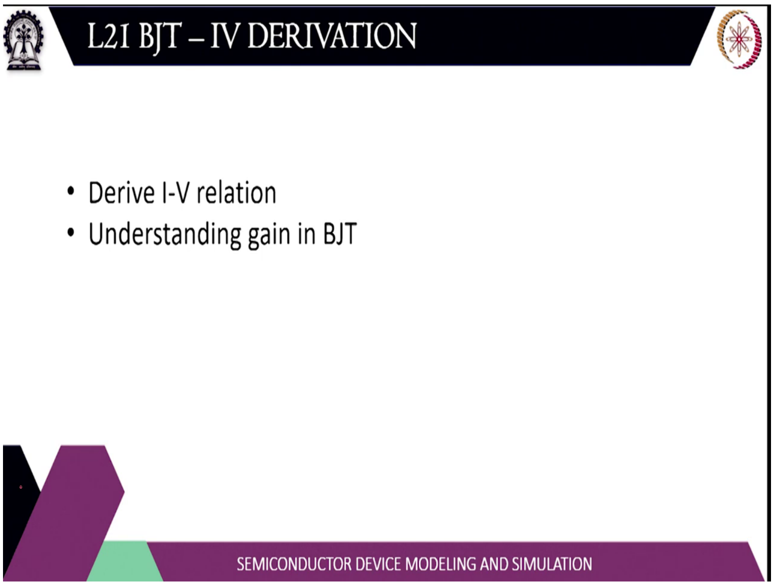
Semiconductor Device Modelling and Simulation
Prof. Vivek Dixit
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology - Kharagpur

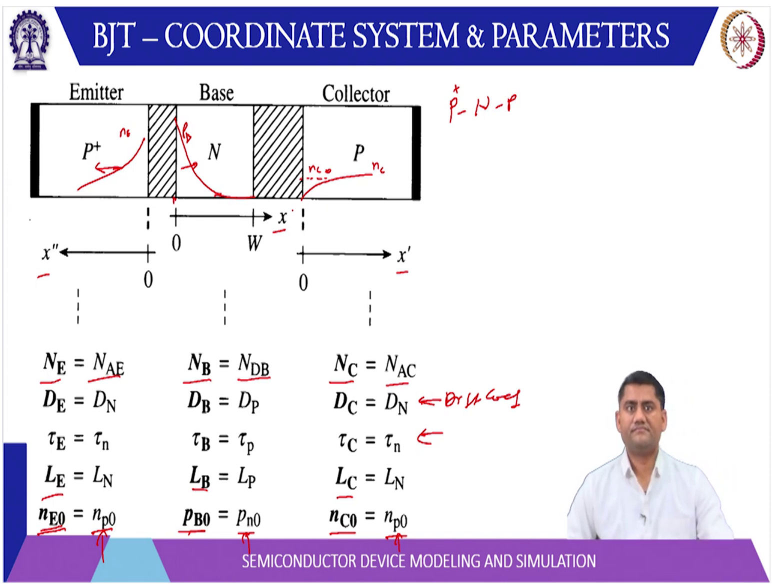
Lecture – 21
Bipolar junction Transistor

(Refer Slide Time: 00:31)



Hello, welcome to lecture number 21 on Bipolar Junction Transistor. In last class we discussed about the bipolar Junction IV characteristic and the naming conventions and all. Now today in this lecture we will drive the IV relationship for the three terminal device and we will try to understand the gain in BJT.

(Refer Slide Time: 00:48)



So, let us have a look at a view of a cross sectional view of a BJT it is a P N P BJ T emitter is P type basis N type and collector is again P type but there is a difference between the emitter and collector emitter is usually highly doped. So, we call it P plus and you can also note here to drive the current equation we will consider the diffusion of the carriers and their profile as we did in the case of PN junction diode first for base.

So, we are starting at the end of this depletion region and we have this x axis here and for collector we have x prime for emitter we have x double prime and you note their directions also because the injected carriers will somehow decrease like this. So, we will find what is the slope. So, what is the diffusion current similarly for p plus also and similarly we are collector because this is usually reverse bias.

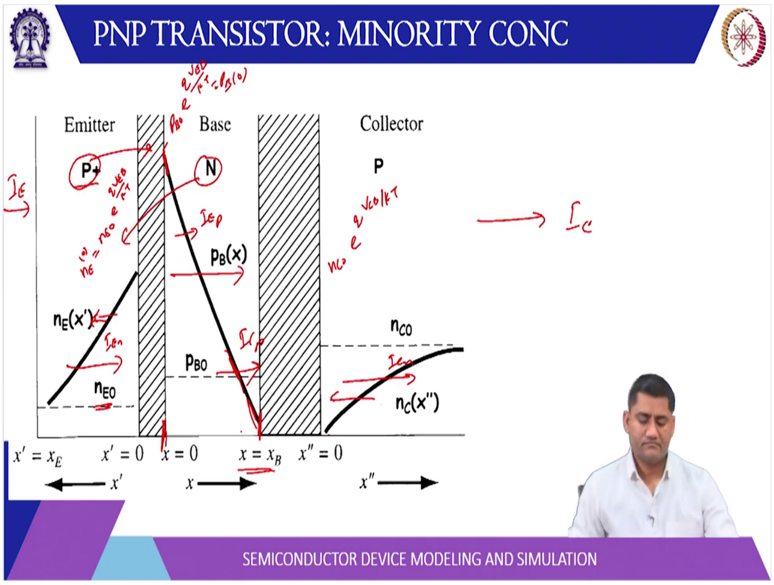
So, the profile goes like this because there is a depletion of carriers. The other parameters that are used in the derivation what is a doping level. So, doping level is this an acceptor enemy term and we can just call it N AE. Similarly number of donors in the base we will call N B and similarly number of acceptor in collector is simply N C the D are the diffusion coefficients Tau is the carrier lifetime.

So, Tau n Tau p and Tau n we are assuming that they are same but they may be slightly different though not that much but for derivation purpose we are assuming same Tau n in both the regions emitter and collector. Then root D tau is a diffusion length. So, this L n and L n they are called L cf, L e here and for holes in the base we call it L V. So, here basically we there is no particle name of the particle. So, which is the carrier whether it is a electron or hole and this is a minority carrier concentration.

