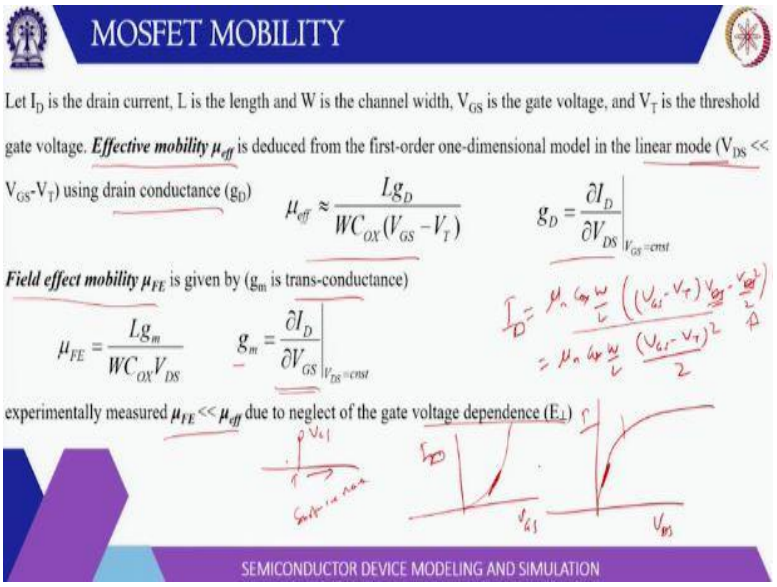

So, another way to measure the mobility is basically through the Mosfet Iv characteristic. So, in mosfet, you have this current equation that is valid for long channel mosfet. So, I = mu n C ox W by L, then V GS – V T times V DS – V DS square by 2. So, that is linear region and in saturation region it is mu and C o x W by L then V GS - V T square by 2. So, this is basically V DS = V GS – V T in saturation.

So, this you got this equation. So, one way to calculate the mobility is called effective mobility and that is measured in linear region so, where V DS is much less than V GS - V T. And then if you take the derivative in linear region, you can ignore this part. So, your g D is del I D by del V DS. This is I D. So, that will come out to mu and C s W L by V GS – V T and then if you rearrange it. So, mu n can you turn as Lg D by WC ox V GS – V T. So, you get this one in linear region.

So, this is your Iv curve either says V DS for different V GS you have different one or you have I versus V GS. So, it goes basically something like this. So, g D is this curve here it is in linear region and you get mu effective from the slope of this region del I D by del V DS that is

your g D and another mobility is called field effect mobility that is obtained from the slope of del I D by del V GS.

So, you have this slope here**.** And this just g m is called trans-conductance and g D is called basically drain conductance**.** So, again, again in the same linear region here. So, if you take the derivation, which is V GS, then you will have V DS here basically. So, this V DS comes in the denominator. And experimentally measure that field factor mode is smaller than the effective mobility.Now, what is happening here? Here we are basically wearing the gate voltage.

So, where is the gate voltage, then if you increase the gate voltage, then this carriers in the channel they will come closer to the surface or the interface of silicon and oxide and there will be a surface scattering. So, n surface scattering will reduce the mobility. So, due to this gate voltage or the perpendicular field dependence perpendicular in a sense, this is the horizontal field and this is a perpendicular field due to the gate voltage. So, this reduces the mobility. So, that is why field effect voltage is smaller than the effective mobility.

