

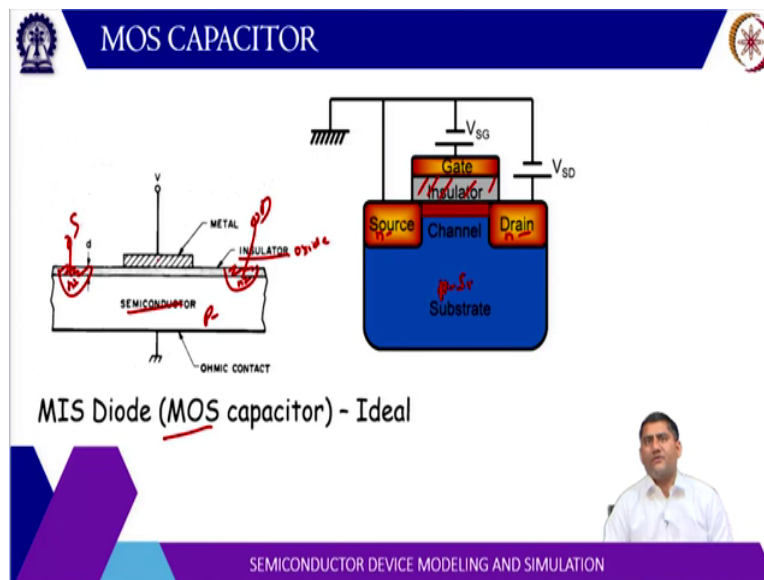
Semiconductor Device Modelling and Simulation
Prof. Vivek Dixit
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology – Kharagpur

Lecture – 28
MOS Capacitor

Hello welcome to lecture number 28, so, before taking some problem on the MOSFET, JFET, let us complete the all the FET devices using MOS Capacitor and the MOSFET. Then we will take up some problem. So, let us discuss the MOS capacitor first. So, as the name suggests, it is metal oxide semiconductor, so, these are three layers. Now, although this gate material has been replaced with poly silicon and so on.

But the name actually persist so, we still call it MOS basically. But instead of metal there can be poly silicon also but the structure is MOS metal oxide semiconductor. So, what we will do in this lecture? We will discuss this structure, solve the poisson equation and try to get MOS capacitor voltage characteristic.

(Refer Slide Time: 01:23)

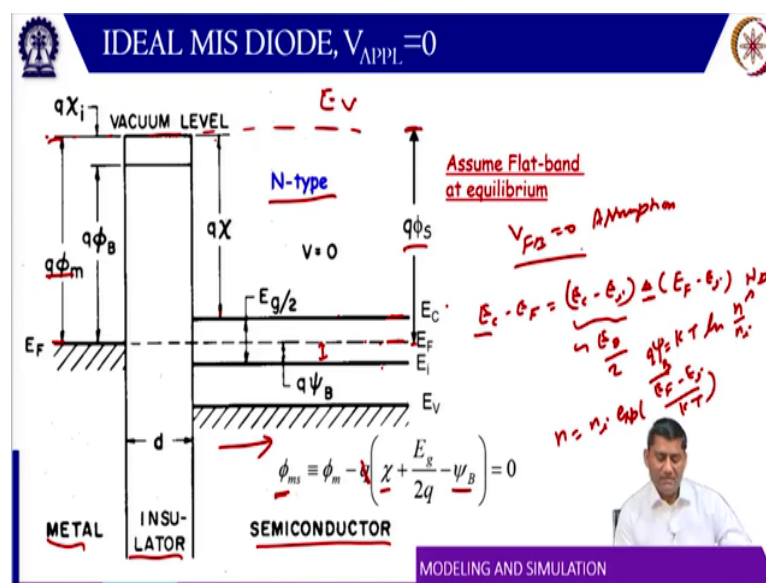


So, if you look here, there is a semiconductor then this is a small oxide layer which is insulator and then there is a metal layer. It is further enlarged here this insulator is actually, quite thin order of few nanometre or tens of nanometre and we are actually, it is decreasing year by year to have a better control on the gate. Then but this is a mass structure, metal oxide semiconductor.

On right side what is done here? Then some contacts are made here you ETCH away this oxide ETCH this oxide and let us say this is p-type. So then you will have N + contact here, N + region here and then we make a source here and the drain here. So that is what is done here. So, this is, let us say, n type substrate, so, this will be there is a p-type substrate. Then it will be n type source and then n type drain and then this gate voltage basically controls the conductivity of this channel region.

If you do not apply any voltage then this channel will not be found. And this is p-type this is n-type, so, no current will flow.