So, number of electrons in P plus emitter region we are calling it N E0. Similarly here p and 0 is called p B0 and N P0 is called n C0. So, with these naming conventions we will try to drive the IV characteristic equation for the visit we can have a preview here this is emitter. So, it will inject holes here and they are in the base. So, we call it p in the base. So, it will be P B and P B naught is a background whole concentration.

Similarly electrons are injected to the emitter. So, we will have N E and N E not the background electron concentration and here will be depletion basically. So, again the multi carries are it is P type. So, n C and n C naught is the background concentration. So, this will be somewhere n C naught here and N will be less than that at the junction because this is usually reverse biased.

(Refer Slide Time: 04:18)



So, this have already discussed. So, the carrier profile will basically look like this and you can recall from the law of Junctions in case of PN Junction diode we discussed that whole concentration in the base will be given by P B naught exponential U V E B by kT and if we take the difference Delta P then we will subtract minus P B naught so this will look minus one similarly for the electrons here n e will be n E, n E naught exponential q V E B by kT.

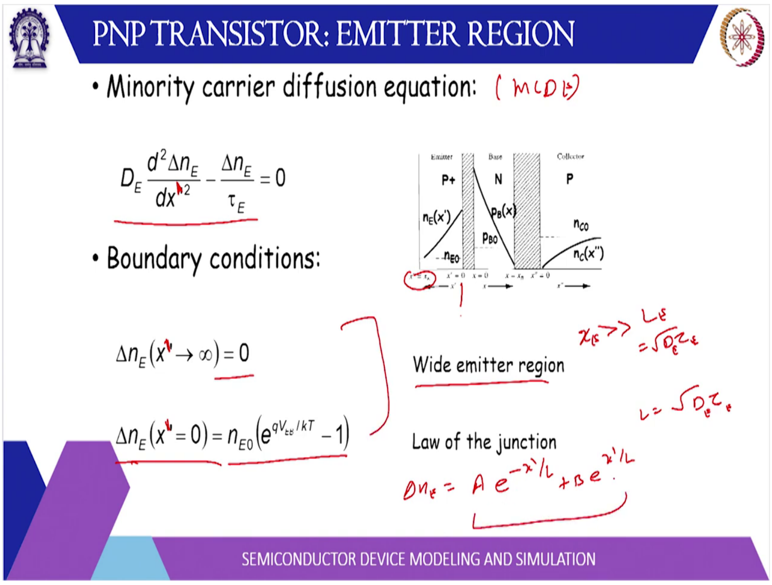
You can also notice because this is highly doped so, n E naught will be less. So, if n E naught is less then n E at 0. So, this is at x equal to 0 will also be less P B naught is more. So, this P B naught this is P B at x equal to 0 will also be more here or you can understand because this is a p plus High doping. So, it is injecting more carriers compared to the base which is less doped same thing here at this Junction but here this B series this is a reverse bias.

So, your V CV is positive or V CV is negative or V VC is positive. So, here the doping is n C naught exponential q V cb y kT. So, V cv is negative. So, this is actually equals to 0 and if you take Delta then you will subtract this n C naught here from this expression. So, from this we can write the equation. So, if you look at the emitter current which is by definition the current entering the emitter terminal IC is the current exiting the collector in the collector terminal and I En, I E will have 2 components one due to these holes.

So, we call it I EP another due to these electrons this is I En and the direction of current will be same because the I EP the whole concentration decreasing. So, they will diffuse in forwarding forward direction. So, I EP will flow like this. Now I En electrons are diffusing this way. So, the current will be opposite. So, both I E and I EP are in the same direction. Similarly I C you can find from this derivative.

So, this is decreasing. So, that means current will be in this direction here also this is decreasing. So, current will be in this direction. Now you notice I EP and this is I CP and this is I CN, I EP and I CP appears to be same but there is little difference here I EP is the derivative at x equal to 0 and I CP is the derivative at x equal to X B or the end of the base region or the base region near the collector. So, 2 at 2 different points we will take the derivative of this consultation profile.

(Refer Slide Time: 07:45)



So, from this we can write if you recall the diffusion equation that d 2n by dx square is equal to and Delta n by d Tau. So, this is a minority carrier diffusion equation M C D E again we discussed in P-N Junction and boundary condition you can note here at this is x prime. So, at x prime equal to 0 this is n E naught exponential q V B by kT and because it is Delta. So, we are subtracting any naught from here.

So, it is exponential q V B by kT minus one and at large x prime this is again 0. So, here of course we are assuming that emitter is wide enough. So, why the emitter means the width of the emitter which is x E here is much larger than the diffusion length L E or which is root of

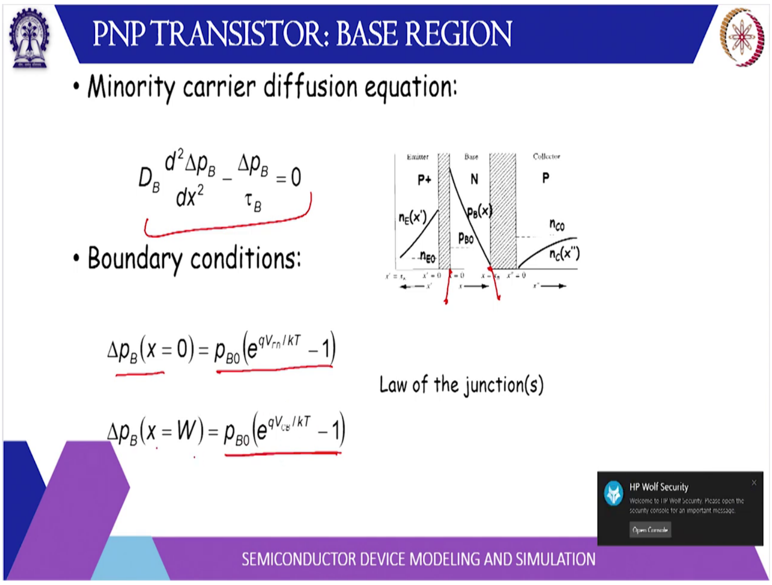
D Tau, so D in the emitter region and Tau in the emitter region or D for the electron in the emitter region and Tau for electron in the emitter region.

