

Semiconductor Device Modelling and Simulation
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Lecture – 33
Problem Session-5

Hello, welcome to lecture number 33.

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PROBLEM-I

A GaAs MESFET is fabricated using an $0.2 \mu\text{m}$ GaAs epitaxial layer doped to $N_D = 10^{17} \text{ cm}^{-3}$. The Schottky barrier metallization has a barrier height of 0.75 eV . Assume $\mu_n = 5000 \text{ cm}^2/\text{Vs}$ and $v_s = 1.3 \times 10^7 \text{ cm/s}$, $N_C = 4.7 \times 10^{17} \text{ cm}^{-3}$.

(a) $L = 10 \mu\text{m}$ and $Z = 100 \mu\text{m}$
 (b) $L = 0.2 \mu\text{m}$ and $Z = 100 \mu\text{m}$

Calculate

(a) built-in voltage V_{bi}
 (b) pinch-off voltage V_p
 (c) What is the depletion layer thickness with zero gate voltage?
 (d) Plot I_D vs V_D

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Today, we will discuss some problems, whatever concept on FET and the MESFET and MOS devices we have discussed. So, let us consider first problem, so, it says a gallium arsenide MESFET is fabricated on a 0.2 micrometre gallium arsenide epitaxial layer. So that is basically your structure here. And then so, this is semi insulating gallium arsenide then on top of that there is a gallium arsenide layer whose thickness is 0.2 micrometre.

Now, it is doping is 10^{17} per cubic centimetre. Now, this is a Schottky context, so, there is a metal layer here. And it is barrier height is so that is $q\phi_{Bn}$ is 0.75 electron volt. So, if you recall the band diagram, so, this is metal Fermi level and in equilibrium this Fermi level is constant throughout the metal and semiconductor and this is basically $q\phi_{Bn}$ this is the barrier height.

And then this is the conduction band E_c and then something happens here. This is E_v the barrier height or the built-in voltage is this value this is qV_{bi} . So, as you can see in the picture, qV_{bi} is basically ϕ_{Bn} minus this gap $E_c - E_f$. So, $-E_c - E_f$ and $E_c - E_f$ you

can find from the doping because we know that here what is given is N_C , N_C is the effective density of a state for the conduction band.

And we know the relationship $n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$. So, from this we can find out $E_F - E_C = kT \log\left(\frac{n}{N_C}\right)$. Now, this is doped with N_D , so, n can be replaced by N_D so, this can be written as $kT \log\left(\frac{N_D}{N_C}\right)$. So, this is $E_F - E_C$. So, effectively this is $E_F - E_C$ so, $\phi_{Bn} + kT \log\left(\frac{N_D}{N_C}\right)$. So, this will be the built-in voltage. Then the pinch-off voltage, pinch-off voltage you can easily find.

Pinch-off voltage, the voltage required where the depletion region goes and touches this lower side. So, this depletion width is exactly equal to the width of this **(0) (03:49)** layer. So, we know what is the depletion layer thickness? The depletion layer thickness, so, $W_{depletion}$ is square root of $2 \times \epsilon \times (V_{bi} - V)$ divided by $q \times N_D$. And the way to remember is basically because the charge consume, if you assume that the depletion region is uniform charge concentration is abrupt change in this:

So, the charge concentration is qN_D plus so, this charge concentration will give or the fixed charge will give a linear change in potential electric field and that will be quadratic change in potential. So, you can find out that electric field will be linear so that will be qN_D plus times the width this is the width here by ϵ . So, this will be the electric field.

