Semiconductor Device Modeling and Simulation
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Lecture - 45
Generation-Recombination (Contd.)

Hello, welcome to lecture number 45.

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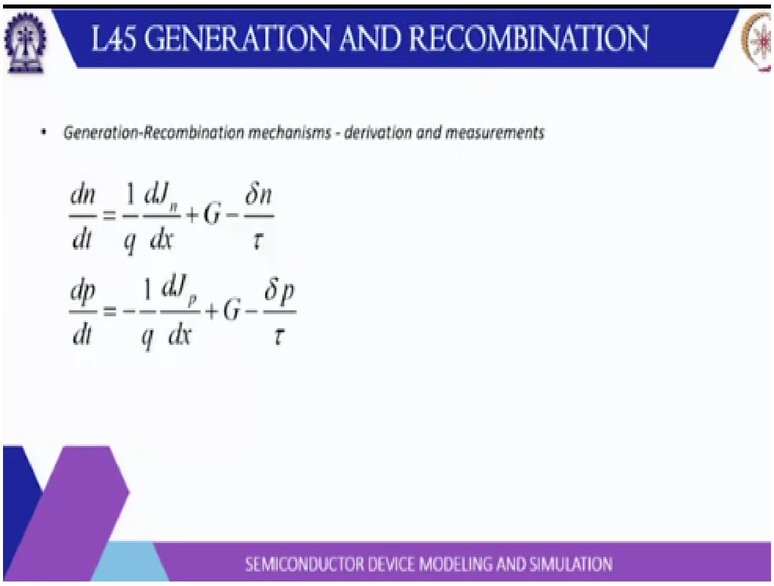

L45 GENERATION AND RECOMBINATION

- Generation-Recombination mechanisms - derivation and measurements

$$\frac{dn}{dt}=\frac{1}{q}\frac{dJ_n}{dx}+G-\frac{\delta n}{\tau}$$

$$\frac{dp}{dt}=-\frac{1}{q}\frac{dJ_p}{dx}+G-\frac{\delta p}{\tau}$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Let us continue our discussion on generation and recombination.

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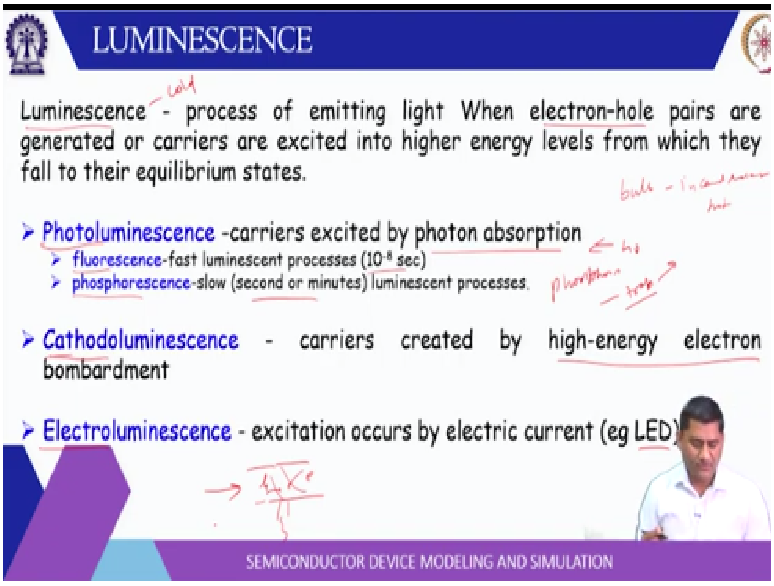

LUMINESCENCE

Luminescence - process of emitting light When electron-hole pairs are generated or carriers are excited into higher energy levels from which they fall to their equilibrium states.

cold

bulb - incandescence heat

- Photoluminescence -carriers excited by photon absorption ← hν
  - fluorescence-fast luminescent processes (10^{-8} sec)
  - phosphorescence-slow (second or minutes) luminescent processes. phosphor - trap
- Cathodoluminescence - carriers created by high-energy electron bombardment
- Electroluminescence - excitation occurs by electric current (eg LED)

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So we have discussed basically we have summarized all the five recombination processes. So here let me tell you about first this luminescence process. You have seen the tube light, you have seen different phosphorescent materials, you have seen the light bulb also. So the light bulb is basically we also call it incandescent bulb, which is actually usually a hot process.

And this luminescence is the generation of light, usually a cold process. So this is usually a cold process. Now what happens in luminescence the light is emitted by the electron hole pair recombination. So these carriers are excited to high energy level and when they fall down, they generate the light. So in photoluminescence there are different luminescence process called photoluminescence, cathodeluminescence, electroluminescence. And photoluminescence, it is assisted by photon. So that means a photon or light will come.

It will excite the carrier and when they fall down, they will emit further light. So that is called photoluminescence. So in photoluminescence there is fluorescence. Now fluorescence usually a fast process which is order of 10 is to -8 second and then there is a phosphorescence. Now phosphorescence actually is a slow process. It comes from let us say there is a phosphorus.

They have some trapped energy level which can hold these carriers for quite some time up to seconds or minutes also. And when it releases after the carrier lifetime, it generate light basically, so that is called phosphorescence. Now cathodeluminescence is basically there is a bombardment of high energy electron from the cathode. And then this generate the carriers.

And when the carriers fall down give the luminescence. Electroluminescence is usually seen in light emitting diodes, where electric current is supplied. So this electric current usually supply the carriers. So let us say hole and electron from here and at the pn junction, they recombine here and then they give light. Now it is not seen in silicon because it is indirect bandgap.

It is seen in direct bandgap semiconductors. So where the current flow allows these carriers to pile up and when they recombine they give away the photon.

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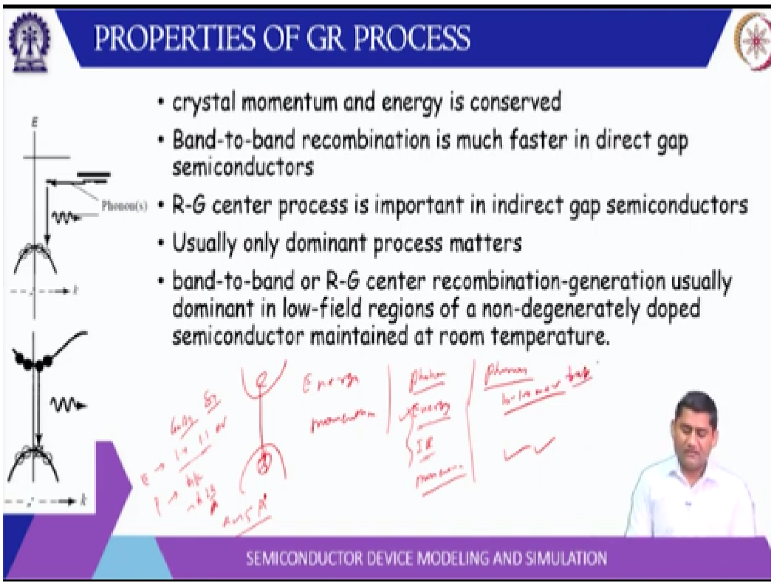

PROPERTIES OF GR PROCESS

- crystal momentum and energy is conserved
- Band-to-band recombination is much faster in direct gap semiconductors
- R-G center process is important in indirect gap semiconductors
- Usually only dominant process matters
- band-to-band or R-G center recombination-generation usually dominant in low-field regions of a non-degenerately doped semiconductor maintained at room temperature.

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now let us discuss the properties of this generation recombination process. From the conduction band electron is combining with a hole in the valence band. So there are two things here, one is the energy another is the momentum. So in any process in any physical process, the energy and momentum has to be conserved. So for example, let us consider silicon or gallium arsenite.

Their bandgap is order of 1.1 or 1.4 electron volt, this is the bandgap. So this much energy has to be conserved. Then momentum. Momentum is related to h bar k and where k is h bar 2 pi by lambda and lambda is somehow related to the latest constant let us say A. And in semiconductor this A is around few 5 something angstrom, okay. So this will have certain momentum.