So, this is Y at emitter and of course if emitter is narrow then of course we will have to use the same condition as the narrow based diode that we discussed at the end of the P-N Junction diode and this expression we obtained from the law of Junctions. Now we know the boundary condition we know the equation we can easily solve it. You can find out that this double derivative is proportional to the function itself.

So, the solution will be of the form solution value of the firm that Delta n E will be some coefficient times e to the power minus x. Now here is x is x should be this is x prime this is double prime. So, this would be x prime this is x prime. So, e to the power x prime by L plus b times e to the power x prime by L and then of course we will apply 2 boundary conditions where L is root of D Tau D E Tau E.

So, if you take the double derivative it will be 1 by L Square. So, L is root D Tau then by using 2 boundary condition we can find out these 2 coefficients A and B.

(Refer Slide Time: 10:25)

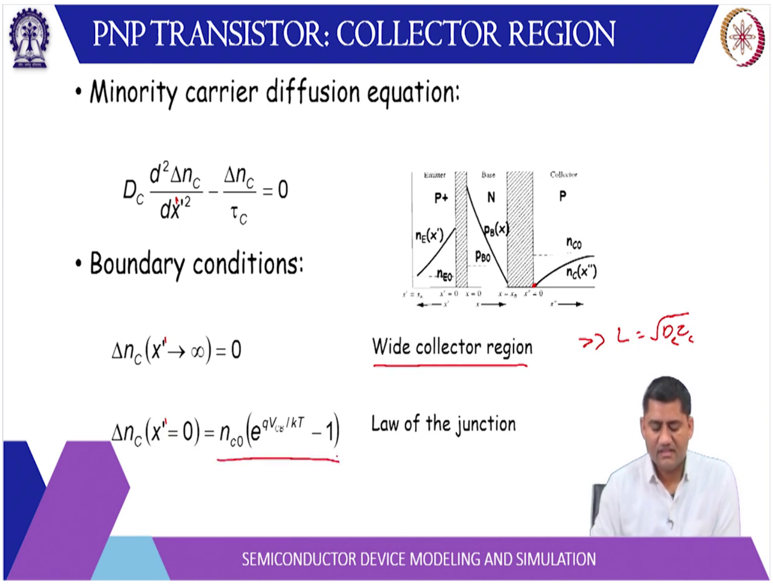


Similar equation we can write for the base. So, here is some holes. So, diffusion coefficient times d 2 P Delta p y Delta x square is equal to Delta P by Tau and here the boundary condition are at x equal to 0 which is at the emitter connected to the emitter and at x equal to x B which is connected to the collector which is near the collector. So, again we at the both the ends we can use law of Junction.

So, P B at x naught is P B naught exponential q V B by kT and then minus the background concentration P B naught and at the base character Junction is P B naught exponential q V c v by kT minus one. So, when it is forward bias then V cv will be positive when this is forward wires then v e b will be positive but in normal operation this is collector basis reverse bias. So, V cb is negative.

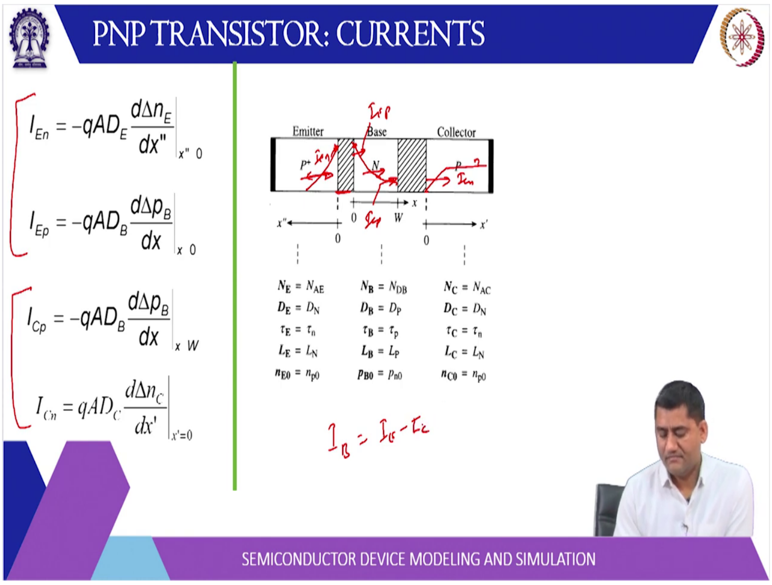
So, that is why it goes to 0 because if V CB is even few kT more than because kT at room temperature is 26 millivolts. So, even if it is one or 2 volt this number will be very small. So, it will be minus P B naught. So, Delta P B is minus P B naught. So, that means P B is almost 0 here.

(Refer Slide Time: 11:51)



Similarly we can write the magnetic array diffusion equation for the collector diffusion coefficient times d 2n by d x square is equal to Delta n by Tau and again here also we are assuming that collector is wide enough that means the length of the collector is much larger than L that is root of D Tau TC times Tau C and at x double prime equal to 0. So, this is in the figure it is double prime. So, accordingly you adjust. So, it is n C naught exponential q V bc by kT minus one.