(Refer Slide Time: 02:40)



Now, let us look at their band diagram, so, this is a metal site. This is the oxide insulator. Now, this is a semiconductor site. You can see that Fermi level is close to the conduction band, so, it is a n-type semiconductor. And right now, let us assume that the Fermi level inside the semiconductor and inside the metal they are equal. So, this is not really true but to simplify our discussion to begin with, we are assuming that.

The flat band condition is that V_{FB} flat band is actually 0 N that what we have assumed so, this is the assumption. Now because of this assumption this all these bands will be actually flat if it had been little away from the Fermi level of the metal then these bands would have bent to equalize the Fermi level. And you can compare this one this is, let us say, the vacuum level. Let us say this is E_v vacuum level.

So, ϕ_m is the work function of the metal. So, there is a gap between the metal Fermi level and the vacuum level and then inside the semiconductor ϕ_s is the work function of the semiconductor, the difference between the inner vacuum level and the Fermi level. In the semiconductor and this ϕ_s is basically χ that is the electron affinity plus the gap between E_C and E_F .

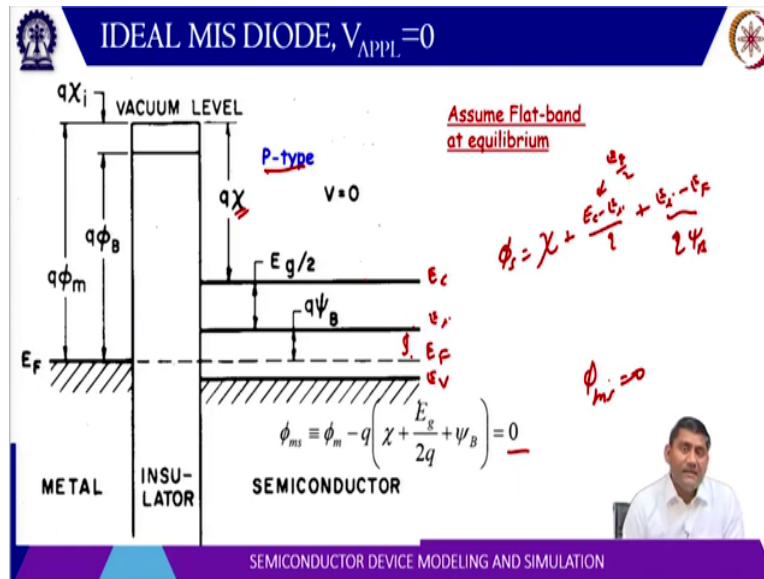
So, the gap between E_C and E_F can be written as $E_C - E_F$ can be written as $E_C - E_I + E_F - E_I$ sorry $E_C - E_I - E_F - E_I$ so that is $E_C - E_F$. Now, $E_C - E_I$ it is approximately E_G by 2 and $E_F - E_I$ can be written as in terms of $n = n_i \exp(E_F - E_I / kT)$. So, $E_F - E_I$ is basically $kT \ln n / n_i$. Now, here $n = N_D$, N_D is a doping concentration and it is also written as sometimes $q \psi$ the potential.

So, this is $q \psi$ is also called bulk potential. This example because the interface when you apply the voltage, these bands will bend away. So, this characteristics will change but away from the interface, the $E_I - E_F - E_I$ is constant and we call it $q \psi_B$. Now, note is one difference here. This E_C , E_F , E_G these are the energies. And ϕ , χ and ψ these are the voltages.

So, when we write an expression containing the energy and voltages, we have to multiply with the corresponding charge. So, ϕ_m is basically $\phi_m - \phi_s$. So, this $\phi_m - \phi_s$ so, it should be actually q , q should be there. So, q we have divided this thing by q so, $\phi_m - \phi_s$, so, ϕ_s will be χ so, this q should not be there. So, $\chi + E_G$ by 2 here q should be there because this is now this energy.

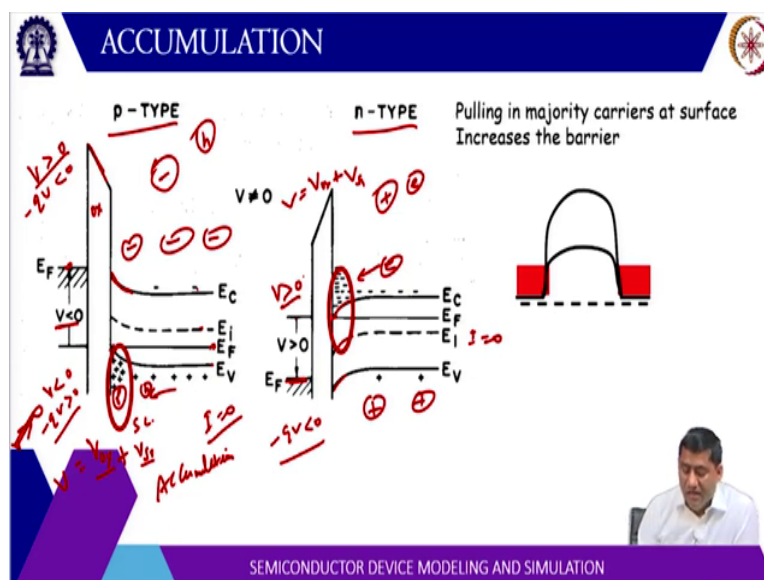
So, divided by q will be potential $- 2 - \psi_B$. So that is ϕ_m and that is equal to 0 that we have assumed that flat amount bend is 0 here.