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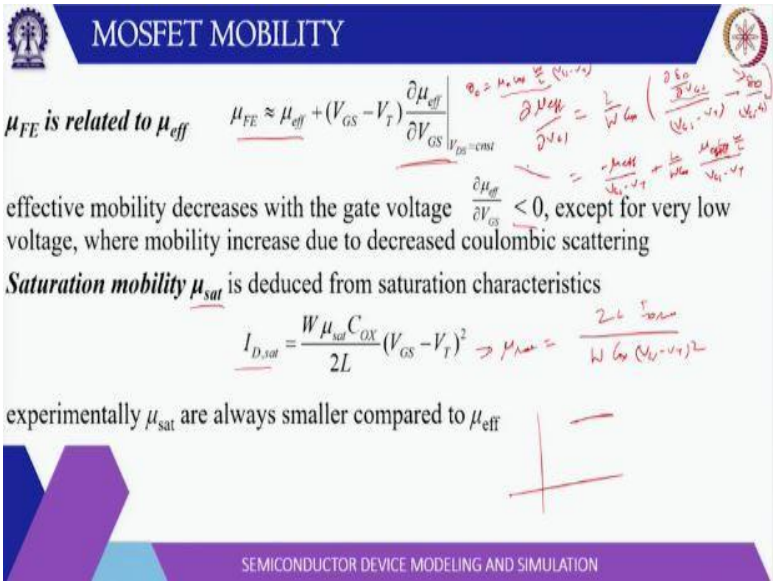

Another relation with that can be found between the field effect and effective mobility is that mu field effect is mu effect a + V GS – V T times the derivative of mu effective. So, that of course, you calculate that generally if del mu effected by del V GS is less than 0**.** So, that means effective mobility decrease with the gate voltage, except for very low voltage, where mobility increased due to increased coulombic scattering.

Another mobility you can define is the saturation mobility. So, there we have the expression for I D sat then we rearrange that, mu sat will be basically 2L I D sat divided by W C ox V GS – V T square. So, this mu sat will be even smaller, because you are already in a region, where there is no change in the current. So, regarding this d mu effective by del V G S you can write expression, that because if you see here, mu effective is Lg D by W C ox**.** So, and Lg D is del I D by del V DS.

So that you just work out, I will give you the expression that I have derived. So, this will come to L by W C ox times del g D by del V GS divided by V GS – V T, then minus g D by V GS – V T square and close the bracket. So, that will basically come out to be minus mu effective, that is the second term due to g D by V G S – V T and plus L by W C ox times V GS – V T. And del g D by del V GS, if you apply this to the expression of g D and that g D is del I D by del V DS. So, this is mu and C ox W by L V GS – V T.

So, if you take del g D by del V GS you will get mu n C ox W by L. So, mu n C ox W by L. So, we can say mu effective, then if you arrange it, you can relate mu effect with the mu field effect velocity mobility. So, this is mu field effect. So, you get basically this relationship from here.

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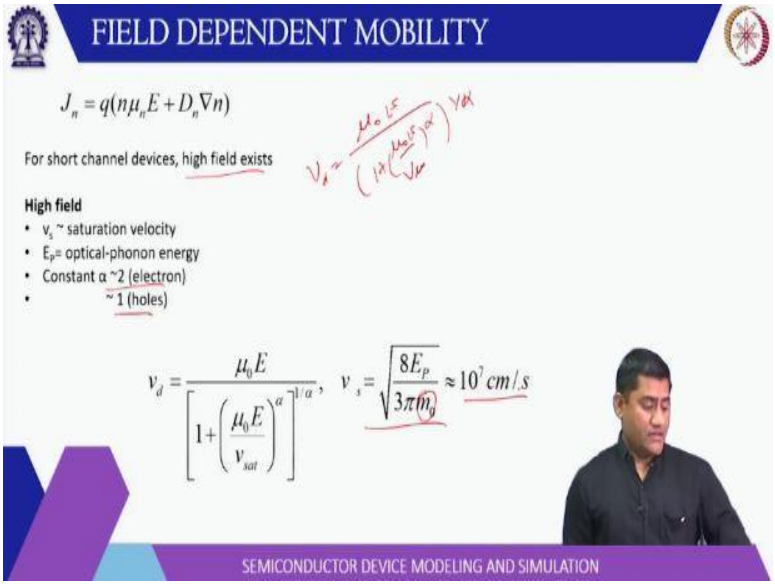

Then, if you go from mosfet mobility to account for higher electric field we can define the field effect or the field dependent mobility. So, there is a model in (()) (18:07) also which called field mop. So, that basically account for the field effect dependent mobility. So, for

short channel devices, high field actually exist. So, in high field of course there is a velocity saturation effect and then there is involvement of optical phonon.

