

Semiconductor Device Modelling and Simulation
Prof. Vivek Dixit
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology – Kharagpur

Lecture – 12
Mobility

(Refer Slide Time: 00:31)

L12 MOBILITY

- a quantity relating the drift velocity of electrons to the applied electric field across a material
 $v_d = \mu E$
- Two factors
 - acceleration due to the electric field
 - deceleration due to collisions and lattice scattering events (phonons, crystal defects, impurities)
 - over the mean free path between scattering events results in the electrons
- Random Thermal motion

Random scattering events

$$\frac{1}{2} m v_{th}^2 = \frac{3}{2} k T$$

$$v_{th} \approx 10^7 \text{ cm / sec}$$

$$\lambda \approx 10^{-5} \text{ cm (100 nm)}$$

$$\tau_c \approx 10^{-12} \text{ sec (THz)}$$

Handwritten notes: $V = v_d \cdot t$, m^{14}

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

Hello, welcome to lecture on mobility. So, in this lecture we will discuss the concept of mobility. Now mobility is basically a quantity, which relates the drift velocity V_d to the applied electric field. So, the drift velocity is directly proportional to the electric field and that constant of proportionality is called mobility. Now, there are 2 factors that are affecting the motion of the electrons, these electrons are in random thermal motion at a given temperature.

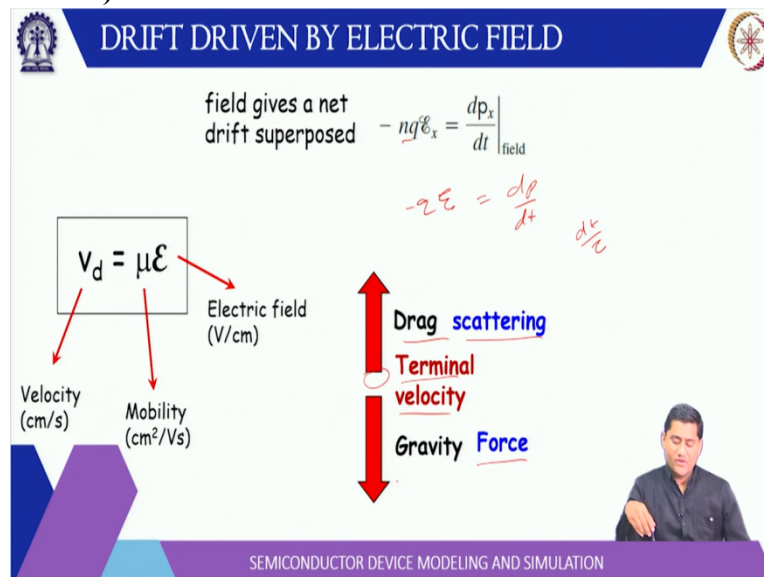
So, if you consider room temperature, let us $T = 300$ kelvin, then the energy of these electrons is $\frac{3}{2} kT$ based on the degrees of freedom and we can compare that with $\frac{1}{2} m v^2$ that tells us the thermal velocity = 10^7 is to power 7 centimeter per second. Here case the Boltzmann constant T is 300 kelvin, m is the mass of free electron and from this thermal velocity and if we estimate that mean free path is around 10^{-5} centimeter so that is a distance traveled by the electron between 2 successive collisions.

And because this well distance is travel with the velocity with thermal, so we can say the mean collision time is around 10^{-12} second, so this is a typical scenario for electron which

is under random thermal motion inside a crystal at room temperature. So, now when we apply the electric field then there are 2 things that are happening this electron is accelerated by the electric field. So, let us have forces minus e times E and this electron gets accelerated.

So, if this electron is accelerated, its velocity will keep on increasing v is equal to $u + at$ or $V_{naught} + at$ whatever. So, this velocity will keep on increasing but there is another opposing force because electron does not have an infinite velocity. So, there is a collision with other lattice atoms surrounding it, that hinders the acceleration of the electron or rather we put it provides a deceleration, so deceleration due to collisions and the lattice scattering event. Now, this lattice scattering can be done by 2 phonons which are basically the lattice vibrations or the defects or from the impurities.

