Semiconductor Device Modelling and Simulation
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Lecture – 19
Problem Session - 3

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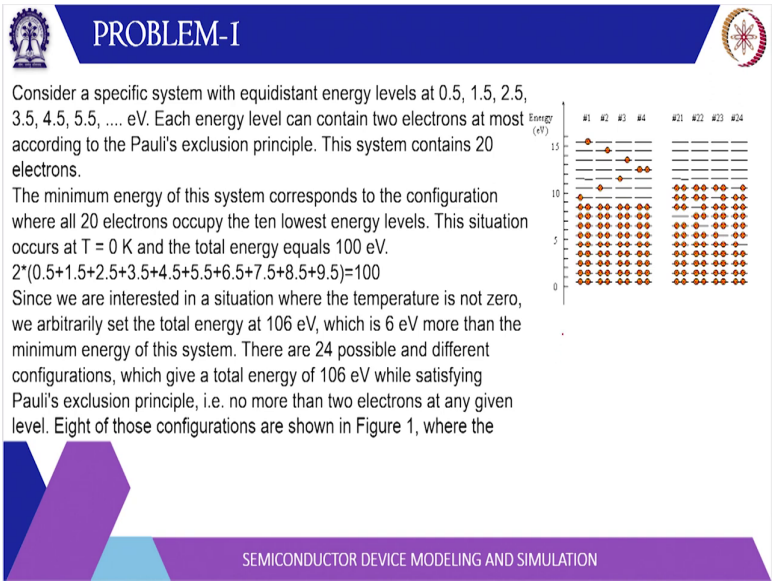

Welcome to the problem session we have discussed up to the PN Junction diode. Now let us consider some problems this problem is related to the Fermi direct distribution. So, now let us consider a situation consider a system where energy levels are 0.5, 1.5, 2.5. So, the difference is one basically and it goes up to 15.5 it can go to 16.5 also and so on. The difference of one now each level can have 2 electron one spin up one spin down one spin up one spin down and so on.

So, these are the states basically these are the x states. Now consider system that this system contains 20 electrons. So, these 20 electrons at zero Kelvin will be accommodated in the first 10 levels. So, for first level energy of 2 electron will be 0.5 + 0.5 that is one for second level 1.5 + 1.5 will be 3 and so on.. So, you can add 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 and then when you add them you get 100. So, this is the energy at zero Kelvin.

So, a system consisting of discrete step States in the with the difference of one electron volt the lowest being 0.5 electron volt and upward there is no limit now lesson of temperature is increased and it is non zero and let us say total energy of the system is 106 electron volt. So,

it is a general situation in this problem they have assumed 106 electron volt you can assume 105 or 1010 minus 110.

So, the idea is that if energy is more then of course this electron will not remain in this lowest energy levels they have to move upwards. So, let us say one possible configuration is that the electron with energy 9.5. So, one electron here goes to energy 15.5 so that is one possible configuration. So, now energy increase by 6 and it is 106 or one electron goes 0.5 up other goes 5.5 up or one electron goes one up other goes 1.5 up other goes 2.5 up is 1 upright one up and this is 2 up so, 224 and 112.

So, total 6 so 2 is straight up and its course 31 straight up and then both can move 3 straight up so, 336 and so on. their total 24 configurations that of course you have to manually find out of course you can write a program also to calculate these distributions.

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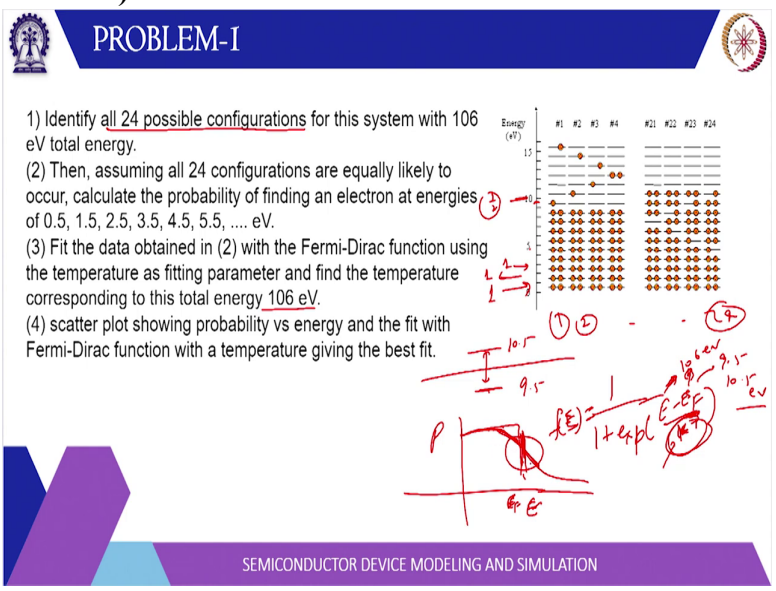