So, you can say that velocity is modelled as drift velocities is mu naught E that is the general relationship and then there is a factor here. So, which is 1 + mu naught E by V sat. Then this power alpha and this power 1 by alpha. So, this alpha r basically the fitting parameters and it is found that alpha value is around 2 for electron and 1 for hole. And the saturation velocity is again determined by the phonon scattering. So, it is founders around 8 E p by 3 pi m. So, around 10 is power 7 centimeter per second.

So, that is the fixed velocity. Now, here I have used m naught, but in different semiconductor, there is a different effective mass. So, in gallium arsenide this m naught is basically some with the factor, which is much smaller. So, this actually saturated velocity is larger. In silicon short option is for 7 only. So, the range is basically same, but if semiconductor has a smaller effective mass, then this saturation velocity actually increases. And these semiconductors also have higher effective mobility.

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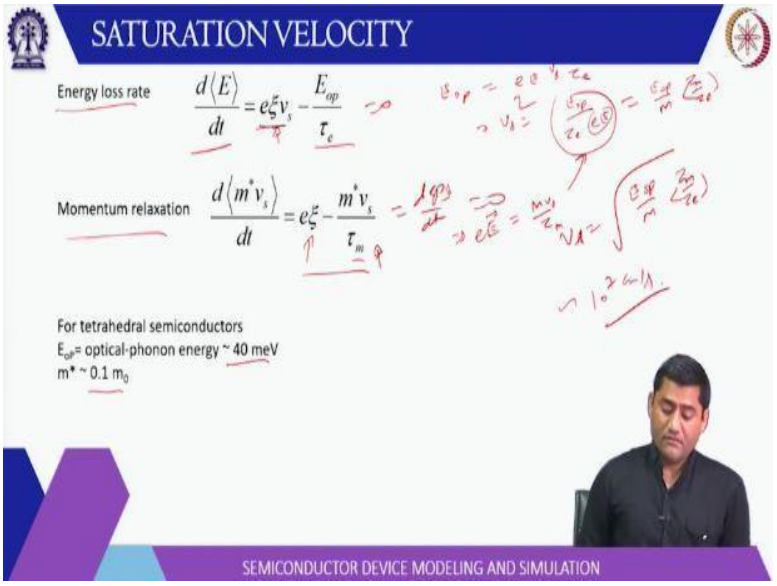

So, if you go here the saturation velocity is obtained from their characteristic. So, you can consider the energy loss rate or the momentum relaxation. So, if you consider the momentum relaxation. So, this is basically d p by dt**.** Now, d p by dt we have discussed earlier also the rate of change in the average momentum. So, this momentum is coming from the field. So, there is an electric field, so which is basically enhancing the momentum.

Then there is retardation due to scattering, so that is p by tau or m v by tau. So, these two facts are balancing and they give rise to a constant momentum rather than increasing momentum. Similarly, the energy loss rate the rate of change of increase of energy is the force times the velocity. So, the force into distance is the change in energy. So, if you apply a certain force f2 object and it travels distance x, so the energy gained by the particle is f times x.

And if you calculate the rate of change, then it becomes either rate of change of the force or rate of change of the position. So, because force is field is less affixed. So, you can write dx by dt, which is basically velocity. So, this is the increase in the energy due to the field, because field will accelerate and as it go ahead the energy will increase. Then of course it will interact with the optical phonons.

Then this phonon scattering will have some kind of will try to reduce the energy of this particle. So, this is E optical by tau e. So, tau e is the energy relaxation time and tau m is the moment of relaxation time. So, in steady state you can say this is 0 and this is 0. So, this tells you that E optical is around for steady state is e times electric field times V s times tau e and so from this you can basically calculate that V s is E optical by tau e times electric field.

