Semiconductor Device Modelling and Simulation
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Lecture - 35
Semiclassical Transport (Continued)

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Hello, welcome to lecture number 35. So, today we will continue our discussion on the distribution function. As we have mentioned that to counter the problem of raking all the carriers, we came up with the concept of distribution function. Now, by solving for the distribution function, we can calculate all the required parameters that is a carrier concentration the current density, the energy density from the distribution function itself.

So, we can either solve the equations governing, semiconductor equations, in terms of these parameters or directly in terms of distribution function. So, let us discuss two distribution functions in this lecture, one is the Fermi-Dirac distribution function other is the displaced Maxwellian function.

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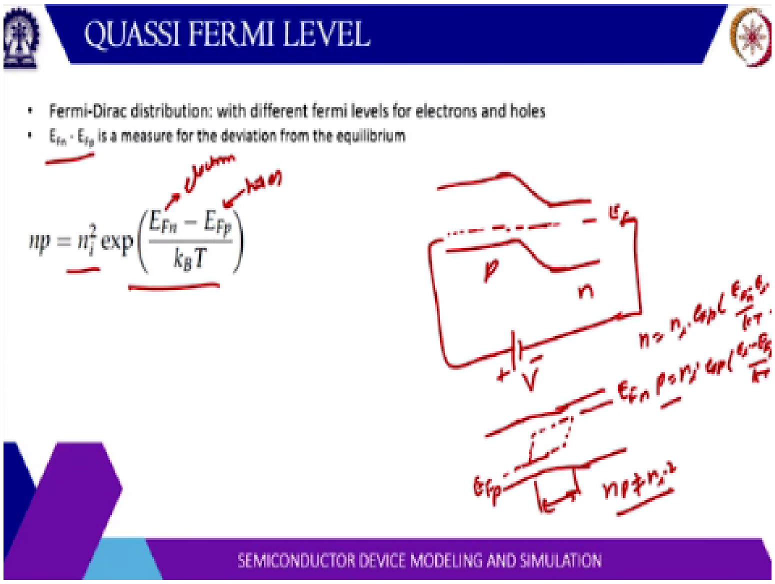

Now, Fermi-Dirac distribution function we know, let us recall the P-N junction. So, in case of P-N junction for let us say P this is the fermi level is here for N time fermi level is here, and then we add them for the transition like this here and this is anti-equilibrium the fermi level is constant. So, in this case, both the fermi levels are same whether for electron or for both. But this is not a frequent situation. What we do, actually we apply some bias here.

So, let us say we apply some bias here. So, what will happen when we apply a bias so let us say this side is positive, that is negative. So, when we apply positive bias, then the energy will reduce, so this fermi level will actually come down. So, what we can do? We can write it like this, fermi level here. This is a fermi level here and then the sub conduction band valence band it is a conduction band.

And let us say this is a conduction band, the valance band and there is some drop here. So, this is E F p we call it E F n. So, that we call the discussion on P N junction. So, now we have two fermi levels and we call it quassi fermi level. So, there is a concept of quassi fermi level. Now, if you see here away from the junction, the n or p are given by in terms of this fermi level here, but in between these two fermi level actually exist.

So, this of course it will be like this so suppose and then it depicted. So, in this reason, that two fermi levels so electron consultation is given by n i exponential E F – E i by k T and all conception is given by n i exponential so this is F n this is E i - E F p by k T and many, multiply the 2 you get n p = n i square times E F n - E F p by k T. So, this is true for in this region. That means here n p product is not an i square, is that is larger than i square if you apply point of bias to the p side.

So, E F p becomes less so this because more than n i square and we apply the reverse bias, it becomes less than n i square. So, what we can do? We can define a separate fermi level for electron that is for E F n and we can divided a separate fermi level for holes. That is E F p and then we are at the equation for n and p and that is one distribution function. Where the difference E F n – E F p is a major of deviation from the equilibrium. This is obviously a sub, kind of extension, the discussion on fermi level.

