Digital Control in Switched Mode Power Converters and FPGA-based Prototyping
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 05
Frequency and Time Domain Digital Control Design Approaches
Lecture - 47
Time-Optimal Control of a Buck Converter and Identifying Performance Limits

Welcome; in this lecture, we are going to talk about the Time-Optimal Control of a Buck Converter and we need to we wanted to identify what are the performance limit.

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So, in this lecture, we will first talk about what is the time optimal recovery in a buck converter under load step transient, under reference step transient, and what are the performance limit and the design conflict.

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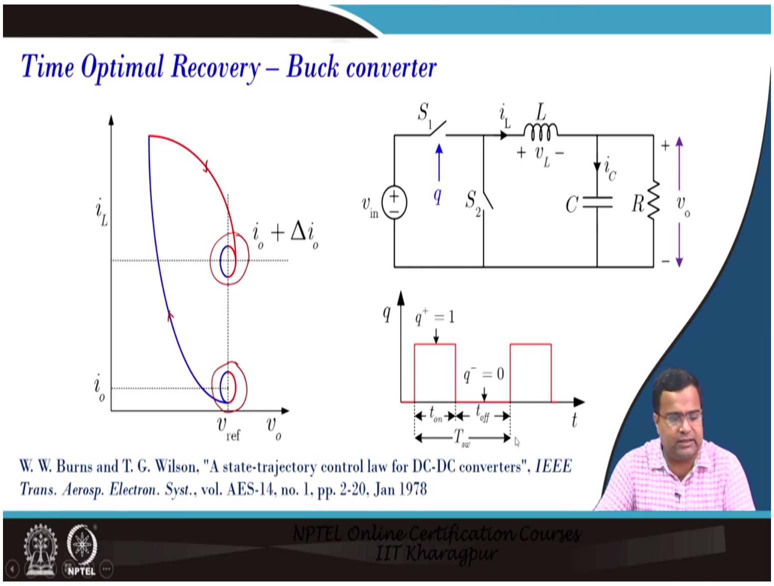

So, here if we take a buck converter, you know we know about the turn on, turn off time.

And their equivalent current I mean under on time when the switch is on high side switch; then how does the inductor current look like, when the switch is off how does the inductor current on the output voltage look like we all know? As we have discussed in our earlier NPTEL course in lecture number 48 if we want to move from one operating condition to the other load the current condition in one switching action.

This blue color is the turn-on trajectory and the red one is the turn-off trajectory; then how to achieve that we have discussed and this optimal trajectory control was discussed in this paper 1978.

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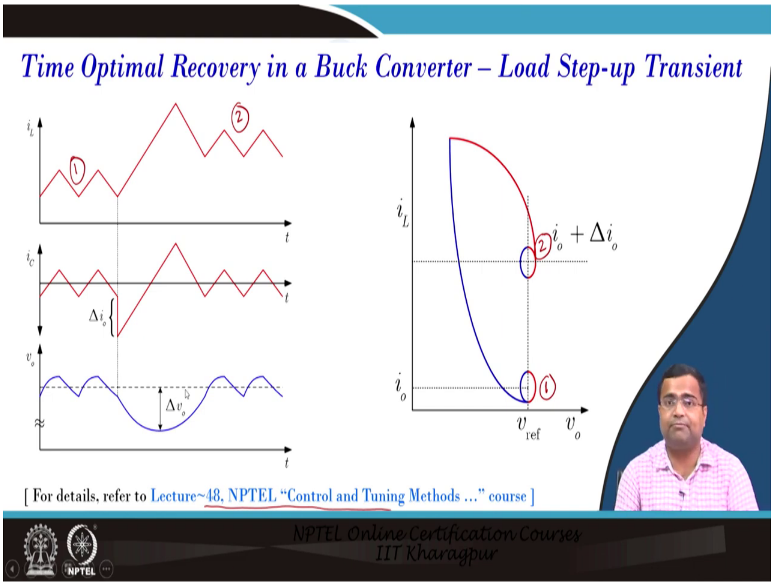

So, then we have also discussed in lecture number 48 in our earlier NPTEL course that if we consider this one load current condition to the other load current. So, this is like operating condition 1, this is operating condition 2 and these are their corresponding time domain waveform.

And to achieve the one switching action, then we know what how does the switched capacitor current look like; then what the output waveform looks like and that is the fastest transient response.

