



**Semiconductor Device Modelling and Simulation**  
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**Lecture – 23**  
**Bipolar junction Transistor (Contd.,)**


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L23 BJT




- BJT GUMMEL POON MODEL
- HBT
- BJT TRANSIENT BEHAVIOUR
- BJT SMALL SIGNAL MODEL




SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Hello welcome to 23rd, lecture we will continue our discussion on bipolar junction transistor. So, in today's lecture we will discuss further about the considerations how to improve the variety we will discuss Hydro Junction bipolar transistor called HBT and we will also discuss the transient and small signal model behaviour of BJT.

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GUMMEL-POON MODEL




- Ebers-Moll model: second order effect not incorporated
- Gummel-poon model: charge control model
- typical graded doping profiles in the base, there is a built-in electric field that causes drift of minority carriers

$$I_{EP} = qA\mu_p p(x_n) \xi - qAD_p \frac{dp(x_n)}{dx_n}$$

• assuming  $n(x_n) = N_D(x_n)$

$$I_{EP} = qA\mu_p p \left( \frac{-kT}{q} \frac{1}{n} \frac{dn}{dx_n} \right) - qAD_p \frac{dp}{dx_n}$$


$$I_{EP} = -\frac{qAD_p}{n} \left( p \frac{dn}{dx_n} + n \frac{dp}{dx_n} \right) \quad \frac{d}{dx} (np)$$



Hermann-Gummel

$\xi = -\frac{kT}{q} \frac{1}{n} \frac{dn}{dx}$

$\frac{dn}{dx} \rightarrow$   
 $\frac{dp}{dx} = \frac{kT}{q}$



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So, compared to Eber-Moll model this is a Gummel-Poon model it takes into the consideration the second order effect. So, which were not considered in the Eber-Moll model. Now Gummel-Poon model is based on the charge control method. So, if there is a gradient base profile we have slightly discussed about it the graded base profile there is a built-in electric field called E.

So, Now the current in the base region will have 2 components the drift component due to the built-in electric field and the diffusion component diffusion compound component is same as  $q A D_p \frac{dp}{dx}$  and the drift component is  $q A \mu_p E$ . Now we assume let us say electron concentration is same as the doping profile in the base that means all the depend ions are ionized in the base region.

And if we substitute the expression for this electrical field which was minus  $kT$  by  $q$   $1$  over  $N_B$  times  $T$   $N_B$  by  $dx$  and we replace  $N_B$  by  $N$ . So, we have this expression here and another relationship you can recall that  $d$  by  $\mu$  is  $kT$  by  $q$  is also called Einstein relationship. So,  $\mu$  times  $kT$  by  $q$  can be written as  $D$ . So, we can take  $q A D$  by  $N$  outside. So, this is  $P$  times  $dn$  by  $dx$  and this is  $n$  times  $dp$  by  $dx$ . So, this can written as  $d$  by  $dx$  of product  $n p$  times  $q A$   $dp$  by  $n$ .

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**GUMMEL-POON MODEL**

- Integrating from emitter-base junction (0) to base-collector junction ( $W_b$ ), assuming  $I_{EP}$  is more or less constant in the narrow base

$$-I_{EP} \int_0^{W_b} \frac{ndx_n}{qAD_p} = \int_0^{W_b} d(pn) = p(W_b)n(W_b) - p(0)n(0)$$

$n_i e^{qV_{CB}/kT}$       $n_i e^{qV_{EB}/kT}$

Denominator is integrated majority carrier charge in the base, and is known as the base Gummel number,

$$I_{EP} = - \frac{qAD_p n_i^2 (e^{qV_{CB}/kT} - e^{qV_{EB}/kT})}{\int_0^{W_b} ndx_n}$$

**QB**

$$Q_B \equiv \int_{0(V_{EB})}^{W_b(V_{CB})} n(x_n) dx_n$$

-----This can take care of Early effect

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Now if we integrate this from base emitter Junction to the base collector Junction. So, let us say this is the base emitter Junction this is the base collector Junction. So, from 0 to this  $w$ . So,  $I_{EP}$  times  $n$  by  $q A d$  is this thing  $dn$   $P$  by  $dx$ . So, it is basically  $I E$  times  $n$  by  $q A d$   $dx$  is

equal to  $dn_p$ . So,  $dn_p$  on right side is basically  $p_p A$  times  $n$  at  $w_b$ . So,  $p_n$  product here minus  $p_n$  product at emitter base Junction and because through the base the current has to be constant IEP of course assuming that ignoring the recombination. So, IEP we can take outside.

So, it is  $n dx$  by  $q$  by  $dp$ . Now if we take it to the right side we can write  $I_{EB}$  is equal to  $q A dp$  on the numerator side and this is  $n_i^2 \exp(qV_{CB}/kT)$  this is  $n_i^2 \exp(qV_{EB}/kT)$  from the law of Junctions. So,  $\exp(qV_{CB}/kT)$  minus  $\exp(qV_{EB}/kT)$  and  $I_{EB}$  outside  $q A dp$  by integral and  $dx$ . So, this is the expression for IEP. Now if you look at the denominator here this we call the base Gummel number and this we are integrating from 0 to  $W$ .

And this 0 and  $W$  both are function of the bias voltage across the 2 junction. So, 0 position depends on the base emitter bias and  $W$  position depends on the base collector bias. So, this takes care of the early effect and you can notice here if the base collector reserves bias then this IEP will be positive because this minus sign and minus sign will be positive and if you operate in reverse active mode then  $V_{CB}$  is positive then this IEP will be negative.