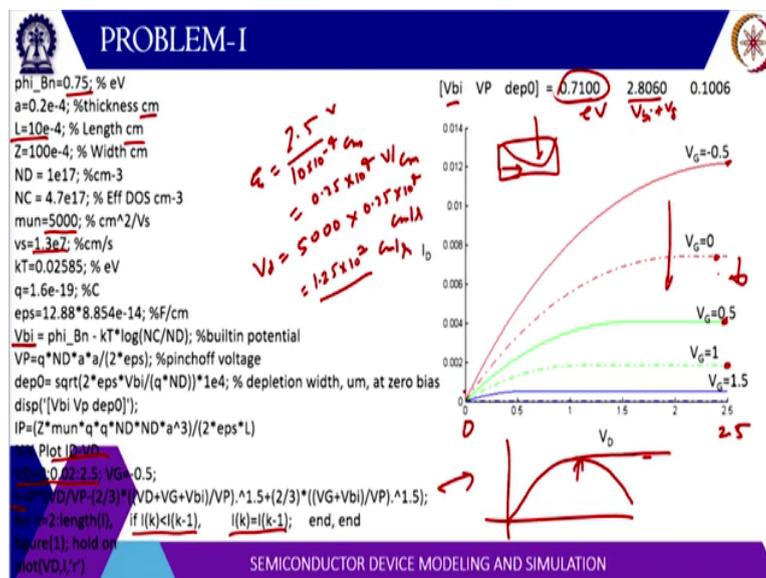
And the potential will be the area of this thing so, half base into height, so, potential will be V is basically $qN_D W$ by ϵ times half W . So, this is E_{max} . So, this is E_{max} into half W because E looks like this, so, this is the barrier and now this barrier is actually V_{bi} this barrier is equal to V_{bi} minus forward applied voltage or if this is reverse voltage then it will come plus a basically. So, this is V for what you can say this is V forward.

So, what did we do here actually in case of MESFET, this gate voltage is always reverse bias, so, this will become basically plus v_{gate} . So, $V_{bi} + V_{gate}$ is equal to this. And you can find from here that $W = \epsilon \times qN_D$ times this. So, this is the depletion layer thickness at 0 bias we will just use V_{bi} so, V_{bi} we have calculated already from expression 1. Then pinch-off voltage is when this $W_{depletion}$ is equal to let us say this is a when this is equal to a .

So, you can find out $V_{bi} + V_{gate} = a \text{ square times } qnD \text{ divided by } 2 \text{ epsilon}$. So, this is basically the pinch-off voltage generally what we do? We ignore this V_{bi} then we say this is pinch-off voltage. So but this is small but not insignificant so, actually we should include it and that will give you the pinch-off voltage. So, what I have done here basically, I have written a Matlab code to calculate all these parameters.

And then use the expression that we derive for $I_D V_D$ to calculate the I-V curve. Now, here the concept that I wanted to introduce we have to compare this input two cases one this is the length so, in one case length is 10 micrometre. In other case, the length is 0.2 micrometre and Z is basically the width into the paper. So, this is into the paper that is a Z so that will be the cross action area so W times the a .

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So, this I have written here the ϕ_{Bn} is 0.75. The unit is electron volt then a is 0.2 micron here I have used centimetre for all this values expressed in centimetre because our carrier concentration all those parameters are in terms of centimetre. So, they will effectively manage out and N_D is a doping N_C is the effective density of a state for conduction vent. Then μ_n is the mobility centimetre square per volt second.

And then this is the expression for V_{bi} so, this $\phi_{Bn} - kT$ and the value we get around 0.71 electron volt. Then V_P is $q N_D a \text{ square by } 2 \text{ epsilon}$ so that is around 2.8. So, this is basically $V_{bi} + V_{gate}$. So, V_{gate} will say 2.1 basically, you subtract 0.7 from here. So, the corresponding to V_{gate} of 2.1 you will get the pinch-off. Then to calculate $I_D V_D$ what we do? We select some range of voltage V_D from 0 to let us say 5 volt.

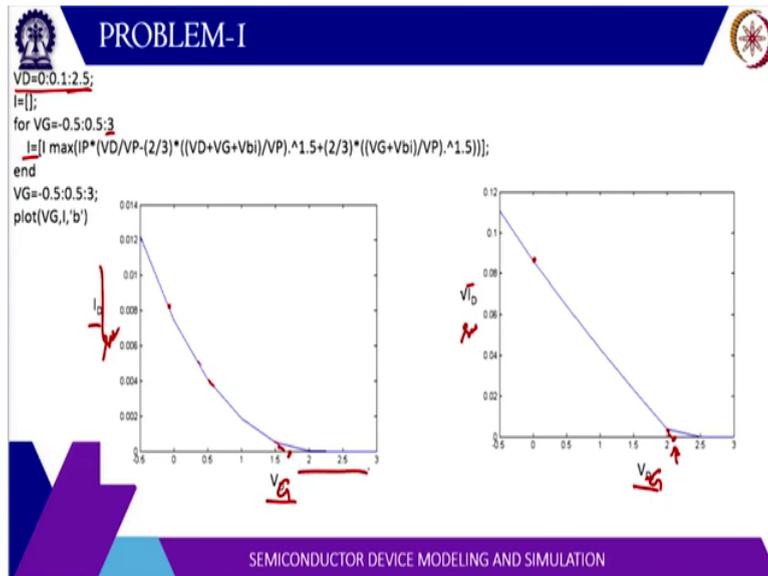
So, 2.5 volt so, this is 0 to 2.5 volt then use that expression that $I_D \propto V_{DS}^{2.5}$ by $V_{GS} - V_{th}$ to the power 3 by 2. So, this expression then what we do here? V_{DS} is a vector and therefore, this I_D is also a vector. Then because the curve, actually if you use this equation, the curve you will get, will be something like this like MOSFET, you know very similar.