Now what you have to do you have to identify all 24 possible configurations for this system you can pause the video here and calculate these things do on your own write a code whatever and identify those 24 configurations. And once you have done your part you can continue from this time onwards. So, now if you consider all these 24 convocation let us say this is configuration one this configuration 2 this is 24.

So, in all the 24 convocations you see this first level is always occupied. So, the probability of occupation this level is one second level is also also occupied the probability is one third level is also occupied the probability is one. And then of course as you go up the probability

will slightly reduce and somewhere it will be half and then above it will be less than up now can you guess from here where the probability will be half the highest occupied orbital or energy level State at zero Kelvin is 9.5 this highest then after 9.5 next level is 10.5.

So, you can think of this as a bandgap. So, highest occupied is 9.5 and next is 10.5. So, your permeable some will be somewhere in between this these 2 levels and there are probability of finding electron will be half because 9.5 is occupied 10.5 is empty and it in between there are no States. So, then when you plot this probability versus the energy level you will get some permit rate distribution like curve.

So, from the fitting you can find out you know the Fermi level distribution function f is equal to one over one plus exponential e minus E f by kT. So, here e you know is 106 electron volt sorry E f is 106 electron volt and then if you change the energy. So, this is somewhere E f kT is some energy corresponding to the temperature t and the effect of T will be that if T 0 this will be sharp as you T increase this slope will actually increase.

So, the matching of the slope will be done by the T and matching of the sub energy point will be done by the E f. So, you can actually find out what will be the E f in this curve for one zero6 electron volt total energy. So, this is the probability and which is a function of energy. So, as energy changes the probability changes and as we calculated between 9.5 and 10.5 somewhere there Fermi level will lie. So, your E f should be somewhere around 9.5 to ten 0.5 electron volt ok. So, we have 2 fitting parameters ok now let us go at.

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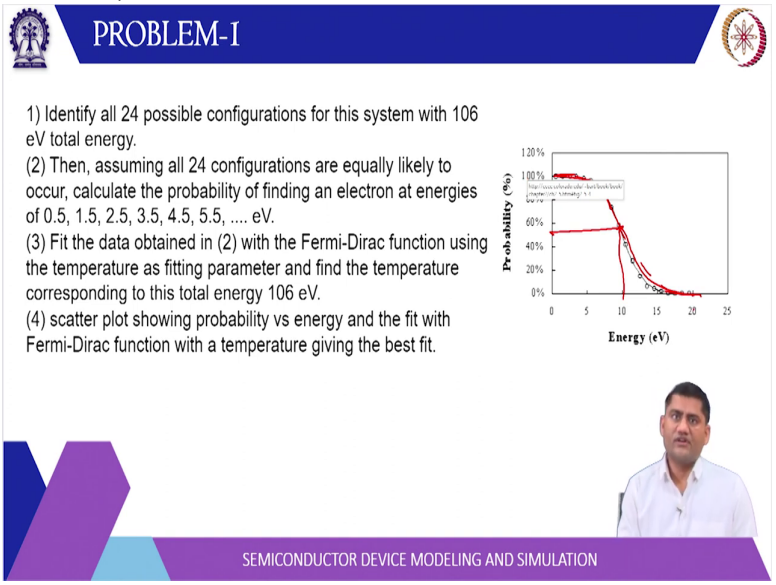

So, this is the typical curve that you can find out and you can compare with it. So, around 10 this probability is half and up to this point it is one then it reduces. Now here you notice one more thing because the gap is one electron volt. So, it will not be that sharp you can try to interpolate and get these points but this will be not this will not be. So, smooth.