(Refer Slide Time: 06:52)



Similarly. We can write the band diagram for the p-type semiconductor. So, for p-type the Fermi level is here it is close to balance band E_V and this is E_C . So, here ϕ_{is} will be χ this + E_C this is E_I so, $E_C - E_I$ by q so, this is roughly E_G by 2. So, E_G by $2q$ + $E_I - E_F$ this and this is $q\psi_B$ in this case for p-type. So, this is E_G by $2q$ + i_B . So, again if we are assuming the flat band at equilibrium this will be equal to 0 at no bias. So, $\phi_m = \phi_s$. So, we are assuming that ϕ_{ms} is 0.

(Refer Slide Time: 07:59)



Now, what we can do when we apply the bias, so, let us say this is for p-type, so, p-type Fermi level is close to the balance band. So, now, when we apply a voltage greater than 0 then what will happen? $-qV$ is energy, so that will be less than 0. So, a positive potential applied to the metal will lower down the energy and a voltage greater than 0 upon sorry, voltage less than 0 a negative voltage, so, $-qV$ will be greater than 0.

It will pull up the energy level. So, here the voltage applied is negative, so, this will pull up the Fermi level so now, compared to the semiconductor Fermi level. Metal Fermi level is pulled up because of this negative voltage. Because negative voltage times minus q is positive, so, energy is higher here. Now, these bands will bend in such a way that this applied voltage will drop across these two regions.

One is oxide region, other is semiconductor region, so, this energy is up here, so, some potential is dropping here and some potential is dropping here. So, this is called V_{oxide} and this is V inside the depletion region silicon. So, total V will be V in oxide plus v in the silicon. Now, what will happen? Because of this potential these bands are bending like this. So now, this Fermi level is constant because I is 0.

So, no current is flowing, so, Fermi level has to be constant, so, permeable is constant here. Then other bands are basically bare bending up because this energy is pulled here. So, you are pulling it so, other energies will also be it will also try to pull other energies. So, other energy is also getting pulled in the process. This E_V is going close to E_F so that means when E_F goes to close to E_B . What will happen?

This E_I is the intrinsic level Fermi level near the E_C gives electron, Fermi level close to the valence band as holes, so, this speed of semiconductor, so, what it has. Basically, it will have, a donor ions and holes. And there is other way round this should be ions are distributed throughout this region. When this bend is pulled up here, these holes which are positive here these are free also basically these holes.

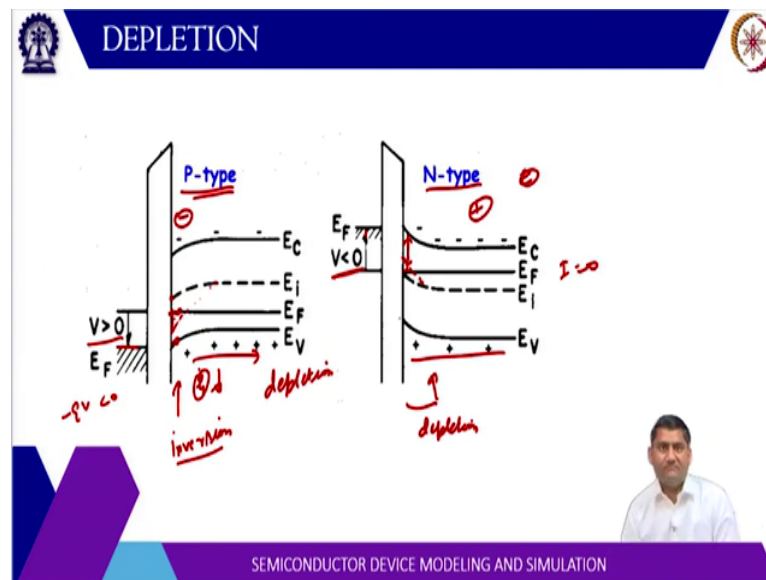
These holes will accumulate here. So, what happens? These holes actually, move to the interface, so, there is a accumulation of holes. Similarly, in case of n type. When you apply a positive voltage to the metal, the energy is $-q$ times V which is less than 0 so, this energy is pulled down. Here the current is 0, so, the Fermi level is constant and due to pulling up this energy level in on the metal side.