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**GUMMEL-POON MODEL**

- Forward active mode:
 
$$I_{Ep} = \frac{qAD_p n_i^2 e^{qV_{EB}/kT}}{Q_B} \quad I_{En} = \frac{qAD_n n_i^2 e^{qV_{EB}/kT}}{Q_E}$$
- High level injection:
 
$$\int_0^{W_p} n(x_n) dx_n > \int_0^{W_n} N_D(x_n) dx_n$$

$$I_C \propto I_{Ep} \propto e^{qV_{EB}/2kT} \quad \text{--- Recall diode theory}$$

$$I_B \propto I_{En} \propto e^{qV_{EB}/kT} \quad \text{--- Emitter is highly doped}$$
- Thus,
 
$$\beta = \frac{I_C}{I_B} \propto \frac{e^{qV_{EB}/2kT}}{e^{qV_{EB}/kT}} \propto e^{-qV_{EB}/2kT} \propto I_C^{-1}$$

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So, for forward active mode IEP can be written as you can ignore this  $V_{CB}$  by  $kT$ . So, your simple  $q A dT dp$  and  $I$  Square exponential  $q V$  by  $KT$  by  $q B$  as a base Gummel number and similarly  $I_{EN}$  can be written as by same logic  $q A d$  and  $n_i^2 \exp(qV_{EB}/kT)$  by  $q B$  by  $q$ ,  $q$  is the emitter Gummel number. So, this is again integral in the emitter region. Now these are the equation for the diode with a base Gummel number and the emitter Gummel number.


So,  $\int n \, dx$  in the base region this is a base gamma number  $Q_E$ . Now  $n$  is the doping level here. So, according to open level there is a electrons. Now at high level injection when the current is high this  $N$  is not equal to  $N_D$  at high level injection  $n$  is more than  $N_D$ . So, that means the carriers that are injected into the base they are more than the doping carrier concentration. So, if you recall that at high level injection what happens your current  $I_E$  will be exponential  $q V_B$  by  $2kT$ .

But now if you compare this thing with a on the emitter side so, on the base side there is high level injection but on emitter side there will not be high level injection why because this is less doped this is base is less doped. So, number of carrier it can inject will be much less emitter is highly doped. So, number of current injected can number of carriers it can inject will be more.


So, your  $I_E$  will have exponential  $q V_B$  by  $2 kT$  but  $I_E$  and will have exponential  $q V_B$  by  $kT$  that is due to the difference in the doping of emitter and base. So, if you take the ratio of  $I_C$  by  $I_B$ . So,  $I_C$  is proportional to  $I_P$  and  $I_B$  is proportional to  $I_N$ . Now again here we are making some assumption because  $I_B$  consists of three components that  $I_E N$  plus recombination plus reverse saturation current at a collector base junction.

So, we ignore these 2 we say that  $I_B$  is  $I_N$  we get this expression and if you take the ratio you will have  $E$  to the power minus  $q V_B$  by  $2 kT$  that is  $1$  over  $I_C$ . So, at high level injection the gain reduces at  $1$  over  $I_C$  that we can obtain from the Gummel Poon model.

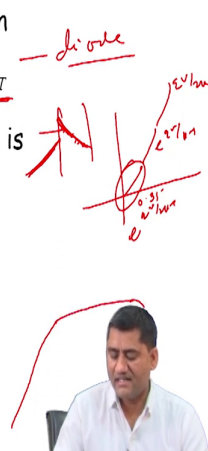
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## GUMMEL-POON MODEL




- Low current levels: generation-recombination in the base-emitter region
- large emitter current injected into the base is mostly unaffected by generation-recombination
- Thus, for low  $V_{EB}$  or low  $I_C$ , the current gain

$$\beta = \frac{I_C}{I_B} \propto \frac{e^{qV_{EB}/kT}}{e^{qV_{EB}/nkT}} \propto e^{(1-1/n)qV_{EB}/kT} \propto I_C^{(1-1/n)}$$




SEMICONDUCTOR DEVICE MODELING AND SIMULATION



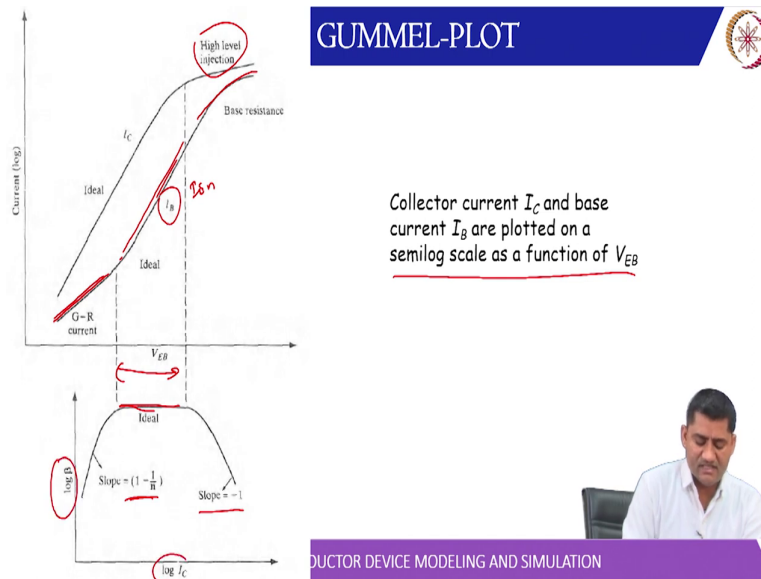
At low current levels generation recombination in the base emitter region that we ignored is becomes invalid. So, this at low current if we compare this with the diode PN Junction theory that we discussed for real diode your  $I_E$  is now proportional to exponential  $qV_B$  by  $\eta kT$  or  $N kT$   $N$  is ideality number. So, this is the carriers that are injected from the base and base is less doped and in case of emitter from the emitter large number of carries will be injected. So, at low current levels this will be normal injection.

So, this is basically kind of normal it is still low level injection but not like high level injection. So, you can still say that its  $q$  exponential  $qV_B$  by  $kT$  proportional to exponential  $qV_B$  by  $kT$  and because base is less doped. So, there is a small injection level here to the emitter. So, this is basically having some you can recall that for the base bias that up to 0.35 it was exponential  $qV_B$  by  $2kT$  then it exponential  $qV_B$  by  $kT$ .

Then again exponential  $qV_B$  by  $2kT$  so, this region here because of less doping, so, the current is limited the  $I_{en}$  is limited and the expression is exponential  $qV_B$  by  $N kT$ . So,  $n$  can be half or 2 or. So, if you take the ratio  $I_B$  by  $I_C$  by  $I_B$ . So, you will have exponential  $1 - 1/n$  by  $qV_B$  by  $kT$ . So, that is  $I_C$  to the power  $1 - 1/n$ . So, that means as  $I_C$  increase your beta actually increases at lower current levels.

