Semiconductor Device Modelling and Simulation
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Lecture – 39
Problem Session - 6

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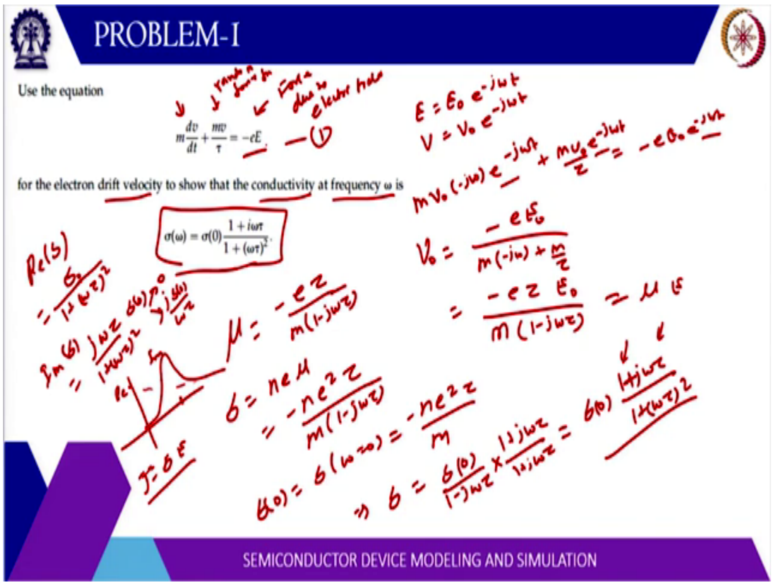

Hello, welcome to lecture number 39. So, we will take up some problems today. Use this equation governing the drift velocity of electron to show that the conductivity at frequency omega is given by the following expression. Here, it is basically a force balance equation where this is the force due to electric field then this is m dv by dt which is mass into acceleration, so which is f = ma.

Then there is another force here which is called random force function. So, this is basically a force valence equation. Now, we have to calculate the drift velocity at frequency omega. So, let us assume that we are applying electric field E = E naught e to the power – j omega t. And let us say that the drift velocity follows the same expression. So, V is also V naught e to the power –j omega t. Now, let us substitute this equation number 1.

So, it becomes mv dv by dt becomes mv naught – j omega e to the power –j omega t + mv naught e to the power –j omega t by tau= –e times e naught e to the power –j omega tau e to the power –j omega tau is common, so, it will cancel out. Then we can write V naught = –e divided by – E naught divided by m times – j omega + m by tau. So, this we can further write.

We can multiply and divide by tau, so, – e tau times E naught by m into 1 – j omega tau. Now, we know that V = mu times E. So, from here we can find out the mobility mu = – e tau by m into 1 – j omega tau. And this mobility is related to the conductivity in semiconductor. That sigma = ne mu. So, it becomes –ne square tau by m times 1 – j omega tau. Now, sigma naught, we can define, is sigma at omega = 0.

This is sigma at omega = 0, so that is –ne square tau by m. So, from this we can write sigma as sigma naught times 1 over 1 – j omega tau. Then we multiply it by 1 + j omega tau by 1 + j omega tau to get the expression. So, this is sigma naught times 1 + j omega tau divided by 1 + omega tau whole square. So, this basically tells you that the conductivity is a function of frequency.

Then you also notice here there is a real part of conductivity and there is imaginary part of conductivity. Now, real part of conductivity is basically a real part of sigma is sigma naught by 1 + omega tau whole square. That means that omega = 0 sigma is simply sigma naught. And at high frequency this conducted is actually decreasing. Then you can note the imaginary part of sigma, so that will be j omega tau sigma naught by 1 + omega tau square so, at 0 frequency or DC.

This is basically goes to 0 and at high frequency it actually goes to j sigma naught by omega tau. So, it high frequency also it actually goes to 0. So, in between it peaks up at some frequency, so, your imaginary conductory will look something like this. So, this will be some frequency relatively peak and then real conductor actually is always goes down with the frequency. So, this is the real part. This is imagery part.