And from this moment of relaxation, you can find what is the value of e times electric field, that is m v s by tau. So, if you substitute here you get e can be replaced as m v by tau. So, this becomes basically E optical by m and then so, this is tau m by tau e. So, momentary relaxation time by the energy relaxation and this v s goes here basically, because there is m v s. So, you can say this becomes v s square.

So, your v s is basically square root of E optical by m times tau m by tau e. So, this saturation velocity is dependent on the optical energy, so optical phonon energy by m. So, typical optical phonon is order of 40 millielectron volt. Let us say m star is 0.1 m naught, so that will give you some velocity order of 10 is to power 7. So, that will give the saturation velocity around 10 is to power 7 centimeter per second.

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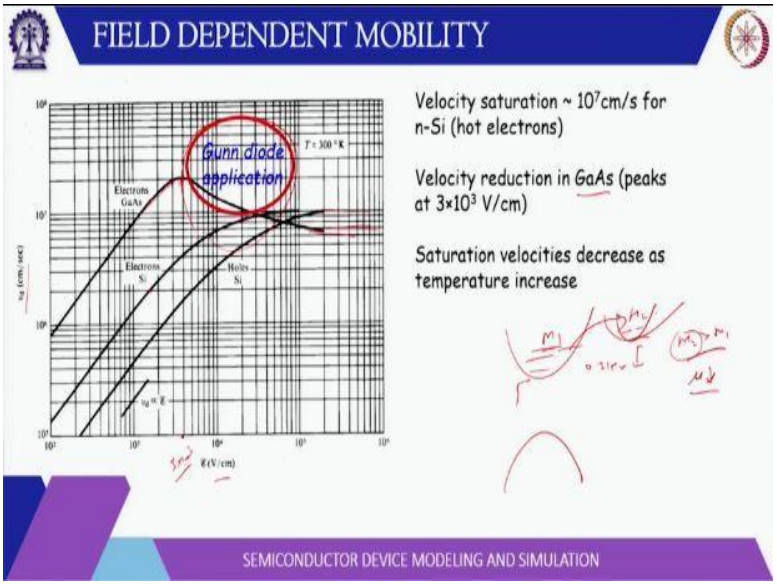

Now, in some semiconductors such as gallium arsenide, there are multi values basically. So, if you see the band structure of gallium arsenide is a direct band gap. And if you go and increase in the field then this will basically get filled, but once it crosses certain limit I think the gap is around 0.31 electron volt, it will go to other values also. So, this is the gap between these two. So, once it crosses this limit they will start to fill other values also.

Now, if you check these values here, let us say mass is m 1; here mass is m 2. So, they are more actually flat. So, this m 2 is more than m 1. So, if electron is in the second valley, then the mass is more therefore mobility is small. So, what happens? As you increase the field, some of the electron transfer from m 1 to m 2 and their mobility actually decrease. So, that is why you see this kind of effect.

So, as you increase the field they are in this gamma valley; this is called gamma valley so there this thing mobilities or velocity is increasing, but at certain field let us say this is around you can see 1, 2, 3 so around 3 into 10 raise to the power 3 volt per centimeter, so, 3 kilovolt per centimeter. They probably migrating to the second valley and then mobility is actually decreasing.

So, this reason gives rise to a negative gradient of the mobility or the velocity. So, we can say that you have negative differential velocity. So, you cannot say that is a negative mobility versus a negative differential mobility. So, then, of course again it saturates and because the mass is more. So, this saturation actually goes to a lower value. And of course, for silicon electron hole there is no such this thing; the gap is actually quite large.

So, you generally have saturation in the same valley. So, they basically increase and saturate to some saturation velocity and you can see order of difference. So, here if you see that let us say this is around 2 into 10 power 6 and corresponding this will be around 1.5 into 10 raise to power 7. So, there is one order higher velocity or the mobility in case of gallium arsenide**.** And this negative differential velocity region we use for gun diode application. So, the therefore gallium is also used in case of gun diode application.