So, it will have something like this beta increases becomes constant and at high injection it decreases. So, that is called Gummel plot.

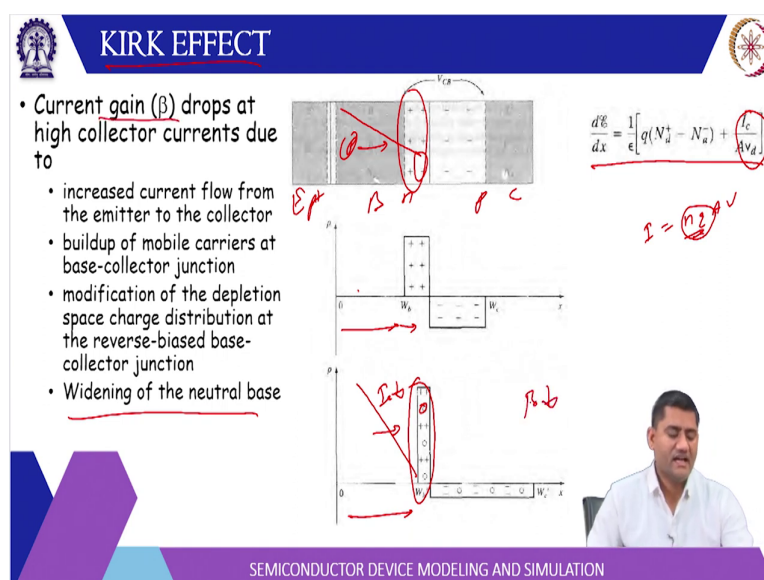
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So, here  $I_C$  increases and at high level injection this becomes the slope changes. So, you recall this  $I_B$  characteristic for a diode in case of  $I_B$  which is basically  $I_E N$  there is a low level generation recombination current then this ideal region and then there is a base resistance region. So, this is the middle region where beta is constant below the beta increases as  $I$  to the power  $1 - 1/n$  and at high level injection beta decreases at  $1 - 1/n$ .

So, so this Gummel plot is basically when  $I_C$  and  $I_B$  are plotted on a semi log scale as a function of  $V_{EB}$ . So, Gummel plot tells basically about the the gain characteristic of a bipolar junction transistor and we can identify what is the typical range that we can operate it in. So, this tells you the  $I_C$   $I_B$  versus  $V_{EB}$ . So, the range of  $V_{EB}$  and then how the beta changes versus log of  $I_C$ .

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There is another effect called Kirk effect. Now Kirk effect is observed at high currents. So, what happens let us say this is your emitter region this is the base region this is the collector region. So, this is positive ions. So, that means N type doping P type doping here E plus doping here. So, this will send the holes here and because of large current these holes may tend to accumulate here.

Now, because holes are accumulating here large due to large number of holes here so, this depletion region the charge record is supplied by these holes. So, what will happen the number amount of charge has to be same. So, this depletion region will actually width will actually reduce here because. Now it has ions plus holes. So, effectively what has what has happened that the base width has increased.

So, at high current base with effective basement actually increases an effective ways with increase basically causes the current to drop. So, you can write this expression  $d$  by  $dx$  is  $\frac{1}{\epsilon \mu}$   $q$  and  $d$  plus minus  $N_A$  plus  $I_C$  by  $A V d$ . So, this is the charge accumulated there  $I_C$  is the current. So,  $I$  is equal to  $n q$  times  $A$  times  $V$ . So,  $n$  by  $A V d$  is  $n q$ . So,  $n q$  is the charge carrier concentration times charge on each carrier.

So, this widening of the neutral base is called Kirk effect. So, this causes that because of widen the current will reduce. Now this  $I_C$  will reduce and your gain will actually drop. So, beta drops through to this region at  $I$  current gain. So, this is another second order effect.

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## HETEROJUNCTION BIPOLAR TRANSISTOR

- Dilemma for bipolar transistors:
  - For high frequency operation want low base resistance - high base doping
  - For high current gain want to minimize hole injection into emitter (npn) - low base doping
- Solution HBT - heterojunction bipolar transistors
- For CMOS integration use  $Si_{1-x}Ge_x$  system
  - Bandgap difference (1.12 eV Si, 1.0 eV,  $Si_{0.8}Ge_{0.2}$ )
  - 80%  $\Delta E_g$  in VB
  - 0.1 eV additional barrier for holes to emitter
  - Higher base doping w/same gain
- Selective growth of pseudomorphic Ge on Si substrate

Handwritten notes:  
 $Si \rightarrow 1.12 \text{ eV}$   
 $Ge \rightarrow 0.62 \text{ eV}$

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

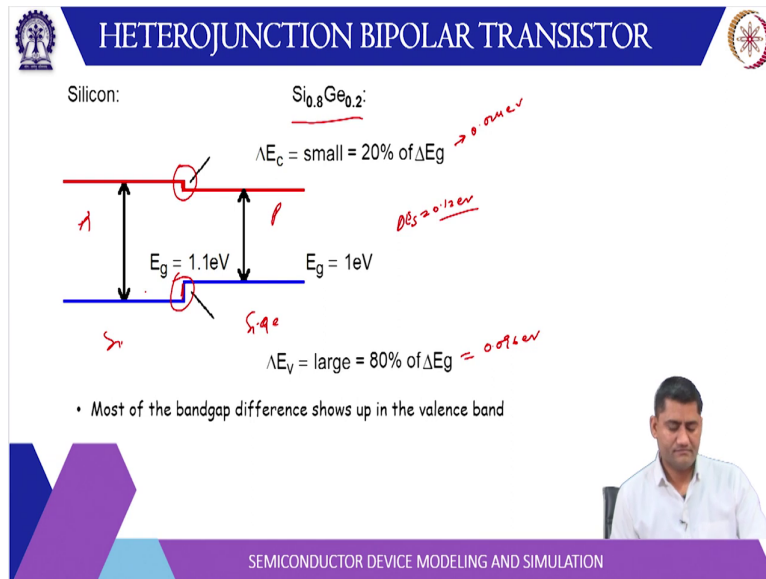
Now there are contrasting requirements in case of bipolar junction transistor for base region we want less doping to increase the emitter injection efficiency we want height opening to reduce the base resistance. So, is it possible that we can achieve both of them there is a solution that instead of using silicon here we can use some hetero junction bipolar transistor. So, we know that silicon bandgap is around 1.12 electron volt germanium is around 0.67 electron volt.