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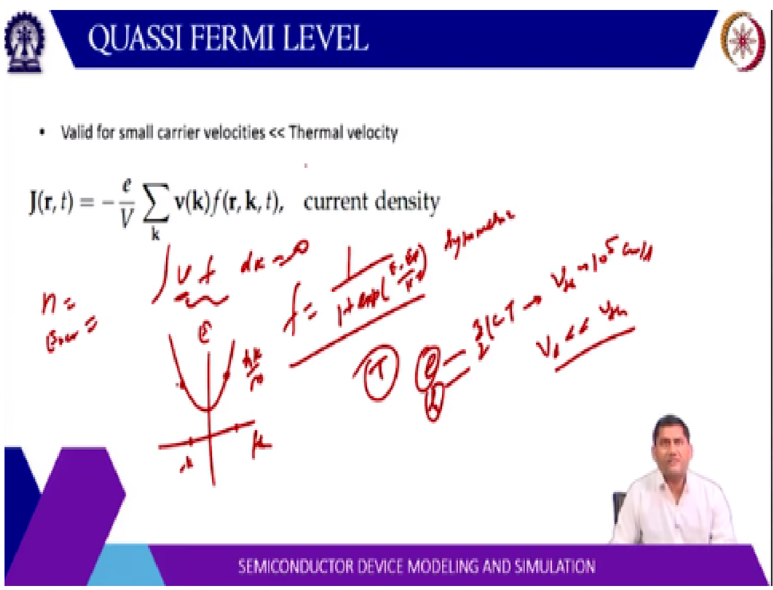

And then of course once we know the fermi level, we can find out the carrier density. We can calculate we can electron density, energy density and the current density. Now, if you notice here, when drawback with this fermi level concept is that if you integrate v times f dk. So, v is basically your E by E versus k. So, there is a v f h bar k by n. So, for parabolic point, this is symmetric. So, for let us get exist for - k.

We are v n – v n equal but if you notice the fermi level distribution based on fermi level. So, this is 1 over 1 + exponential E - E F by k T. So, this is symmetric, that means if you integrate, you will always get 0. So, that means this fermi level of distribution is valid only when these velocities are very small. Now, very small what does it mean? See at a given temperature these carriers they are randomly moving with some energy k T or one d half k T is 2 k T or for 3 d 3 by 2 k T and that gives you some thermal velocity.

Some 10 is to power 5 centimetre per second. So, if you drift velocity is much less than this thermal velocity, then you distribution function will not be altered much. And you can use this perimeter distribution because the velocities are much smaller than the thermal velocity. But if this is not the case or for a general scenario, we have to look for a better distribution function.

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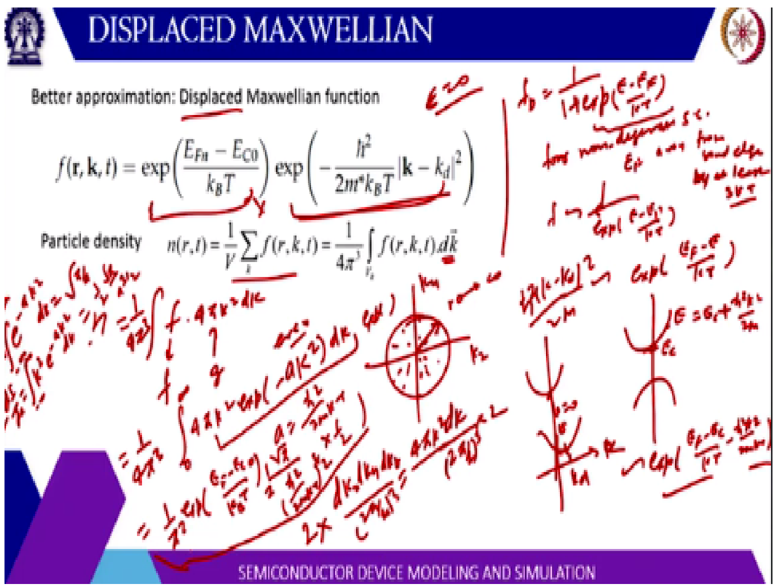

So, the better distribution, there is another option is call displaced Maxwellian function. So here, what is done here? This can be drag from the permittable distribution function also. So, if you recall the permittable distribution is 1 over 1 + exponential E - E F by k T. Now for nondegenerate semiconductor, your fermi level is away from band edge by at least 3k T. So, what we can do? This is much larger than one.

So, this f can we have approximated x 1 over, this is larger than so it is going to exponential E - E F by k T that is exponential E F - E by k T. Now, if you remember that this equation here. So, for electron, the energy is this is let us easy the bandage energy plus kinetic energy. So, for given k, h bar is square k is square by 2m. So, we substitute here, what you will get? You will get exponential E F - E c by k T - h bar square k square by 2 F times k T.

