Semiconductor Device Modelling and Simulation
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Lecture – 24
Problem Session-4

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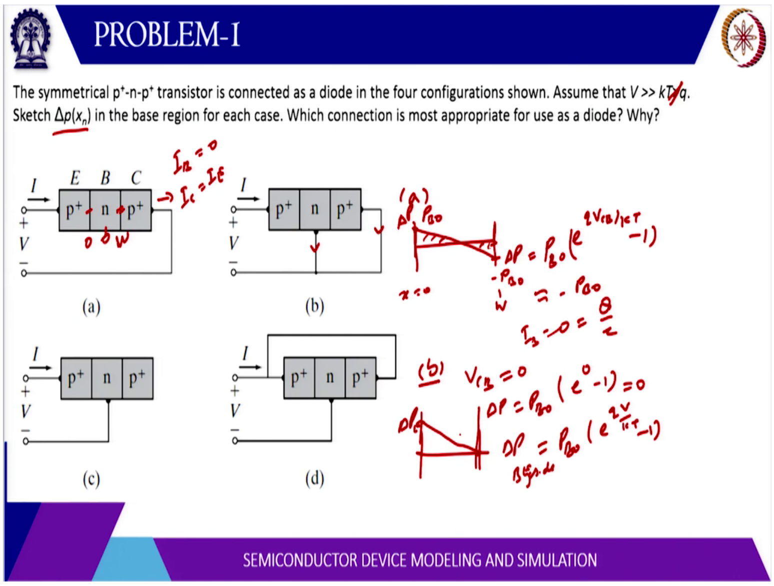

PROBLEM-I

The symmetrical p⁺-n-p⁺ transistor is connected as a diode in the four configurations shown. Assume that $V >> kT/q$. Sketch $\Delta p(x_n)$ in the base region for each case. Which connection is most appropriate for use as a diode? Why?

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Hello, welcome to lecture number 24. We will discuss some problems related to BJT in this problem session number 4. The first problem is the symmetrical P plus N P plus transistor is connected as diode in the four configuration zone. Assume that voltage applied is much larger than kT by q this will be kT by q. Then sketch minority carrier consultation access minority carrier consultation in the base region and out of this configurations which is considered best to use as a diode.

Now there are four configurations. So, let us look at the first configuration. In this configuration a voltage is connected between the emitter and collector and the base is over. So, that means I B is equal to 0 and the collector current I C is equal to I E. Now you notice one more thing here this applied voltage will fall across two Junctions emitter base Junction and the base collector Junction.

Now emitter based junction being forward bias it will take a small voltage drop because current will not be that large. So, remaining voltage will fall across the base collector Junction. So, the voltage across a base collector junction will be more. So, that means this base collector Junction is reverse biased. So, we can say that in the base collector junction let me plot here this is for a part let us say this is the excess carrier consultation Delta P.

Now because this Junction is always bias. So, we know from the law of Junction that Delta p is equal to p in the base naught base exponential q V CB by kT minus one where P B naught is the magnetic array concentration equilibrium monetary carry concentration in the base region. So, because V CB is reverse bias is negative large. So, you can say this will be almost 0. So, this is will be roughly minus P B naught.

So, you can say this is that say minus P B naught. Now this is the carrier concentration of the emitter region collector collect base collector junction. Now I E is equal to I C that means the slope at x equal to 0 and at x let us say this width of the base is W this is 0 this is w. So, the slope will be same and we also know that I B has to be 0. So, the excess carrier concentration here the total Access consultation because this is q by tau is the carrier lifetime.

So, this has to be 0. So, if you see connected in a straight line this should be P B naught. So, the total area under this should be 0 and slope should be same because I C is equal to I A that is one and ok for B part the voltage is applied across a meter and base collector assorted. So, for B part this is a part for B part V CB is equal to 0 V CB equal to 0. So, that means your Delta P B at collector site. So, base collector Junction Delta p is equal to again P B naught exponential 0 minus one.