Now if the process is radiative, so it can involve particle such as photon or it can involve a particle called phonon. Now photon has energy. If you consider any photon in visible light, it will have energy close to some value and this around 1.1, 1.4 it will come under infrared region. So these photons will have, photons in infrared region will have energy similar to the bandgap of these materials.

But the momentum will fall short because these photons have very small momentum. So if it is direct bandgap then energy is conserved and momentum is not required. So electron can take care of a small momentum. In case of indirect bandgap it can conserve the energy, but momentum it will not conserve because it does not have sufficient momentum.

So if you consider the phonons they will have sufficient momentum because they are the latest vibration of the same host semiconductor. So the momentum will be okay here, but they have a small energy, few 10 to 100 millielectron volt. So they cannot conserve the energy. So this is the issue. So if there is a phonon related process it has to be in multiple steps, usually assisted by a trap energy level, okay. So let us see.

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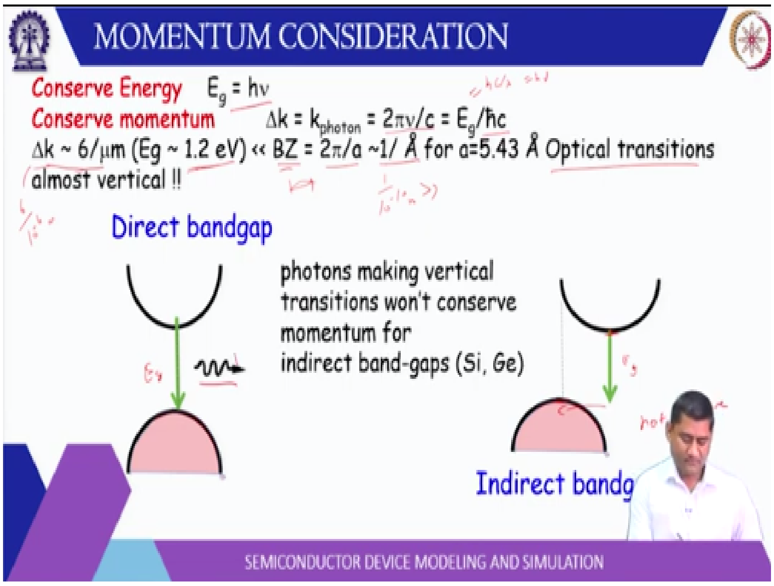

MOMENTUM CONSIDERATION

Conserve Energy $E_g = h\nu$

Conserve momentum $\Delta k = k_{photon} = 2\pi\nu/c = E_g/\hbar c$

$\Delta k \sim 6/\mu m$ ($Eg \sim 1.2$ eV) << $BZ = 2\pi/a \sim 1/$ Å for $a=5.43$ Å Optical transitions almost vertical !!

photons making vertical transitions won't conserve momentum for indirect band-gaps (Si, Ge)

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

This is the direct bandgap semiconductor. So the bandgap is E g and if photon estimated this E g should be equal to h nu and because there is no as such change in the momentum here so if you compare the momentum of a photon, which is 2 pi frequency by c or E g is equal to hc by lambda. So if you compare or equal to h nu. So you can write it as 2 pi E by h. So e by h bar c.

You can write this thing like this. And that will give you around for 1.2 electron volt. That will be pretty small basically. It comes around to be 6 per micrometer. So this is the change in the wave vector k for photon with energy 1.2 electron volt. You can substitute the value. E g let us say 1.2 electron volt, h bar is the modified Planck constant, c is the velocity of light 3 into 10 to the power 8 meter per second.

So you substitute you get around 6 per micron. Then for the brilliant zone, so where A is the lattice constant of the material. So you can say that lambda will be some order of this lattice constant. So the wave vector k is 2 pi by lambda. So it is around 2 pi by

A. And that comes out around 1 by angstrom. So if you see here this is quite high because this is 1 by 10 raised to power -10 meter.

This is 6 by 10 raised to power -6 meter. So this is much larger because this is 10 raised to the power 10. So optical frequencies the photons, they cannot supply this much momentum. So in case of indirect bandgap the momentum has to be conserved because in the indirect bandgap the wave vector k for the minimum of the conduction band and the wave vector k for the maximum of the valence band they are at different locations.

So they are different wave vectors. So when electron make a transition it has to you know of course this energy and the momentum also. So both has to be conserved. So this the both cannot be supplied by the photon. So this indirect bandgap process is not radiative and direct bandgap process is radiative.

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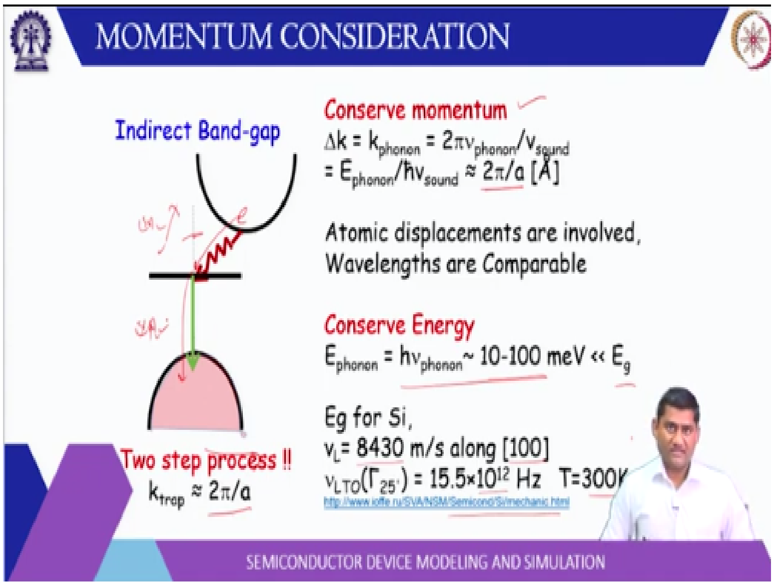

MOMENTUM CONSIDERATION

Indirect Band-gap

Conserve momentum

$\Delta k = k_{phonon} = 2\pi\nu_{phonon}/v_{sound}$
$= E_{phonon}/\hbar v_{sound} \approx 2\pi/a$ [Å]

Atomic displacements are involved,
Wavelengths are Comparable

Conserve Energy

$E_{phonon} = h\nu_{phonon} \sim$ 10-100 meV $\ll E_g$

Eg for Si,
v_L = 8430 m/s along [100]
$\nu_{LTO}(\Gamma_{25'}) = 15.5\times10^{12}$ Hz T=300K

Two step process !!
$k_{trap} \approx 2\pi/a$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now here you can see now for the indirect bandgap, you can conserve the momentum using the phonon. So phonon is basically is 2 pi by a, same order. So that much displacements are involved here in the phonon. So there the phonon wavelength and this wave vector k they are comparable. So using the phonon this momentum can be conserved easily.

But if the, if you see the energy of the phonon, which is around 10 to 100 milli electron volt, it is much smaller than the bandgap. So you will need multiple phonons and that is not possible. So if you compare this thing uh, let us say there is a website here www.ioffe.ru where different properties of these materials are listed basically.

So here you can find out that velocity of this phonon along 100 direction is around 8430 meter per second. And the frequency of this phonon is around 10 is to power 12 hertz. And if you compare, convert that into energy you will get around 10 to 100 millielectron volt at 300 Kelvin. So what is required here is a two step process.

So in one step it will go to a trap level okay, with the help of phonon and then it will make a transition from here. So because the trap will supply this much momentum. And then this will make a transition to the trap level. From the trap level it will make to the valence band. Now if it is somewhere here, then this process again inefficient. So this trap level has to be somewhere in the middle.

So then it is most efficient. When the trap level is the middle of the bandgap then it has to basically overcome half energy here and half energy here. And that way, in indirect bandgap there is a dominance of the trap assisted recombination.