So, we start separating these two to we have the product exponential E m - E c by k T exponential minus h bar square by 2m k T. Now displaced Maxwellian means this is symmetric around k = 0. You can write it as a h bar square k - k T square by 2m. So, that means now it is **(()) (09:11)** k d. So, instead of 0 it is symmetric round k d so this is k d, so this is E versus k this is around k d is the bottom **(()) (09:24)** here.

So, this is the displaced Maxwellian the function distribution function. For this we can get calculate the particle density. So, the particle density, of course sigma f by volume or if you write in terms of the integration, then this n = integral f this dk is in 3 d. So, if you want to calculate the density in 2d you can calculate it in 2d, 1d of 3d. So, let us do it for 3d. So, in 3d the volume is given by this is a space k x k y and the dimension not on your k z.

And if you recall this density of a state in case always uniform, because the state is dk divided by the volume of state there is 2 pi by L. So, if you do 3d, then this is 2 pi by q times volume in 3d is 4 pi k square dk or you can write it as a dk x dk by dk z divided by 2 pi by m whole cube. So, then you have to include a spin factor of 2 because each state can of 2 electron. So, this is 2 pi cube divided by 2 because 4 pi cube. So, how you get fermi pi 4 by 2.

Then increase the volume this n is per unit volume so we can this is taken care of because this n is per unit volume. So, this 1 by b then f times 4 pi k square dk. Now if you recall it this is fermi rate distribution and this is g the density of state 4 pi k square dk which we converted into energy and integrated it. We can do it in terms of in the k also and we use the same expression. So, let us substitute let us says assume that k T= 0 here.

No field is applied let us elective field applied is 0. So, if you integrate it, what will you get? 1 over 4 phi cube integral this exponentially outside. So, and this is 4 pi k square exponential this character is minus a times k square dk and where a is h bar square by 2m k T. So, this is a standard integration of times this exponential function so k square, exponential - a k square dk. If you remember exponential e to the power - a k square dk is root pi by a.

And then so, this is integral I you would differentiate dI by da what you get - dI by da you get integral k square e to the power – a k square dk and that will be 1 by 2 root pi by a to the power 3 by 2. So, you can substitute here this is 1 over pi 4 pi cube because 1 over pi square that exponential we can write here E F – E c divided by k T this is k Boltzmann constant times root pi by 1 by 2. So, this is limit is minus infinity to infinity.

Here, the limit will be 0 to infinity because what we are doing it. This is we are assuming it a sphere. So, sphere for the radius of a sphere will go from 0 to infinity. So, this will become half because this is even function. So, minus infinity integration is 2 times 0 to infinity integration. So, again we would have another factor of 1 by 2 times root pi by 2 times a to the power 3 by 2 and a is h bar square by 2 m k T power 3 by 2.

So, if you look at it, it is same expression we drive for the particle density in the beginning. So, before the simplified here and so let me some part that I leave it to you homework and that you simplify this expression and compare it with the Dirac density function for the carrier density.

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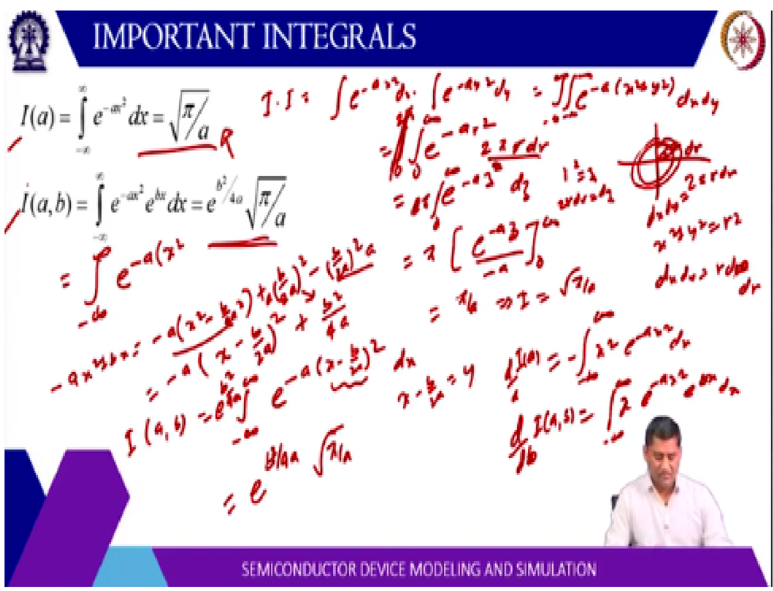

Now before we proceed for the calculation of other parameter, let me formally introduce the two integrals that are very helpful in calculating these parameters. So, one is of course e to the power – a x square dx is root pi by a. So, this, I already used it in the previous derivation, so how do we get basically, what we do? We multiply I with another I and be changed the variable x to y so e to the power - a x square dx time e to the power - a by square dy it becomes I square.