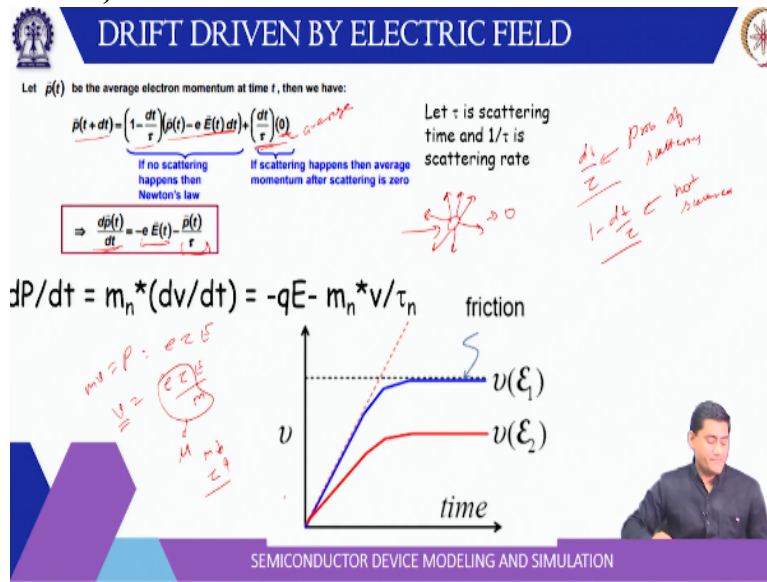
(Refer Slide Time: 03:26)



So, visually if you can see it like this, if an object is falling, so, gravity is applying a force and there is a drag. So, which can we compare to the scattering and the gravity can we compare to the electric field induced force and then overall it basically achieves certain terminal velocity. Now, let us compare let us say $-q$ is a charge on the electron and E is electric field so that is equal to dp by dt the change in the momentum.

The n electrons then the force on this group of electrons will n times qE and the drag basically proportional to the scattering. So, if the mean collision and time is τ , then during that time dt the probability of scattering will be dt by τ .

(Refer Slide Time: 04:32)



So, if we write the momentum equation in terms of these 2 parameters, so dt by τ is the scattering and so that means, dt by τ tells you the probability of scattering that means number of electrons that are scattered. So, $1 - dt$ by τ is the number of electrons that are not scattered. So, those are not scattered they will achieve the acceleration due to electric field. So $-e$ times dt . So, their momentum will change by this. So, P minus force times dt for those that are scattered the scattering is such that it randomizes the momentum.

So, that means electron moving in certain direction when it is scattered there is it has equal probability of getting scattered in all the directions. So, that is if you take the average momentum after scattering that will be 0. So, this is what is written here. So, momentum after scattering now this is the average momentum not the individual momentum. So, average momentum of electron is 0.

So, change in momentum is 1 minus probability of a scattering times the change in momentum due to the electric field plus probability of a scattering times average momentum which is 0. So, if you rearrange these terms, what you get dp by dt that is an actual force is equal to $-e$ times E and $-P$ times τ . So, $-e$ times E is the force on the electron due to the electric field and $-P$ times τ is the decelerating force due to the random collisions.

Suppose a steady state they will be this dp by dt will be 0. So, what we can say we can write $P = e \tau \text{ times } E$ and $P = mv$. So, V can say $e \tau E$ by m . So, this factor will be representing μ . So, that is $e \tau$ by m so that means the drift velocity will be more or the mobility will be more if mass is less and the mean collision time is more that is there is a greater difference between the 2 successive collisions and the mass is less. So, depending on these 2 parameters the saturation velocity or the drift velocity can be different in different materials.