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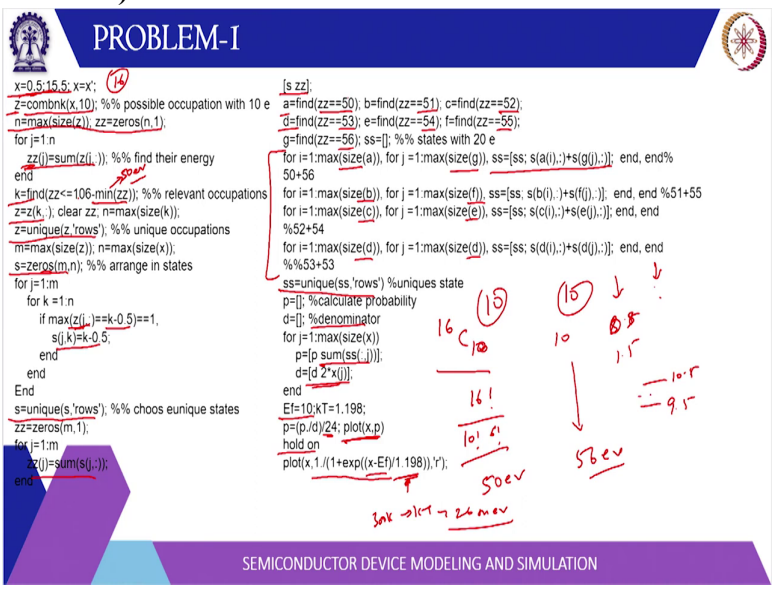

So, what I have done I have written a simple code here let me explain that I have considered this state from 0.5 to 15.5 because 15.5 is the highest occupied State here for first configuration. So, up to 15.5 I have considered. Now so, there are total 0.5 to 15.5. So, there are total 16 states are there. Now we have to arrange 20 electrons in them. So, what I have done basically I have divided into 2 half 10 each because each state can have maximum 2.

So, I consider all the configurations arranging these 10 electrons in the 16 sets and again same set. So, all these possible will be all these possibilities are 16 C 10. So, that is done using this combination n k x, 10. So, this is x all the combination of x with 10 at a time so, with 10 State at a time. So, that will give me the all possible combination of the states and that actually runs becomes quite large because this is basically 16 factorial by 10 factorial times6teen minus 10 that is 6 factorial.

So, that becomes quite High you can calculate now what I do I calculate this number of states. So, that I can find out from the size of this z. Then what I do I calculate the total energy corresponding to all the states. So, for one to n sum them find the total energy. So, that I put in the one vector called z, z now because the sum should be 106. So, what I can do I will find out I state which has minimum energy and here I found out that it was 50 electron volt.

So, that was a minimum energy for all these states in Z the minimum bus 50 electron volt. So, if you subtract so, 106 minus 50. So, if one combination is minimum 50 electron volt others other combination a second set with 10 electrons with 10 electron here can a maximum 56 electron volt. So, I find all the state with energy less than 56. So, that basically drastically reduces my number. So, I identify all those states with the index K and then I set z is equal to z k.

So, only those stress I have considered and that is actually a small number not a big number. Then I find out unique because there may be repetition because I have taken 16 C ten. So, I can select the unique occupants then of course with that because now it is basically having the numbers 0.5, 1.5, 2.5 like this. So, when I will add them let us say this is 0.5 here let us say some status start with 0.5 less other state start with 1.5.

So, the first column is not the same. So, to arrange them according to the column for the state what I have done I have rearranged them. So, that rearrangement is done in the form of in a vector s where if the number is 0.5 it will come to first column if number is 1.5 it will come to Second column and so on and if there is no 1.5 then that will be 0 basically. So, there is zero electron there. So, either one electron or zero electron there.

So, that is done using this thing. So, if maximum of this z j that particular state is not equal to k minus 0.5 or equal to k minus 0.5 then that is assigned and if it is not equal to that means that state is not there. So, that is automatically zero because any layers initialize it to zero. Then of course again I select unique rows ok then what I do this again I sum and find the energy.

So, this is basically just a check command where I list all the states as and their energy in z then I find all the state with 50 energy 51 energy 52 energy 53 54 55 and 56 there are 6 possible combinations. So, I add 50 with 56. So, this is one for a to g for a and g and I add them these states then 51 and 55 that is b and f. So, for b and f again I do the same thing I include those States and then 52 and 54 that is c and e.