(Refer Slide Time: 12:33)

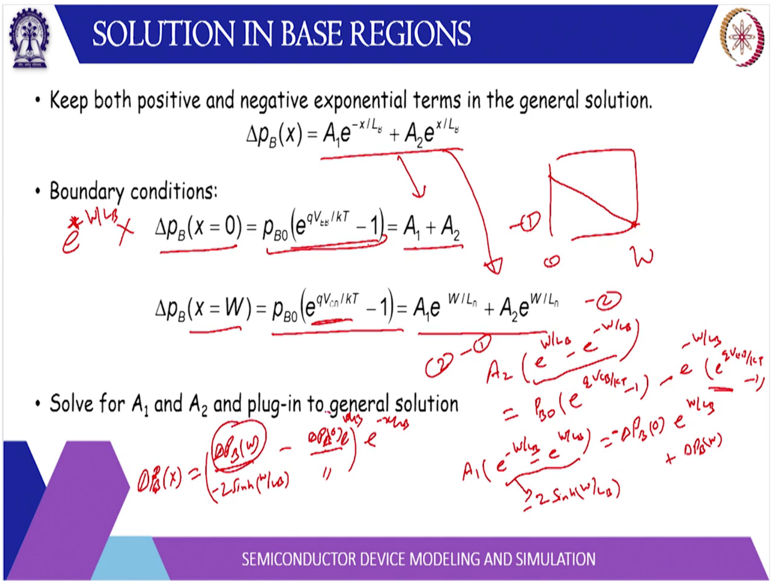


And now since we already know the profile we can write the expression for the currents. So, I En is minus q A D E times d n D of Delta n E by dx. So, that is for emitter region. So, this profile you can see here this will be due to this will give rise to I En and in this direction then I EP this will be I EP. So, the slope is here then for I CP slope is taken here. So, this will give you ICP this will give you I EP this will give you I EN.

So, slope you please note the slope is taken at this terminal only and the assumptions are similar to the ideal P induction diode. We assume that no generation recombination takes place in the depletion region. So, all the carriers that are entering the depletion they are coming out and similarly for I C n so, this is the direction of I C n. So, all these four components are presented here.

So, I E is basically the sum of I E n plus I B I C is the sum of I C n plus I C p and I B is I E minus I C. So, that way we can find out all the three currents.

(Refer Slide Time: 14:10)



Now this we have already discussed that because we can find out all the four components I E n, I E P, I C n, I C P. So, we have defined that gamma is emitter injection efficiency emitter injection efficiency that is I E p by I E. Similarly Alpha is base transport Factor. So, it is I C P by I E P and Alpha is the common base current again. So, that is gamma times Alpha transport and beta is common emitter current CAN so that is alpha y one minus Alpha.

Now we can express the solution for electron concentration for both the emitter region and the collective region for the meter region we can write this n E naught exponential q V B by kT minus one times e to the power minus x by L E. Now why is that. So, A 2 is to be 0 according to boundary condition because as x increases the concentration cannot go on increasing because at x equal to Infinity the concentration has to finite.

So, the A 2 is 0 from the second condition and first condition at x equal to 0 this n E Delta n is equal to A 1. So, A 1 is the carrier consultation is the junction itself. So, that is n E naught exponential q V B by K T minus one times e to the power minus x by L and then if you take the derivative this thing. So, the current is q A D by times dn by dx. So, dn by dx is this thing then dx will receive it will divide by 1 by L is right. So, this one by L is there. So, this is the expression for I E n.

Similarly very similar expression you will have I for I C n dc by L C times n C naught one minus exponential q B C by kT minus one right. The sign differences basically coming because if you notice here in case of emitter this drop is like this and in case of collector this is the slope is basically different and the x are in the basically c s in the same direction for both the x. So, this is x prime this is x double prime.

So, so the slope is different basically. So, therefore this difference is coming because B CB is actually quite base characterized reverse bias. So, this term will be very small. So, it will be most dominator this one term here. Now let us find the I E P and I C P using the carrier slope minority carrier concentration slope in the base region. So, Delta p is again A 1 e to the power minus x by L E plus A 2 e to the power A 2 e to the power x by L E.

So, we know the boundary condition that x equal to 0 it is controlled by the base meter Junction at x equal to w, w is the width of the base region. So, this is 0 this is w and it is basically controlled by the at w it is controlled by a collector region that is P B naught exponential P BC by kT minus one. So, if you substitute this x equal to 0 in this equation you will get this is equal to A 1 plus A 2 and if you substitute x equal to W you will get a one exponential minus W by L E plus a 2 exponential W by L E.

So, from this we can find A 1 and A 2 the process is something like this what you will do you will multiply this equation by e to the power minus W by L B. So, you will have this A 1 e to the power minus W I will be and then you subtract. So, when you subtract what you will have you will have A 2 e to the power that let us say you subtract equation 2 minus equation one so, e to the power W by L B minus e to the power minus W by L E.

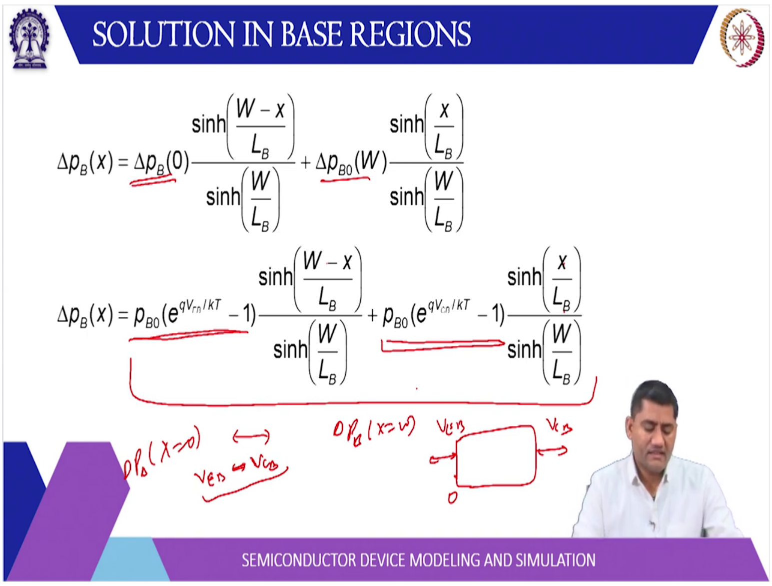
So, this is right side and this is equal to P B naught exponential u V C V by kT minus one minus e to the power minus W by L exponential q V E V by kT minus one. So, from this you can get a 2 and similarly instead of minus U multiplier Plus e to the power plus W by L E. So, then if you subtract so, same thing you will get a one e to the power minus W by L B minus e to the power W by L B is equal to let us call this term as P B 0 let us say call it P B at 0 and this is P B let us call it P B W so, Delta P B 0 e to the power W by L B. So, this is subtracted and this is Delta P B at W.