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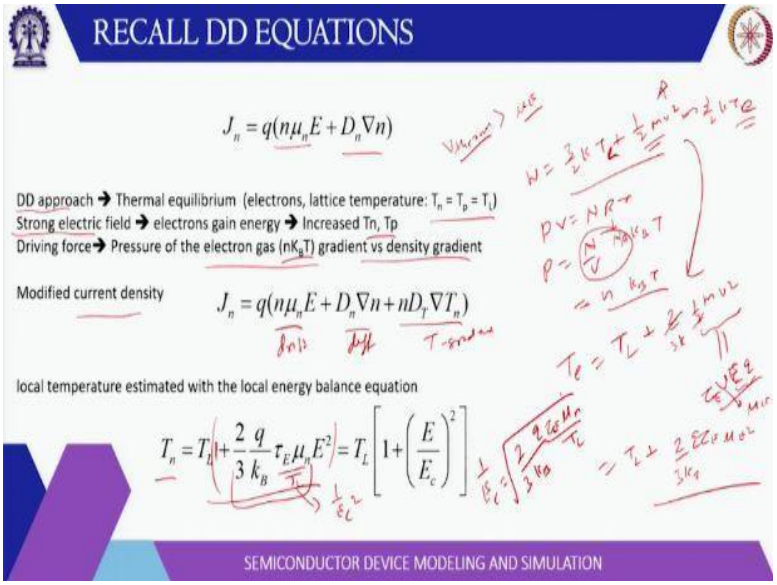

Now, let us recall the drift diffusion equation. So, the drift diffusion equation, basically is a drift term and there is a diffusion term. So, the drift diffusion approach basically assumes that electron and holes are in equilibrium with the lattice. That means, electron temperature, hole temperature and lattice temperature are equal. What does it mean? This temperature will give some velocity. So, that velocity we call thermal velocity**.**

Now, thermal velocity is much larger than the drift velocity, which is mu times electric field. So, that means the component of the drift velocity is quite a small. So, the velocity of the carriers in a semiconductor is basically predominantly the thermal velocity. So, that means their temperature is basically same as the lattice temperature or you can write that the energy of a electron let us say, w is basically 3 by 2 k T.

That is a thermal energy plus half m v square because half m v square is quite small, so let us call this is the electron thermal energy. This is same as 3 by 2 k T times lattice temperature. If I have m v square is small. So, now what happens? If you apply a strong electric field, then

this half m v square component will increase. So, electron gas energy and now the T e, because the energy of electron is thermal energy plus kinetic energy and the total energy of electron can be written as 3 by 2 k T where, T e is the temperature of electron and same thing you can write for all.

So, at high electric field, the energy will increase. And therefore, this T will be increased and T will be or T n or T p there will be more than the lattice temperature and such electrons are called hot electrons or such holes are called hot holes and so on. Now, these electrons can be considered like an electron gas**.** So, because these are the basically like gas molecules travel in empty room. Similarly, these electrons move in the crystal.

So, if you recall there is a relationship called P v = NRT and you can write P = N by V times RT and R can be written as Avogadro numbers times volts when constant times T. And this whole thing is basically, you can say is electron density times k T. So, you have this relationship, the pressure of this electron is n k T. N is the electron density. And here this capital N is the molar concentration and each mole is Avogadro number of species.

So, N times N a by volume is basically the carrier density. So, this pressure basically of electron gas will have gradient. So, this will give rise to some kind of density gradient. So, the modified current density can include the temperature gradient. So, there is a drift term plus diffusion terms. So, this is a drift term; this is the diffusion terms and this is a temperature gradient.

Now, the electron temperature T e or T n can be estimated using this equation. So, we have this let us say, let us write this equation here. So, T e = 3 by 2 k T same so, you can write T n plus then 2 by 3 k T equals 2 by 3 k times half m v square. Now, half m v square is the kinetic energy and it can be obtained from the electric field. So, electric field times velocity is the energy rate. So, you multiply by that energy relaxation time.