(Refer Slide Time: 07:25)

DRIFT DRIVEN BY ELECTRIC FIELD

Let $\bar{p}(t)$ be the average electron momentum at time t , then we have:

$$\bar{p}(t + dt) = \left(1 - \frac{dt}{\tau}\right) \bar{p}(t) - e E(t) dt + \left(\frac{dt}{\tau}\right) (0)$$

If no scattering happens then Newton's law
If scattering happens then average momentum after scattering is zero

Let τ is scattering time and $1/\tau$ is scattering rate

$$\Rightarrow \frac{d\bar{p}(t)}{dt} = -e E(t) - \frac{\bar{p}(t)}{\tau}$$

$$dP/dt = m_n^* (dv/dt) = -qE - m_n^* v / \tau_n \quad \text{friction}$$

$$\mu_n = q\tau_n / m_n^*$$

$$\mu_p = q\tau_p / m_p^*$$

time

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So, overall we can say the mobility for electron is $q \tau$ by m effective mass of electron and mobility for holes is $q \tau$ times effective mass for holes.

(Refer Slide Time: 07:43)

DRIFT CURRENT

Current density $J_n^{\text{drift}} = \sigma_n E = q n v = q n \mu_n E$ (A/cm²)

$J_p^{\text{drift}} = \sigma_p E = q p v = q p \mu_p E$

Conductivity $\sigma_n = n q \mu_n = n q^2 \tau_n / m_n^*$

$\sigma_p = p q \mu_p = p q^2 \tau_p / m_p^*$

$\rho = 1/\sigma$

$\sigma = \sigma_n + \sigma_p$

$R = \rho L / A \quad \Omega$

Which mass should we use? $= \frac{1}{6} \frac{1}{A}$


SEMICONDUCTOR DEVICE MODELING AND SIMULATION

And once we know the mobility we can write the corresponding drift current, so that J is the current density which is σ times E and that is basically qn times v and v is a drift velocities so it is μ times e . So, qn times μ times E . Similarly, the current density for hole is qp times μ times E we are n and p is the carrier density for electrons and hole respectively. And if you compare $J = \sigma E$ you can find out the conductivity σ which is conductivity is nq times μ_n and use substrate the expression for μ_n which is $q\tau$ by m .


So, you get $nq^2\tau$ by n and σ_p is $pq^2\tau$ by m . So, the conductivity increases if you increase the carrier concentration, if you decrease the effective mass and if you increase this time between 2 successive collisions and of course, overall conductivity will be $\sigma_n + \sigma_p$ and the resistance you know is expression $\sigma\rho$ times L by A or 1 by σ times L by A . This is a general expression that I think you should be familiar with.

Now, the question is here this mass is appear in the denominator, so, which mass we should be use, we have already discussed that DOS effective mass that is basically calculated by comparing the number of states that the corresponding ellipsoid with n number of such valleys as the same number of states in a equivalent spherical constant energy surface that is the DOS effective mass here what we will use that mass we call it conductivity effective mass.

(Refer Slide Time: 10:05)



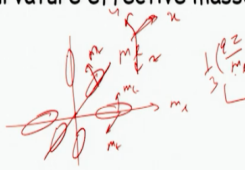
CONDUCTIVITY EFFECTIVE MASS

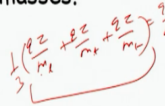



- different from DOS effective mass
 - number of carriers in bands
- conductivity effective mass for charge transport problems.
- For Silicon, 6 equivalent X CB valleys, conductivity effective mass is harmonic mean of the band curvature effective masses.