So, that is 0. So, Delta P 0 is p is equal to P B naught. So, let us say this line is basically P B naught. So, it is 0 here and all the voltage Falls across base emitter junction so, Delta p on emitter side. So, this is on emitter side base emitter Junction is equal to P B naught exponential q V by kT minus one. So, it will fall something like this. So, this is Delta P emitter side. Now in this case the both I C and I B current will flow and their sum will be equal to I A and you notice here the slope is larger here in the B case. So, current will also be larger here.

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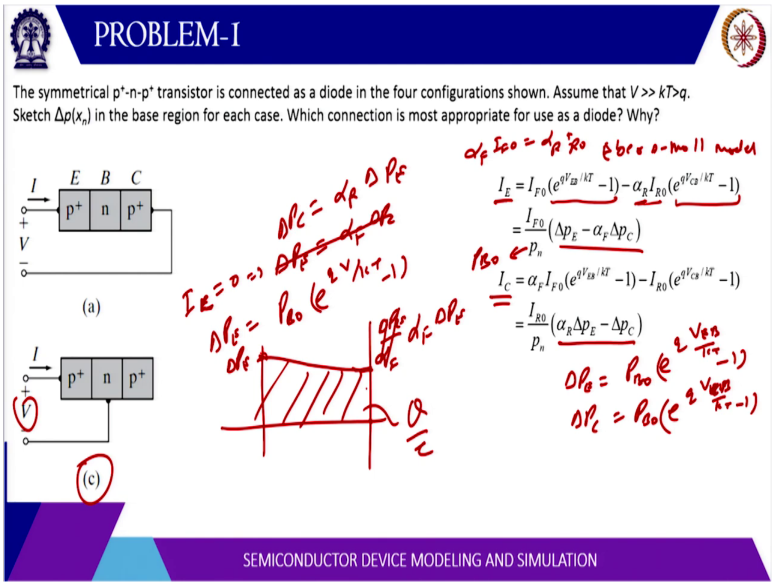

PROBLEM-I

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$\alpha_F I_{F0} = \alpha_R I_{R0}$ ebers-moll model

$$I_E = I_{F0}(e^{qV_{EB}/kT}-1) - \alpha_R I_{R0}(e^{qV_{CB}/kT}-1)$$

$$= \frac{I_{F0}}{p_n}(\Delta p_E - \alpha_F \Delta p_C)$$

$p_{B0} \leftarrow p_n$

$$I_C = \alpha_F I_{F0}(e^{qV_{EB}/kT}-1) - I_{R0}(e^{qV_{CB}/kT}-1)$$

$$= \frac{I_{R0}}{p_n}(\alpha_R \Delta p_E - \Delta p_C)$$

$\Delta p_E = p_{B0}(e^{qV_{EB}/kT}-1)$

$\Delta p_C = p_{B0}(e^{qV_{CB}/kT}-1)$

$I_E = 0 \Rightarrow$ ~~$\Delta p_E = \alpha_F \Delta p_C$~~

$\Delta p_C = \alpha_R \Delta p_E$

$\Delta p_E = p_{B0}(e^{qV/kT}-1)$

$\alpha_F \Delta p_E$

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Now let us consider C part before that this is C part you recall the equation for the current I E is equal to I F naught exponential q V B by kT minus one minus Alpha R this is from ever small model model i r r naught exponential q V CB by kT minus one. You recall one more condition that Alpha F I F naught is equal to Alpha R naught. So, we can replace Alpha R with Alpha R I R naught with Alpha F I F naught then we can take I F naught common. So, I F naught is taken as common and Delta P you know Delta P E is equal to P B naught exponential u V EB by kT minus one.

So, this P N is actually P B naught so, the whole concentration in the base. So, it can written as Delta P E by P N this term minus this can be written as Alpha F I F naught. So, I F is outside solve here and this is Delta P C right Delta P C is equal to P B naught exponential q V EB by V CV by kT this is V CB by kT minus one. So, this form can return as. Now you notice here in C part I E is equal to 0.