The real frequency because j = sigma E. So, it is in the same phase that j and d are in the same phase. For imaginary part, they will be 90 degree out of phase. So, this conductivity if we consider a more general case of this because now we have considered only the electric field. We can consider the effect of magnetic field also because that also exerts a force.

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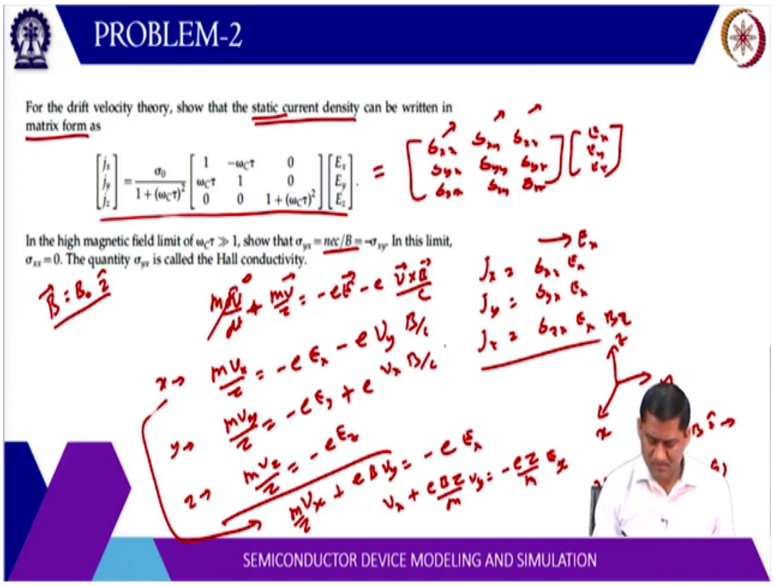

So, for that we have another problem here. It says from the drift velocity theory so that the static current density can be written in the matrix form as this. Now, if you look here, j = sigma E. And this can be written as sigma xx, sigma xy, sigma xz, sigma yx, sigma yy, sigma yz, sigma zx, sigma zy, sigma zz times E x, E y, E z. So now, it means that electric field is in x direction. If electric field is in x direction it can give component in the current component in x direction and y direction and z direction.

So, let us write here in terms of E x J x = sigma xx E x. J y is equal to that is a second here. So, sigma yx times E x and J z will be sigma zx times E x. So, just E x is able to give the current density in all the three directions. Now, how is it possible? Because that is a general case and it happens when there are certain constraints. So, let us write the same equation again m dv by dt = + mv by tau = –E.

Now, let us consider there is not only force due to electric field. Let us say there is a magnetic field also. So, we can write minus e times E is a vector minus e times V cross B. So, this will be more general case when both the electric field and magnetic field are present. Now, in this problem magnetic field is in z direction. So, let us say, B vector is some B naught in z direction and we have to find this relationship.

It is also shown here that static current density, so that means it is in is a static situation. That means time derivative are 0 so, this will be 0 here. So, we will have mv tau by – e – E times V cross B and let us say, B is in z direction. So, let us write for individual components because V is a vector. So, for x vector we can write mv x by tau = –e times E x because there is accept direction – e times V cross B.

So which component of B will give force in x direction. So, let us say this is x, y, z and B is in this direction B z direction. So, we have to get the force in y so, the velocity has to be in y direction. So, let us say, v is in y direction cross B in z direction. So, y cross z will give a component in x direction. So, it will be V by times B. Similarly, for y axis, we can write mv by tau = –e times E y – e times.

Now, for y your V has to be x in x direction, so, V x cap, cross B z cap, will give. Now, note the direction x cross z will give – y cap. So, we have to write minus here so, minus, minus will become plus. So, it will be minus V x times B similarly, for z cap mv z by tau will be minus e times E z. And for z direction the force due to B will not exist. Because when we take V cross B it will always be perpendicular to the B.