And that can be written as e to the power - a times x squared that y square times dx dy. Now, this is now area integration so - infinity is infinity, minus infinity to infinity area integration. Now, we can change the parameters plus we considered area so that is the area will be 2 pi r dr. So, instead of dx and dy, we can write 2 pi r dr. So, at distance, are we are taking a another region here and r will be from 0 to infinity and x square + y square is actually r square.

So, this integral is square is basically e to the power - a r square 2 pi r dr and limited 0 to infinity for r and 0 to 25 for theta, but we have already taken this single so about done here. This not required now because if you write this area as dx dy s r d theta times dr. There of course when you integrate this d theta because this is r d theta times d as so this is more area. When you integrate this data 0 to 5, you get this factor. So, this is not required here.

So, 0 to infinity e to the power - a r square 2 pi r dr when you integrated now, here r square can be substituted as the z so, e to the power - a z at 2 pi can we take outside 0 to infinity r dr s. So, one, this two configures to 2 r dr is dz so, e to the power – a z is square z. So, r square = z, 2 r dr = dz. So, this is simple now. So, pi times e to the power - a z y - a 0 to infinity. So, at infinity this 0 at 0 is only a, so this really pi by a.

So, that means your I will be root of pi by a, so that we get this integral. Now, our both in the form, sometimes it has some power x also, instead of x square. So, this is another form e to the power - a x square e to the power dx dx. Now, this condition is minus infinity to infinity E to the power - a times x squared, what we can do here? Just look at the power – a x square + b x. This can be done as - a times x square then - b by 2 a x this is b by a x.

Then what we can do, we can plus b by 2 a square - b by 2 a square. So, this is - a times x - b by 2 a whole square so b by 2 a times a so, this will be a times x - b by 2 whole square. So, now we will have plus b square by 4 a square multiplied by b square by 4 a this minus is absorbed here about this is making up whole square. So, your I a, b is – infinity to infinity e to the power - a x -

b by 2 a whole square times e to the power b square by 4 a is taken out e to the power 4 a as dx.

Now if we substitute x - b by 2 a = **(()) (20:39)** y. So, this integral becomes e to the power - a by square dy and limit remains same. So, this is integral is root pi by a so, this is e to the power b square by 4 a stands root pi by a. For these to remember now so I am just these two factors we can calculate other integer also because first is a special case for b = 0. If you put b = 0, you will get the first integral. Another one you can also get integral with coefficient x or x square.

So, if you do let us say I as a so d by a, what you will get you let x square e to the power - a x square dx. If you know d by da db for I a, b that will be x times e to the power - a x square e to the power b x dx and then of course, you can differentiate this one. So, you can get all the possible integrals just by using these two integrals. So, you remember these two integrals then the rest of the things will follow in line. Now this may valid drive basically.

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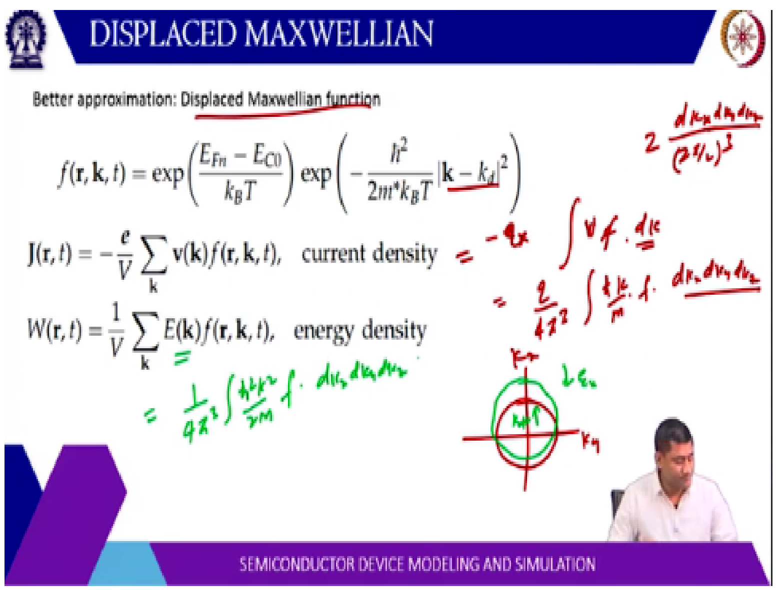