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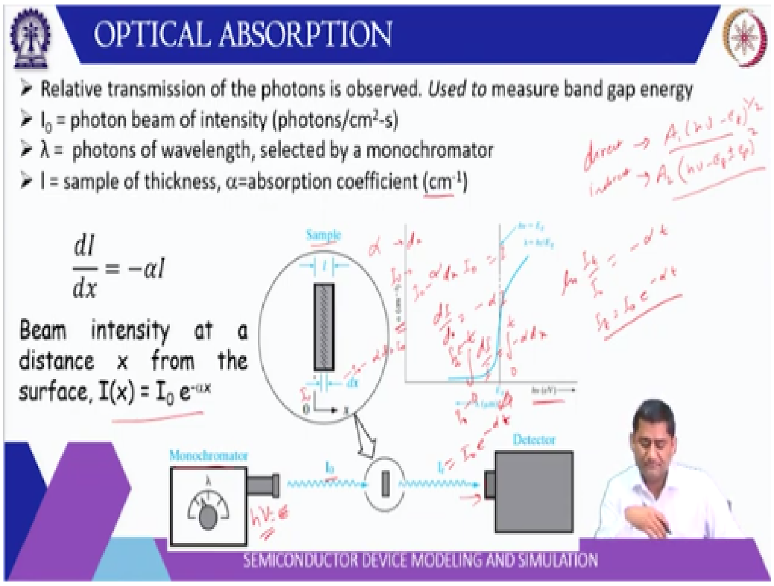

OPTICAL ABSORPTION

- Relative transmission of the photons is observed. *Used to* measure band gap energy
- I_0 = photon beam of intensity (photons/cm^2-s)
- λ = photons of wavelength, selected by a monochromator
- l = sample of thickness, α=absorption coefficient (cm^{-1})

$$\frac{dI}{dx} = -\alpha I$$

Beam intensity at a distance x from the surface, $I(x) = I_0 e^{-\alpha x}$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now how do we measure these properties? So for example, using optical absorption, so we have this monochromator here. From this we select certain frequency. So e is equal to h nu will be the energy of this photon. And then we will sign it on a sample and see the transmission characteristic. So how much light is transmitted and that will also tell you how much light is absorbed.

So if alpha is the absorption coefficient, so if alpha is the absorption coefficient of this piece of semiconductor, then when it travels through a distance dx the absorption will be alpha dx. If i is the intensity of this light then the absorption is alpha dx times i naught. So this much is absorbed here. So what is remaining is i naught minus alpha dx i naught is the output intensity.

So here is i naught and here is i naught minus alpha dx times i naught. So if you write this in differential form, so then d i by dx will be minus alpha times i. So of course, you can integrate over a length. So if it is uniform, then d i by i will be minus alpha dx and then if you integrate 0 to thickness t 0 to thickness t so this will be, let us call it this is let us say i naught here and this is let us say i transmitted.

So this is log of i transmitted by i naught will be minus alpha t. So this will be i transmitted by i naught will be minus alpha t. So this will be i transmitted by i naught e to the power minus alpha t. So this is how it will look like or for general x i naught e to the power minus alpha x. So this intensity is basically exponentially decaying.

So depending on the thickness of this piece of semiconductor the transmitted intensities, let us say this is i naught. So i t is i naught e to the power minus alpha t, where t is the thickness of the sample, alpha is the absorption coefficient. If we do not know the absorption coefficient, so from the major intensity we can estimate what is the absorption coefficient.

And you can also notice this absorption coefficient should have a unit 1 over length because it is multiplied by the thickness here. And we can measure this alpha for different values of photons or different wavelengths or the frequency of the photons, okay. Now this is basically, let me clear it. This is the curve which shows alpha versus a frequency.

So here you can notice below the bandgap the absorption is quite low and above the bandgap absorption increases basically. So there are two types of this semiconductor band structure. There is a direct bandgap and there is indirect bandgap. So in case of direct bandgap this curve will basically can be fittted with some constant A times x nu is the energy of the sub photon minus bandgap to the power half.

And in case of indirect bandgap because indirect bandgap it is usually photon assisted so there is no role of phonon. In case of indirect bandgap let us say it is some coefficient a 2 times h nu minus E g, and then it takes a phonon then plus or minus a phonon energy e phonon power 2. So in bandgap it is basically like this, indirect bandgap. So it is more gradual in case of indirect bandgap.

In case of direct bandgap it is more sharp basically. So there is a sudden change, just blow the bandgap and just after the bandgap. So just from the measurement of this absorption coefficient, we can determine whether the piece of semiconductor is direct bandgap or indirect bandgap and we can also measure what is the bandgap energy for that say piece of semiconductor because, after that bandgap the energy will basically, the absorption will be more.

Because now photon have sufficient energy to excite these carriers from valence band to conduction band.

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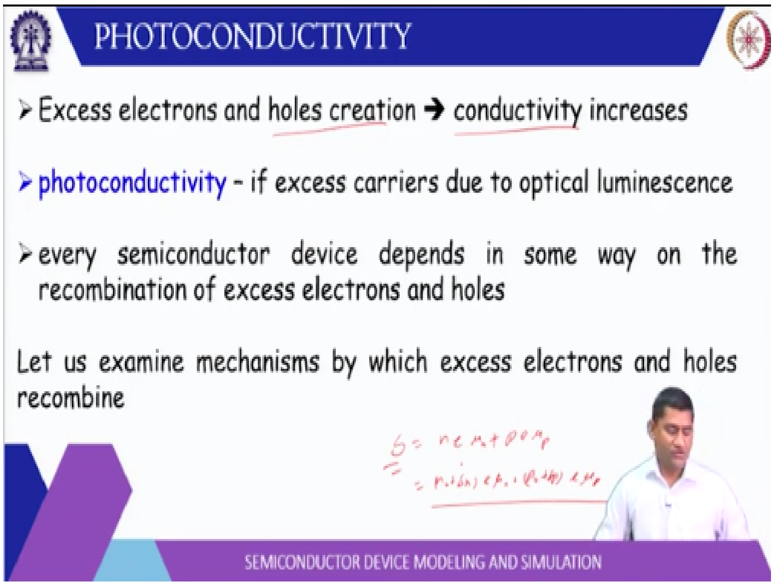

PHOTOCONDUCTIVITY

- Excess electrons and holes creation ➔ conductivity increases
- photoconductivity - if excess carriers due to optical luminescence
- every semiconductor device depends in some way on the recombination of excess electrons and holes

Let us examine mechanisms by which excess electrons and holes recombine

$\sigma = n e \mu_n + p e \mu_p$

$= (n_0 + \delta n) e \mu_n + (p_0 + \delta p) e \mu_p$

Now how do we measure the excess carriers? So when you shine light on the piece of semiconductor, it will be absorbed and the number of carriers will increase. So when the number of carriers electron hole will increase, obviously the conductivity will increase. Conductivity you can write sigma is equal to n e mu plus p e mu. So this is mu n, this is mu p.

So if you have some excess carriers so this will become n naught plus delta n times e mu n plus p naught plus delta p times e mu p. So these excess carriers will increase the conductivity of the piece of semiconductor. Now the semiconductors once there is excess carrier they will try to recombine also.

So as long as the light is on, then some carrier concentration of n p which is more than n naught p naught will adjust and when you turn off the light, they will again recombine and come back to their equilibrium value, let us say n naught and p naught.