So, what we do here? Up to this region where there is a peak there is a linear region and then this goes to saturation. So, beyond this value, where it start to decrease, we maintain a constant value. So that is what is done here. So, if I_D is less than $I_{D,sat}$ then we set I_D to $I_{D,sat}$ and so on. So, this way for remaining values we make it constant and we get this curve, so, this is I_D versus V_{DS} curve.

Now, if you see here, the potential is 2.5 maximum that V_{DS} are applying and the length is 10 micrometre. So that is 10 into 10 is to power -4 centimetre. So, the electric field that comes out to be around 0.25 into 10 is to power 4 volt per centimetre. And correspond drift velocity will be μ_n times E is 5000, so, 5000 centimetres of a volt second times 0.25 into 10 is to power 4 volt per centimetre.

So that will be basically centimetre per second, so that is around 1.25 into 10 is to the power 7 centimetre per second. And if you compare this thing with the saturation velocity is around 1.3 10^7 is to power 7. So, it is below saturation velocity. So, therefore, we can use this expression with $V_D = \mu_n E$ because this expression was derived with the substitution that $V_D = \mu_n E$.

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And for the same case, we can also calculate I_D versus V_D . This should be V_G . This is V_D and this is V_G so, this should be V_G actually this should be V_G . So, this is I_D , this is square root of I_D versus the technique is same. What we do basically? For different V_G we find out what is the maximum value? So, this is basically what we have plotted here is the maximum value or I saturation.

This is I saturation, so, maximum value of I for a given V_G from the range of 0 to 2.5 value of V_D . So, what we have done here? Let us say if you see the previous figure, this is the maximum value, this is the maximum value, this is the maximum value, this is a maximum value. So, $V_G = 1.5$ the I_D is around is pretty small for $V_G = 1$. It is around 0.002, $V = 0$ at around 0.008 you can also notice a trend here, as the V_G is increasing and the current is decreasing.

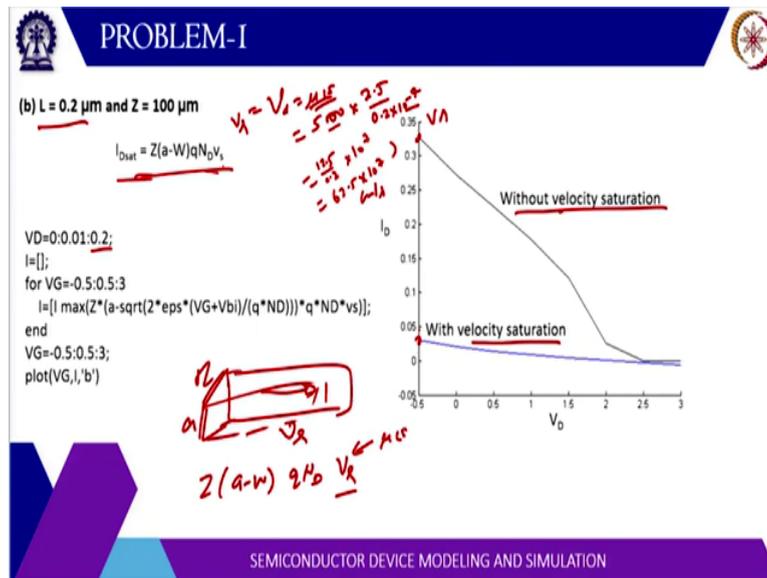
Because this is reverse biasing, basically, reverse biasing means your channel is getting narrowed basically. So, this is your channel region and this is getting narrowed basically. So, the current that is flowing through this channel is getting reduced basically. So, we cannot go below -0.71 because beyond this it will come forward bias. So, we go to 0.05 although this is also not a region of operation, we typically operate it below 0 volt.