So, I equal to 0 means Delta P E is equal to Alpha F times Delta PC and Delta P we know from the bias Delta P is equal to P B naught exponential U V V EB is equal to applied voltage V so, q V B by kT minus one. So, it will be something like this. So, this is Delta P E and here is at the collected terminate is not 0 but Alpha F times times Delta p is equal to Alpha times Delta P. So, this is Delta P E by Alpha Delta P E by Alpha F.

So, this is I E. So, this is I C. So, second equation will be applicable. So, second equation basically tells you. So, this say some slot change here this is Delta P C is equal to Delta P C is equal to Alpha R times Delta P E because I C is 0. So, this is I C is 0 not I E, I C is 0. So, this will be it will be not one by Alpha but it will be Alpha F times Delta P E. So, now you notice here in this case there is some slope here but the area is quite large why because collector current is 0.

So, that means this slope is basically not contributing to the collector current. So, it should actually slope should go almost 0 here and there may be some slope here and this area is basically the total charge divided by Tau will be the base current. So, if you see here this base emitter is forward bias and the base is actually narrow here. So, charge is getting accumulated here and if you want to use this as a diode its transient characteristic will not be very good because it will be slow.

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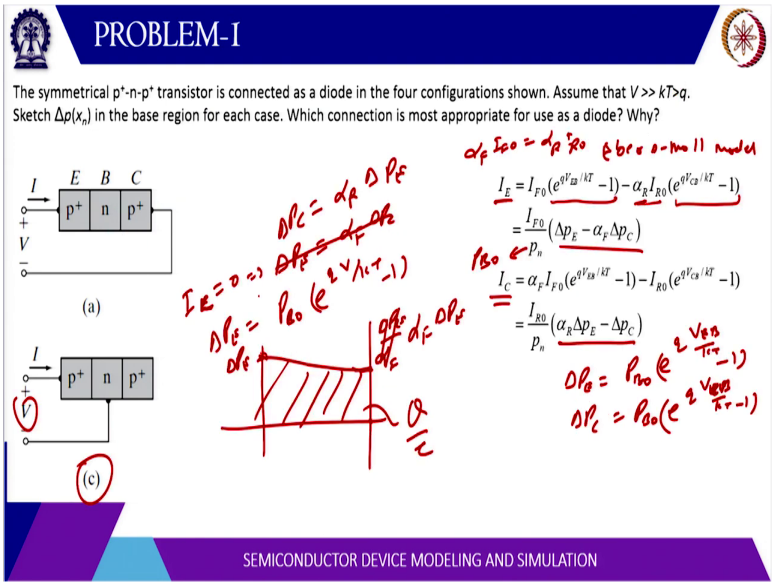

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$$= \frac{I_{F0}}{p_n}(\Delta p_E - \alpha_F \Delta p_C)$$

$p_{B0} \leftarrow p_n$

$$I_C = \alpha_F I_{F0}(e^{qV_{EB}/kT} - 1) - I_{R0}(e^{qV_{CB}/kT} - 1)$$
$$= \frac{I_{R0}}{p_n}(\alpha_R \Delta p_E - \Delta p_C)$$

$\Delta p_C = \alpha_R \Delta p_E$

~~$\Delta p_E = \alpha_F \Delta p_C$~~

$I_E = 0 \Rightarrow$

$\Delta p_E = p_{B0}(e^{qV/kT} - 1)$

$\Delta p_B = p_{B0}(e^{qV_{EB}/kT} - 1)$

$\Delta p_C = p_{B0}(e^{qV_{EB}/kT} - 1)$

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So, this is C part in case of D part this is a D part here base emitter. So, this is emitter and this is base and collector is there applied with the same voltage. So, that means your Delta P E is equal to P B naught exponential q V EB which is B applied by kT minus one and that is same as Delta P C. So, Delta p and Delta P C are equal but if you notice here the direction of current is the slope of this carrier concentration. So, from emitter current is flowing.

So, the slope should be something like this and that is Delta P E and from collector also the current will flow like this because this base collector Junction is. Now forward bias. So, here also it will be something like this. So, overall it will look like this and this cons this carrier concentration here excess calculation is also P or which is equal to Delta P C. So, in this case also the store charge will be quite high because this is narrow base Junction and it will act slowly basically.