So, if you see here this term this term is almost same only sign is different. So, accordingly you can adjust the sign. So, what you will have basically A 2 is P B naught. So, maybe I can write it like this ah. So, Delta P B as a function of x can be written as. Now A 1 A 1 is basically Delta P B w Delta P B W divided by. Now this is basically 2 minus 2 sine hyperbolic W by L.

So, minus 2 sine hyperbolic W by L B then minus Delta P B 0 divided by same denominator minus 2 sine hyperbolic W by L B multiplied by e to the power minus x by L B and same thing we can write for other. Now when you simplify it what you will get here basically because Delta P B W and Delta P B 0 they have this exponential ah q V C by kT minus one q V B by kT minus one times e to the power minus x by L E and this has exponential W by L E so, this P B 0 as exponential W by L B.

So, for P B naught this term is there and similarly in case of A 2 P B w the C to the power minus W by L B is there. So, when you add the basically 2. So, you can separate out Delta P B w and Delta P B 0 terms.

(Refer Slide Time: 22:10)

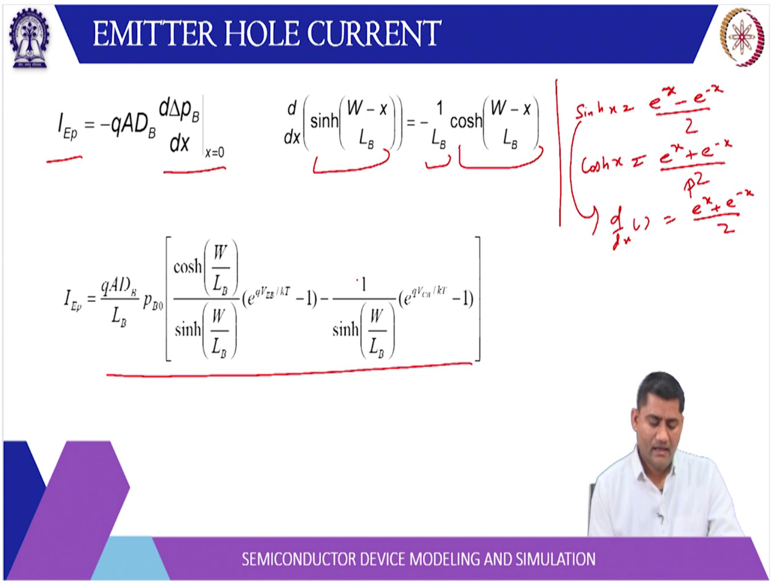


So, overall expression will look like this. So, Delta P B 0 times sine hyperbolic W minus x by L E divided by sine operator W by L B. So, this W minus x is basically coming from e to power W I L B multiplied by e to the power minus x by L B. So, this term is there other we are subtracting. So, that will come from this a 2 term basically. So, here is basically x minus W its W minus x right.

So, that will give Sun a probability W minus x by L B and similarly for Delta P B 0 you will have sine hyperbolic x by L B divided by sine hyperbolic W by L B. So, now P B at 0 is P B naught exponential q V B by kT and P B Delta P B w is P B naught exponential q V C by kT minus one. Now you see this another thing you can notice here x equal to 0 the Delta P B at x equal to 0 and Delta P B at x equal to w they should be same if we exchange V E V with V C V because at x equal to 0 you have applied voltage V at x w the problem with V C B as per base is concerned it does not distinguish.

So, at x equal to 0 this is sine operation hyperbolic. So, you will adjust Delta P B 0 and this will be 0 and at x equal to w this will be sine hyperbolic W by L B. So, this will become 1 and at w this will become 0 sine hyperbolic is 0 is 0. So, you will get this 2 boundary condition you can check. Now take the derivative the derivative of sine hyperbolic is the cos hyperbolic.

(Refer Slide Time: 24:04)

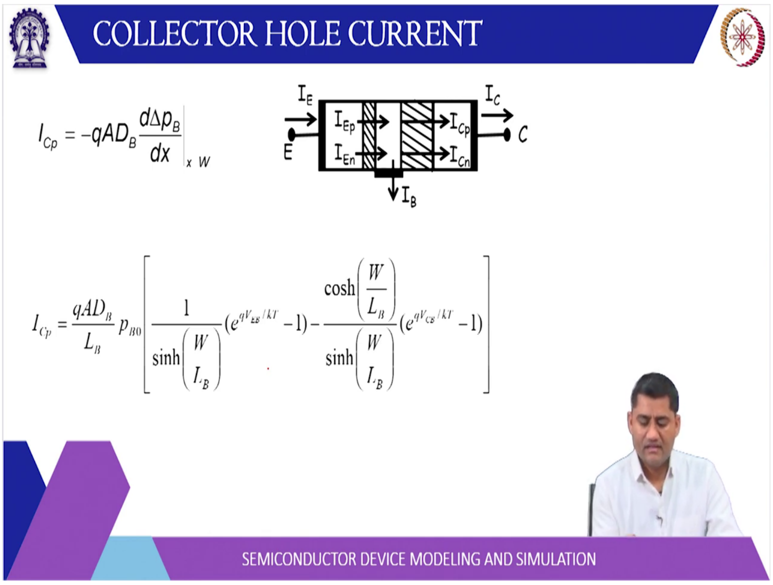


So, just to define sine hyperbolic axis e to the power x minus e to the power minus x divided by 2 cos hyperbolic x is e to the power x plus e to the power minus x divided by 2. So, if you take the derivative of sin upper over x. So, d by dx is e to the power x is M minus the e to the power minus x is minus. So, they become plus divided by 2. So, that is basically cos hyperbolic x. Now this term in the bracket is w minus x.

So, derivative of w minus x is basically minus one and one by L B. So, that is why minus 1 by L B is coming outside. So, if we take the derivative at x equal to 0 we will get term I E P. So, this is the expression for I E P. So, it is directly coming from the expression for Delta p in the base region from this one. Derivative of sine hyperbolic is cos hyperbolic but cos hyperbolic at x equal to 0 so, that that is basically cos hyperbolic 0 is one.