So because B is in z direction, so, it will not exert any force in this direction. So, with these three equations we can find out sigma xx, sigma xy, sigma yz. So, let us take first equation due to this x here, let us say this is the first equation. So, here, V x m by tau + e B v by is equal to minus e times E x. So, we can write V x + e B tau by m times V y = –e tau by m times E x. So, this is basically the right side. So now, we can write this thing for all the three cases.

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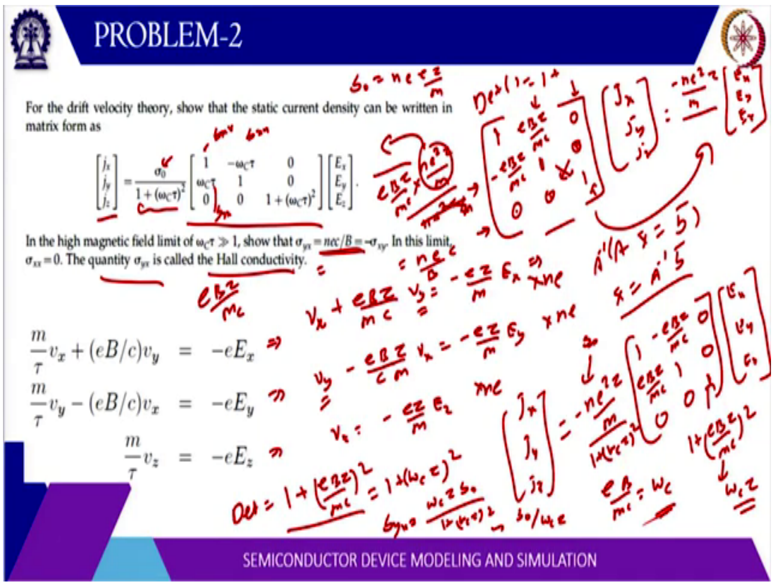

So, we can write here V x. o One more thing here you can notice the force is V cross B. This is in q times V cross B. This is an SI unit but in this problem the unit they have chosen is Gaussian units. So, this force basically becomes q times V cross B divided by C which is the velocity of light. So, this is nothing but a scaling factor relating the Gaussian unit to SI unit. So, this will appear here in this problem. So, it has to be e V by C.

Now, V x + e B tau by mc V y = –e tau by m times E x. Similarly, from this one V y – e B by C tau by m V x = –e tau by m times E y. And this is of course, simple V z = –e tau by m times E z. Now, we know that j = –j = n times e times V. So, J x = ne V x. So, we can substitute here so, V x can be written as J x by ne. So, if you further substitute here so, what we get here? Let us write in matrix format is form itself.

So, let us say, J x, J y, J z and V x is J by ne. So, all the V x can be replaced by J by ne or what we can do? We can multiply this thing by ne so, times ne times ne here. So, ne V x is J so, this becomes 1. Then it becomes E ne square B tau by mc. So, ne square B tau by mc. So, this becomes or we can just write e B tau by mc times J y. So, this is J y. So, it will now e B tau by mc plus this is e B tau by mc times J y and there is no z component 0.

So, this is 0 and this is equal to this has to multiply by n so, –ne square tau by m. So, this is constant for all of them. So, you can write here E x, E y and E z. Similarly, for second equation this is y is 1 so, y is in the middle line, so, this is one. For x you have minus e B tau by mc. There is no z component 0 and here x and y both are 0 z is 1. So, this also multiplied by ne so, ne is J z so, this is 1.

So, a coefficient times J is equal to a constant times E x, E y, E z. Now, what we have to do? We have to write this in terms of so, we have to take this matrix here. So, let us say this is something like A then x vector = B so, we can write. We multiply by A inverse to both sides. We get x = x vector is equal to A inverse times B vector. So that is what we will get. So, let us rearrange it.

So, we can write J x, J y, J z is equal to inverse of this matrix times ne square tau by m. So, if we take the inverse of this matrix, so, this we can keep as it is minus ne square tau by m times E x, E y, E z and this is the inverse of this matrix here. That say this matrix is A. So, if you take the inverse so, first, you have to take the determinant. So, inverse is basically the inverse has to be taken, as is a transpose of cofactor matrix divided by the determinant of the matrix.