So, and this is basically let us say let us consider N P and BJT and P N. So, this is supplying the electron here this is supplying the holes here. So, if we somehow tailor that the barrier for the electrons because. Now band gaps are different. So, the barrier for electron is less and barrier for hole is more. So, this would be small. So, barrier for electron is less barrier for hole is large then we can improve the emitter injection efficiency.

So, now let us consider I think this is the first time we are encountering hetero junction. So, hetero junction is a structure where 2 different materials with different bandgaps are attached. So, when they come together these bands have to align. So, the bandgap difference they say this is  $E_g 1$  this is  $E_g 2$ . So, now these bands have to align because in case of silicon base and silicon emitter bandgap or Aqua. So, the band edges were continuous. So,  $E_C$  and  $E_V$  were continuous.

Now in case of hetero junction these bandgaps are different. So, these bandgaps have to be discontinuous. So, if let us say 1.1, 2 volt for silicon this is a bandgap 1 electron volt for the Silicon germanium. So, the difference is 0.12 electron volt and out of this 80 percent falls across a valence band and 20 percent falls across the conduction band. So, that means there will be additional extra barrier for holes to emitter. So, that means we will have a higher emitter injection efficiency.

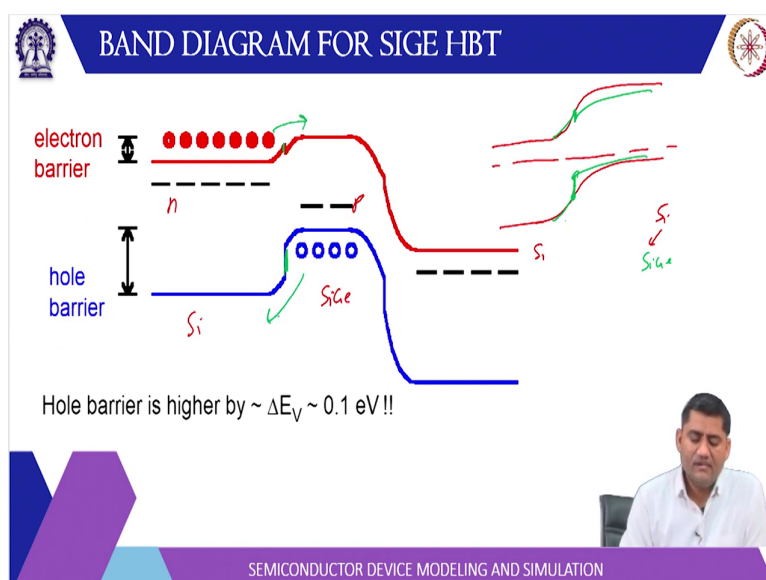
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So, let us look at this Band diagram this is silicon this is silicon germanium. Now we cannot have pure germanium here because of lattice mismatch issue if we you know have too much lattice mismatch then there will be defects in the silicon germanium layer and these defects will basically tend to reduce your BJT operation and they will have some leakage issues electrons carriers will be trapped there.

So, it will not be properly operational and then the difference Delta E g is 0.12. So, Delta E B is 80 percent. So, that is 0.096 electron volt and this is 20 for 20 percent. So, this is 0.024 electron volt. So, this barrier is small for the electrons barrier is large for holes. So, if we use this as a emitter with n type this has a whole base with P type. So, here the barrier will be small here the barrier will be large you can see here.

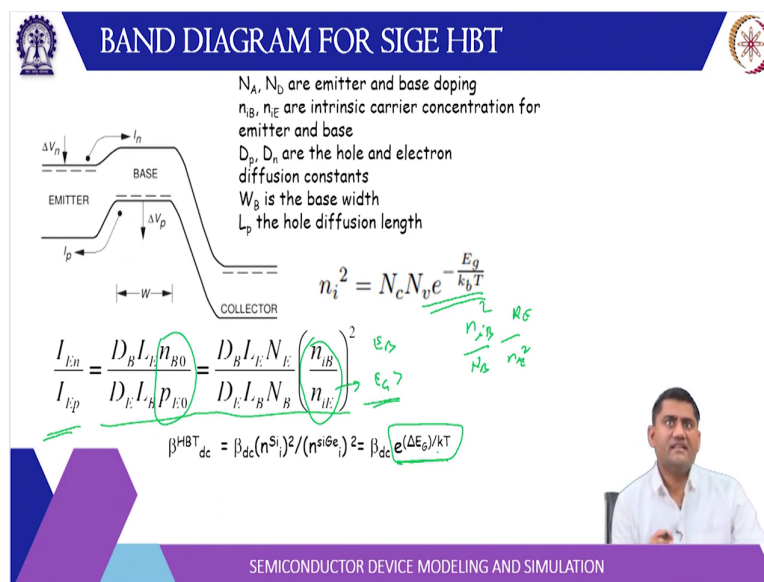
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So, this is your silicon this is silicon germanium this is N type this is P type. So, in case of equilibrium the fermi level available is same and this will go like this will be somewhere here. So, if both were let us say both were silicon ok but if this is silicon germanium that means different colour here I say this is silicon Germanium. Now the bandgap is not this much. So, 20 percent falls across here 80 percent falls across here.

So, now this is up to middle point then this distance will continue and this 1 will continue. So, you see here, this just continue is same this is going down this is going up. So, this is going down and this is going up. So, this is going down here this is going up. So, now you see here for the electron this very small for all this barrier is large. So, regardless of the doping or you know we have greater flexibility in terms of doping the emitter injection efficiency will be quite high.