$$\frac{1}{m_n^*} = \frac{1}{3} \left(\frac{1}{m_l} + \frac{2}{m_t} \right)$$

$$= \frac{1}{3} \left(\frac{1}{m_l} + \frac{1}{m_t} + \frac{1}{m_t} \right)$$







SEMICONDUCTOR DEVICE MODELING AND SIMULATION

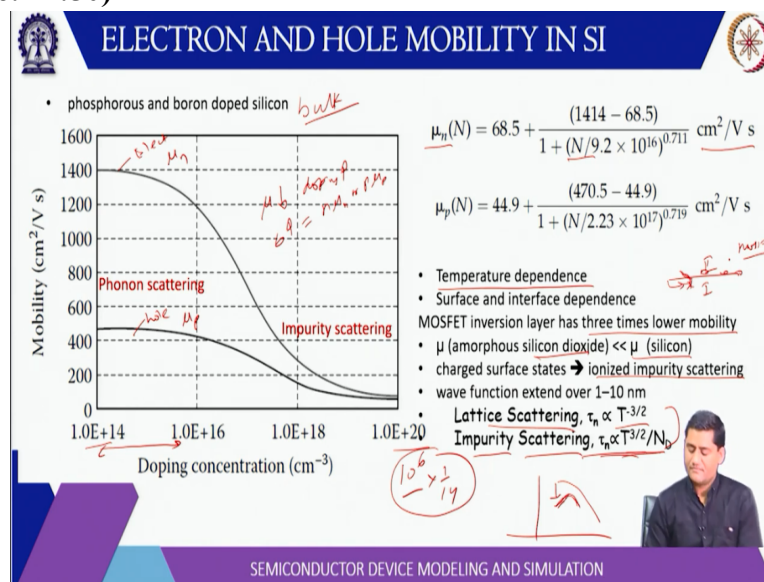
Now, if you notice in case of in silicon there are 6 such valleys. So there is m_l longitude effective mass there is m_t there is a transverse effective mass. So, if it is moving in one direction

its mass is m_l , if it is moving in another direction the effective mass is m_t , if it is moving another direction the effective mass is m_t . So, similarly you can write for all these 6 such valleys, here the mass will be m_l mass will be m_t here mass will be m_t .

So, the corresponding mobility will be $q\tau$ by m_l , $q\tau$ by m_t , $q\tau$ by m_t . So, if you take the average of these 3 because these electrons are equal probability whether they can move in x direction y direction or z direction. So, if you add them that will be $q\tau$ by $m_{\text{effective}}$. So, 1 by $m_{\text{effective}}$ is basically 1 by $m_l + 1$ by $m_t + 1$ by m_t . Now, what we have done we have added 3 terms here so we have to divide by 3 also. So, this is the average of 1 by m in all the direction.

So, because we are m_t are equal because this is ellipsoid for general case we can write 1 by 3 1 by $m_l + 1$ by $m_2 + 1$ by m_3 . So, that will be the general explanation for conductivity effective mass.

(Refer Slide Time: 11:50)



Now, here is a picture showing the actual values of electron and hole mobility in silicon. So, this curve is for the electron mobility that is μ_n this is for the hole mobility that is μ_p you notice here at the doping is smaller than 10^{16} order is 10^{14} . The conductivity is high and as you increase the doping, the mobility decreases. So, mobility is decreasing as you increase that doping borrow the conductivity. Conductivity will increase because it is n times μ_p or p times μ_n .

So, although the mobility is decreasing, but the number of carriers increases significantly see, it is from 10^{14} to 10^{20} . So, there is an increase of 10^6 order here and your mobility is decreasing from less than 1400 to 100. So, this is 10^{14} and for the hole again 500 so 10^5 power 10. So, the conductivity increases significantly although mobility decreases slightly.

So, this is basically these values are actually measured and tabulated and based on the measured values, some fitting is done to relate this mobility with the doping concentration in the unit of mobility centimeters square per volt second, this mobility also depends on temperature as you increase the temperature there will be more scattering. So, due to the lattice scattering, the mobility will decrease in case of impurity scattering interestingly, the mobility slightly increases that is because at high temperature their thermal velocity is high.

And when the thermal velocity is high, they spend less time in the vicinity of these impurities that is they get a smaller time to get deflected. So, there is scattering is reduced so that is when due to the impurity. The mobility actually due to impurity scattering, mobility tends to increase with temperature, but overall effect is basically is that as temperature increases, mobility decreases and if you increase a doping concentration, then also mobility will decrease.