So, this for 0 volt this is around 0.008. So, you can hear for 0 voltage around 0.008 and then square root of 0.008 will be around 0.08. And then you can identify here, especially with square root of I_D if you extend this linear line, it will enter somewhere here. So, this is the

basically intersection of root I D versus V G and same thing if you extend somewhere here, this is the voltage you will get.

So, this is how concept of the threshold voltage that should we beyond this voltage basically, current will not flow because depletion region will cover the whole of the channel layer.

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Then for a smaller length, where length is 0.2 micron here, if you use $V_d = \mu E$. What you will get, basically μ is 5000 times electric field is let us say we apply 2.5 volt V_d divided by 0.2 micron so, 0.2 into 10 is to the power -4 centimetre. So, the value will be around 0.2 so that will be 10 is to power $7 \ 3 + 4$ is 7 then that is 12.5 divided by 0.2. So that will be around 62.5 into 10 is to power 7 centimetre per second.

Now which is actually much larger than the saturation velocity so that means we cannot use this expression $V_d = \mu E$. So, in this case, we will assume that throughout this channel region, the velocity is V_s because now electric field is pretty high. And you can also find out at what point this is equal to V_s ? So, you can find the corresponding voltage that will be pretty small, so, here we go up to 0.2 volt only so that is also in the saturation only.

So, when the field is high enough, electron move with the saturation velocity. So then current equation will be so, the width of this Z so, this is a width and then this is the height, let us say a and that the depletion width is W . So, the area of cross, section Z times $a - w$ times charge that is qN_D times the velocity. So, here we are not using μE . We are using saturation velocity because length is small.

And therefore, the field will be high here and the velocity value saturation velocity. So, with this expression, if we calculate then you will get this blue curve with velocity saturation. And if you do not include velocity saturation without velocity saturation, so, using the previous these equations, this equal this code with this equation, only changing the length equal to 0.2 micron you will get this black curve.

So, you see, this basically kind of over estimating by a large amount. If you see here that it is around say 0.05 here, this is around 0.3 something so, it is pretty large, basically. So, at high field, basically we have to modify the expression and same thing applies to MOSFET, also. So, in MOSFET also, we have to use the similar expression when there is a velocity saturation scenario.

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PROBLEM-2

Calculate depletion width at inversion.
 $N_A = 10^{17}/\text{cm}^3$, at room temp $kT/q = 0.026\text{V}$
 $n_i = 10^{10}/\text{cm}^3$
 $\epsilon_s = 11.9 \times 8.85 \times 10^{-14} \text{ F/cm}$

$$W_{\text{max}} = \sqrt{\frac{4\epsilon_s kT \ln\left(\frac{N_A}{n_i}\right)}{q^2 N_A}} = \sqrt{\frac{4 \times 11.9 \times 8.85 \times 10^{-14} \times 0.026 \ln\left(\frac{10^{17}}{10^{10}}\right)}{1.6 \times 10^{-19} \times 10^{17}}}$$

$W_{\text{max}} = 10^{-5} \text{ cm} = 0.1 \mu\text{m}$

Handwritten notes in red include:
 $W = \sqrt{\frac{2\epsilon_s (V_{gs} - V_{th})}{q N_A}}$
 $N_A = P = n_i \cdot \exp\left(\frac{E_i - E_F}{kT}\right)$
 $\psi_s = q\phi_A = \epsilon_s \cdot \epsilon_F = \frac{kT A_n \left(\frac{N_A}{n_i}\right)}{L}$

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Now, a problem 2 is basically we have to calculate the depletion width at the inversion. So, this is basically a MOS structure, so, there is a metal then there is a oxide here or insulator here and then there is a semiconductor here. Now, if you draw the band diagram, let us say this is oxide and this is N_A so, this is p-type semiconductor. So, let us say this is p-type silicon.