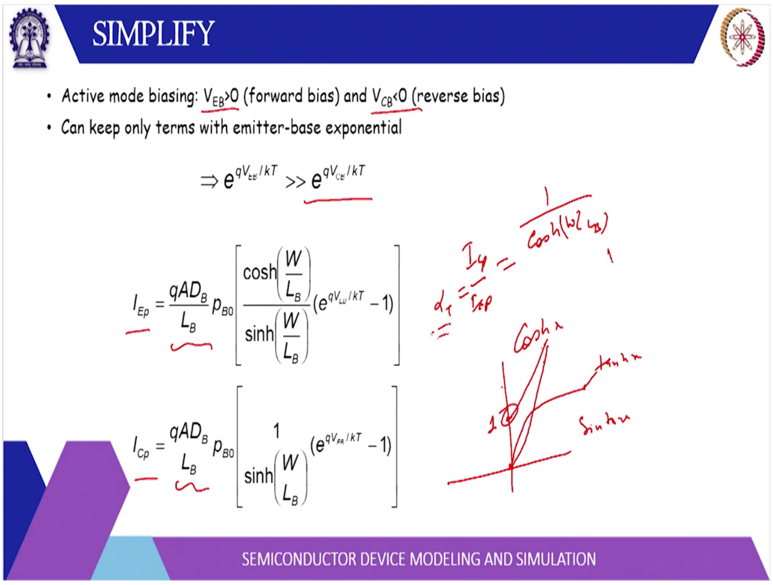
So, that is why one is coming here similarly I CP you can understand from this one the this position will basically x get exchanged. So, I CP is one by sine hyperbolic here and cos hyper will be sine hyperbolic here. So, instead of x this is w this is W L B this is W L B.

(Refer Slide Time: 25:48)



Similarly we can get the expression for ah. So, we can put all these four expression together I E P, I C P, I E n and I C n.

(Refer Slide Time: 25:59)



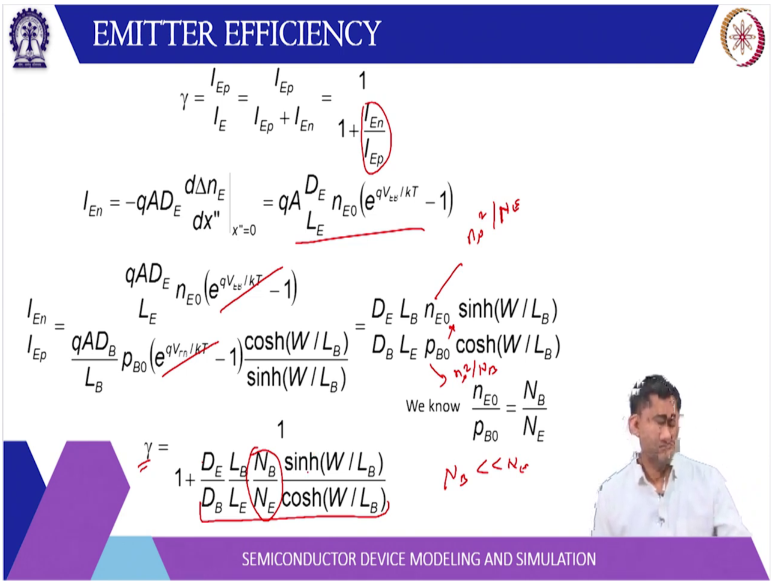
Now we can simplify this expression further in forward active mode base emitter bias voltage is more than 0 is positive more than point seven typically and V C B is reverse bias. So, V C B is less than 0. So, if V C B is less than 0 we can ignore this continuation because exponential q V C V by kT will be very small. So, we can ignore that term then we can write

I E P is this I C P is this. Then you can take the ratio of I C P by I E P and that will give you the alpha transport to the base.

So, that will come out to be around this coefficient is same this is same so, only a COS hyperbolic W by L B one over this thing. So, this I C P by I E P. So, one by cos I probably like W by L B. Now cos hyperbolic W by L B cos hyperbolic function is like this cos hyperbolic x. So, the minimum is at x equal to 0 and that is one and then it goes exponentially like this sine hyperbolic x menu is 0 and then again it has goes like this and tan hyperbolic x is the ratio.

So, here it is 0 and then it goes to one basically because they are both exponential function here. So, this is ten hyperbolic. So, one by cos hyperbolic means it will it will be less than one because cos high probability is always more than one and if we want it to be close to one then we should be somewhere close here. So, that means your W should be is smaller than l b. So, that it is close to one.

(Refer Slide Time: 28:10)

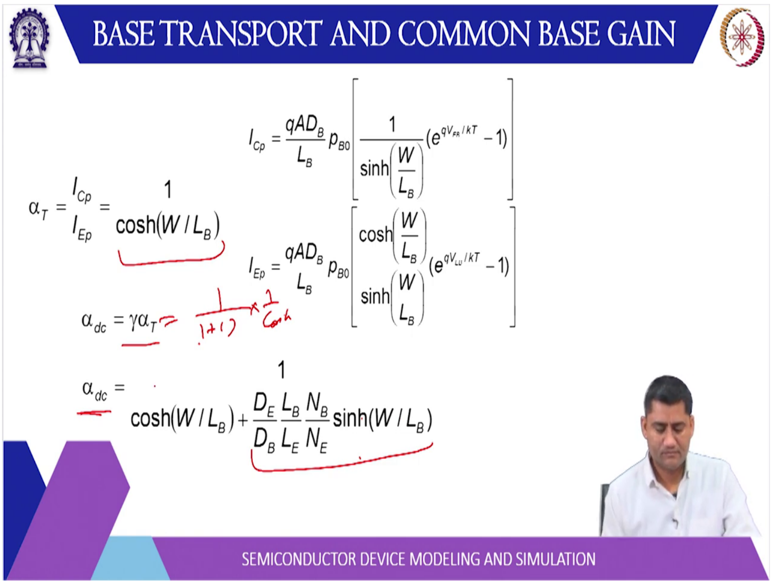


Now we can also define the; calculate the emitter injection efficiency here. So, that is I E P by I. So, when we substitute this is actually one over one plus ratio of I E n by I E P and I E n we know this is the I n and I E P is consisting of this cos hyperbolic terms. So, when we take the ratio exponential V B by kT minus one they will cancel out and we will have this sine operator like cos hyperbolic term here and then ratio of D E times n E divided by D B times P B. Now P B n E is a minority carry concentration.