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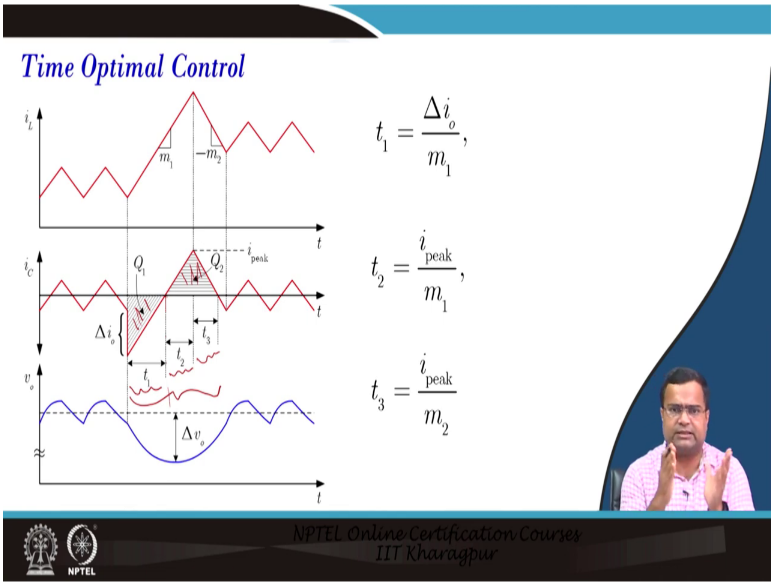

Now, in the time optimal control under this, so it is the capacitor charge balance; that means whenever there is a load step transient, the sudden inductor current cannot change immediately.

So, the whole load step will appear across the capacitor current and because the capacitor current is negative for a load step-up transient, the output voltage will dip as long as the capacitor current is negative. Once the capacitor current starts crossing the zero current, then the voltage will start rising. So, to get the output voltage to its previous operating point, we need to balance this negative charge with the positive charge and that is the whole concept of time optimal control.

And if we want to find out what is this time t 1, what is this time t 2, and what is this time t 3 so that we can get the total recovery time from here to here, that is the sorry the total recovery time here, here is nothing, but it is a sum of t 1 plus t 2; that means this is my total recovery time, the point where the output voltage again will reach steady state is the earlier operating point.

So, you can find t 1, t 2, and t 3 from this expression, and these things are detailed discussed in detail in lecture 48 in our earlier course.

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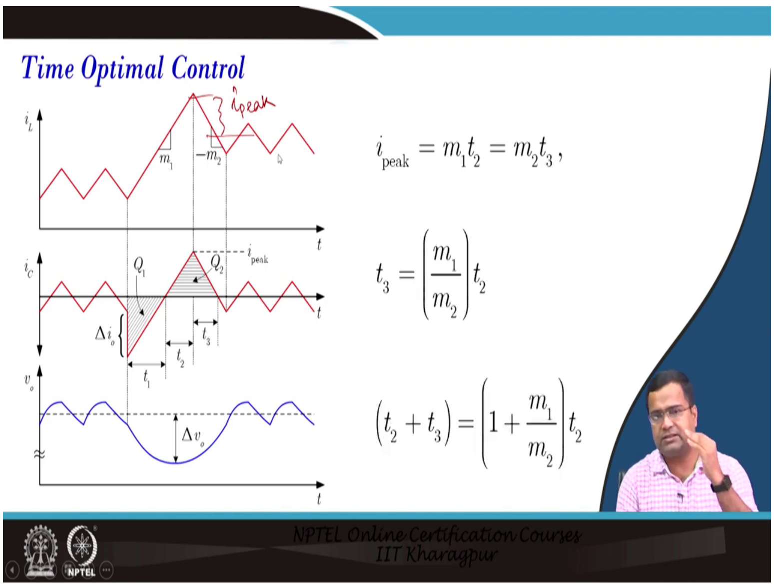

So, then we can find out the peak current; peak current means if we consider the peak current with respect to the capacitor, so this is nothing, but with respect to the next operating point.

So, this is my i peak, this difference. So, peak means that the additional current goes over the new operating steady state point.