So, this much time they get between the scattering losing the energy or gaining the energy. So, this will be your tau v is energy basically. So, now q e, so this electric field is applied on a charge q. So, you can multiply by q. So, q is the force, trans-velocity times tau. So, this will be the energy. So, you can replace this half m v square by this energy, because the electrons are getting energy from the electric field.

So, if you replace, then you get T L plus, so this 2 will cancel, but this 2 will come here, so because you have to write 2 by 3 only. So, 2 by 3 k is a Boltzmann constant, then tau e now v you can write q here. Now v and e the velocity is mu times e you can write. So, you can write mu times e square. So, you have this expression and then of course you can take this constant here and then you can write this as 1 by e c square.

So, you can write T L is 1 + 1 by e c square; of course, you have to take T L out. So, if you take T L out, then this becomes 1 plus this by T L**.** So, this whole thing, suppose 1 by E c square. So, you can write from here E c becomes the critical electric field becomes 2 q by 3 k times tau E mu n by T L. So, this is 1 over E c. So, this is 1 over E c is square root of this thing. So, that will be the critical electric field.

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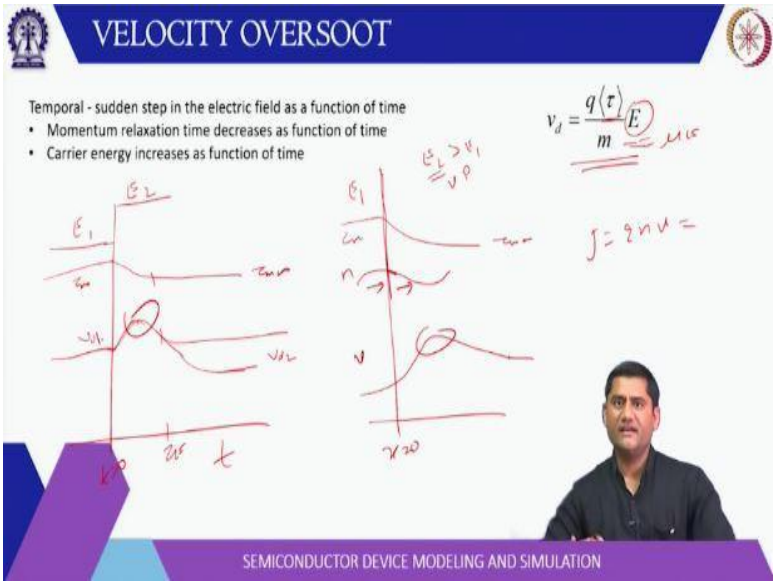

Now, the velocity oversoot. So, there is a temporal or sudden step in the electric field as a function of time. So, what happens? Let us say you consider region of space and then at T = 0. You change the electric field from E 1 to let us say E 2 a high electric field. So, what will happen? Now, you see here this is q tau by m is the mobility. So, this is mu times E. So, q tau by m is the mobility.

So, what will happen to the tau? So, this tau will actually, if you increase the energy this tau will decrease; the moment of relaxation time will decrease. So, what you can do? But, this will take some time. So, if you can plot this versus T, this tau m will take some time. So, this is tau

m lets say not here and it decreases basically and then that is called tau m infinity. So, now if you consider mobility so, this will be v d 1, this is let us say v d 2.

So, if tau m is less, then the drift velocity will also be less so, because it is q tau by m. Now, what will happen? The electric field is more here so, that means it will increase the velocity. So, initially at T = 0 tau is same, electric field is more so it will basically increase the velocity. And then of course, this tau has increased so, it will decrease the velocity but due to higher electric field this may actually go somewhere here**.**

So, there is a certain increase in the velocity for certain time. And then we can say this is up to some tau E energy relaxation time**.** So, this phenomenon is called the velocity oversoot in time domain. Similarly, you can have velocity oversoot in a special domain so this is x = 0. So, there is a electric field even here there is electric field E 2, which is larger than even, so same thing will happen here**.**

So, there is a tau m here then it will decrease tau m in free, then you can also see that j = q times n times v. So, if E is more, then v will be more so, because current has to be constant. So, your electron concentration will also have a dip here or you can think like this is the region here, so electrons are moving, here they are moving fast. So, that means, there will be some dip here in the electron concentration.