So, for p-type silicon Fermi level is close to the conduction band balance band so, this is Fermi level. Now, just applying the voltage to metal gate here it does not allow any current to flow, so, your Fermi level will be constant. But when you apply the potential, these bands

will actually bend here and they will bend down. And that say this is the intrinsic level in between so, this will also bend.

Now, at inversion this bending is such that if this is ψ_B or $q\psi_B$ this will also be $q\psi_B$ so that means total, so, the total band bending is $2\psi_B$. So and ψ_B is what ψ_B is $q\psi_B$ is $E_i - E_F$ and for semiconductor now here is E_i and E_F so, we will write expression $p = n_i \exp\left(\frac{E_i - E_F}{KT}\right)$. So now, this is doping is N_A so, p we can equate to N_A and we can find $E_i - E_F = KT \log$ of N_A over n_i .

So, this is your ψ_B $q\psi_B$ so, 2 times $q\psi_B$ and again for depletion region you know the expression W is equal to square root $2\epsilon_{si} V_{bi} + V$ divided by q . Here we have N_A instead of N_D . So, you can write $q n_i$ here. Now, the band bending here, so, this potential here will be replaced on the band bending and that is $2\psi_B$. So, $2\psi_B$ will be $2KT \log N_A$ by n_i . Now, this is ψ_B so, this is $q\psi_B$ so, Q will go here in the denominator.

So, it will come create E by q that will be ψ_B . So, you will get this expression for the maximum width of the depletion region. Now, why this width is the maximum? Because once it reaches this level where the band bending is such that from ψ_B it becomes minus ψ_B . So, beyond this, small change in the band, this potential energy will result in large change in the inversion charge. So, Q_I here and Q depletion charge here.

So, if you change it slightly, most of the charge will appear as inversion region here instead as depletion region here. So, the depletion width effective does not change beyond this voltage, so, this width can be said as the maximum depletion width. However, having said this, if you do fast sweeping so that means, if you quickly drop the voltage to a large, lower value.

In that case, we can get a depletion width larger than this that is possible. But that is basically testing procedure and normally that is not used in device operation. But if you suddenly change the voltage to high voltage, we can get a depletion width bigger than this and because this inversion charge will take some time to get created there. So, in first sweeping it is possible but in normal operation this is the maximum depletion width.

Now, from this we go to third problem and we calculate the C-V relationship for this MOS structure.

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PROBLEM-3

Draw C-V curve for a MOS capacitor. $d=5$ nm thick oxide, $\epsilon_i=3.9 \times 8.85 \times 10^{-14}$ F/cm, $N_A = 10^{17}/\text{cm}^3$, at room temp $kT/q = 0.026$ V, $n_i = 10^{10}/\text{cm}^3$, $\epsilon_s = 11.9 \times 8.85 \times 10^{-14}$ F/cm

Handwritten notes on the slide include:

- $\phi_{ms} = \frac{\mu}{\epsilon_0 \epsilon_s}$
- $\phi_{Si} = \frac{\mu}{\epsilon_0 \epsilon_s}$
- $C_{ox} = C_{in} = C_{ox}$
- $C_{ox} = \frac{3.9 \times 8.85 \times 10^{-14}}{5 \times 10^{-9}} \text{ F/cm}^2$
- $C_{min} = \frac{1}{\frac{1}{C_{ox}} + \frac{1}{C_d}}$
- $C_d = \frac{\epsilon_s}{d}$
- $C = \frac{dQ}{dV}$
- $C_{ox} = \frac{\epsilon_i}{d}$

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So, this is a metal, this is oxide, this is a p-type silicon, so, for this C-V and we know that this is p-type. So, for negative voltage the capacitance will be same as oxide capacitance because this will lead to the accumulation of these holes negative voltage. So, your C will be C oxide or C insulator and that will be epsilon oxide by d the thickness of the oxide. Then at flat band voltage so, here we are assuming ideal oxide and we have no information is given or the phi ms.

So, we can assume it to be 0 so then flat 1 voltage will be here at the 0 volt. This is negative, this is positive so, this will go down here. And then at threshold voltage so, this is some threshold voltage. So, this is the flat band voltage, V FB and this will be the minimum. And again it will be if it is low frequency, it will be again the C ox or C insulator. Now, if we find the key voltages the key capacitances here then we can basically draw the curve.