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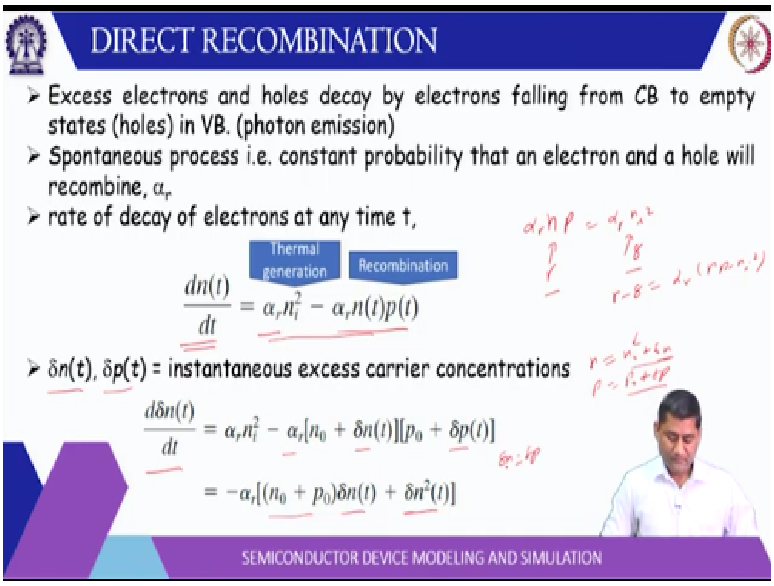

DIRECT RECOMBINATION

- Excess electrons and holes decay by electrons falling from CB to empty states (holes) in VB. (photon emission)
- Spontaneous process i.e. constant probability that an electron and a hole will recombine, α_r
- rate of decay of electrons at any time t,

Thermal generation

Recombination

$$\frac{dn(t)}{dt} = \alpha_r n_i^2 - \alpha_r n(t)p(t)$$

$\alpha_r n p = \alpha_r n_i^2$

r

g

$r - g = \alpha_r (np - n_i^2)$

- $\delta n(t)$, $\delta p(t)$ = instantaneous excess carrier concentrations

$n = n_0 + \delta n$

$p = p_0 + \delta p$

$$\frac{d\delta n(t)}{dt} = \alpha_r n_i^2 - \alpha_r [n_0 + \delta n(t)][p_0 + \delta p(t)]$$

$\delta n = \delta p$

$$= -\alpha_r [(n_0 + p_0)\delta n(t) + \delta n^2(t)]$$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So let us look in detail the rate of change of these carriers for example, electron will be generation minus recombination. So we have already pointed out that recombination is proportional to np product and then some coefficient let us say alpha r times np. And in equilibrium, this is equal to the generation rate and this np is equal to n i square. So you can say this is alpha r times n i square.

So this is your generation rate, this is your recombination rate, and they can be separated out. And then if you want to write the net recombination or net generation, so you can write g minus r will be the net generation rate or dn by dt or r minus g will be net recombination rate and that will be alpha r times np minus n i square and this we have already discussed. Then based on this dn by dt, we can write let us say we have some extra carriers delta n and delta p.

So your n can be written as n naught plus delta n, p can be written as p naught plus delta p. So for dn by dt n naught is a constant. So dn by dt can be written as d by dt of delta n. So this is written here. And then of course, n i square is kept as it is; n and p they are replaced with n naught plus delta n, p naught plus delta p. And n naught p naught is equal to n i square. So this will cancel out.

So what you will have, alpha r times delta n plus delta p and plus delta n delta p. And here it is assumed that delta n is equal to delta p. Because this when you shine the light on it, the generation process does not discriminate. So that means, a number of electrons that will be generated will be equal to the number of holes that are generated. So both will be equal in number.

So you can write delta n equal to delta p. So n naught delta p plus p naught delta n can be written as n naught plus p naught times delta n plus delta n square and because delta n is equal to delta p.

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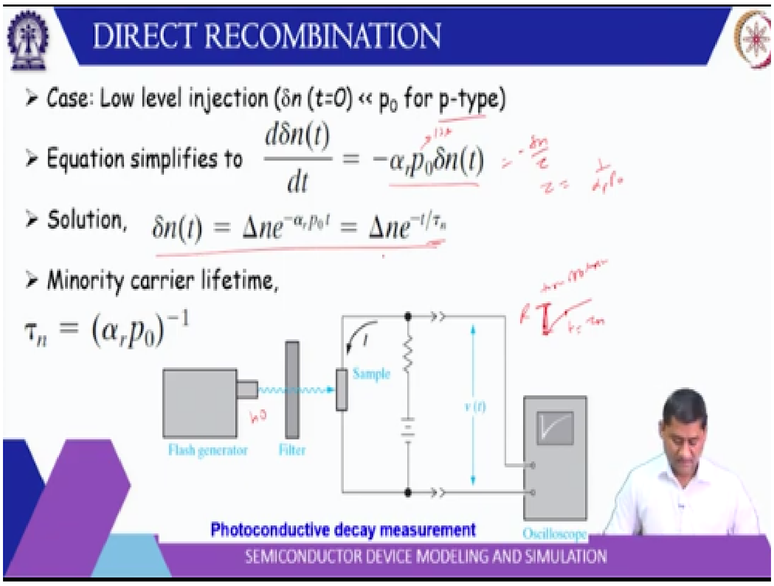

DIRECT RECOMBINATION

- Case: Low level injection ($\delta n\,(t=0) \ll p_0$ for p-type)
- Equation simplifies to $\frac{d\delta n(t)}{dt} = -\alpha_r p_0 \delta n(t)$ $= -\frac{\delta n}{\tau}$ $\tau = \frac{1}{\alpha_r p_0}$
- Solution, $\delta n(t) = \Delta n e^{-\alpha_r p_0 t} = \Delta n e^{-t/\tau_n}$
- Minority carrier lifetime,

$\tau_n = (\alpha_r p_0)^{-1}$

Photoconductive decay measurement

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now based on this expression, this low level injection high level injection we have discussed that in low level injection this you can ignore delta n square. High level injection you can ignore this one. So for low level injection delta n by dt is equal to minus alpha r times p naught delta n for p type and p naught is equal to na and of course this can be written as delta n by tau and tau is 1 over minus delta bar, 1 over alpha r times p naught.

And then if you measure the conductivity of this piece of semiconductor with time you can measure this lifetime tau n and is called minority carrier lifetime because the change in the majority carrier concentration is pretty small for low level injection. It is the change in the minority carrier that is significant relative to the original value of the minority carrier concentration.

So what is done here basically, some light is there which is generating the number of carriers to the sample and then we are measuring the resistance of the sample. So as you shone the light on it, its resistance will actually decrease. So the resistance will actually decrease, that is more current will flow. Then once you achieve this level you turn off the light, turn off the light.

So when you turn off the light now there is no more extra generation. So whatever these excess carries are they will start recombining. So when they will start recombining the resistance will again increase and from this plot of the resistance we can find out the value of tau. So at t equal to tau n the resistance will become you know 1 by e or something like that. So this is the way to measure the carrier lifetime in a piece of semiconductor.

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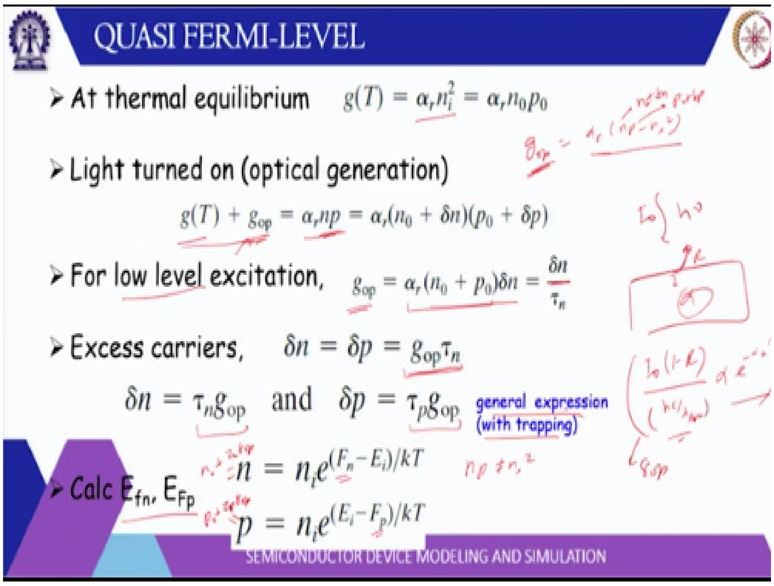

QUASI FERMI-LEVEL

- At thermal equilibrium $g(T) = \alpha_r n_i^2 = \alpha_r n_0 p_0$
- Light turned on (optical generation)

$$g(T) + g_{op} = \alpha_r np = \alpha_r(n_0 + \delta n)(p_0 + \delta p)$$

- For low level excitation, $g_{op} = \alpha_r(n_0 + p_0)\delta n = \frac{\delta n}{\tau_n}$
- Excess carriers, $\delta n = \delta p = g_{op}\tau_n$

$\delta n = \tau_n g_{op}$ and $\delta p = \tau_p g_{op}$ general expression (with trapping)

- Calc E_{fn}, E_{Fp} $n = n_i e^{(F_n - E_i)/kT}$

$p = n_i e^{(E_i - F_p)/kT}$

$g_{op} = \alpha_r(np - n_i^2)$

$np \neq n_i^2$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Then let us say we shine the light continuously. So during that process that the total generation will be thermal generation plus optical generation and that will be equal to recombination between alpha times np. And we know that thermal generation is alpha r times n i square. So now we have added extra term to the generation. It is called optical generation g optical.