So, the determinant of the matrix here will be 1. If we expand it then we can see actually for of diagonal element. It is always coming out to be 0. So, the determinant is 1 only. So, we do not have to divide by anything because the determinant is 1 only. So, determinant of this matrix is 1. So now, we have to take the cofactor matrix so, for the first element 1 into 1. So, this is 1 only so, we can write 1 here.

For this what is the term? So, this 1 will be –e beta by mc. Now, this will come here because this is a transpose. So and this is basically with a inverted sign so, this will be e B tau by mc. So, you can write here e B tau by mc and the third one if you take so, this will be 0, so, this will be 0 here. Now, for this element will be minus e B tau by mc. So, this will be minus e B tau by mc. And for this element will be equal to 0.

Now, for the middle element, 1 – 0 so, this will be 1 only. Then for this right side element, it will be 0, so, it will be 0. Then for this element it will be 0, so, this will also be 0. And then this for corner element this is 1 + e B tau by mc whole square, so, this will be 1 + e B tau by mc whole square. Now, this can be multiplied inside. So, if you notice here, this is ne square tau by m this be a value determined in the previous problem.

And this e B by mc is cyclotron frequency, so, this term can be written as simply omega C times tau omega C is the cyclotron frequency times tau. So, we are exactly getting this matrix and this coefficient ne square tau is same as sigma 0 by 1 + omega C tau square. So, we get this relationship relating the all the x, y, z component of the current density with the electric field.

So, when the magnetic field is present then the electric field in x direction can give the current in x direction current in y direction. May electric field in y direction can also give the current in x direction current in y direction. An electric field inject direction it can only give the electric field in z direction because the magnetic field is in z direction, so, there is no such force.

Now, at high magnetic field omega C tau will be much greater than one because this resonance frequency, the cyclotron frequency is basically e B by mc. So, at high field, this omega C will be quite large, so, omega tau will be quite large. Then this is basically, if you compare, this is sigma xx, this is sigma xy and so on. So, this is sigma yx, sigma yx so, sigma xy n sigma yx is e B tau by mc and this is e B tau by mc.

And we have to multiply this thing by ne square tau. So that is how you are getting it so, e B tau by mc times ne square tau by m. So, here we take this is a product but this sigma 0 has to be taken out. Sigma 0 is ne mu which is n times e times e tau by m. So, this has to be divided by sigma 0 which is ne square tau by m and this is simply sigma 0. So, this is e B tau by mc. So, we have e B tau by mc. Then what you have done here?

We have divided and multiplied by this is 1 + omega C tau square. So, this 1 here o I think we made a mistake here. Determinant is not 1. Determinant is 1 + this is 1. Let me create some space here determinant is 1 – e B tau by mc. So, this is the first column. The second column will be, it will be 0 and this will be 1. So, again – e B tau by mc so, it becomes 1 + e B tau by mc whole square. The third column is 0.

So, determinant is this 1 + e B tau by mc whole square, so, this is basically 1 + omega C tau whole square. So, omega C is e B by mc. So, this is divided by 1 + omega C tau whole square. Then now, it is and this is of course sigma naught. Then a sigma yx is omega C tau times sigma naught y 1 + omega theta whole square, so, this is basically sigma yx = omega C tau times sigma naught 1 + omega C tau whole square.

So, at high frequency 1 can be ignored, so, this will be sigma naught y omega C tau. And omega C tau is e B tau by mc. So, this is basically e B tau by mc in denominator and this is a numerator. So, if you find out e will cancel m will be cancel so, e tau will come cancel. So, you will simply have ne then e tau cancel by B and then C will go up. So, ne is C by B. So, this is what you are getting?

Now, the significance of this is that at high frequency in this limit, the conductivity sigma xx is 0. So, there is the conductivity to the electric field itself is going to 0 at high frequency. So, only the conductivity due to the magnetic field is, basically existing there. And this quantity sigma by x is called the whole conductivity. So, we could discuss two problems in this session. Thank you very much.