So, if you compare all the four configurations D, C, A and B. So, B by F R gives you the good result it gives you the good current and store charge is also less in this case.

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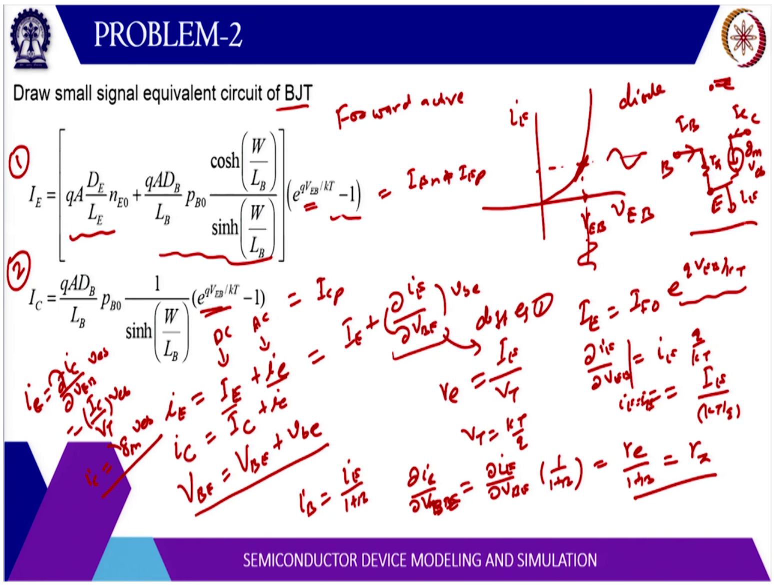

PROBLEM-2

Draw small signal equivalent circuit of BJT

Forward active

①
$$I_E = \left[qA\frac{D_E}{L_E}n_{E0} + \frac{qAD_B}{L_B}p_{B0}\frac{\cosh\left(\frac{W}{L_B}\right)}{\sinh\left(\frac{W}{L_B}\right)} \right]\left(e^{qV_{EB}/kT}-1\right) = I_{En}+I_{Ep}$$

②
$$I_C = \frac{qAD_B}{L_B}p_{B0}\frac{1}{\sinh\left(\frac{W}{L_B}\right)}\left(e^{qV_{EB}/kT}-1\right) = I_{Cp}$$

$$i_E = \overset{DC}{I_E} + \overset{AC}{i_e} = I_E + \left(\frac{\partial i_E}{\partial V_{BE}}\right)v_{be}$$

$$i_C = I_C + i_c$$

$$V_{BE} = V_{BE} + v_{be}$$

$$i_e = \frac{\partial i_C}{\partial V_{EB}}v_{eb} = \left(\frac{I_C}{V_T}\right)v_{eb} = g_m v_{eb}$$

$$I_E = I_{E0}\, e^{qV_{EB}/kT}$$

$$r_e = \frac{I_E}{V_T}$$

$$V_T = \frac{kT}{q}$$

$$i_B = \frac{i_E}{1+\beta}$$

$$\frac{\partial i_B}{\partial V_{BE}} = \frac{\partial i_E}{\partial V_{BE}}\left(\frac{1}{1+\beta}\right) = \frac{r_e}{1+\beta} = r_\pi$$

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So, now consider a second problem. Now it is say draw the small signal equivalent Circuit of BJT. So, we all know the equation of the current for a BJT. So, let us recall that now this equation I have purposely left out the term containing exponential q V CB by kT we are assuming forward active mode. So, forward active mode and we are ignoring the contribution from the reverse reverse saturation current for the base collector junction.

So, the I is basically I En plus I EP. So, this is I E N plus I EP where I En is first term exponential q V B by kT and I EP is the second term in the base this term and I C is basically I CP that is coming from the base and other reverse saturation current coming from The Collector we have ignored it. Now what we can say that small signal equal circuit is basically a superimposition on a DC signal.