So now this np product has to be more than n i square to accommodate this optical generation, because this optical generation and the recombination of these excess carriers they have to balance out. So this is written as something like this that gT plus g optical is equal to total recombination or you can write optical generation is equal to alpha r times n p minus n i square.

Because this is a net recombination, alpha r times np minus n i square should be equal to the g optical. So when we again substitute and we can calculate what will be the excess carrier concentration due to this g optical. So g optical, what will be the g optical? If you think like this, let us say this is your piece of semiconductor and you are shining the light on it with certain intensity i naught.

So first at the interface, some of the light will be reflected, some of the light will be transmitted. So we can write let us say this i naught is the intensity of light times 1 minus reflection coefficient. So this is the reflection coefficient. So that means the light that is transmitted. Then the energy of this light which is causing the electron hole per generation, so that is x c by lambda light.

That much energy will be absorbed by one carrier generation. So this has to be generated times, now this has to be absorbed by the piece of semiconductor with absorption coefficient alpha. So then you can write this intensity is absorbed by alpha into e to the power minus alpha x. And of course, this x c by lambda is to be more than the bandgap.

So this absorption will be related this g optical. So the intensity can be related to this g optical. So this g optical is function of the light intensity plus the reflection on the surface plus absorption coefficient alpha. So let us say you know overall absorption is let us say g f, g optical is equal to alpha times np minus n i square. Now we can find out from here what is excess carrier?

So let us say n becomes n naught plus delta n and p become p naught plus delta p. And when we equate it, so again for low level excitation delta n delta p is much smaller than majority carrier concentration. So this can be approximated n naught plus p naught times delta n. And then if you express this as delta n by tau n. So tau n will be 1 over alpha r times n naught plus p naught.

And we can also calculate delta n the excess carrier which will be tau n times g optical. So if there is a light energy falling with some rate g, then the excess carrier that will be generated with g optical times tau n. And similar for delta n and delta p. So tau p g optical tau n g optical. Now what will happen? Now this n and p product is not equal to n i square.

That means, n concentration is n naught plus tau n g optical, p is equal to p naught plus tau p g optical. So this is a concentration. So then we have to define that different Fermi level. So this is EFN for the electron and EFP for the hole Fermi level. So we will be calculating basically these quasi Fermi levels and from that we can have a general expression and calculate the in general the optical generation rate for this kind of system.

So we have discussed this optical generation and optical absorption and the change in the optical conductivity, the conductivity of a semiconductor due to optical measurement.

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INDIRECT RECOMBINATION

William Shockley

Robert N. Hall

Also called
Shockley-Read-Hall (SRH) Recombination

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now we will discuss about the indirect recombination. It is atributed to these scientists William Shockley, Robert Hall and Read. So after their name it is named as SRS recombination.

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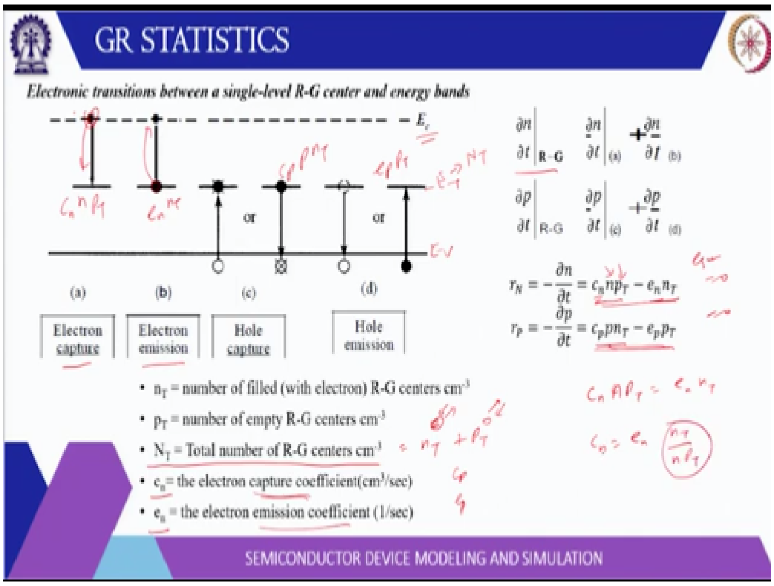

GR STATISTICS

Electronic transitions between a single-level R-G center and energy bands

Electron capture | Electron emission | Hole capture | Hole emission

$$\left.\frac{\partial n}{\partial t}\right|_{R-G} \quad \left.\frac{\partial n}{\partial t}\right|_{(a)} + \left.\frac{\partial n}{\partial t}\right|_{(b)}$$

$$\left.\frac{\partial p}{\partial t}\right|_{R-G} \quad \left.\frac{\partial p}{\partial t}\right|_{(c)} + \left.\frac{\partial p}{\partial t}\right|_{(d)}$$

$$r_N = -\frac{\partial n}{\partial t} = c_n n p_T - e_n n_T$$

$$r_P = -\frac{\partial p}{\partial t} = c_p p n_T - e_p p_T$$

$$c_n n p_T = e_n n_T$$

$$c_n = e_n \boxed{\frac{n_T}{n p_T}}$$

- n_T = number of filled (with electron) R-G centers cm^{-3}
- p_T = number of empty R-G centers cm^{-3}
- N_T = Total number of R-G centers cm^{-3} $= n_T + p_T$
- c_n = the electron capture coefficient(cm^3/sec)
- e_n = the electron emission coefficient (1/sec)

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So SRS recombination we have mentioned but in this lecture we will derive the expression. So you consider a semiconductor band structure with conduction band E c and valence band E v. And there is a trap energy level here, let us call it E t. Now please pay attention. Let us say number of states is capital N T, capital N T is the total number of states in the trap level.

Out of them small n T are filled and small p TR n T. So that means they have electron here and they do not have electron. So that means they can take electron and they can give electron or they can take a hole or they can give a hole. Then of course, these trap energy state have the ability to capture electron or capture hole. So hole capture coefficient is will be c p and e p.

So hole capture will be c p and c p. And electron capture coefficient will be c p, c is the capture coefficient and e p will be the electron emission coefficient. Certainly they can emit a hole and they will be limited quantities. Now if you want to write dn by dt. So dn by dt in what way this electron concentration can change. One is that it can capture electron. So that will reduce.

It can emit a electron. So there is electron emission there is electron capture. So dn by dt will be electron emission minus electron capture. So electron emission minus electron capture. So dn by dt is electron emission minus electron capture. So net recombination will be minus dn by dt. So that is capture minus emission. So capture will depend on the of course the capture coefficient.

Then to capture electron should be there. So multiplied by the electron concentration. Then there should be state which will capture the electron, so multiplied by P T. So electron capture is basically the capture coefficient times the electron concentration times this state which can capture the electron. Emission, it only requires the emission coefficient for electron. Now number of states that have this electron.