So, for this displaced Maxwellian function we can also calculate the current density at the energy density. So, if you write the expression for the current density in terms of integral so this is - e times this is integral v f dk in 3 d so that is e times v times f times dk and **(()) (22:41)** started q. So, this kind written as q times the dk is basically volume in **(()) (22:50)**. So, what we do basically rewrite? dk divided by 2 pi by L whole cube this is dk x, dk y, dk z or divided by 4pi square dk.

So, what we write here? This is q by then there is a factor coming from Pauli Exclusion Principle. So, you have this factor here 1 over 4 pi cube. So, this also a per unit area so this L cube and this will be the unit of area, then b can written as h bar k by m times f times dk x, dk y, dk z. So, I am writing dk x, dk y, dk z because now is not same in x y z direction, whether it is displaced. So, if you look at some so this is your k x k y map, k x, k y and k z.

When you apply the electric field let us say you apply the electric filed in this direction. So, wave form will be displaced in less than x direction. So, this is the displayed Maxwellian function. So, now you see there is a net movement in k x direction. So, there is electric field here that says in negative x direction then these carriers will move in e x direction. So, this centre is instead of 0 k d. So, this value is k d. So, it is better to use dk x, dk y, dk z.

Similarly for energy density of we can write 1 over 4 pi cube times E k is h bar square k square by 2m times f dk x, dk y, dk z you can integrate. So, maybe I will give you some of steps so that you can do it.

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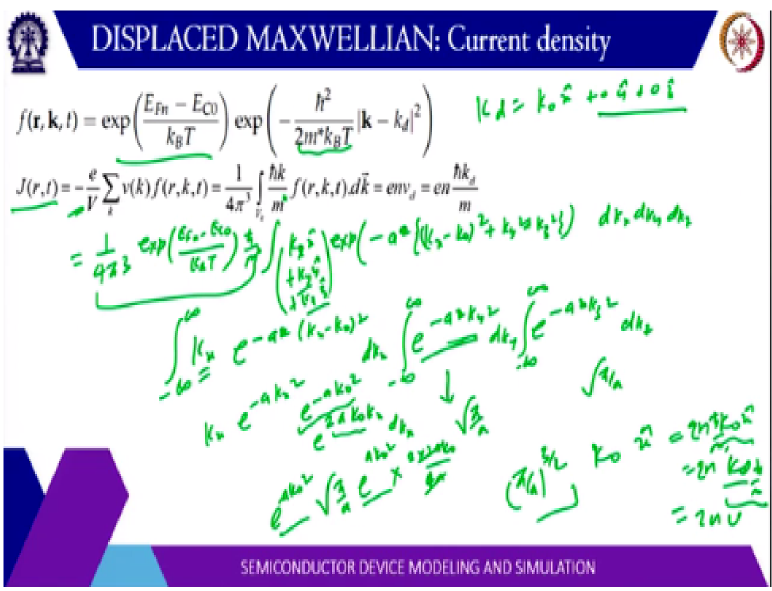

So, this is the expression for the current density which is 1 over 4 pi cube h bar k m is the velocity then f dk. So, what you get here, basically now this let us say without the loss of generality let us say k d is basically some k 0 in x and y the direction 0, basically so 0, in y direction the 0 in z direction. So, we can choose in any direction, **(()) (25:48)** and but our life will be easy but this is general basically.

See the direction of electric field or the fellow can **(()) (25:57)** any direction, so y 0 in x direction. So, let us assume k d is k 0 in x direction. So, this integral 1 over 4 pi cube times exponential this term can be taken out because this is not dependent on k so, E f n – E c 0 by K b times T then h bar by m then also take about the m is in effective times k. Now this k 0 in x direction times e to the power or exponential minus let us call this as a so - a square.

Now k - k d square can be written as k x – k 0 square + k y square + k z square times dk x, dk y, dk z. Now this can be integrated you will notice here other components are 0 here so, this current flow will also in the x direction only so that is the direction of k d because this is k d. Here, this is not the k d this will be k x x cap + k y y cap + k z z cap does this exponential. So, what we have to? We have to expand it for all three dimensions.