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We can calculate it also if you recall the expression for  $I_{En}$  by  $I_{Ep}$  this expression. Now we have this ratio  $N$  in the base by  $P$  in the elect emitter. So, this is a meiotic carrier concentration in the base divided by meiotic carrier concentration the emitter. So, this is  $N_i$  Square by base doping this is  $N_i$  Square by emitter dropping. Now this is  $N_i$  Square in the base this  $N_i$  Square in the emitter.

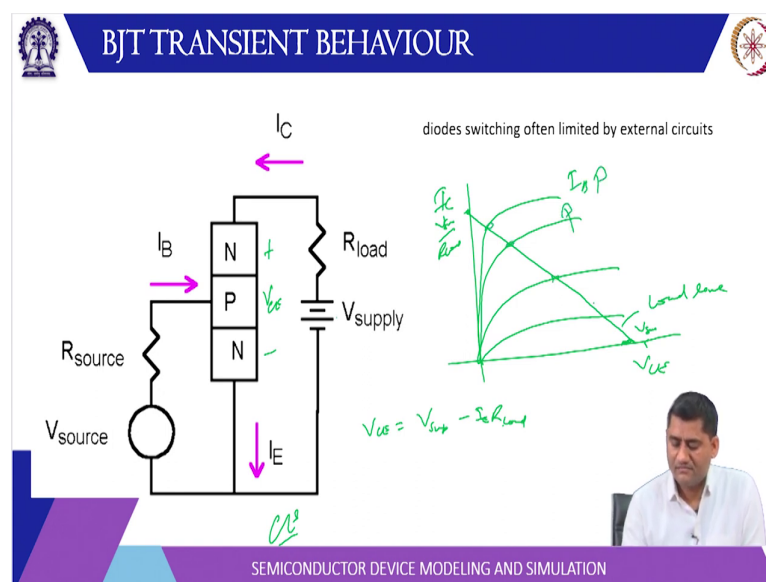
So, this is a ratio  $n_i$  square base by  $N_i$  Square emitter. Now emitter bandgap is more than the base bandgap. So, this  $n_i$  Square will be less the denominator  $n_i$  Square will be less than the numerator. So, this number will be proportional to  $E_g$  by  $2 K T$ . So,  $n_i$  Square will be  $E_g$

by  $kT$  minus  $E_g$  by  $kT$ . So, if you take the difference these 2  $\Delta E_g$  by  $kT$ . So, now your  $I_{EN}$  by  $I_{EP}$  is same thing multiplied by exponential  $\Delta E_g$  by  $kT$ .

So, this is more by  $\Delta E_g$  by  $kT$  so this extra factor of  $\Delta E_g$  by  $kT$  appears here. So, this expression is same as that we derived for the base emitter Junction in the previous lecture the only difference is the  $n_i$  square is different now in case of base and the emitter. So, we have given the some preview of hetero junction bipolar transistor and through the band diagram we have shown that using HBT we can achieve higher gain and simultaneously we can have higher base doping to reduce the base resistance.

So, generally these HBT can be operated for higher frequencies and at higher speeds and they give us good bandwidth.

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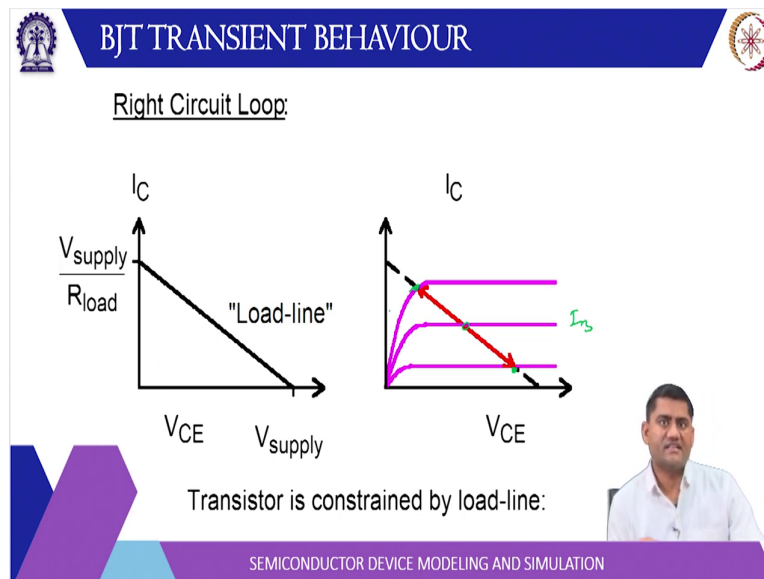


Now let us look at the behaviour to the signal variation. So, you can have transient behaviour. So, if you look at this diagram there is a base emitter voltage and there is a collector emitter voltage here. So, it is a common emitter configuration then at the common emitter you can see there is a voltage here there is  $R_L$  and this voltage is  $V_{CE}$ . So, if you plot this curve  $I_C$  versus  $V_{CE}$  you can write that  $V_{CE}$  is equal to  $V_{Supply}$  minus  $I_C$  times  $R_{load}$ .

So, you can draw a load line here that at  $V_{CB}$  equal to 0 we  $V_{supply}$  equal to  $I_C R_{load}$  so that means your  $I_C$  is  $V_{Supply}$  by  $R_{load}$ . So, this is  $V_{Supply}$  by  $R_{load}$  and then  $I_C$  is 0 your  $V_{CB}$  is equal to  $V_{Supply}$ . So, this is  $V_{supply}$ . So, this is your load line then you can also write  $I_C$ ,  $V_{CB}$  from the characteristic of this device. So, that you can recall it appears like