So, it can written as n i square divided by doping in the emitter. So, that is P yeah doped diameter is. So, that is a hole basically. So, let us call it any and P B naught is the n i Square Times the doping in the base E n. So, this is basically comes from n v by ND. Now if you look at the emitter injection efficiency how can we increase it these are diffusion coefficient. So, we cannot do much about it L B and L E are the diffusion lengths n B by n is basically the doping ratios that we can control.

So, if we want gamma to be close to one this number should be small and that number will be small if we select n b is much less than n e. So, this number will be small and sine hyperbolic W by L by cos are this ten hyperbolic W by L that is between 0 and one. So, to have a good emitter injection efficiency your n B should be less than much less than n E.

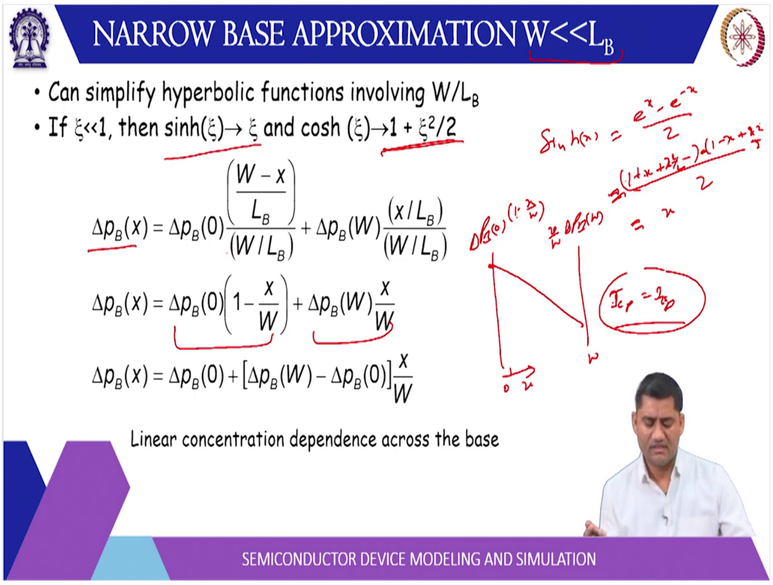
(Refer Slide Time: 30:06)



Now the base transport factor is one over cos hyperbolic W by n. So, that we have discussed that your width should be the base width should be less than the diffusion length will be and then when you multiply this Alpha T and Gamma we get common base current gain and that is basically you can say here gamma is one over some one plus x term. So, this is basically let us say 1 over 1 plus some big term that is gamma times one over cos right hyperbolic.

So, this is basically cos hyperbolic plus the bracket term is containing sign up probably cos hyperbolic cos hyperbolic cancel. So, you will assign hyperbolic here. So, that is common base current gain Alpha.

(Refer Slide Time: 31:02)



And of course beta you can get from alpha. So, it is alpha by one minus Alpha and you can rearrange it you can write one over one by Alpha minus one. So, one by Alpha is this denominator here. So, minus one this denominator minus one right so, one over the denominator of alpha minus one. So, that is beta. So, this if Alpha is close to one then this number will be close to one. So, then your beta will be large enough. So, we try to keep Alpha close to one.

So, typically 0.9 or 0.95, 0.99 like that. Now total current is I E P plus I n and we can add the 2 terms similarly total current I C is I C n plus I C P. So, this looks little complicated but you can easily see the Symmetry here. So, this is cot hyperbolic times exponential V B by kT in I E same thing core hyperbolic times exponential V C B by kT. So, this is exactly same you see if you know the expression for I E you can just by observation you can write the expression for I C.

So, to get the expression for I C from I E you just write replace V E B by V C B and replace V C B by V B E. So, when I sign up all this and these are the terms basically this is I C n and this is I E n. So, they are also similar ok. So, from this we can write the expression for the common emitter again and when the base is small. So, that means base width is much less than the diffusion length in the base we can approximate.

So, sine hyperbolic x for a small x can be approximated as. So, this is e to the power x minus a to the power minus x divided by 2. So, e to the power x is basically one plus x plus x square by 2 then this is one this minus x minus x plus x square by 2 and so on divided by 2. So, this is 2 x by 2. So, that is basically X. So, the approximation sine hyperbolic x is x. Similarly for cos hyperbolic this is plus year.

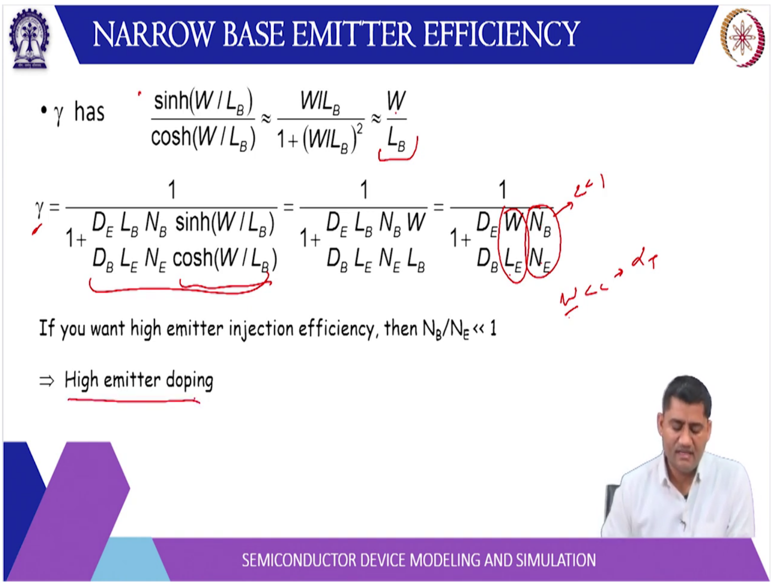
So, x and minus x will cancel the second term x square by 2 and x square by 2 they will add there. So, it will be one plus x square by 2 Zeta Square by 2. So, cos hyper log Theta because in that case this because this is a plus here this minus will become plus F. So, this x will cancel. So, one plus x is square by 2. So, if we substitute this approximation in the for the profile of the base in the holes in the base.