But if you see the curve versus temperature, so, you may see something like this. And for high doping, you may see some curve like this. So, high doping of course, the mobility is less but in low temperature region. It is slightly in cases for certain temperature then of course at high temperature it again follows a letter scattering. So, there is temperature dependence for this mobility, mobility also depends on other scattering mechanism and let us say there are 2 materials side by side so material 1 and material 2.

So, even though electron is moving in this region, you can compare this thing with a MOSFET channel. So, this wave function has finite probability of entering into the nearby layer, let us say silicon dioxide also. So, then what happens although classically electron is moving here, but the wave function can penetrate to the upper other layer where the mobility is less. So, what happens

the overall mobility is basically reduced because it is the average of these 2 mobility and in the neighboring material silicon dioxide mobility is small. So, overall mobility is basically small.

So, in MOSFET universal layer it is 3 times lower mobility is there because in the neighbouring layer silicon dioxide which is MFRS the mobility is much smaller than in the silicon then there may be some charges tapes also here and they may induce the ionized impurity scattering so that further reduces the mobility. So, these are the values for the bulk silicon, you may not achieve these mobilities in case of inversion layer or any other 2d or 1d structure. So, this mobility is basically varies widely with respect to that temperature doping and other considerations such as surface or the interface.

(Refer Slide Time: 16:47)

FACTORS AFFECTING MOBILITY

Scattering Mechanisms

- Lattice Scattering, $\tau_n \propto T^{-3/2}$
- Impurity Scattering, $\tau_n \propto T^{3/2}/N_D$

Temperature dependence

Surface and interface dependence

MOSFET inversion layer has three times lower mobility

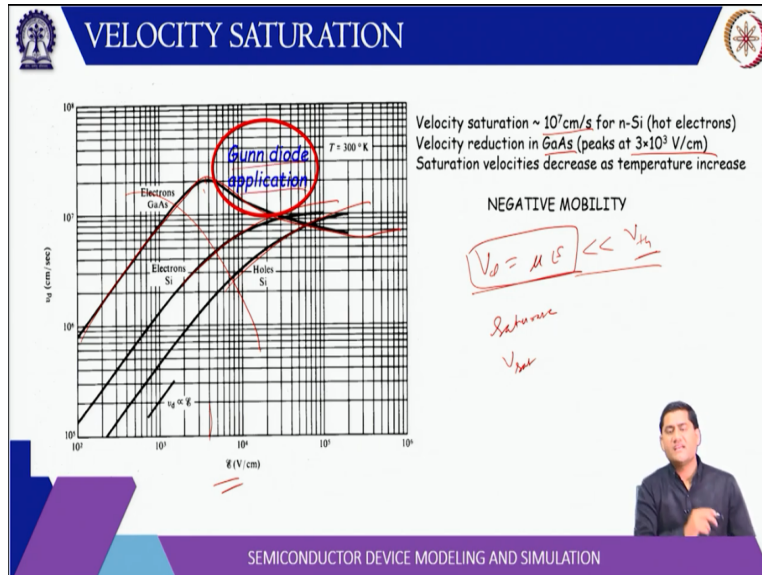
- μ (amorphous silicon dioxide) $\ll \mu$ (silicon)
- charged surface states \rightarrow ionized impurity scattering
- wave function extend over 1–10 nm

SEMICONDUCTOR DEVICE MODELING AND SIMULATION

The slide features a blue header with the title 'FACTORS AFFECTING MOBILITY' and two institutional logos. The main content is a list of factors affecting mobility, including scattering mechanisms (Lattice and Impurity), temperature dependence, and surface/interface dependence. A specific note mentions that the MOSFET inversion layer has three times lower mobility. A list of factors includes the low mobility of amorphous silicon dioxide compared to silicon, charged surface states leading to ionized impurity scattering, and the wave function extending over 1–10 nm. A red arrow points from the text 'wave function extend over 1–10 nm' to a diagram of a wave function. A video inset in the bottom right corner shows a man in a dark shirt speaking.