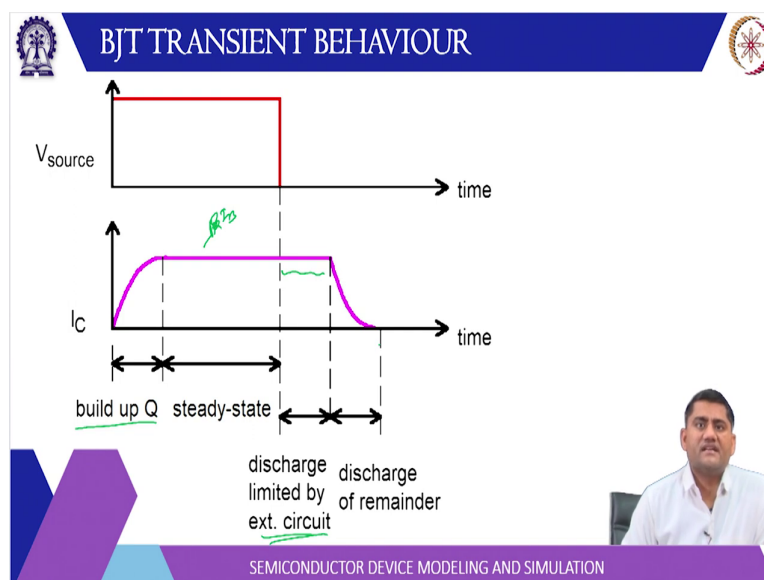
this as you increase the  $I_B$   $I_C$  increases it as you go up right. So, you will have different solutions.

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So, depending on the  $I_B$  you will have different solution. So,  $I_B$  is less you will have larger  $V_{CE}$  if  $I_B$  is more you will have a smaller  $V_{CE}$ . So, transistor characteristic is constrained by the load line.

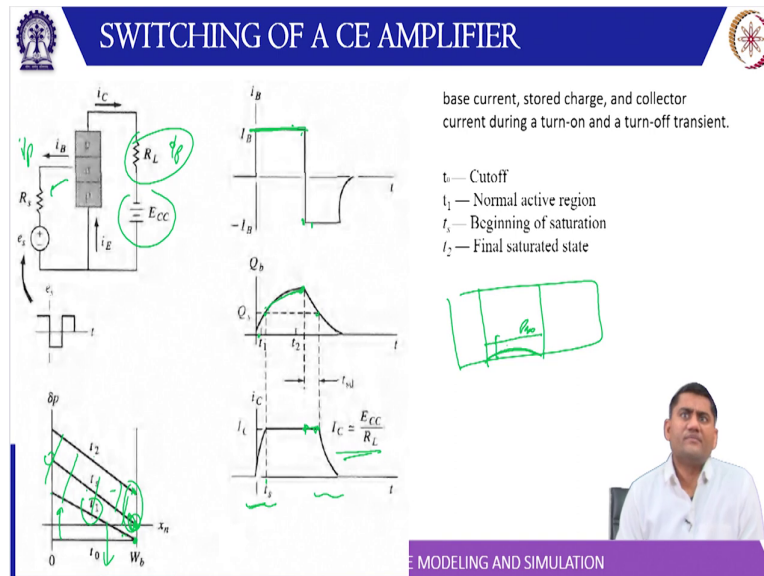
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Then if you look at the transient behaviour the source is connected to the base emitter junction and when you switch on the source you apply a some forward bias  $I_C$  will increase. So, the charge will be built up and then it will once it reaches ICT State you will have constant  $I_C$  which is beta times  $I_B$ . Then when you switch it off then initially this charge will be discharged through the external circuit.

So, it will depend on the external circuit or the load then of course it will remain the charge will be remaining charge will be discharge exponentially.

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So, we can further look into the diagram this is your base emitter input. So, this is the input side this is your let us say output side the load side then when you apply a positive voltage here then some IV will be there. So, let us say we have some positive  $I_B$  here then for a point of  $I_B$  the charge will keep on building up. So, if you consider this region. So, the charge will actually this is the background concentration.

So, in the reverse bias let us say this is P this is P V naught. So, when is reverse bias it is 0 here right then when forward bias it small by  $I_B$  flows this will go up. So, you see this will go up because the base characteristic reverse bias base emitter is forward bias. So, here the concentration increases here is still same then. So, this linear region is called active region. So, in this region it is forward active if  $I_B$  you know is slightly large or more than the forward active region current.

Or it can drive into the saturation region then what will happen more charge will charge will be accumulated here. So, here this has increased. Now you see the slope is same but this charge has increased at the collector terminal also. So, this means the  $V_{CB}$  is 0 m here  $V_{CB}$  has reverse bias here  $V_{CB}$  is 0 and then further increase basically it tells you that  $V_{CB}$  is now slightly positive. So, this is the characteristic for the saturation region.

So, from  $t_1$  to  $t_{\text{saturation}}$  and  $t_2$  so you can see here this is  $t_1$  this is  $t_{\text{saturation}}$  after the saturation current builds up but there is no change in the current. So, this  $I_V$  is constant and then up to  $t_2$  it goes here then when you switch off then this switch of the base current then this charge extra charge has to discharge. So, when it discharging the current will still continue. So, it assumes the same current but the direction is now reversed.

So, this is basically discharging. So, from the base region this charge is going back this charges going back discharging and till this charge reduces to the saturation level and once it reduces the saturation level this exponential decays and this  $I_C$  is  $E_{CC}$  by  $R_L$  that is a collector current. So, if you look at the characteristic there is a limit or there is a speed limit up to which we can operate this BJT transistor and that is limited by this time constant. So, the switching of AC amplifier is limited by this transient characteristic.

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Now let us spend some more time to just understand how the small signal response looks like.

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## BJT SMALL SIGNAL RESPONSE

- Assume the transistor can follow AC voltages and currents quasistatically (frequency not too high). Also neglect capacitances of pn junctions and other parasitics

(a)

(b)

Common Emitter equivalent circuit model

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So, for a BJT let's say this is a common emitter. So, your  $I_B$  is proper function of  $V_{BE}$  and  $I_C$  is function of  $V_{CE}$  and small signal is basically each current component  $I_{B,DC}$  plus a small  $i_b$  and a small signal current. Similarly  $I_C$  is  $I_{C,DC}$  plus a small  $i_c$  that is the AC component this can be written like this.

And when you substitute this equation that  $I_B$  is some coefficient times exponential  $V_B$  by  $kT$  and so on and when you differentiate with respect to  $V_{BE}$  and  $V_{CE}$ .