So, c and e and then of course this is 53 which is D. So, D will combine with itself. So, I do for D all the D's and then I got all the possible combinations of states having 20 electrons and

then of course again I select the unit once because if there is repetition that will be discarded out and here I get actually 24 rows only and then of course I can calculate the probability and probability is calculating by adding all these energies in particular state.

So, that will give me the total energy and they will be arranged in each column. So, each columns are basically getting added then what I have to do I have to divide by the because energy is also getting added. So, I have to divide by the number of states that is 24 and the denominator is basically the energy. So, 2x j I am just adding this 2x because energy of each state is X and 2 electrons are 2x.

So, this will give me the probability P will be the probability and then I am plotting x versus p versus p. So, x is the energy of the state and P is the probability and then for the fitting I plot another curve hold on is basically is a command. So, that 2 plots are plotted on the same figure. So, here I am plotting one by 1 + exponential x minus E f by k T here kT have taken around 1.198.

So, that is a now if you come see this thing this is actually very large because at room temperature 300 Kelvin the kT is around 26 Milli electron volt and this is fairly large actually. So, that means it signifies a very high temperature and E fi have taken around 10 because that basically gives me the good fitting and as expected because the highest occupied level was 9.5 and the next was ten 0.5. So, it has to in between. So, 10 for 10 gave me a good fit.

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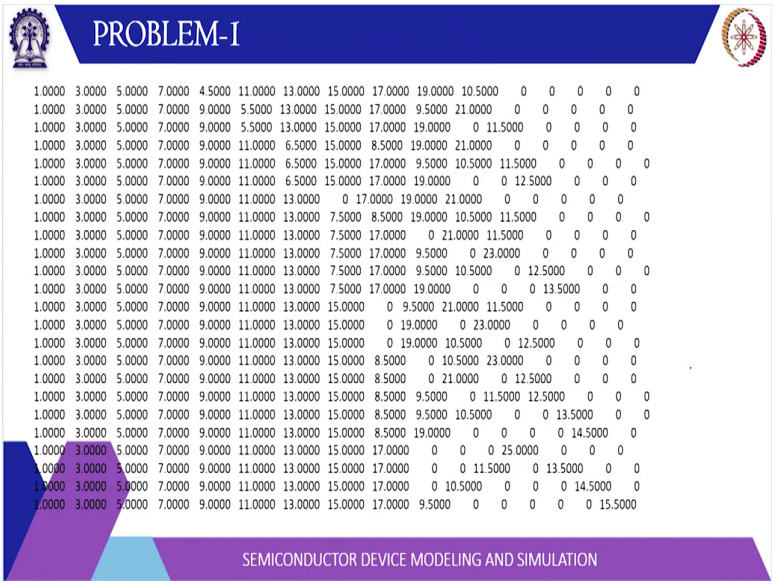

So, these are all the state that I have listed here. So, you see for this is the combination 1 2 3 this is 24. So, for all the combination first is occupied 0.5 + 0.5. So, this energy is one second is 1.5 1.5. So, 3 all are occupied this third is also occupied 2.5 fourth is also occupied fifth is one unoccupied a rest are occupied then 6th one you see there are 2 are unoccupied then for thirteen one 3 are unoccupied then for this one.

So, 1 2 3 4 5 are unoccupied actually half occupied and one is totally unoccupied. So, the probability will be in this case a probability occur assembly these are 24. So, there are 24 is a total and occupation probability is how much? So, 24 into 2 because there are 24 states with 2 electrons. So, 48 and out of them unoccupied are occupied minus unoccupied right. So, 1 2 3 4 5 and 2 so 7. So, 41 by 48 that is the probability at the synergy level that is seven 0.5 and that way we have actually calculate the occupancy.

Then this is 8.5, 9.5 this is 10.5 then you see here there is only one state like this this actually is not arranged properly but this is the last one so this is a basically last step this is the last one. So, here the probability is one by forty eight and here will be maybe I think 2 by 40 or something.