So, let us this your coefficient of it take it away. So, we have three integrals so, one integer will be k x times exponential - a square k x - k 0 square then e to the power – a square k y square e to the power - a square k z square then dk x we can write here dk y we can write here dk z we can write here and you can integrated. The integrated minus infinity to infinity now we are not assuming to be spherical. So, this is for dk x, dk y, dk z.

Now, here we can use those integer formulas. So, this is simple e to the power so this is a **(()) (28:16 -28:18)** so e to power - a x square dx is root pi by a this is root pi by a this is also root pi by a, this is k x e to the power - a k x - k 0 square. So, this one we can recall that expression dI by d a dI by db.

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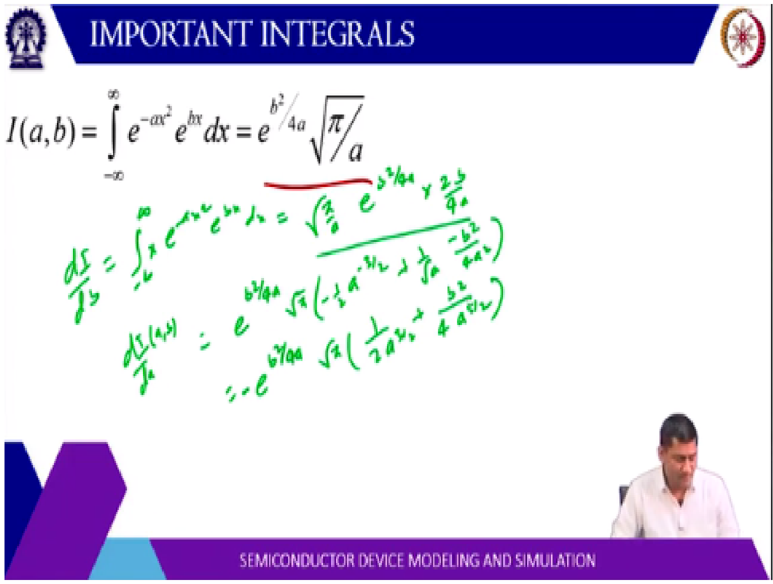

So, that will be - infinity to infinity e to the power – a x square x times e to the power b x dx and that will be if you differentiate with respect to b so root pi by a will be same and e to the power b square by 4 a times 2b by 4a. And it is k x - k 0 whole square we can substitute here. So, these integrals what we can do? We can write it separately like this k x time e to the power – a k x square then e to the power – a k 0 then e to the power - 2 k 0 k x so that becomes 2 times a k 0 k x dk x.

So, a cannot taken out so what we have? This a and this can we tell as b, so e to the power if you release compared to e to the power - a x square e to the power b x. So, we can substitute this expression here now root pi by a e to the power b square by 4a so b square by 4a is 2a k 0 square by 4a. So, that is a k 0 squared and what is the third term 2b by 4a, so 2b by 4a is 2b is a times k 0 so 2 times 2 a k 0 by 4a so, this 4a will cancel.

And e to the power – a k 0 square so this two term will cancel out. So, what you have basically? This integral is root pi by a 3 times of power 3 by 2 times k 0 in x cap direction. So, this term, basically if you recall, these are the same as for n. So, if you calculate the carrier concentration so this is basically n so it can written as some n times k 0 x which is k d that is n times k d. And of course, the charge is outside so, q n times k d times h bar by m 0.

So, times h bar by m 0 so this is velocity. So, this is q n times v you will get. Now what about other component k by n k z. For k by n k z if you look here so for k y there is no k 0 here. So, k by times, e to the power - a k y square dk y will be 0. Because this is even function, even function multiplied by odd function integer will be 0. So, the y component z component will be 0. So, this is a expression for current density from displaced Maxwellian function. We can find out that the velocity is e n times h bar k d by m this e n times due to velocity v, d.