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## BJT SMALL SIGNAL RESPONSE

$$I_B = I_B(V_{BE}, V_{CE})$$

$$I_B(V_{BE} + v_{be}, V_{CE} + v_{ce}) = I_B(V_{BE}, V_{CE}) + \frac{\partial I_B}{\partial V_{BE}} v_{be} + \frac{\partial I_B}{\partial V_{CE}} v_{ce} = i_b$$

$$I_C = I_C(V_{BE}, V_{CE})$$

$$I_C(V_{BE} + v_{be}, V_{CE} + v_{ce}) = I_C(V_{BE}, V_{CE}) + \frac{\partial I_C}{\partial V_{BE}} v_{be} + \frac{\partial I_C}{\partial V_{CE}} v_{ce} = i_c$$

$$i_b = g_{11}v_{be} + g_{12}v_{ce}$$

$$i_c = g_{21}v_{be} + g_{22}v_{ce}$$


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So,  $I_B$  is  $I_{B,DC}$  component plus  $\Delta I_B$  by  $\Delta V_B$  plus  $\Delta I_B$  by  $\Delta V_{CB}$  times  $V_C$ . So, these are basically  $G_{11}$ ,  $G_{12}$  and similarly this  $I_C$  this is  $G_{21}$  and this is  $G_{22}$ . So, if


you only consider the AC equivalent circuit then these 2 components are basically small I small Sub C these 2 components are a small I this is the small sub b this is small i small Sub C. So, the small Sub c is  $g_{21} V_B$  plus  $g_{22} V_C$ .

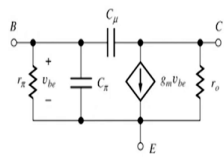
So, because I C is at  $V_{CE}$ , so, this  $g_{22}$  is basically conductance so this  $g_{22}$  is the conductance and I C is  $g_{21} V_{BE}$ . So, this is  $g_{21} V_B$  similarly I B is  $g_{11} V_{BE}$  right. So, this  $V_{BE}$  and I B there are the same terminal. So,  $g_{11}$  is the conductance. So, you see this is  $g_{11}$  and then plus  $g_{12} V_{CBE}$ . So, that is how this equivalent circuit is drawn.

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### BJT SMALL SIGNAL EQUIVALENT CKT





$C_\mu$  is the capacitance of the reverse-biased collector-base diode:

$$C_\mu = \frac{C_{j0}}{\sqrt{1 + (V_{CB}/\phi_j)}}$$

The frequency dependence of the BJT in forward-active region can be modeled by adding capacitors  $C_\mu$  and  $C_\pi$  to the hybrid- $\pi$  model.


$C_\pi$  models the change in base minority carrier charge as the base-emitter voltage of the transistor changes:

$$C_\pi = g_m \tau_F$$

where  $\tau_F$  is the forward base transit-time of the transistor, the time a carrier takes to cross the base region. For BJT,

$$\tau_F = \frac{Q}{i_T} = \frac{W_B^2}{2D_n}, \quad \text{where}$$

$W_B$  is the base width,  
 $D_n$  is the diffusion coefficient



SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Now for a frequency response these cap stains also play an important role. So, this is basically R Pi which we call  $g_{11}$  this was basically  $g_{22}$  and this was  $g_{21}$  there is another current source here called  $g_{12}$ . So, that is basically a small. So, we ignored it then we have this cap stance  $C_\mu$  and  $C_\pi$   $C_\pi$  is the base emitter capacitance and  $C_\mu$  is the base collector capacitance and that again base collector capacitance can be obtained from the base collector reverse bias diode that is see some constant divided by  $1 + V_{CB}/\phi_j$  this if I just is a built-in potential and  $V_{CB}$  is the reverse bias.

So, it varies inversely as I square root of  $V_{CB}$  and of course  $C_\pi$  is modelling the change in the base moiety carrier charge as the base emitter voltage of the transistor changes. And  $C_\pi$  is related to the base transit time  $\tau_F$  and the  $g_m$  and the transit time is given by  $q$  by  $i_T$  which is  $W_B^2$  square by  $2D$  you can understand like this. This is your base region let us say this is your current.

So, this  $q$  is integral  $I dx$  then divide by  $I$ . So, if you recall it is basically  $q A D$  times let us say  $dp$  by  $dx$ . So, which basically comes as  $W$  by  $L$  so now if you integrate with respect to  $dx$  you will get multiplied by  $W$  here and this  $I$  is proportional to  $D$  times  $t$   $p$  by  $dx$ . Now this is  $1$  by  $W$  is basically coming here. So, this  $W$  get multiplied here. So, when you integrate you get  $W$  Square by  $2D$  there.

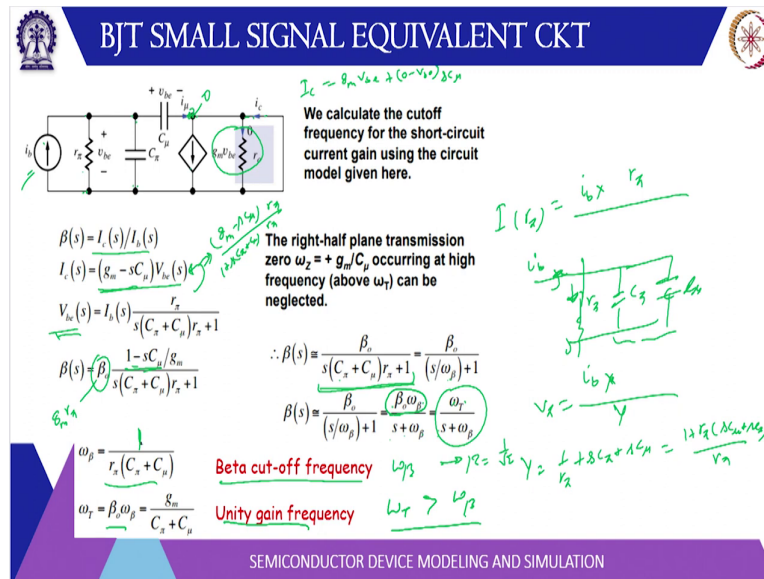
So, where  $W$  is the base width and these are diffusion coefficient or you can look at like this if there is a concentration Gradient in the the majority carrier concentration gradient here due to the injection. So, if higher the diffusion coefficient lesser time it will take to diffuse through higher the width more time it will take through the diffuse through the base Junction. So, it is proportional to the  $W$  the transit time and it is universally proportional to the  $D$ .