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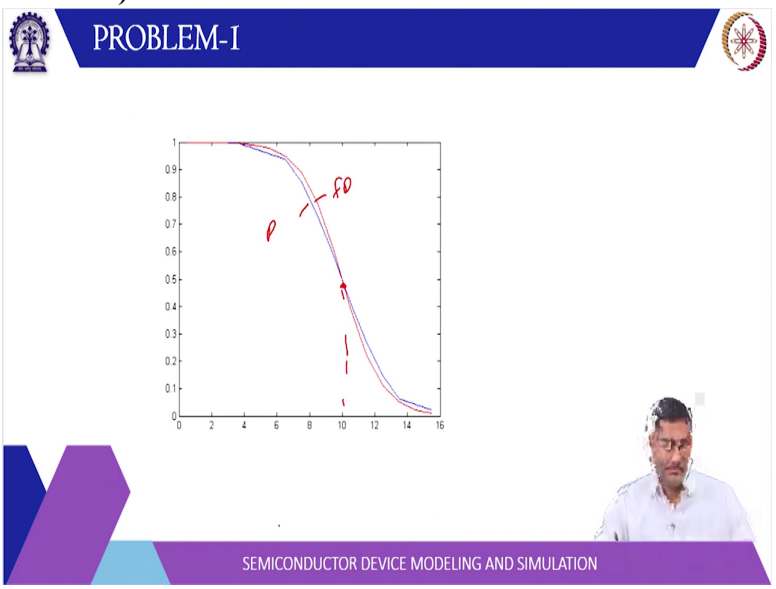

So, this I plotted here. So, this blue curve is basically p and this is the Fermi direct distribution function and they fairly match here and this is around half. So, this is around ten. So, I hope with this example you will have a greater appreciation for the Fermi drag distribution now this is an important concept and we will consider it later also when we go to the numerical procedures where we will discuss the methods to calculate the distribution

function carry consultation and the current. So, this is an important concept. So, that is that is why I took this example.

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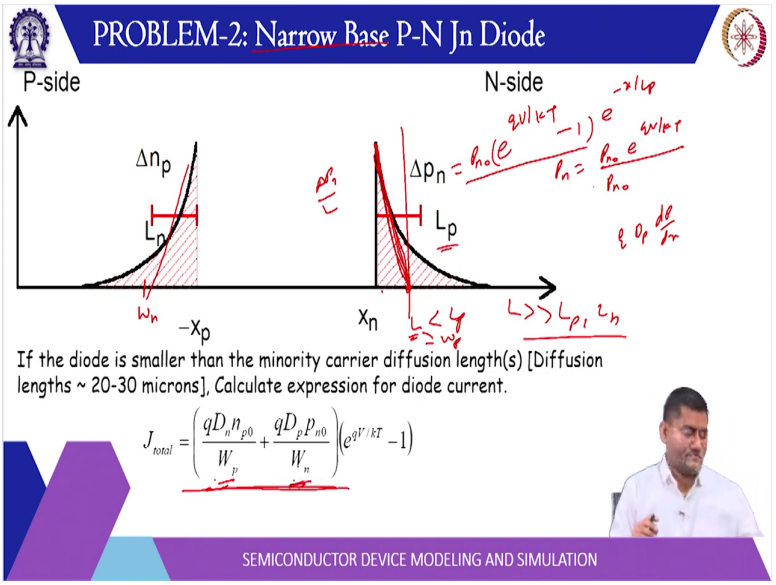

Now second one is related to the PN Junction diode. Now what we did we assumed that the length of the diode on either side was much larger than the diffusion length for whole or electron right. So, we could basically assume that this was exponential decay and you know that Delta p is given by in N region is p n naught exponential u b by kT minus one. So, p n is basically p n naught exponential q v by K T and there is a some background this minority carry concentration p n naught.

So, p a naught is already there and due to the injection of the carriers this many carries have come. So, Delta p n is a difference of the 2. So, that is why we have this expression and this is the gradient basically because this is changing error. So, e to the power minus x y l p or do root of D Tau. So, what we do because this is large here. So, these exponential decays and we calculate the derivative and that gives us the diffusion current.

So, that is q D p times dp by dx right and that gave us this one by L P times this thing now consider a situation it is a narrow base p and Junction diode. So, narrow base means this length is small. So, this length is less than or comparable to L P. Now what will happen now at the boundary condition or the contact with the metal this magnetic carry consultation has to go to zero. So, thus it will be forced to go to zero here.