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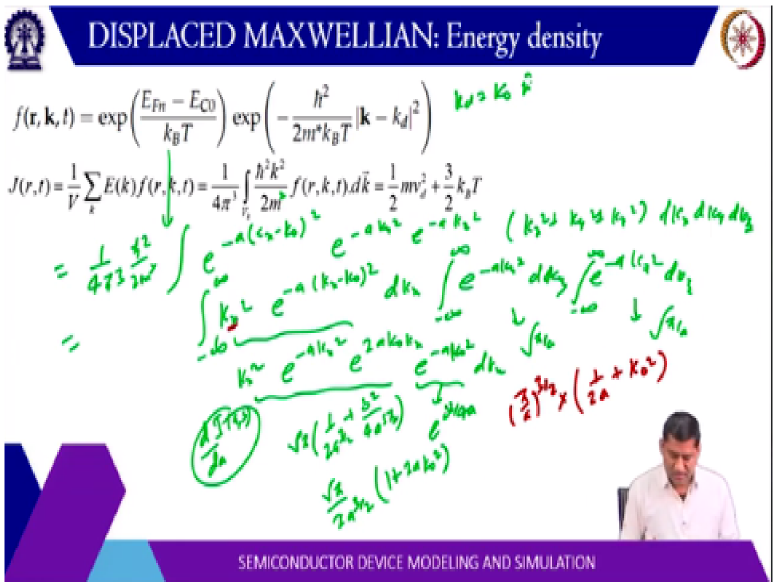

Same way we can find out the energy density. So, here again, we have to with the same integral so this is 1 over 4 pi cube times h bar is square by 2m. This is m star effective mass times f dk. So, let us say this exponential we write as it is and e to the power - a k x – k 0 again we assume that k d = k 0 in x direction k 0 square e to the power – a k y square e to the power – k z square times this k square here k x square + k y square + k z square times dk x, dk y, dk z.

So, now I will integrate basically you keep this term outside the again you have current density of k x square and k y square and k z square. So, there is 3 integral so one will be k x square times e to the power - a k x – k 0 square dk x e to the power - a k y square dk by e to the power of – k z square dk z and integrated minus infinity to infinity. So, this will be the root pi by a this will also root pi by a and this is k x squared e to the power - a x k 0 square.

This is basically, this will be double derivative. So, if you look at this equation here, this is dI by db so for you can take with respect to k also so that can also return, this is k x square. So, this can be written as k x square e to the power – a k x square e to the power 2 a k 0 k x e to the power - a k 0 square dk x. So, this can be taken out, k 0 square. So, this is a and b same power**.** Now you have this a square here.

So, what you have to do you to take dI by da for I a, b and that we can calculate here dI by da a, b you will get e to the power b square by 4a times root pi so 1 by 2 that is - 1 by 2 e to the power - 3 by 2 that is 1 + 1 over root a times derivative of this exponential, that will be - b square by 4a square e to the power b square by 4a and root pi by 1 over minus we can takeout e to the power 3 by 2 + b square by 4 a to the power 5 by 2, this is 2.5 or 5 by 2.

So, we substitute this one there. So, e to the power b square by 4a is same as –a square that will cancel. This will cancel out e to the power b square by 4a and other terms are root pi times 1 over 2 to the power 3 by 2 + b square by 4a to the power 5 by 2 and that will substitute here. What you will get root pi times 2a to the power 3 by 2 can be taken out. So, this is 1 plus this is b square by 2 a. So, b square by 2 a is square by 2 a times k 0 square k x square.

This becomes root pi 1 over 2 3 by 2 + b square by 4 by 5 the root pi by a. So, this can be written as pi by a to the power 3 by 2 times one power 2a + k 0 square. That is for k x squared.

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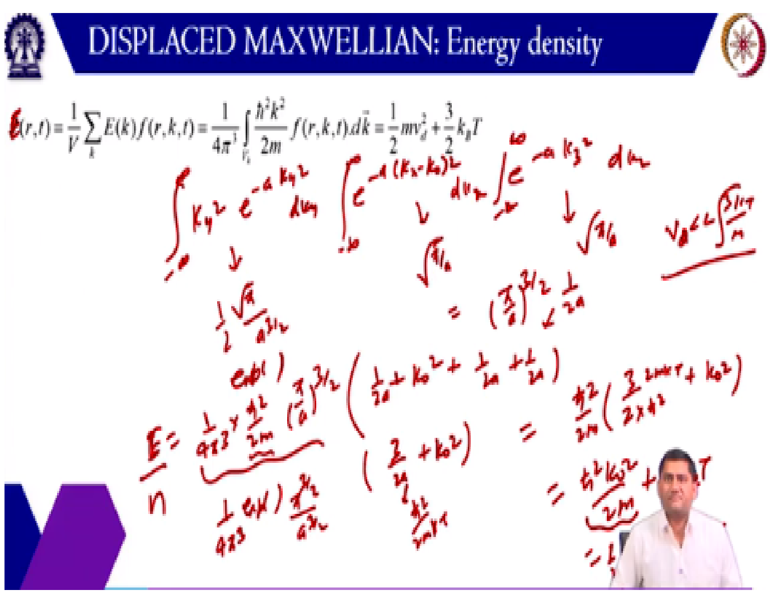