So, and so or you can think like this let us say  $W$   $B$  is the width. So, the length divided by time. Now time will be the velocity  $\mu$ ,  $\mu E$  or the diffusion coefficient. So, this is  $d$  by  $W$ . So, the transit time is proposed the transit velocity is proportion to  $D$  by  $W$  and length is  $W$ . So, this is  $W$  by  $D$ . So, the transit time for a BJT through the base is  $W$  Square by  $2D$  where  $W$  is the base width.

Now you can see another thing here the speed of this BJT will be limited by this Transit time. So, unless these carriers entering the base emitter Junction they exit at the base collector junction if you apply a higher frequency then this  $1$  then they will take longer time than the time duration of the signal this will not function properly. So, the frequency of the applied signal has to be smaller than by  $1$  by  $\tau$   $f$ .

Then of course there are other parameters they will limited the  $C_{\mu}$  and  $C_{\pi}$  they will also control the RC time constant. So, these three factors actually determine the speed of the BJT small signal equivalent circuit operation.

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Now we can further look at the different cutoff frequencies. So, if we consider that we have applied a base current and we can find out the maximum collector current that will flow through this circuit. So, this is sorted out here and this is  $I_B$  there. So, this is sort you know. So, this is also actually grounded actually. So, this  $I_B$  is in parallel with  $C_\pi$  parallel with  $C_\mu$ . So, by current division we can say that  $I_C$  is a sort of (FL).

So, here you can see the BJT small signal equivalent circuit now Beta is the ratio of  $I_C$  and  $I_B$ . So,  $I_C$  is the current here  $I_B$  is the input current if you write KCL node equation here, then the current because this is sorted here. So, no current will flow through this  $R_{naught}$  so,  $I_C$  is equal to  $g_m V_{BE}$  plus 0 minus  $V_{BE}$  times  $s C_\mu$ . So,  $I_C$  can be written as  $g_m V_{BE}$  minus  $s C_\mu$  times  $V_{BE}$ .

Now  $V_{BE}$  is basically is the voltage across  $R_\pi$  and  $R_\pi$  is in parallel with  $C_\mu$  and  $C_\pi$  because this is also ground here. So,  $V_{BE}$  is  $I_B$  times  $R_\pi$ . So, the current through  $R_\pi$ , so, current through  $R_\pi$  will be  $I$  through  $R_\pi$  will be  $I_B$  times current division. So, current division will be  $R_\pi$  divided by nothing like this is  $R_\pi C_\pi$  this is  $C_\mu$  this is the current  $I_B$ . So,  $I$  through  $R_\pi$  will be  $I_B$  times this impedance divided by overall impedance or admittance.

So, this will be the total admittance is  $1$  over  $R_\pi$  plus  $s C_\pi$  plus  $s C_\mu$ . So, that will come in the denominator so that is total  $P_i$  and times. So, this is this can be simplified as  $1$  plus  $R_\pi$  times as  $C_\mu$  plus  $s C_\pi$  divided by  $R_\pi$ . So, we do not have to calculate the current through

the  $R_{Pi}$  we just  $I_B$  by  $y$  that will be  $V_{pi}$  right. So, this is  $V_{Pi}$  or  $V_{BE}$ . So, that is  $I_B$  times  $R_{Pi}$  by  $1 + R_{Pi} s C$  by  $S_{mu}$ .


Now we can substitute this  $V_{BE}$  in the second equation that is  $I_C$  this. So, from this we can get  $I_C$  by  $I_B$  and that will be  $g_m$  minus  $s C_{pi}$  times  $R_{Pi}$ . So, this will be  $g_m$  minus  $s C_{mu}$   $R_{pi}$  divided by the denominator. So,  $1 + s C_{pi} + C_{mu} \times R_{Pi}$ . Now  $g_m R_{Pi}$  is  $\beta_{naught}$ . So, we can take  $g_m$  out. So, we can write  $1 - s C_{mu}$  by  $g_m$  multiplied by  $\beta_{naught}$  which is  $g_m R_{pi}$  divided by the denominator which is same.

Now you notice here  $C_u$  is  $C_{mu}$  is actually quite a small. So, we can say that  $\beta$  is  $\beta_{naught}$  by  $1 + s C_{mu} + C_{pi} \times R_{pi}$ . So, this will reduce to half when  $R_{pi} \times C_{pi} + C_{mu}$  is 1 and that is called  $\beta$  cutoff frequency. So,  $\beta$  cutoff frequency is  $\Omega_{\beta}$  which is  $1 / (C_{pi} + C_{mu} \times R_{pi})$ . At this frequency the  $\beta$  is one over root 2 at unit gain cut off frequency you can further simplify in simplify it.


So,  $\Omega_{\beta}$  you can take up, so,  $\beta_{naught} \Omega_{\beta}$  by  $s + \Omega_{\beta}$ . So, at unity gain frequency this  $\Omega_T$  should be quite large compared to this  $\Omega_{\beta}$  and  $\Omega_{\beta}$ . So, this at  $\Omega$  equal to  $\Omega_T$  Which is less than larger than  $\Omega_{\beta}$  this will be 1 basically. So,  $\Omega_T$  is given by  $\beta_{naught} \times \Omega_{\beta}$ . So,  $\beta_{naught} \Omega_{\beta}$  is the unit again frequency.

So, unity gain frequency is usually larger than the  $\beta$  cutoff frequency. So, this can be calculated in terms of this small signal parameters.



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## CONCLUSION



- Discussed Gummel-Poon model, HBT and BJT's response to time varying signal



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So, in summary in this lecture we have discussed the Gummel-Poon model then we also discussed heterojunction bipolar transistor which can enhance the gain of the bipolar junction transistor with the possibility of reducing the base resistance. And then we also discuss the BJT's response to the time varying signal both the transient response as well as the small signal response, thank you very much.