The energy level on the semiconductor side will also be pulled down, so that voltage drop will be V across oxide and V across the silicon. So that will add up to applied voltage. So because of this pull in this conduction level, it is going close to the Fermi level so, again it is

n-type. So that means there are electrons here and there are positive ions here. So, this conduction energy level, is close to the Fermi level.

So, these electrons have come here and this point of ions distributed throughout. So, this is the case of accumulation because this electrons are getting accumulated here.

(Refer Slide Time: 12:42)



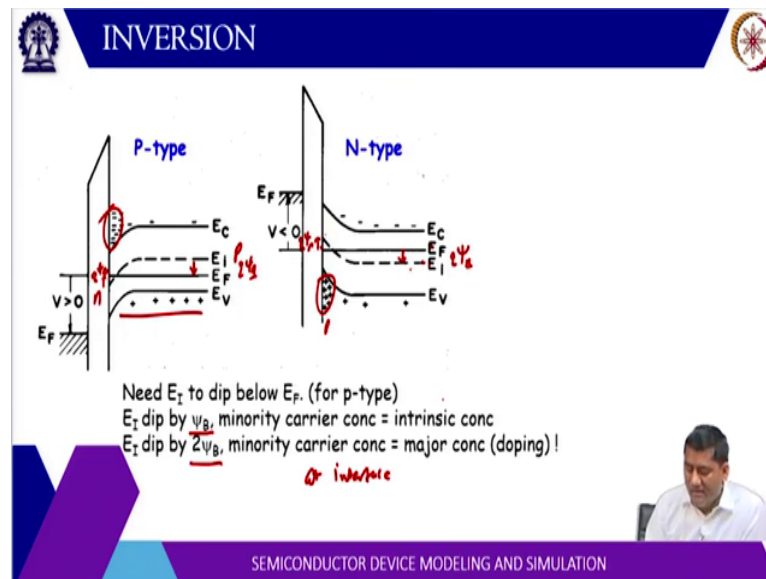
Now, if you change the polarity. So, for P-type, if we apply a voltage greater than 0. So, $-qV$ is energy, so that means this will be negative. So, this energy is pulled down. So, all the bands are pulled down now this Fermi level is away from the valence band. That means number of holes is reduced here but this is still below E_I so that means it is still P-type only. If you increase this voltage further, it is possible that this may come down further.

So, this may change the polarity here. And when it changes the polarity here, this region is called inversion. So, what is happening here? Now, holes are reduced, so, it will have more ions here. So, P means negative ions. So, it will have more negative ions here and holes will be less in number. So, they have moved away from the interface, so, this is called the depletion region.

So, as long as E_I is above E_F . This is depletion region. When E_I goes below E_F we call it inversion. Similarly, on N-type it has electrons here and the positive ions. So, when you apply a negative voltage that means energy increases. Fermi level goes up, so, it tried to pull other Fermi levels also because the current is 0 Fermi level is constant. So, E_I approaches E_F and E_C goes up, so, this gap is now more so, this region is less N-type.

So that means electrons are less here. And these ions are equally distributed throughout because these ions cannot move so, this is called depletion region. And of course, if you further decrease the voltage then this E_i may go above E_F and then this we call it inversion region. Because now E_i is above E_F so, this becomes P-type because now E_F is close to E_V .

(Refer Slide Time: 14:45)