So, you will at P B 0 W that is sine hyperbolic is replaced by the number the term in the bracket that is W minus x by L B and cos hyperbolic is basically one plus this zeta square by 2. So, you have this is sine hyperbolic wise let me see what is expression Delta P B okay sine hyperbolic sine hyperbolic. So, all are sine hyperbolic there is no cos hyperbolic there right for Delta P B expression.

So, w one is x by L B, w by L B this is x by L B w by L B. So, L B L B will cancel. So, this will be x by w this is one minus x by w. So, you see this is a linear profile basically. So, this is basically this is Delta P B 0 this is Delta P B at w and this is a linear relationship basically. So, this is x. So, here the coefficient is one minus x by w. So, at x equal to 0 this is one and this is x by w. So, x equal to W this is one and of course if you in this case if you check the slope, slope will be same at both the places.

So, this will not differentiate between the I C P and I E P this will give I E P equal to I C P. So, your some information is lost in this approximation and this is a good approximation in a case where the recombination in the base is negligible or you can ignore the recombination in the base or the base current.

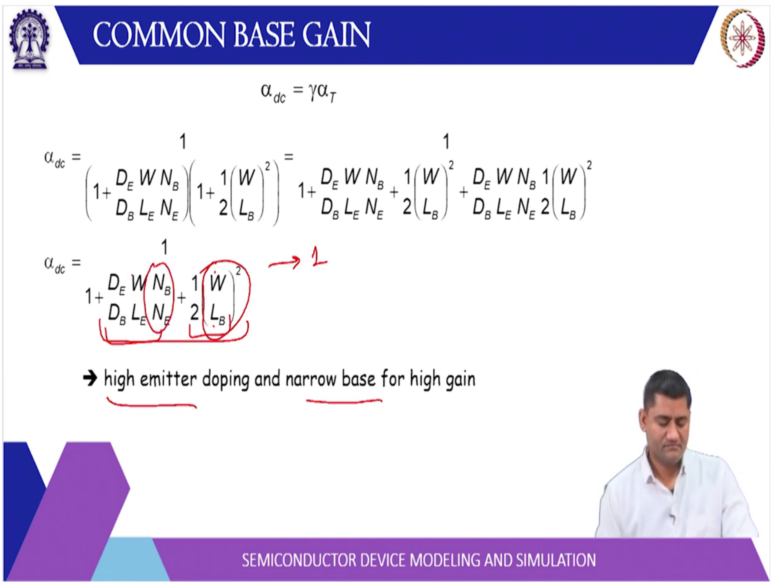
(Refer Slide Time: 36:07)



So, in this case the gamma is sine hyperbolic cos hyperbolic. So, W I will be divided by 1 plus W by L B square. So, that is roughly by W by L B and this is the sine hyperbolic cos hyperbolic. So, gamma will be one over one plus this denominator term and this can be represented replaced by W by L B this time hyperbolic w by L B. So, you will have basically one over one plus d by db w by L B times n B by n E.

So, now we have 2 more parameters here one is n B by n E. So, if n B by n E is small then we will have good emitter injection efficiency and if W by L is small. So, both are actually favourable because W is small also favour Alpha t. So, it is also favouring the gamma the emitter injection efficiency. So, that means that n b is less that means your base should be less doubt compared to emitter. So, it implies High emitter doping.

(Refer Slide Time: 37:19)

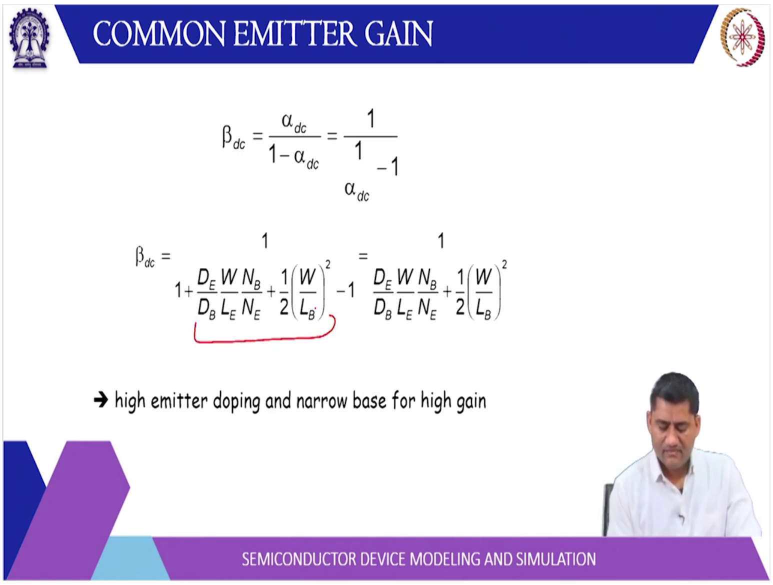


And Alpha T is basically one minus half W by L B square that is coming from one approximation of one by cos hyperbolic W by L B. So, this is basically coming from the approximate one over one plus x for a small x can be approximated as one minus x. So, this is coming from there. So, if you want High base transport then this W by L B should be small this should be small or alternatively if you have large L B that is through large Tau p but this is difficult to control content compared to controlling the width.

So, generally we choose a smaller base width and of course the common base current gain again you can approximate this is cos hyperbolic term. So, this is replaced by one plus one by 2 w by L B square and then you multiply you have Alpha is basically one over one plus this coefficient Plus half W by L B Square. So, high meter doping and narrow base weights apply the allow us to have a good common base gain.

So, W should be narrow because this is one plus a number this number should be small to get Alpha close to one. So, W by L B should be small and N B by N E is so, to be small.

(Refer Slide Time: 38:55)



And similarly beta same terms are appearing here so, same condition high emitter doping and narrow base.

(Refer Slide Time: 39:07)

So, in this lecture we have derived the IV relationship in BJT and we have understood different gains. Common base gain common emitter gain and the base transport factor and the emitter injection efficiency using those expressions, thank you.