So that is n T and they can easily go to the conduction band, so that is not required. So this is the expression for minus dn by dt which is electron recombination rate. Similarly expression we can write for the holes. That will be this is hole capture. So hole capture is again c p times there should be hole here, then this should be empty.

Then hole emission. Of course, emission, hole emission coefficient times this should have a hole so that is P T the, so this is the expression for r p. So recombination rate for the holes. Now if you see here these rates under equilibrium they should be zero. So at equilibrium both should be zero. So this will give us a relationship between the emission coefficient and the capture coefficient.

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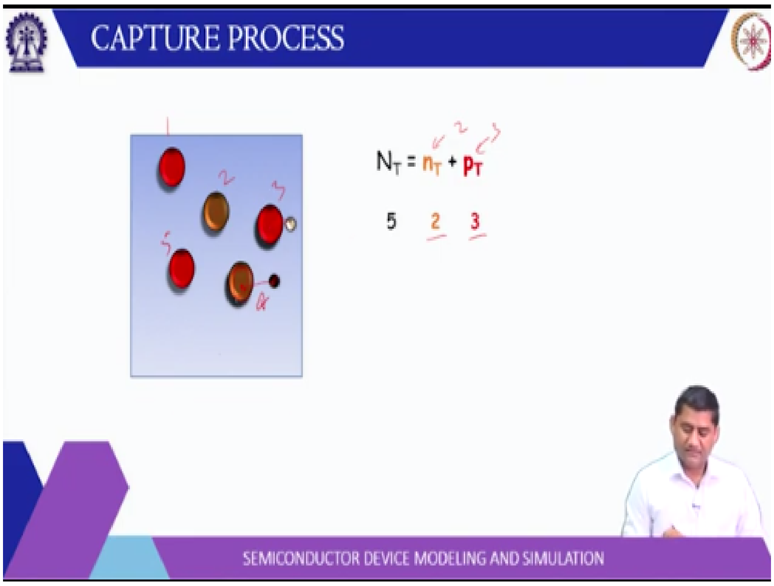

CAPTURE PROCESS

$N_T = n_T + p_T$

5 2 3

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So before that let us look at this process here, how this actually works out. So there are let us say 1, 2, 3, 4, 5 states are here. So there are 5 trap states. Out of this these red ones they are filled and these two are empty. Now let us see what happens. Then it basically one hole okay it should actually go here. So this goes here basically, so this hole goes here.

Then this becomes again this electron goes here, it becomes filled basically. So now there are three filled and two empty. Then there is another particle here, which again enters here. So this electron has entered here. So this will become three. So empty become three and this become two basically.

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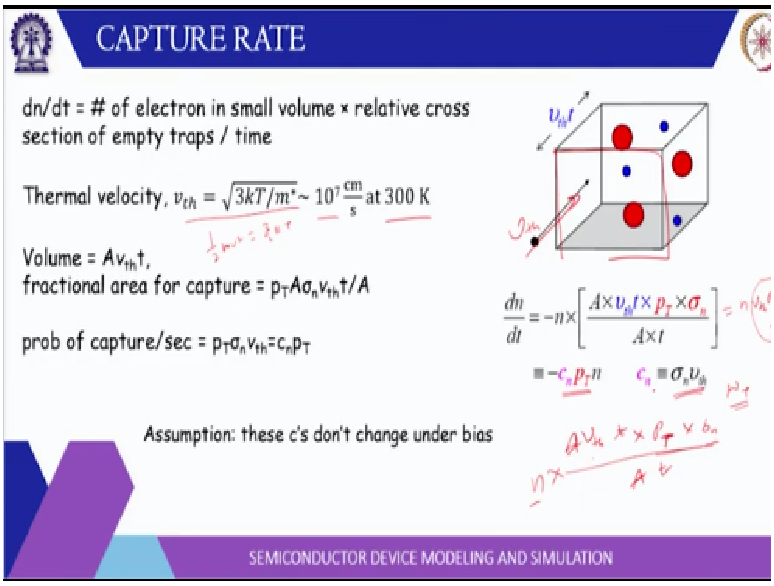

CAPTURE RATE

dn/dt = # of electron in small volume × relative cross section of empty traps / time

Thermal velocity, $v_{th} = \sqrt{3kT/m^*} \sim 10^7 \frac{cm}{s}$ at 300 K

$\frac{1}{2}mv^2 = \frac{3}{2}kT$

Volume = $Av_{th}t$,

fractional area for capture = $p_T A \sigma_n v_{th} t / A$

prob of capture/sec = $p_T \sigma_n v_{th} = c_n p_T$

$$\frac{dn}{dt} = -n \times \left[\frac{A \times v_{th} t \times p_T \times \sigma_n}{A \times t}\right]$$

$$= -c_n p_T n \qquad c_n = \sigma_n v_{th}$$

$$n \times \frac{A v_{th} t \times p_T \times \sigma_n}{A t}$$

Assumption: these c's don't change under bias

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So this is the process basically you can see. Then let us write the capture rate, how these carriers are captured. So you can piece a, you can consider a piece of semiconductor here. Let us say we consider one area of cross section and this is the velocity of this carrier. So velocity can be assumed at room temperature is a thermal velocity. So half mv square is equal to 3 by 2 k t.

So you can write v thermal is 3 k t by m. So around 10 is to power 7 centimeter per second. So this electron is moving with this thermal velocity. Now the distance traveled by this electron is v thermal times t, in time t. And let us say this is the area of cross section perpendicular to this velocity is A. So this is the volume traveled by this particle in time t. Now in this volume, how many trap states are there?

So to capture a trap, the density of these trap states is p T. So p T why are we taking p T because p T are the trap states which can absorb electron. N T is the total trap state, but out of them p T are empty, so they can capture this electron. Now this is the density of the states. This is per unit volume. So we have to multiply this thing by the capture cross section of individual state.

So the sigma n is the capture cross section of this individual state. So this is the total kind of probability that in this volume A times with thermal times T it can impinge on this p sub t states with cross section sigma n. So let us say p T is 1. Then there is only area is sigma n. If there are two, then this is two times sigma n and so on. Now this is a total capture rate divided by the area times t.

So that is per unit area per unit time. So this is a capture per unit area per unit time. Now what is the probability? If there are n such electrons, then n times this is the probability. So if you simplify this one, A and t will cancel out. Then, so this dn by dt simply becomes n times v thermal times p T times sigma n. So this is basically n times v thermal times p T times sigma n.

So this can be written as v thermal p T times sigma n can be a coefficient and which is written as C n times p T. And C n is the capture coefficient. And capture coefficient if you compare is basically sigma n times v thermal. So thermal velocity times the capture cross section is your capture coefficient. And of course, we are assuming that these capture coefficient are not changing under the bias.

Because these impurity atoms they will be affected by the bias, but effect is not significant, so we can assume that c n are constant.