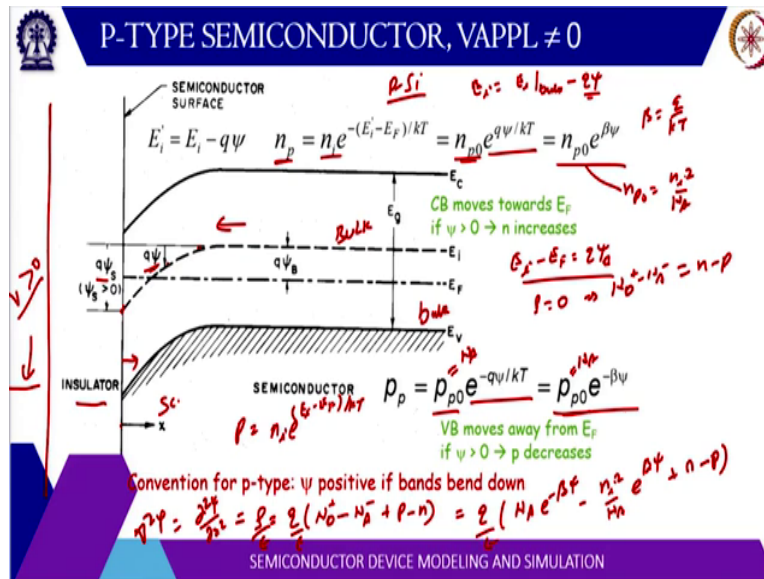


So, here is the picture showing the inversion. Now, you see here, this E_F is below E_i but at the surface E_i is below E_F . So, here it is P-type here it is N-type. So, there are lot of electrons here basically. The ion concentration is same throughout, so, this is called inversion, so only for N-type. This E_i is here below E_F here E_i is above E_F . So, this region becomes P-type.

There are lot of holes here, so, this region is inverted. So, as we have defined, this is ψ_B , $E_F - E_i$ is ψ_B here $q\psi_B$ here also it is $q\psi_B$. So, the definition of strong inversion is this is also equal to $q\psi_B$. This is also equal to $q\psi_B$ So, when these two are required, so that means there is a depth of $2\psi_B$, so that is called the depletion concentration at interface is equal to majority concentration of the doping.

That is a condition for a strong inversion. So, when $E_i = E_F$ so, there is a dip of ψ_B . It becomes intrinsic, so, we call it just the verge of inversion. And below this is the depletion region.

(Refer Slide Time: 16:18)



Now, how do we calculate or can we get a more quantitative expression for describing this scenarios? So, what we do here? Let us consider this P-type semiconductor $E_i - E_f$ is $q\psi_B$. Then let us say there is some band bending so, this band is bending that means the E_i is going down. That means on the metal side the Fermi level is pulled down here. So, Fermi level is pull down means the energy is lowered.

So, $-q$ times B is less so that means B is positive. So, on the metal side a point voltage is applied then the bends will bend like this. With respect to the bulk. Let us say this is the bulk region as we move towards the surface these bands are bending unless it is band bending is represented by $q\psi$. So, ψ is a potential drop as a function of position X . At the surface we call it $q\psi_s$.

So, ψ_s is the surface potential at the metal at the semiconductor and the oxide or insulator interface. So, at the insulator and semiconductor interface ψ_s is the surface potential. Now, we know that n electron concentration is n_i exponential $E_f - E_i$ by KT , if you look at this position $E_i - E_f$ by KT this E_i is actually, changing. So, as you move towards the surface, your E_i is basically E_i in bulk $- q\psi$, so, this can be written as $E_i - q\psi$.

So, if you take E_i you separate $q\psi$, so, you can write e to the power $q\psi$ by KT multiplied by exponential E_i bulk $- E_i$ by KT that is the bulk electron concentration. So, the n concentration as a function of position can be written as n_0 n_p which is the bulk magnetic reconcentration times e to the power $q\psi$ by KT . So, here beta is written as q

by kT . Similarly, for the whole concentration it will be P times E to the power $-q\psi$ by kT because this will increase.

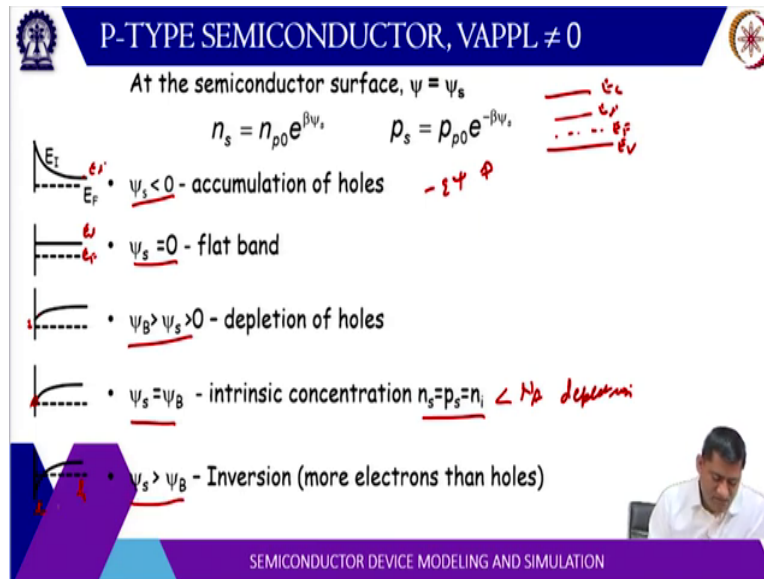
Let us say this electron concentration is increasing here. So, similarly, the whole concentration will decrease because whole concentration is $P = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$. So, E_i is $E_i^{\text{bulk}} - q\psi$, so, this is $E_i^{\text{bulk}} - \exp\left(\frac{q\psi}{kT}\right)$ times p naught. So, this p naught is basically acceptor concentration, let us say n_a and this n naught is n_i^2 by n_a .