So, this is the C in accumulation, C at flat band, C at beyond inversion and this is C minimum. So, C oxide is basically epsilon oxide divided by the thickness and we know that epsilon oxide is 3.9. That is the dielectric constant times 8.85 into 10 is to power -14 farad per centimetre then this 5 into 10 is to power -7 centimetre. So, capacitance will be farad for centimetre square.

So, this term this epsilon by D is actually capacitance per unit area. So, what will be the area? So, if you consider let us say, this is your gate. So, this is the width Z times the length so, Z times L multiplied by C ox will be the overall capacitance of the gate. So that we can find C x and psi s at that point we already know. So, we can find the minimum capacitance also so, C minimum will be when the depletion width is maximum.

So, there will be two capacitance in parallel oxide capacitance and the depletion capacitance. So, 1 over C minimum will be you can write 1 over C oxide plus 1 over C depletion and C oxide is epsilon oxide by d oxide plus C depletion will be epsilon silicon by W. And W we have calculated around 0.1 micrometre. So, C minimum can be obtained from here that will be C minimum and then this is the inversion region here.

So, once you go to inversion region C oxide, only and flat band will come from the debye length. So that will be so, for the flat band this is C ox the flat band occurs at 0 volt here. So, the bands are actually flat there. So but if you change the voltage slightly then there will be some change in the depletion width. So because C is dQ by dV, so that can be found from the debye length.

So that will be C silicon that will be epsilon silicon by the debye length for in silicon for a given doping.

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PROBLEM-3

Draw C-V curve for a MOS capacitor. $d=5\text{ nm}$ thick oxide, $\epsilon_i=3.9 \times 8.85 \times 10^{-14}\text{ F/cm}$, $N_A = 10^{17}/\text{cm}^3$, at room temp $kT/q = 0.026\text{V}$, $n_i = 10^{10}/\text{cm}^3$, $\epsilon_s = 11.9 \times 8.85 \times 10^{-14}\text{ F/cm}$

$$C_i = \frac{\epsilon_i}{d} = \frac{3.9 \times 8.85 \times 10^{-14}}{5 \times 10^{-7}} = 6.9 \times 10^{-7}\text{ F/cm}^2 = C_{ox}$$

$$\psi_s(\text{inv}) = 2\psi_B = \frac{2kT}{q} \ln\left(\frac{N_A}{n_i}\right) = 2 \times 0.026 \times \ln\left(\frac{10^{17}}{10^{10}}\right) = 0.84\text{ Volts}$$

$$C_{\text{min}} = \frac{\epsilon_s}{d + \frac{1}{2}W_{\text{max}}} = \frac{3.9 \times 8.85 \times 10^{-14}}{5 \times 10^{-7} + (3.9/11.9) \times 10^{-5}} = 9.1 \times 10^{-8}\text{ F/cm}^2 \Rightarrow \frac{1}{C_{\text{min}}} = \frac{1}{C_{ox}} + \frac{1}{C_{\text{dep}}}$$

$$\frac{C_{\text{min}}}{C_i} = 0.13$$

$$V_{\text{TH}} = \frac{qN_A W_{\text{max}}}{C_i} + 2\psi_B = \frac{1.6 \times 10^{-19} \times 10^{17} \times 10^{-5}}{6.9 \times 10^{-7}} + \psi_s(\text{inv}) = 0.23 + 0.84 = 1.07\text{ Volts}$$

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So, if we substitute this and we do the calculation then the oxide capacitance we get around 6.9 into 10 is to power - 7 fard per centimetre square. Then of course inversion surface

potential for inversion region that is $2 \text{ times } kT \text{ by } q \text{ lon } N_A \text{ by } n_I$, so that is around 0.84 volt. And C_{minimum} this is basically arranged like this $\frac{1}{C_{\text{minimum}}} = \frac{1}{C_{\text{ox}}} + \frac{1}{C_{\text{silicon}}}$.