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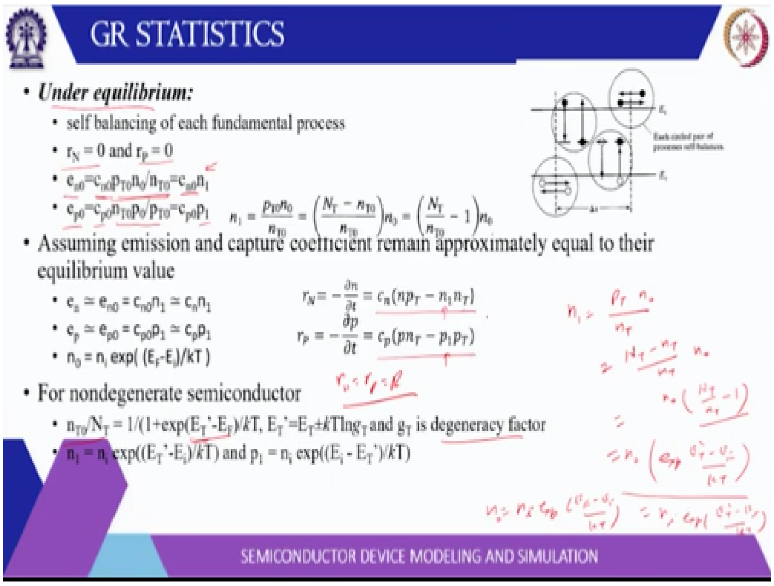

GR STATISTICS

- ***Under equilibrium:***
  - self balancing of each fundamental process
  - $r_N = 0$ and $r_p = 0$
  - $e_{n0} = c_{n0} p_{T0} n_0 / n_{T0} = c_{n0} n_1$
  - $e_{p0} = c_{p0} n_{T0} p_0 / p_{T0} = c_{p0} p_1$ $\quad n_1 = \frac{p_{T0} n_0}{n_{T0}} = \left(\frac{N_T - n_{T0}}{n_{T0}}\right) n_0 = \left(\frac{N_T}{n_{T0}} - 1\right) n_0$
- Assuming emission and capture coefficient remain approximately equal to their equilibrium value
  - $e_n \simeq e_{n0} = c_{n0} n_1 \simeq c_n n_1$
  - $e_p \simeq e_{p0} = c_{p0} p_1 \simeq c_p p_1$
  - $n_0 = n_i \exp((E_F - E_i)/kT)$

$$r_N = -\frac{\partial n}{\partial t} = c_n(np_T - n_1 n_T)$$

$$r_p = -\frac{\partial p}{\partial t} = c_p(pn_T - p_1 p_T)$$

$r_N = r_p = R$

- For nondegenerate semiconductor
  - $n_{T0}/N_T = 1/(1+\exp(E_T' - E_F)/kT$, $E_T' = E_T \pm kT\ln g_T$ and g_T is degeneracy factor
  - $n_1 = n_i \exp((E_T' - E_i)/kT)$ and $p_1 = n_i \exp((E_i - E_T')/kT)$

$n_1 = \frac{p_T n_0}{n_T}$

$= \frac{N_T - n_T}{n_T} n_0$

$= n_0\left(\frac{N_T}{n_T} - 1\right)$

$=$

$= n_0\left(\exp \frac{E_T' - E_i}{kT}\right)$

$n_0 = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = n_i \exp\left(\frac{E_T' - E_i}{kT}\right)$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now once we have idea of this what is the capture coefficient, we can relate them to the emission coefficient. So in the equilibrium, these individual process rate for electron and hole capture is individual to the zero. So we will look at this equation. If you equate this equation, then what you get here c n times n p t is equal to e n times n t. So your c n can return as e n times n T and n pT.

So this ratio is relating the c n and e n. So e n and c n are related by e t times n naught by n T. And similarly, e p naught is related to c p naught n T p naught by p T naught. So this whole term can be defined as some by some constant called n 1. And in case of hole it is denoted by p 1. And if you see here N T p naught by p T is basically, let me write here, let us say n 1. So n 1 is p T times n naught by n T.

Now if you see here p T is capital N T minus small n T. So capital N T total trap states minus small n T divided by N T times n naught. So this is basically n naught times N T by small n T minus 1. So this is very similar to the Fermi's expression. So if you see N T by n T can be given by the Fermi type distribution, where it is 1 over 1 plus exponential e minus E f by k T.

So here E T is basically trap energy level because we are considering the trap energy level to be the filled energy level. So we are filling the trap energy level. So your energy is replaced by E T. So if you write N T by small n T minus 1 that will come out to be, this will be n naught times N T by small n T minus 1. So it will simply exponential E T minus E f by kT.

So this is very similar to the expression for your n because n is equal to n i exponential E f minus E i by k T. So if you substitute here this n naught is equal to let us say n i. So if you substitute here what you get here? You get n i times exponential E m minus E i E T minus E f. So it will be E T minus E i by k T. Now E T is replaced by E T prime to take care of the degeneracy factor.

So like we saw donor level acceptor level there are degeneracy of two for donor level, there are degeneracy of four for acceptor level. So to get care of the degeneracy E T is replaced by E T prime. Now we can substitute this here. So we can express e n in terms of c n. Then the modified expression will be c n times n p T minus n 1 n T and for r p will be c p times p n T minus p 1 p T where n 1 p 1 are the computed constants.

And this n 1 is basically e n by c n naught. Now at equilibrium they are they will, so this is under equilibrium we have derived. And we have assumed that same emission coefficient and capture coefficient are valid when we move away from the equilibrium. So then we can go to the stt state where r n will be equal to r p will be equal to recombination rate.

And from that we can get this relationship between n p and n T p T. Because n T p T are basically the sum is capital N T. So we can find the expression for N T.

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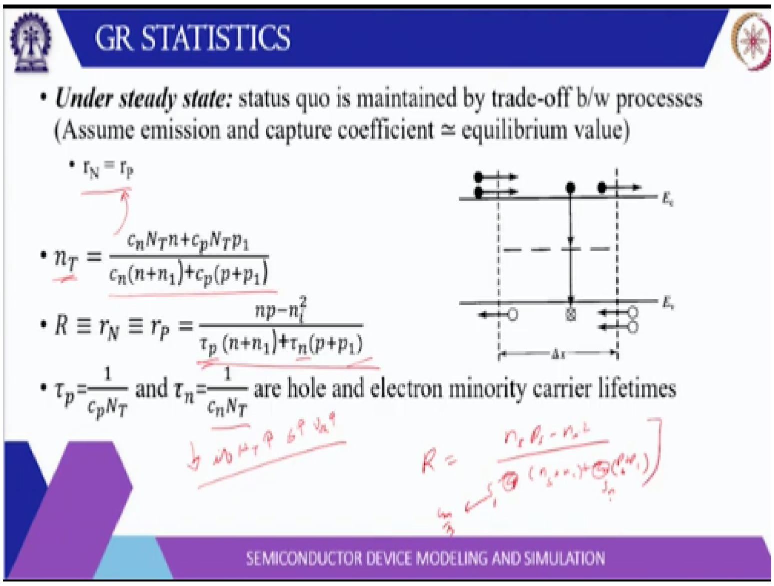

GR STATISTICS

- ***Under steady state:*** status quo is maintained by trade-off b/w processes (Assume emission and capture coefficient ≃ equilibrium value)
  - $r_N = r_p$
- $n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n(n+n_1)+c_p(p+p_1)}$
- $R \equiv r_N \equiv r_p = \frac{np-n_i^2}{\tau_p(n+n_1)+\tau_n(p+p_1)}$
- $\tau_p = \frac{1}{c_p N_T}$ and $\tau_n = \frac{1}{c_n N_T}$ are hole and electron minority carrier lifetimes

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So by algebraic manipulation, we get the expression for n T like this, where we equate this r N, RN and r P. So once you have this expression for N T you can substitute it to this expression for r N r P and then you get this expression, np minus n i square by tau p n plus n 1 plus tau n times p plus p 1. This we have already discussed and here the constant tau p and tau n are basically 1 over c n times N T.

And c n we know it is sigma the capture cross section times thermal velocity times N T. So that means, the carrier lifetime decreases, if trap energy level increases, if the capture cross section increases or the thermal velocity increases. So then the carrier lifetime actually reduces.