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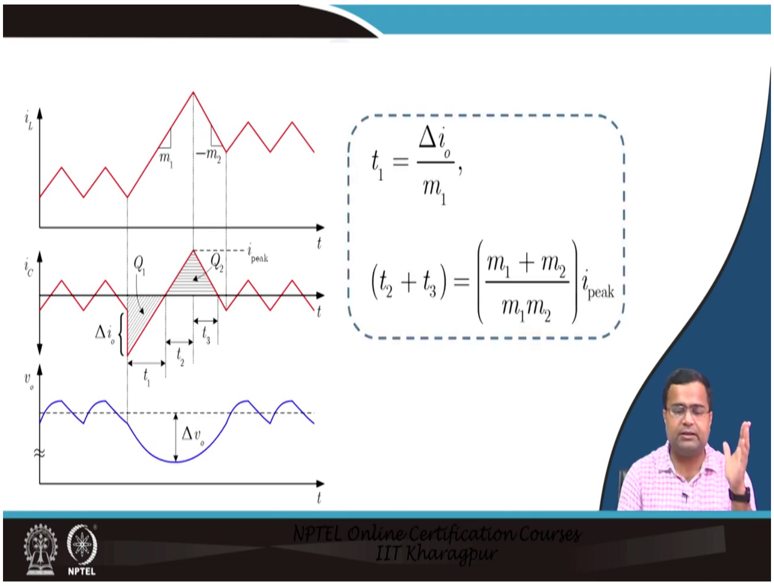

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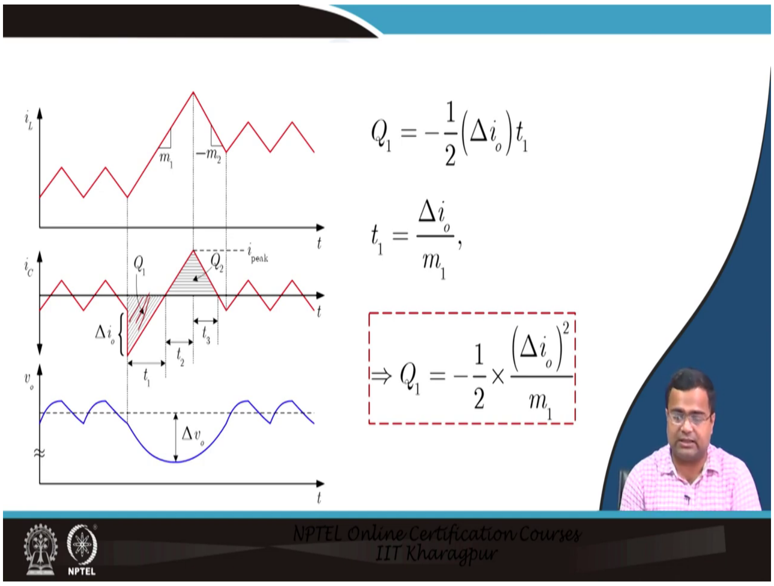

Then we discussed how to find out this peak current and these are the step-by-step derivation; how to find out t 1, t 2, t 3, then finally what is the total time? That means, under t 1, t 2, t 3 these are the step then what is Q 1? So, what area under this curve is how to find out? Then how to find out the Q 1 expression in terms of load step size and the slope?

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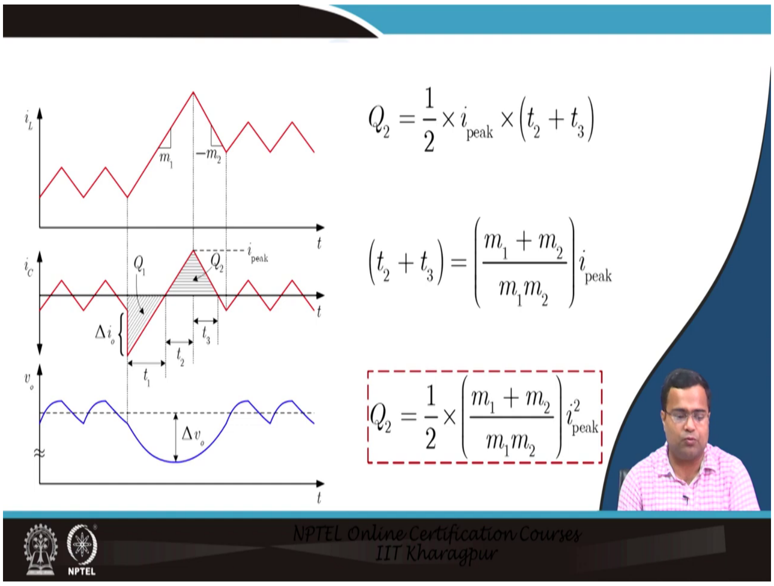