So, all the voltages let us say please pay attention to the notation here time. So, I a small i sub capital A this is a total current and capital I Capital sub E is the DC part and a small i and a small sub e is the AC component. So, total current is a DC current plus AC current. Similarly for I C small i Capital Sub C is equal to Capital I Capital Sub C plus a small i a small Sub c. So, that is DC current and AC current then for V B also right.

So, small v capital B capital E is equal to capital V sub capital B. So, this is a DC part plus a small v be sub. So, this is AC part. So, this is a general notation that textbook follows so, where total current is expressed as a sum of DC part plus a small signal AC part. Now if you want to write I current as a I capital E plus I sub small e. Now if you notice here in the first equation number one this equation number one and this is less equation number two.

So, in equation number one q A D by L times n all these are material parameters basically and the design parameters the applied wires is appearing here V EB. So, we can write as this V EB can be written as something like this V EB. So, so its change basically so, instead of V B you can write V EB. So, your I can be written as I E plus Del I capital E by Del V BE times a small v be. So, I is at DC current value for the emitter and a small v b is a small base emitter voltage changes meter voltage this is a derivative.

So, if you want to see it pictorially let us say this is your I E versus V EB here. So, it will have some characteristics. Now you choose any operating point. So, this is your operator. So, this is capital V sub capital EB. Now if you superimpose a small AC signal on this one. So, the corresponding change in the current will be so if this voltage increases this current will also increase. So, this slope here is Del I by Del V BE.

That we also call the dynamic resistance it is same as the curve of a diode. So, in diode we call it dynamic resistance in this case we can call it the emitter resistance or emitter base junction resistance and looking from different side whether you are looking for the emitter or you are looking from the base the resistance value will be slightly different. So, if you take this derivative Del I by Del V BE.

So, we differentiate equation number one differentiate equation number one you will get because differential of exponential is same thing and if the current is fairly large we can ignore this minus one. So, this is same as let us call it I E is equal to I let us say F naught exponential q V EB by kT. So, if you take the derivative. So, Del I E by Del V EB is equal to same this I E total I E times this is q by kT.

So, you can also write it as. So, at I E is equal to Capital sub I capital I is equal to capital I capital E by kT by q and kT K by q is thermal voltage. So, this is basically you can say this is I E divided by V T, V T is kT by q. So, bit is a thermal voltage it is kT by q. Now this is called R E. So, this is basically looking for the emitter if you want to write looking from the base then you have to write the expression for base.

So, I B now I we will have the same form as I the only difference is that I B is I E by one plus beta. So, the derivative of if we assume that beta is not changing or in the region of operation beta is fixed then Del I E by Del V BE will be Del I B by Del V BE Del I E Del I B by Del V BE EB right or BE is equal to Del I E by Del V BE divided by one plus beta. So, this is R E by 1 plus beta and this we call R pi the current looking from the base.

So, if you recall g m R Pi model. So, this is the resistance looking from the base. Similarly we can find out for the collector. So, your eyes kept sub capital C can be done as capital I C plus a small i. So, here a small i small e is basically Del I C by Del V EB times small v eb. So, if you take the derivative again here it will be I C divided by kT by q. So, I C by V T times V EB. So, this factor is called g m because I C is equal to g m V EB.

So, if you try to make a circuit here let us say this is your base and this is your emitter let us say this is your collector. So between the collector and emitter so, between the collector and emitter there is a current I C. So, this is I B this is I C and this is I E. So, here you can write a voltage source which is dependent on V EB. So, this is g m times V EB and this is looking from I B it is R pi.