Now, this is a carrier concentration as a function of position because this ψ is a function of position. So, at any position at position X we can write the total charge, so, the total charge will be q times. Let us say if there are some donor then this will be N_D which is point of charge if the receptor the which is the case here, so, it is $-N_A$ then $+p - n$. So that is the total charge density and then we can substitute this expression here.

So, you will get q times you can note one more thing here $N_D - N_A$ because in the bulk region, ρ is 0, so that means $N_D + (-N_A) = n - p$. So, this can be written as $N_D - N_A = n - p$. So, this is $p - n$ so, p is basically this is N_A . So, this is $N_A \exp\left(-\frac{q\psi}{kT}\right)$. Then this $N_D - n_i^2$ by $n_A \exp\left(-\frac{q\psi}{kT}\right)$ and then $+n - p$. So, this is a total charge concentration and that if you recall the Poisson equation.

It will be $\frac{d^2\psi}{dx^2}$ or $\nabla^2\psi$ for 3D. In 1D we write $\frac{d^2\psi}{dx^2}$, so that will be ρ by ϵ . So, this is by ϵ then we solve this equation.

(Refer Slide Time: 21:59)




So, before we solve the Poisson equation, this is the summary of the band bending. So, when ψ_s is less than 0 that means the potential is negative. That means energy is going down, so, energy will be $-q$ times ψ_s , since negative, so, $-q$ side will be positive. So, energy is going up actually, so, the energy is going up here. So, this is the accumulation of holes. When ψ_s is 0 it is a flat band, so, all the conduction band, valence band all are flat basically.


So, this is P-type, so, this is E_F this will be E_V and E_F is P-type E_F is here this will be E_V . This is E_i , this is E_C so, this is E_F , this will be E_i so, E_i is going up this accumulation at $\psi_s = 0$, E_i is constant, E_F is of course, always flat because there is no current flow. At the depletion region this ψ_s is positive, so that means this energy actually, goes down as you move towards the surface and this gap is less.

So now, it is E_F is more close to E_i so that means it is more intrinsic and at $\psi_s = \psi_B$, it is exactly intrinsic. So, at the surface $n_s = p_s = n_i$ so, both are less than the doping concentration N_A . So that means it is depletion region. Then, of course we, if you, if it crosses E_V , it is inversion, so that means there are more electrons here than holes here.

(Refer Slide Time: 23:47)



CALCULATE ψ and E-field



• Solve **Poisson's equation** to get E field, potential based on charge density distribution (one dimension)

$$\rho(x) = q(N_D^+ - N_A^- + p_p - n_p)$$

$$\nabla \cdot \xi = \rho / \epsilon_0 = \rho / \epsilon_s = \frac{d\xi}{dx} \rightarrow 1-D$$

$$\xi = -\frac{d\psi}{dx}$$

$$\Rightarrow \frac{d^2\psi}{dx^2} = -\rho / \epsilon_s$$

Away from the surface, $\rho = 0$

$$\Rightarrow N_D^+ - N_A^- = n_{p0} - p_{p0}$$

and $p_p - n_p = p_{p0}e^{-\beta\psi} - n_{p0}e^{\beta\psi}$

$$\Rightarrow \frac{d^2\psi}{dx^2} = -\frac{q}{\epsilon_s} (p_{p0}(e^{-\beta\psi} - 1) - n_{p0}(e^{\beta\psi} - 1))$$

$\frac{\partial \psi}{\partial x} = -\frac{\partial \psi}{\partial x}$

$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x}$

$= -\frac{\partial \psi}{\partial x} \left(\frac{\partial \psi}{\partial x} \right)$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now, this is a Poisson equation, so, $\rho = q$ times the charge density which is the fixed ions $N_D - N_A + P - N$ the movable charges. And if you substitute those expressions for $P - N$ is $N - P$ in the bulk. So, we can solve this equation with the boundary condition now the boundary condition will be basically if you look here. So, one boundary condition we can keep here where ψ is 0 and one we can keep here where $\psi = \psi_s$ and in between this position x .

So, as you move from $\psi = 0$ to ψ_s as x moves from, let us say from the bulk to surface that is $x = 0$. So, $d^2\psi / dx^2 = -q / \epsilon_s (p_{p0}(e^{-\beta\psi} - 1) - n_{p0}(e^{\beta\psi} - 1))$. And to evaluate this one what we can do? We can use a trick we can write we can recall this equation $E = -d\psi / dx$, so, $d^2\psi / dx^2$ is actually $-dE / dx$.