So, due to this boundary condition the slope will actually be more and now if you calculate the diffusion current. So, the slope will be Delta p n divided by this l or this written as w p and w n. So, this L is actually w p here and here the L is w n on N side. So, this is the expression you will get another thing you can get but in this expression we have made one assumption that we have ignored the recombination here there is a possibility of recombination.

So, that I give as homework if you include the recombination please find out how this expression will change because now this curve will not be straight it will be some what curvature will be there that curvature is due to the recombination and that expression we will again drive in BJT and till then if you want you can try this for narrow base diode and get the exact expression that will be again kind of homework for you.

And but if you ignore the recombination in this region then this expression you will get. So, the only difference with the ideal diode equation and the narrow base P-N Junction diode is in the denominator instead of L p and L n you have the width of 2 regions w p and w n. So, we replace the biotic array diffusion length with diode width on either side.

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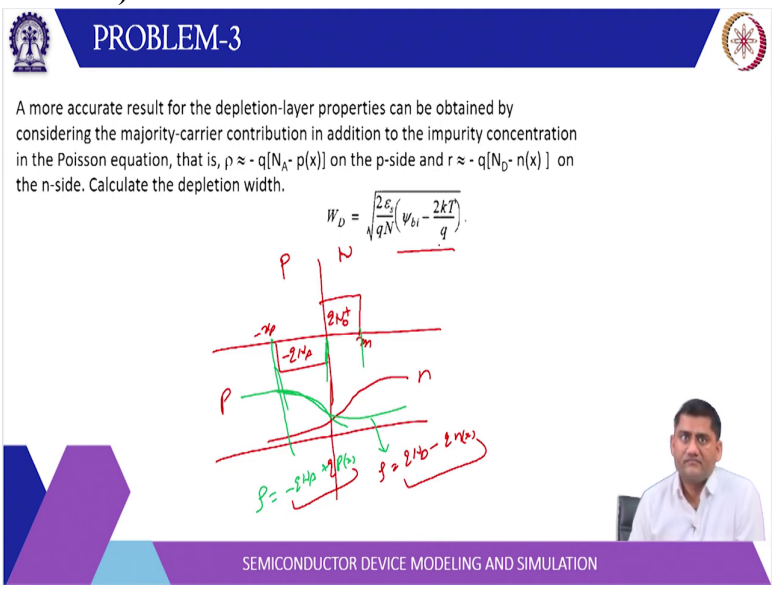

Now let us consider a third problem this is also again related to the PN Junction what we have done basically we assumed approximation depletion approximation where let us say this is PN Junction. So, P side is the dope end acceptors are there. So, they will have a negative charge. So, this is minus q and A on the N side there are donors. So, q N D Plus and we assumed there are no carriers here in the depletion region.

So, this is x p minus x p and this is x n but in reality if you find out there are carriers here. So, on the N side the carries a large air and they go to a small number here on P side on P side the holes are a large number and they go to a small number here. So, in the depletion region these carries are there. So, if you calculate the total charge here if you calculate the total charge here. So, rho will be how much minus q and A + these holes here.

So, that is + p as a function of position x because this p is changing. So, L is up to this region similarly on the N side your row will be q and D minus q n. So, this will be q p as a function of x. So, this will be the exact expression for rho. So, now with this we can solve the Poisson equation on either side and we expect that the depletion weights will be now smaller and that is why factor of kT by q on either side. So, total is 2 kT by q. So, let us take one side and solve the Poisson equation this example is from SMG's textbook.

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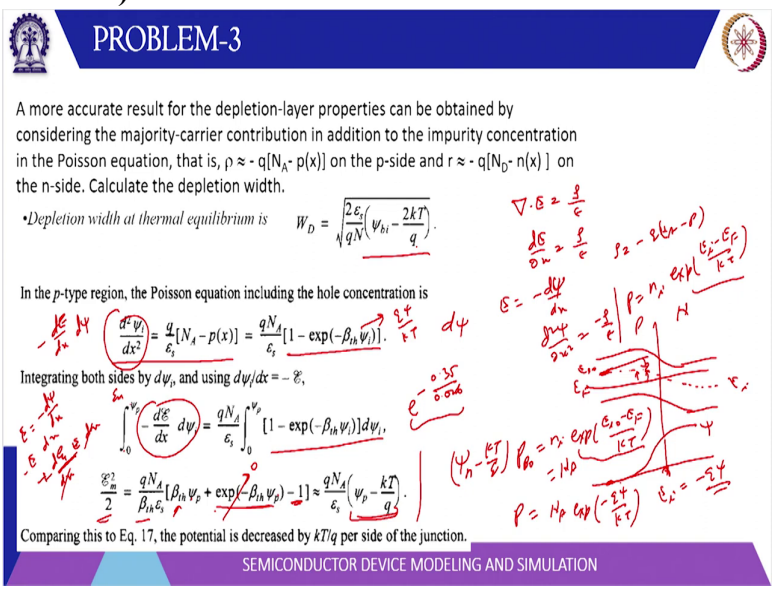