Now, same thing we can do for k y square. So, let us try the integer for k y square e to the power

- a k y square e to the power - a k x - k 0 square e to the power - a k z square dk z, dk x, dk y and these integral from minus infinity to infinity. So, this is of course root pi by a this is e to the power - a k x - k 0 square that will be e to the power this bit again be root pi by a only because you will take a times k 0 square outside, then it is e to the power a x square + b x.

That will also in this plus b square by 4a, so that b square will get cancel. So, it will also root pi by a this one is k y e to the power 1 is k y square so this is a derivative with respect to I. So, that will be root pi with respect this y so this will be 1 by 2 a to the power 3 by 2. Similarly, we can do for k z so this is basically root pi by a that is pi by a to the power 3 by 2 times 1 over 2a. The third term will also be same pi by a cube by 2 + 2 x.

So, total the integer will be now 1 by 4 by 4 by 2 times h bar square by 2m times pi by a to the power 3 by 2 then 1 by 2a + k 0 square + this one 1 by 2a + and another one 1 by 2a. So, what you will get basically it is 3 by 2a + k 0 a square and this term is here so, now if you compare this one, this one is related to the n and a is if you write it again substitute for a is h bar square by 2m k T.

So, a is h bar square by 2m k T. So, this is basically your here divided by n. So, if we divide whole thing by n so n is basically that exponential factor. So, n is 1 over 4 pi q that exponential factor E c - E F by k T times pi to the power 3 by 2 y a to the power 3 by 2, so that was n. So, this will basically cancel here. So, what you have here basically h bar square by 2m will be outside, these things will be cancel out so what you have 3 by 2a a is h bar squared by 2m k T + k 0 square.

So, what you have here? h bar square k 0 square by 2m + 3 by 2 k T. So, this is half m v square + 3 by 2 k this is Boltzmann constant this average energy as often from the displaced Maxwellian function. So, the idea of the lecture was that just from the distribution function, we can get all the parameters, we can get the distribution, the carrier density, we can get the current density we can also get the energy density.

So, here energy density is half mv square by 3 by 2 k T. Now the cases when v d is much less than this three k T by n square root, so that means energy, mostly a thermal energy and the drift **(()) (43:51)** is that is small there.

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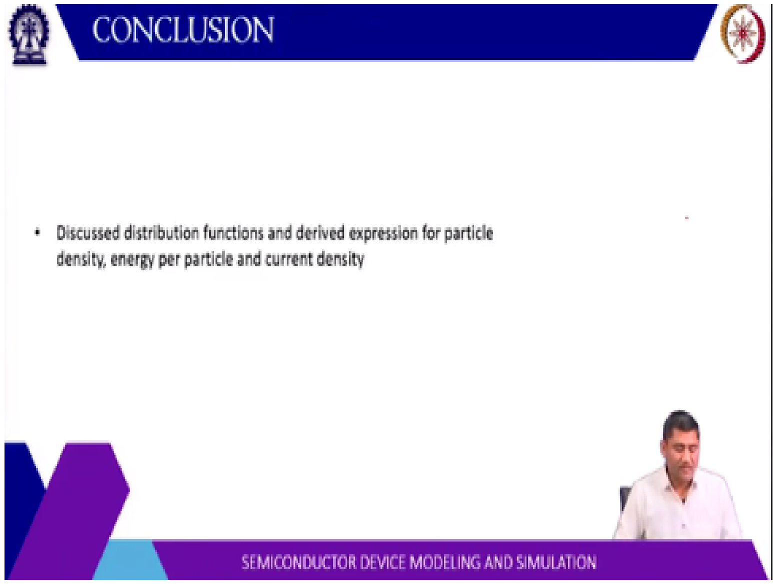

So, in this lecture, we have discussed that distribution function that two distribution functions. One is the quassi fermi level based fermi level distribution function another is the displaced Maxwellian function and then from that the particle density, then energy per particle and the current density. Thank you very much.