So, these are factors that affect the mobility a scattering mechanism broadly you know we have considered to the other scattering mechanisms also. So, lattice scattering and impurity scattering temperature dependence we have discussed surface interface dependence we have discussed and the wave function extend up to 1 to 10 nanometer in that neighboring layer.

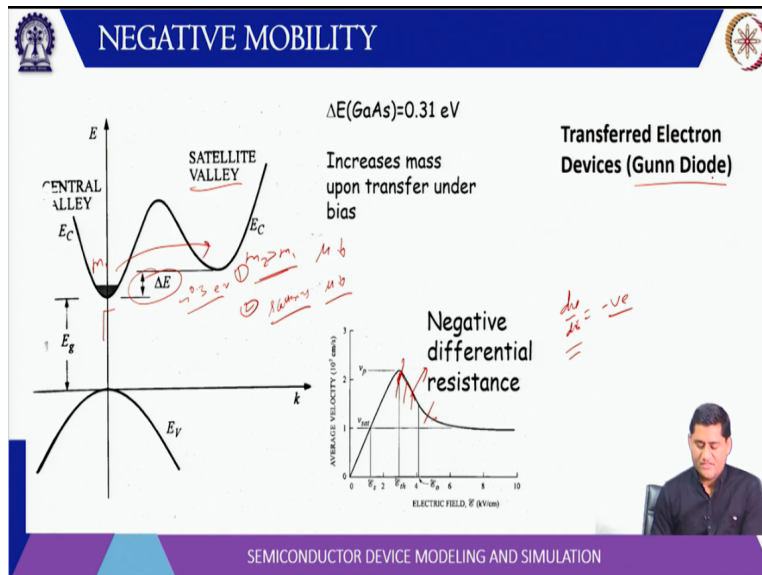
(Refer Slide Time: 17:14)



Then this $v_d = \mu E$ is always valid it is only valid for lower fields. So, when the electric field is small and you can also notice this velocity is much smaller than the thermal velocity as you reach higher fields this velocity does not increase linearly rather it saturates. So, for higher field we have concept of saturation velocity that is around 10^7 cm/sec for n-type silicon which is comparable to the thermal velocity.

And so, first silicon you see this curve here it is saturating basically, but in case of gallium arsenide, you will see some peculiar curve before saturating it goes through a peak and then it saturates basically and the peak is around 3 kilo volt per centimeter. Now, that region can be understood if we look at the band structure of gallium arsenide and this gallium arsenide has application in the Gunn diode it is also called transfer electron effect.

(Refer Slide Time: 18:40)



So, if you see recall the band structure of gallium arsenide there is a gamma valley here there is a satellite valley here and the gap is quite small around 0.3 electron volt. So, at high field these electrons tend to go this valley, go to the satellite valley, where mass is less m_2 here mass is m_1 so m_2 is more than m_1 . So, as they move here, the mass is more. So, the mobility actually decreases and not only the mass the electrons in the satellite valley, they encounter a greater amount of scattering.

So, mass is more scattering is also more so that is also causes a mobility to reduce. So, because of this high field, they move to the satellite valley, they are mobility actually reduces. So, once you cross this peak field, they transfer to the satellite valley where the mobility is reduced. So, this reason is called negative differential resistance. Now, why negative differential assistance? Because if you see the; velocity dv by $d\mathcal{E}$ is negative in this region. So, that means as we increase electric field the velocity is reducing that is the resistance is basically more and this has application in Gunn diode.

(Refer Slide Time: 20:10)



CONCLUSION



- Electron (hole) mobility concept discussed



SEMICONDUCTOR DEVICE MODELING AND SIMULATION

So, broadly we have discussed about electron and hole mobility concepts in this lecture. Thank you very much.