So, this is the Poisson equation if you do not recall the Poisson equation I can tell you that it comes from the ah Maxwell equation where Del dot e is equal to rho by Epsilon. So, for 1D you can write Del E by Del x is equal to rho by Epsilon correct um. So, rho is the charge density and that is q N a minus p as we discussed. So and E is equal to minus dv by dx. So, if you or here psi is use a voltage.

So, size of voltage potential. So, d psi by d x so, we substitute here what you get d 2 psi by dx square is equal to rho by Epsilon with minus sign right yes and rho is for acceptor site is minus q N a minus p. Now please recall the expression for p, p is n i times exponential E i

minus E F by kT. So, this is the expression and this is P side this is N side. So, let us say Fermi level is here this is Fermi level.

So, valence band is here for inside valence conductivity value and then this basically decreases right and in the middle of the bandgap or near the middle of the band cap there is intense Fermi level for inter signature there is E i and this is E f. So, if you see this region here this is E f E i minus a by K T. So, this is reducing here now what we can do here we can write let us say this is E i naught.

So, if you plot the potential profile it will look something like this, this is psi and this is also the built in potential. So, the way this E i is decreasing same base i is increasing. So, you can say that E i is equal to minus q psi. So, if this is let us say E i naught. So, P naught or on P side is basically n i or you can say P p naught P on P side exponential E i naught minus E f by kT and that is equal to the doping N a.

So, P you can write N a times exponential you divide this term by this term. So, what you get here e i minus e i naught. So, e i minus e i naught is this term and that is exactly psi right negative without psi so, minus q psi by kT. So, that is what is done here. So, n is taken out now and you have one minus exponential this is q psi by kT. So, this is the expression in terms of psi and then what we can do we can integrate from x p to the metallurgical Junction and how we will integrate this is d by dx.

So, d 2 psi by dx square is. So, what we do we multiply by this i on both side. So, this is minus d by dx and this is multiplied by d psi and this is also multiplied by d psi and we integrate from 0 to Psi Psi p is the potential at the metallurgical Junction and zero to psi n. Now d psi can be written as because E is minus d psi by dx. So, d psi can be written as E dx minus E dx. So, this is basically minus d by dx times E dx, so this dx cancel.

So, it becomes E times dE. So, that becomes if you in integrate E times dE Square by 2, so, this minus minus + n. So, that is here it is zero here it is because electric field is also zero here will be some Peak electric field that is E m. So, E m Square by 2 on left side on right side if you integrate this I. So, this will be psi minus exponential beta psi by beta right. So, this beta you can take out then you multiply this by Beta and this diameter goes away.

So, this is basically roughly you can see here q A by Epsilon times psi p minus kT by q how q A by PSI this is PSI p this is minus one because the limit is zero. So, PSI P minus zero exponential beta PSI P minus exponential zero that is one year. So, if you see this PSI p is quite large that is around let us say the V bi is 0.7 right. So, half is around 0.35 divided by 0.026 that is milliacron volt and exponential this thing and that will be quite a small compared to one. So, you can ignore it.

So, only one is remaining here, so, that is basically beta this psi minus one by beta. So, one way beta is kT by q. So, you see here this potential is now instead of psi p it is psi p minus kT by q. So, that you can do for both the sides basically same expression will appear for n side. So, you can write psi n minus kT by q. So, total potential will be PSI n + PSI P minus 2 k t by q and that is how you are getting this depletion width approximation.

So, this is a depletion width a thermal equilibrium for a PN Junction using a more realistic expression for the carrier concentration in the depletion region. So, thank you very much and next class we will discuss about BJT.