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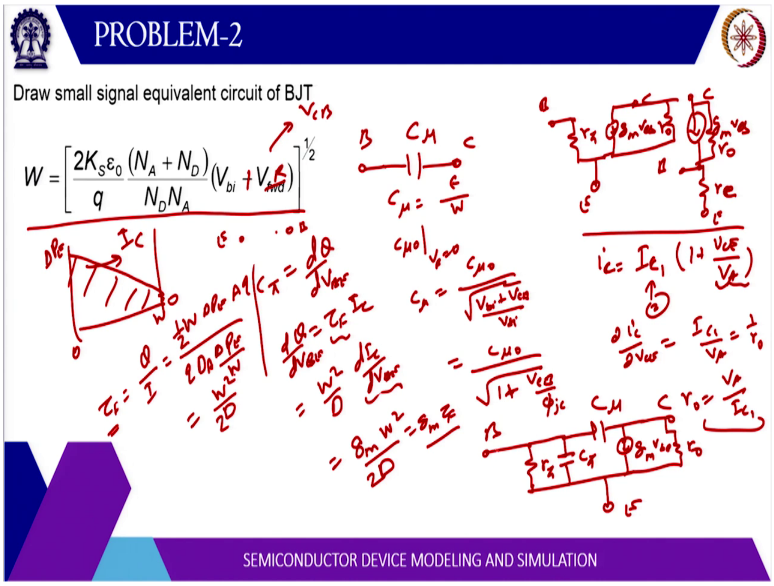

PROBLEM-2

Draw small signal equivalent circuit of BJT

$$W = \left[\frac{2K_S\varepsilon_0}{q}\frac{(N_A+N_D)}{N_D N_A}(V_{bi}+V_{fwd})\right]^{1/2}$$

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Or you can also write this as this is filled. So, we can use another seat. So, or you can write it as this is collector and this is g m V EB and this is base and this is emitter. So, this will be R

E. So, these two models are used basically if current flowing through the resistor is I E then we call it R E and if current flowing through the resistor is I B then we call it R pi this is g m V EB this is the collector this is the base this is the emitter.

So, either model you can use. Now this is what we have done basically we have calculated the resistance for base emitter current and we have also calculated the collector current in terms of base emitter voltage another term is resistance is there at the collector that comes from the early effect. So, if you recall I C you can write those expression this equation number two if you want to include early effect then let us say this term is let us call it V lets say I C one let us call it I C one.

So, your total ice is I C one from equation number two there multiplied by 1 plus v c e by v a. So, V A is the early voltage it is a empirical relationship. So, V A is large it is more close to the ideal BJT. So, if you take the derivative. So, Del I C by Del V CE will be I C1 by A. So, this is called one over R out. So, R out is basically V A early voltage divided by I C1 so, I because this continuity is very small so, I C is almost as good as I C1.

So, what is done here this R naught is also connected here. So, this is R naught or in this circuit this is R naught. So, this is equivalent circuit. Now there are some capacitance also in case of base collector so between base and collector this junction is reverse biased. So, there will be a depletion capacitance here and we call it C mu and if you recall the depletion width in case of P N the junction diode this is the expression.

So, this is for forward wise if it is reverse bias then this sign become negative. So, you can write plus V R the reverse bias voltage. So, in this case V R will be V CB so this expression. So, your C mu is basically Epsilon by this depletion width W and W is basically proportional to square root of V BI plus V CB. So, generally it is expressed as C mu naught which is the C mu at V CB equal to 0 or V res equal to 0.

So, C mu can be written as C mu naught divided by. Now which factor will be there because w at square root of V BI plus V CB and the same unit has already had V BI right. So, we divided by V BI. So, C mu can determine C mu naught divided by a square root of one plus V CB divided by V BI and this V BI generally is returns something like junction voltage for base collector. So, you can write Phi J C.

So, at the junction collector base junction the junction voltage, so, this is the expression for C mu there is another constants between the base and emitter. Now in the base emitter the capacitance is more due to the diffusion because this is forward bias. So, diffusion capacitance is given by C pi and that is d by d V BE of q stored in the base. So, this q can also be written as the time it takes to take the charge away from the base region times the current in the base.

So, that is transiting the base. So, there is a collector current because if you see here I is very close to I C. So, this is basically this is this is I C and very close to I E. So, and this is a charge stored here. So, q is Tau F the time it takes to carry this charge away times I C. So, C pi is derivative of this thing. So, d by d by V BE of q. So, here there are two things. So, Tau F is basically you can calculate Tau F can be written as q divided by the current I.