So, what we can write? We can write $dE / dx = -d^2\psi / dx^2$. So, what we can write? We can write it as $-dE / dx = d^2\psi / dx^2$ so, $d\psi / dx$ is electric field. So, this can be done as electric field times d by dx . And dx we can take to this side d by $d\psi$ times electric field. So, $d\psi$ we can take here and integrate with respect to ψ . This will become e times d .

(Refer Slide Time: 26:19)

SOLVE POISSON'S EQUATION

$$\Rightarrow \frac{d^2\psi}{dx^2} = -\frac{q}{\epsilon_s} (p_{p0}(e^{-\beta\psi} - 1) - n_{p0}(e^{\beta\psi} - 1))$$

$$\xi = -\frac{d\psi}{dx}, \quad \frac{d^2\psi}{dx^2} = -\frac{d\xi}{dx} = \frac{d\xi}{d\psi} \left(-\frac{d\psi}{dx} \right) = \xi \frac{d\xi}{d\psi}$$

$$\Rightarrow \int_0^{\xi_s} \xi \frac{d\xi}{d\psi} = \int_0^{\psi_s} -\frac{q}{\epsilon_s} (p_{p0}(e^{-\beta\psi} - 1) - n_{p0}(e^{\beta\psi} - 1)) d\psi$$

$$\xi^2 / 2 = \left(qp_{p0} / \epsilon_s \beta \right) \left[(e^{-\beta\psi} + \beta\psi - 1) + \frac{n_{p0}}{p_{p0}} (e^{\beta\psi} - \beta\psi - 1) \right]$$

$$\frac{n_{p0}}{p_{p0}} = \frac{e^{\beta\psi_s}}{e^{\beta\psi_s}} \exp\left(\frac{\beta\psi_s}{kT}\right)$$

$$= \exp\left(\frac{2(\beta\psi_s)}{kT}\right)$$

$$\frac{n_{p0}^2}{p_{p0}^2} = e^{-2\psi_s}$$


SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So, when we integrate it from 0 to E_s , E_s is the electric field at the surface and 0 is the potential in the bulk and ψ_s is the potential at the surface, the silicon and the insulator surface interface. So, this becomes $E_s^2 / 2$ is equal to if you substitute these values, you will get qp_{p0} . So, this is $N_A q N_A$ by $\epsilon_s \beta$ times E_s^2 to the power $-\beta\psi_s + \beta\psi_s - 1 + n_{p0}$ by p_{p0} $e^{\beta\psi_s - \beta\psi_s - 1}$.


So, what you have done here. We have taken p_{p0} or N_A on outside this box and then here suppose to p_{p0} here so, we have divided both. So, you have here we have the ratio n_{p0} by p_{p0} . And n_{p0} by p_{p0} . You can recall that n_{p0} is $n_i \exp(E_F - E_I / kT)$ and p_{p0} is acceptor concentration. So, the n_{p0} by p_{p0} or this can also written as $n_i \exp(E_I - E_F / kT)$.

So, if you take the ratio, this will be exponential twice $E_F - E_I$ by kT so, it will be exponential $2\psi_B$. So, as long as this potential is less than $2\psi_B$ this term will be less significant. Because this is already quite a small n_{p0} by p_{p0} because this is N_i square by N_A . So, this is basically N_i^2 by N_A^2 , so, this is e to the power $-2\psi_B$.

(Refer Slide Time: 28:46)



SOLVE POISSON'S EQUATION



Define: $L_D = \frac{\sqrt{kT\epsilon_s}}{\sqrt{q p_{p0} \beta}} = \frac{\sqrt{\epsilon_s}}{\sqrt{q p_{p0} \beta}}$ Debye Length


$$F\left(\beta\psi, \frac{n_{p0}}{p_{p0}}\right) = \left[(e^{-\beta\psi} + \beta\psi - 1) + \frac{n_{p0}}{p_{p0}} (e^{\beta\psi} - \beta\psi - 1) \right]^{\frac{1}{2}}$$

$E_{field} = \pm \frac{\sqrt{2}}{\beta L_D} F\left(\beta\psi, \frac{n_{p0}}{p_{p0}}\right)$