So, this is basically, if you rearrange it then this is $d \text{ by } \epsilon_{\text{ox}} + W \text{ by } \epsilon_{\text{silicon}}$ and this is $\frac{1}{C}$. So, C can be written as $\frac{1}{\frac{d}{\epsilon_{\text{ox}}} + \frac{W}{\epsilon_{\text{silicon}}}}$. So, $\frac{1}{C}$ over this thing, what we can do? We can take ϵ outside. So, this is $d + W \text{ times } \epsilon_{\text{ox}} \text{ by } \epsilon_{\text{silicon}}$. So that is what done here and then you get around $C_{\text{minimum}} \approx 9.1 \times 10^{-8} \text{ farad per centimetre square}$. So, the ratio of C_{minimum} C_{oxide} is around 0.13.

So that means this is your C_{ox} here in accumulation region it goes then similar, so, this is 1 here this will be 0.13. So, let us say this is $1 \text{ times } C_{\text{ox}}$ this will be 0.13 times C_{ox} . And then threshold voltage is basically the dope across the silicon region plus oxide region. So, dope across silicon region is $2 \text{ psi } B$ and the dope x across oxide is $C_{\text{oxide}} \text{ times } Q = CV$. So, voltage is basically $Q \text{ by } C$.

So, Q is the charge here so that you can write qD , now qD will is what? $Q \text{ times } N_A \text{ times } W_{\text{max}} \text{ divided by } C_{\text{oxide}}$. So, add these two you will get the threshold voltage that is around 1.07 volt. Please remember here we have ignored any charges in the oxide and ϕ_{ms} . If there is a ϕ_{ms} charge in the oxide then this threshold voltage will be added with ϕ_{ms} and this will term due to the Fermi level difference between the metal and semiconductor.

And as well as this charge in the oxide so, $Q_{\text{effective}}$ in the oxide divided by C_{oxide} . So, these two term will come. These are usually negative, as we have discussed earlier, for both whether it is silicon is n-type or p-type. The value comes out to be negative for these two.

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PROBLEM-4

The inversion layer in a Si MOSFET can be assumed as a 2-D electron gas trapped in an infinite rectangular potential well of width 100 Å. Calculate the inversion charge per unit area assuming that the Fermi level lies midway between the second and third subbands. Assume low temperature, and effective mass = 0.2 m_0 .

Handwritten solution details:

- Effective mass: $m = 0.2 m_0$
- Well width: $L = 100 \text{ \AA} = 10^{-8} \text{ m}$
- Energy levels: $E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$
- Energy values: $E_1 = \frac{\hbar^2 \pi^2}{2mL^2}$, $E_2 = 4E_1$, $E_3 = 9E_1$
- Density of States (DOS): $DOS = \frac{m}{\pi \hbar^2}$
- Fermi level E_F is midway between E_2 and E_3 .
- Electron density: $n = \int_{E_c}^{E_F} DOS \cdot f(E) dE$
- Final result: $n = \frac{m}{\pi \hbar^2} (E_3 - E_2)$

Now, last problem for this session here it is said that inversion layer in a silicon MOSFET it can be assumed as a 2D electron gas because if you see the diagram here so, this is the band diagram and this region can be filled with electrons. So, it is basically 2D electron gas. Now, this can be assumed, although it is basically triangular but let us assume that it is a rectangular quantum band.

And its width is 100 Å so that is 100 Å is 10 nanometre. Now, we have to calculate inversion charge, so, Q per unit area, as you mean that Fermi level lies midway between the second and third sub band, so because this is a quantum band, so, there will be bands here let us say band 1, band 2, band 3. So, energy levels E_1 , E_2 this is E_3 . This is E_3 so, in the middle of these two bands band 2 and 3.

Now, if you recall, for 2D band structure, please revisit the discussion on the density of a state lecture DOS lecture there we have mentioned or we have derived basically, the density of a state for 2D is constant and given by $\frac{m}{\pi \hbar^2}$ square. So, how it look like? If you cut so, this is a DOS versus energy. So, it will look something like this, so, this is $\frac{m}{\pi \hbar^2}$ square. Then this let us say this is E_1 , this is E_2 so, E_2 again this jump here.

So, this is $\frac{2m}{\pi \hbar^2}$ square plus this is E_3 so, again jump here. So, this is $\frac{3m}{\pi \hbar^2}$ square so, this is a DOS. Now, Fermi level is somewhere here in the middle of E_2 and E_3 . So, from this we can calculate number of electron. So, n will be integral DOS times probability of occupation, dE . So, an integral will be from, let us say E_c to Fermi level E_F . Now, here a condition is given low temperature.