And similarly the velocity also you can see some effect like this. So, your velocity will basically look like some increase here then it will settle down**.** So, this phenomenon is called the velocity oversoot and that is seen. To account for velocity opposite, we need the hydrodynamic model because it cannot be modeled in case of **iv** drift diffusion model, because drift diffusion model considers this electron in thermal equilibrium with the lattice. So, these transient effects are not captured there.

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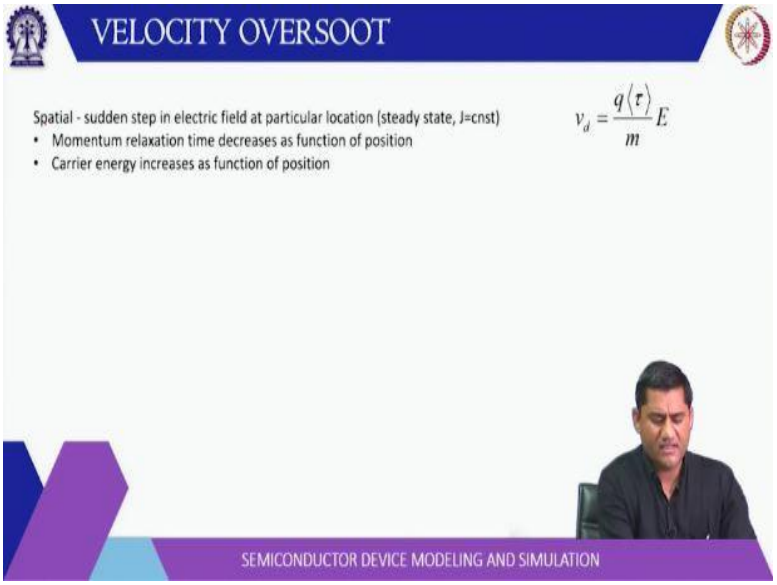

And so, I think we have discussed both the spatial-domain and the time-domain. So, in a spatial domain, momentary lesson decrease as a function of position and carry energy increases as a function of position.

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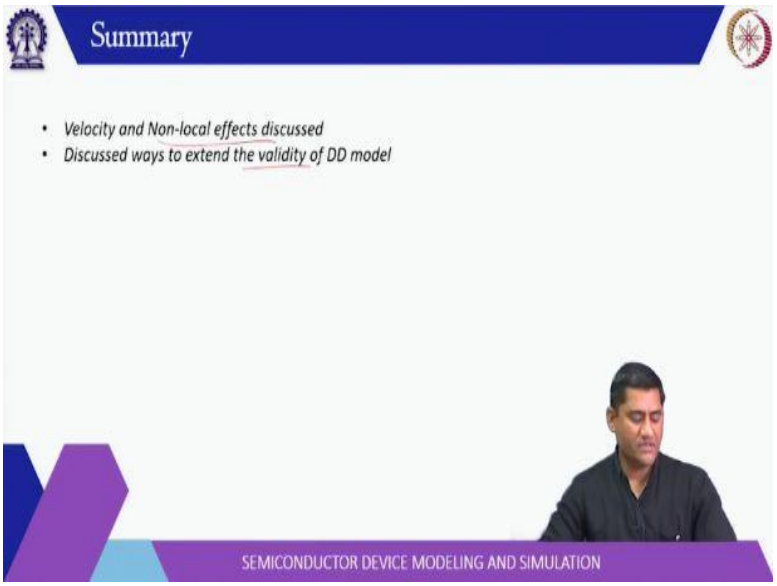

So, in summary, we have discussed the velocity and the non-local effects. And we also discuss the ways to increase the validity of the drift diffusion model. So, thank you very much.