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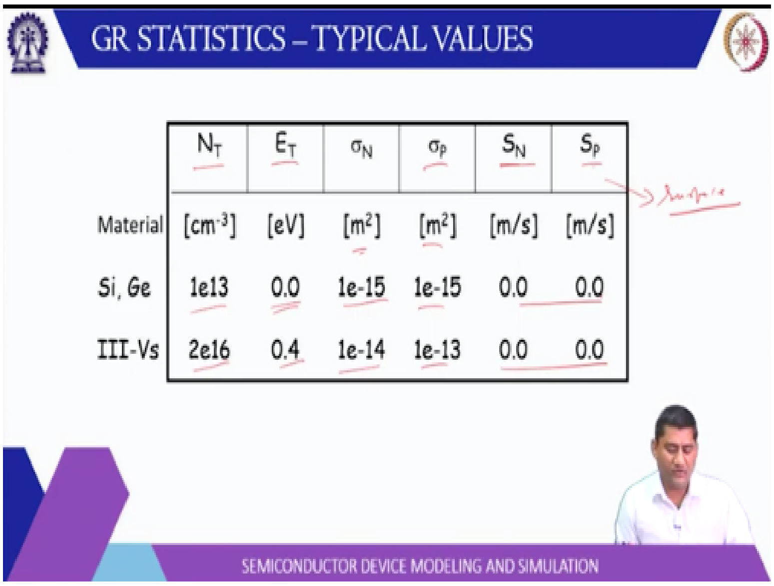

GR STATISTICS – TYPICAL VALUES

	N_T	E_T	σ_N	σ_P	S_N	S_P
Material	[cm^{-3}]	[eV]	[m^2]	[m^2]	[m/s]	[m/s]
Si, Ge	1e13	0.0	1e-15	1e-15	0.0	0.0
III-Vs	2e16	0.4	1e-14	1e-13	0.0	0.0

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Then the typical values that are seen in case of semiconductor that the trap concentration is varies from 10 is to power 13 to 10 is to power 16. And this trap energy levels usually found in somewhere in the middle of the bandgap and slightly above here. And then capture cross section is order of 10 is to power -15 or 14 meter square and same for the holes, roughly same order.

Then S N and S P are called surface recombination velocity. These are applicable at the surface. So this model that we have derived is valid for bulk case of semiconductor. So in case of surface process this R will be replaced by n s p s minus n i square divided by tau p times n s plus n 1 plus tau n times p plus p s plus p 1. So this n s p s will be the surface recombination.

Now you notice here, this surface concentration will be per unit area and earlier this n and n i they are per unit volume. So if this is per unit area, then this tau p and tau n will be replaced by another term called s p and s n. So this now their unit will be surface recombination velocity, so this will be centimeter per second. So this expression, same expression gets modified for surface case also.

So then in that case we have this tau n tau be the replacement surface recombination velocities and they are shown here but they have some finite values. So that can be found from the literature. So with this we complete the derivation of generation recombination model for trap assisted state.

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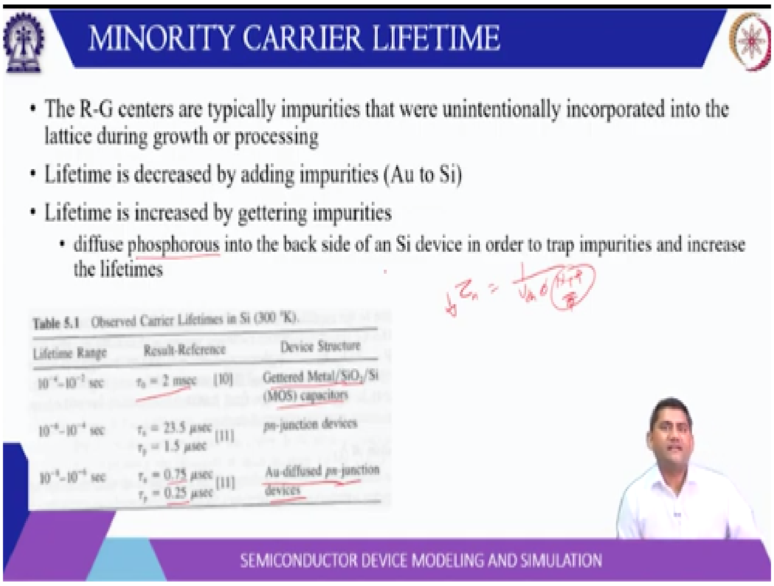

MINORITY CARRIER LIFETIME

- The R-G centers are typically impurities that were unintentionally incorporated into the lattice during growth or processing
- Lifetime is decreased by adding impurities (Au to Si)
- Lifetime is increased by gettering impurities
  - diffuse phosphorous into the back side of an Si device in order to trap impurities and increase the lifetimes

$\tau_n = \frac{1}{v_{th}\sigma}$

Table 5.1 Observed Carrier Lifetimes in Si (300 °K).

Lifetime Range	Result-Reference	Device Structure
10^{-4}–10^{-3} sec	$\tau_g = 2$ msec [10]	Gettered Metal/SiO$_2$/Si (MOS) capacitors
10^{-6}–10^{-4} sec	$\tau_n = 23.5$ µsec $\tau_p = 1.5$ µsec [11]	pn-junction devices
10^{-8}–10^{-6} sec	$\tau_n = 0.75$ µsec $\tau_p = 0.25$ µsec [11]	Au-diffused pn-junction devices

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

And then how do we modify these minority carrier lifetime, because this lifetime we saw let us say tau n was 1 over c n times N T and c n was v thermal times cross section times N T. So if you increase this trap assisted level, then the carrier lifetime will decrease. And if you decrease this trap concentration, then your lifetime will increase.

So when you want to increase this N T, then that is when you decrease the lifetime you diffuse the gold into the semiconductor material, especially the silicon. Then the lifetime will actually will be reduced. So it will be order of microsecond. And when you want to remove this or you want to increase the lifetime especially applicable for solar cell and all what we do we want to remove these impurities.

So this is done by gettering. So the gettering is a process where they basically capture these impurities. So when we diffuse the phosphorus from the back side of silicon device, this will trap the impurities and this will increase the lifetime.

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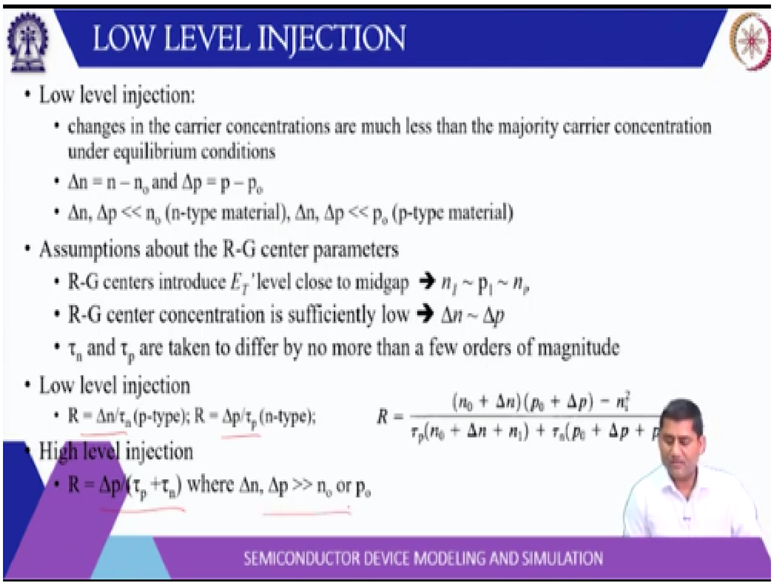

LOW LEVEL INJECTION

- Low level injection:
  - changes in the carrier concentrations are much less than the majority carrier concentration under equilibrium conditions
  - $\Delta n = n - n_o$ and $\Delta p = p - p_o$
  - $\Delta n, \Delta p << n_o$ (n-type material), $\Delta n, \Delta p << p_o$ (p-type material)
- Assumptions about the R-G center parameters
  - R-G centers introduce E_T 'level close to midgap ➔ $n_I \sim p_1 \sim n_i$
  - R-G center concentration is sufficiently low ➔ $\Delta n \sim \Delta p$
  - τ_n and τ_p are taken to differ by no more than a few orders of magnitude
- Low level injection
  - $R = \Delta n/\tau_n$ (p-type); $R = \Delta p/\tau_p$ (n-type);

$$R = \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p}$$

- High level injection
  - $R = \Delta p/(\tau_p + \tau_n)$ where $\Delta n, \Delta p >> n_o$ or p_o

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So this we have seen, we have discussed that for low level injection, this is delta n by tau n for p type and delta p by tau p for n type and for high level injection is delta p by tau p plus tau n, that we have discussed. So with this we complete the discussion on generation recombination. Thank you very much.