So that basically, simplifies our calculation because the Fermi level, basically, if you plot F the Fermi level distribution, it looks something like this. So, as this is low of temperature at high temperature, it becomes more sloppy. So then of course, integration become more complicated but at low temperature, if we assume it is close to 0, it will be rectangular. So that means it is basically 1. So, this F is 1 in this region.

So then this can be simply written as $E_2 - E_1$ because below E_1 this is 0, so, $E_2 - E_1$ times the density of a state so which is m by $\pi \hbar^2$ times 1 because this F is 1 + $E_3 - E_2$ now $E_3 - E_2$ only half of this we have to consider. So and the value is $2m$ by $\pi \hbar^2$ square times half because only half is filled, other half is empty. So that if you simplify what you will get here, this 2, 2 will cancel.

So, this is m by $\pi \hbar^2$ square times $E_2 - E_1 + E_3 - E_2$ so that is $E_3 - E_1$. So, this will be the n so that is a density of carriers in the quantum band. Now, what does this E_1, E_2, E_3 ? That also we can calculate we know $E = \hbar^2 K^2$ by $2m$ now, K^2 written as $K_x^2 + K_y^2 + K_z^2$. If this is the Z direction, so, in Z direction, it will be quantized but in x and y it is free so, it can take any value.

So, now K_z can be written as now, this length is, let us say L . So, the wave function, if you think like this wave function, can have this type of curve or this. So, it is K is basically you can write 2π by λ and λ can be $L/2$ or L like that so, n times $L/2$. So, this will be written as 2π divided by n times $L/2$ because this will be this is λ by half. So, $\lambda/2$ is equal to L . So, λ is $2L$. So, n times $2L$.

So, this will be π by nL so, \hbar^2 is square by $2m$ times $n^2 \pi^2$ by L^2 . This should be L by nL by n so, this is $n^2 \pi^2$ by L^2 so, K is $n\pi$ by L . This requires some background information about the quantum mechanics. So, let me just give a brief overview let us say, the wave function generally, these electrons are described by the wave function. So, this wave function lets here some form $A \sin \psi$ or K times Z .

So, this K is basically the wave vector. So, if you compare this type of situation, where this length is basically equal to this length is L but this is half wavelength, for the second this is 1 wavelength, for third one it will be $3/2$ wavelength and so on. So, your $L = \lambda/2$

times n , $n \lambda$ by 2. So, λ by 2, λ by 3, λ by 2. So, from this we can say, $\lambda = 2L$ by n .

And then K is 2π by λ , so that is then you will get $n\pi$ by L here, if you substitute this here so, this is how we are writing this one. So, the energies will be E_1 these energies are E_n basically. So, these are the energies E_n because in other 2-dimension there is no restriction. So, E_1 will be $\hbar^2 \pi^2$ by $2m L^2$, E_2 will be n^2 so that will be 4 times E_1 , E_3 will be 3 square, so that will be 9 times E_1 , so, $E_3 - E_1$ will be 8 times E_1 .

So that is m by $\pi \hbar^2$ times 8 times E_1 and E_1 is this value. So, we substitute here this n what you have to calculate charge per unit area? So, this is the n is the density per unit area. So, you multiply this thing by Q will be charged per unit area and when you substitute the value, this m is m effective, so that is 0.2 into 9.1 into 10 is to power -31 kg. And then E_1 of course you can substitute here, \hbar you can substitute is \hbar over 2π and the \hbar is 6.63 into 10 is to power -34 few second divided by 2π .

So, we substitute this one then m by \hbar this you can put here and then substitute even here. So, what you will get here? Q times E_1 is \hbar^2 by $2m$. So that is 8 times \hbar^2 will cancel m will cancel so, π^2 will remain here. So, π^2 by $q L^2$ square, so, this q will also cancel out. So, the value will get will be around 20.12 coulomb per metre square. So, you can do the calculation. So, this is a charge density.

So, here the concept that we have discussed the quantization of the energy levels then corresponding density of states to calculate the carrier, concentration that is DOS times F and that simplifies to DOS times $E_3 - E_1$. Then we substitute the value you will get this around 20 coulomb percent metre square as a inverse and charge per unit area. Thank you very much.