So, q is a charge stored. So, charge stored basically this is Delta P E this is almost 0 year because this base collector is reverse bias we are assuming the forward active circuit. So, q is half base that is W of the width of the base region times Delta P E divided by I, I is the I is diffusion current obtained from the slope. So, this is basically q D times derivative of this Delta P. So, derivative of Delta P this is narrow base diode.

So, you can write the derivative is basically the change in the carrier concentration that is Delta P E divided by W anything else. So, there should be area q D A Delta P by W and in top q charge is by half base Delta P here also area will come because this has to be volume. So, bits into areas of volume and Delta p is A times q because q is a charge. So, here if you see q A will cancel out so what will remain here W and W.

So, that becomes W Square by D Delta P will cancel out. So, D w Square by D, so, that is the Transit time. So, the amount of the time the charge is taken to Transit the base. So, so D 2 by d V EB is basically W Square by D times d I C by d V BE. So, that you know is g m. So, that is g m times W Square by this will be factor of two also 2D g m times W Square by 2D or g m Tau F. So, overall equivalent circuit we can write this is R pi this is C pi and this is C mu this is base this is emitter this is collector and this is current source g m V BE and this is R naught.

So, where R naught is V A by I C and R pi is a base emitter resistance looking from the base side. So, this is a small signal equivalent circuit of BJT.

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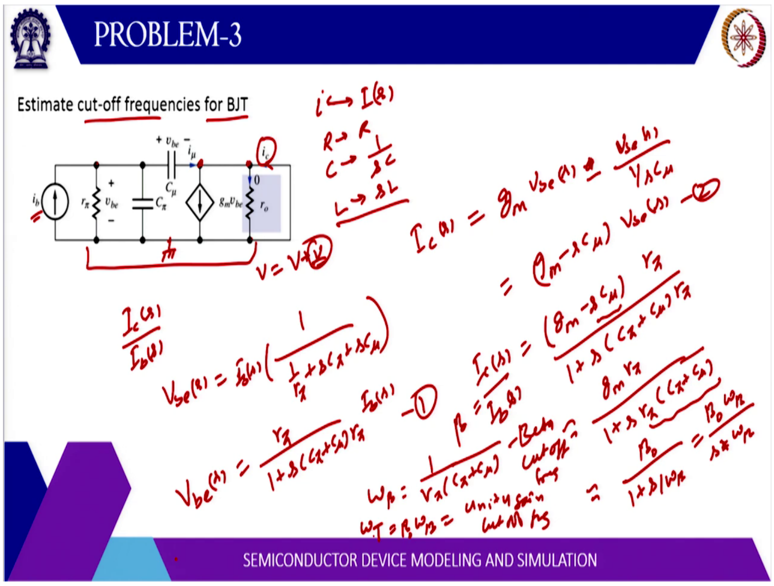

PROBLEM-3

Estimate cut-off frequencies for BJT

$i \rightarrow I(s)$

$R \rightarrow R$

$C \rightarrow \frac{1}{sC}$

$L \rightarrow sL$

$V = V + v$

$I_c(s) = g_m V_{be}(s) - \frac{V_{be}(s)}{1/sC_\mu}$

$= (g_m - sC_\mu) V_{be}(s)$ — ②

$\frac{I_c(s)}{I_b(s)}$

$V_{be}(s) = I_b(s)\left(\frac{1}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)}\right)$

$V_{be}(s) = \frac{r_\pi}{1 + s(C_\pi + C_\mu) r_\pi} I_b(s)$ — ①

$\beta = \frac{I_c(s)}{I_b(s)} = \frac{(g_m - sC_\mu) r_\pi}{1 + s(C_\pi + C_\mu) r_\pi}$

$\approx \frac{g_m r_\pi}{1 + s r_\pi (C_\pi + C_\mu)}$

$\approx \frac{\beta_0}{1 + s/\omega_\beta} = \frac{\beta_0 \omega_\beta}{s + \omega_\beta}$

$\omega_\beta = \frac{1}{r_\pi (C_\pi + C_\mu)}$ — Beta cut-off freq

$\omega_T = \beta_0 \omega_\beta$ = unity gain cut off freq

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Let us look at the problem number three. Now this equivalent circuit we have already derived in the previous problem. Now the task is to estimate the cutoff frequencies for this bipolar Junction transistor. So, let us say excite this thing with a current Source a small current Source I B and we sort this collector terminal and we measure this current IC. So, to find out the cutoff frequencies we have to analyze this circuit in frequency domain.