$\epsilon > 0$
 $\psi > 0$

$\epsilon < 0$
 $\psi < 0$

+ for $\psi > 0$ and - for $\psi < 0$



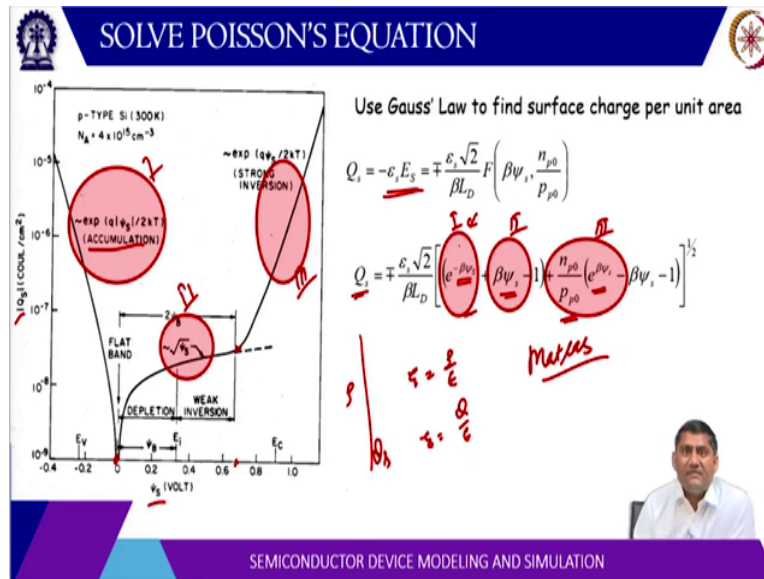
SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Then we can define this $k T \epsilon$ by $p p_{p0} q$ square we call this one as Debye length. So, it is some kind of effective length up to which the screening of the electric field takes place due to the movement of the charges because these charges can move around and they can screen the electric field. In metals these charts are large in number they move quite fast, so, they do not allow an electric field to exist inside the metal.

But in case of semiconductor there is some finite length. So, this is a typical dimension up to which the depletion is not actually, exist and beyond which they screen the electric field. So, electric field is actually, restricted to this reason, as the interface of silicon and oxide. Beyond which there is no electric field, so that means it has been screened up to certain length. So, this is that order Debye length.

And E field can be written as $\sqrt{2}$ by βL_D that is the dividing times some function which is basically e to the power $-\beta\psi + \beta\psi - 1$. So, this expression is very important because this will tell you, what is the charge now, electric field is e so, the charge will be charge is related to this electric field. So, this charge is basically the total charge that is there in the depletion region.

(Refer Slide Time: 30:40)



So that we can write epsilon times E. Now, you can recall basically, if you have a seat of charge, let us say some rho so, the electric field due to this set of charges rho by epsilon. So, here rho is basically charge density per unit area. So, the electric field is rho by epsilon. So, if q s is the charge density of the surface, so, the electric field will be q by epsilon.

Epsilon is the dielectric permeability of the semiconductor here in this case, so, q can be greater than written as epsilon times electric field with a minus sign. So, electric field is basically if you look at this equation, is a square root of this term square root of root 2 by beta l d times f. So, here this e square is two times this term times this term under bracket. So, s I square root. Basically, the electric field is a square root of this term.

Now, if you plot it, basically you can write a Matlab code to plot this expression. Epsilon is around 11.9 for silicon, beta is q by K T. So, at room temperature, K T by q is around or 1 by beta is around 26 milli electron volt then l d of course you can calculate. And for a given doping psi can also be calculated psi B. And so, what you can do? You can plot this q versus the surface potential psi s so, q s versus psi s.

Now what you see here? At 0, the charge is J 0 here because this is a flat band condition because we have assumed that phi m = phi s so, it is a flat band condition. Then, if the potential is negative then it is accumulation, so that means these holes are accumulating here, so, the charge is actually positive there. If so, the term that dominates here is e to the power - beta psi.

Then, if your potential is positive so because for negative potential e to the power $-\beta\psi$ will become positive, so, it will increase exponentially. So, this is actually, increasing exponentially. For positive potential this $\beta\psi$ will dominate and there is a depletion region, so, it dominates up to this region up to 0.7, 0 to 0.7. Now, why this term is not playing at all? Because of this factor n p not by p p naught which is n I square by n A square.

So but once the potential crosses this ψ_B this will start to become important. So, this is the third term, so, this is basically region one. This is region two, the depletion region, this is region three the inversion region because universal region takes place beyond this point, where both electron hole, concentrations are equal. So, it becomes intrinsic. So, this is a depletion region here it will become intrinsic.

Then it becomes basically inverted, so, a strong inversion and here also it is actually, exponential. So, this can be plotted, for a given piece of semiconductor because we know all the parameters here, we can assume some doping. Let us say, is 10 raised to power 15 here and this can be plotted a Matlab court and then you can plot it basically. So, thank you very much we have solve the poisson equations. Then we will continue discussing the same in the next class.