So, for now we can just take that n is current I in frequency domain can be done as I function of S where S is the complex frequency and impedances R will remain R capacitor will become impedance will be 1 over S C and inductor is not here but if there is inductor its impedance will be s l. So, this is the transformation that is used. So, now let us analyze This circuit in frequency domain.

So, we can we have to find out I C divided by I B. Now when this collector is sorted out let us say this emitter is reference potential or let us say ground. Now please remember this ground is AC ground. Now the DC ground ok. So, when a terminal is connected to a fixed voltage or to a DC ground if voltage is not changing. So, the voltage is written as capital V plus A small v so, because it is fixed.

So, this is a small v is 0 basically so any terminal of a BJT which is connected to a fixed voltage or ground will have 0 component for a small signal voltage because there is no change in that voltage. So, this AC signal refers to the small change in the voltage and current. So, I sub c by I sub B you have to calculate now because this collector is grounded to the emitter. So, it will bypass both of them.

So, if you look from the base side this current is seeing three branches R pi C pi and C mu which is grounded again. So, your V Pi or V BE. So, V BE is I B times the admittance of these three elements and the admittance of these three element is they can be added together right. So, V is equal to I times impedance. So, impedance is one over admittance. So, this is one over admittance. So, the advantage they are in parallel.

So, these admittances can be added together. So, this is one over R pi plus S C pi plus S C mu. So, this can be further simplified as R pi divided by 1 plus S C pi plus C mu times R pi times I B. So, these are relationship between V BE and I. Now we have to find the relationship between I C and V BE. So, this is let us say equation one equation two we can find from here if we write the node equation at this point.

So, your I C is the sum of these two currents at this node this I C is defined as moving in. So, this is g m times V BE moving out plus the current here. Now this current will be V BE by impedance of C mu is the current moving to right. So, moving to the left will be minus sign. So, we will have minus sign here so, minus sign so, V BE divided by one by S C mu. So, you can write as g m minus S C mu times V BE.

So, this is I C and if you substitute just V BE from equation one so, if you substitute equation one into equation two you will get I C is equal to g m minus S C mu times V BE, V BE is I B by. So, this is R pi divided by one plus S C pi plus C mu times R pi times I B. So, I B we get out to the left. So, this is I B this is basically you can say beta. Now if you compare these values typically g m C mu this C mu is quite a small and especially at moderate frequencies or mid Band frequencies you can ignore this term.

So, this can be written as g m R pi divided by one plus S R Pi C pi plus C mu and g m R Pi is beta naught if you recall that this is beta naught divided by one plus S times now this can be written as another this can be given another name this is called Omega B. So, if you compare this Omega beta is equal to 1 over R pi times C pi plus C mu. So, this is called beta cutoff frequency. So, when s is equal to or S equal to J Omega.

So, Omega is equal to Omega beta the gain is beta naught by root 2 because this is J Omega and Omega is equal to Omega beta. So, this will be beta by 1 plus J. So, the magnitude will be beta naught by root two. So, that will be again. So, this is called beta cutoff frequency beta cut off frequency similarly we can also find out Unity again frequency. So, you can take this one as beta naught Omega beta divided by S Plus Omega beta.

So, when S is equal to Beta naught Omega beta. So, this is quite large because beta naught is usually you know order of up to hundred. So, this will be one. So, this beta naught Omega beta is called Unity gain frequency unity again cutoff frequency this also denotes as Omega t. So, these are the two cutoff frequencies that are popular for bipolar junction transistor. So, these three problems we have discussed in this lecture, thank you very much.