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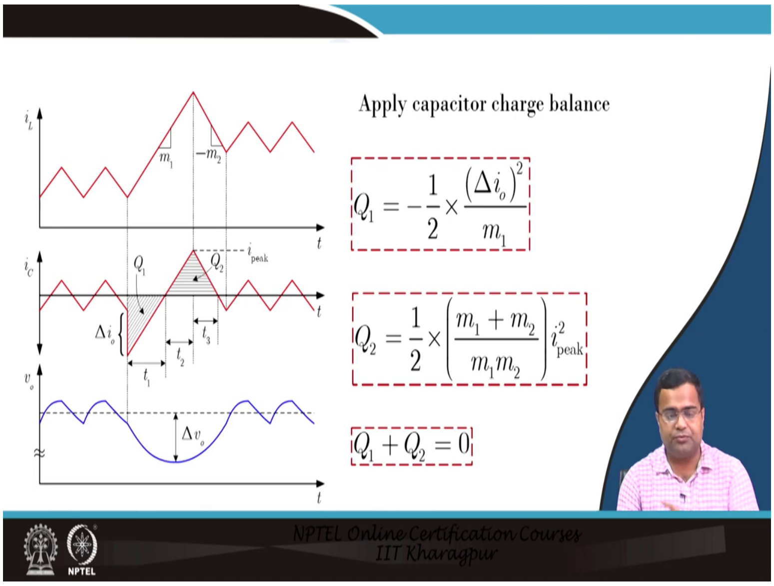

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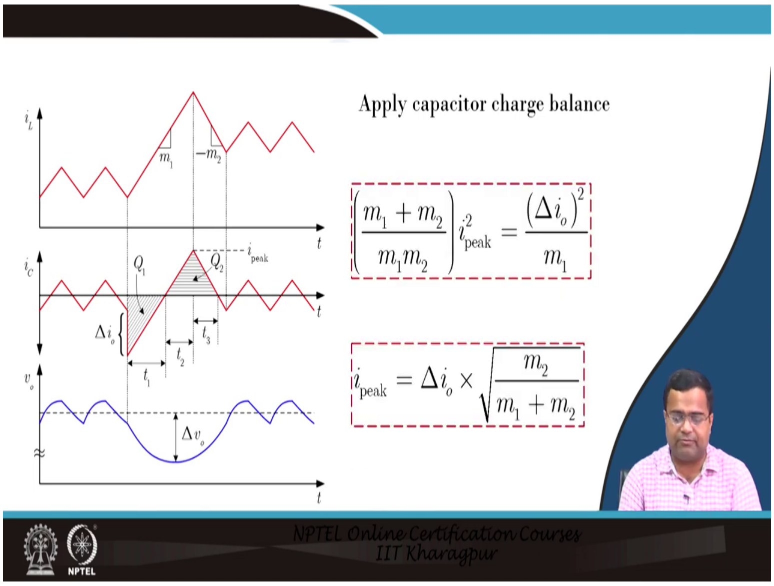

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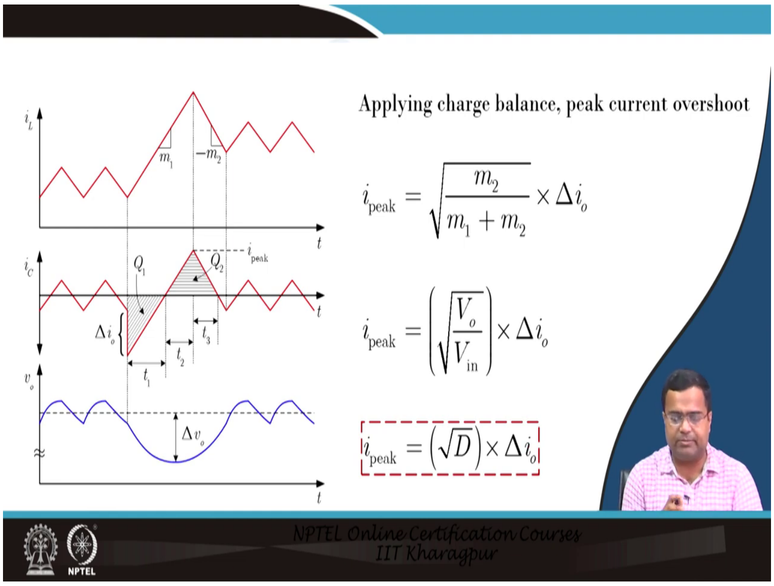

Then how to find out Q 2? So, Q 2 in terms of this peak current m 1, m 2 slopes. Then if you do charge balance, some of the charges will be 0; then from there we can find out the peak current expression and it turns out to be that peak current will be simply duty ratio by the load step size. That means, if we assume you know from this charge balance expression, we are not considering that variation in the output voltage in the slope.

If we consider then the expression will be some it will be more complex. So, under the ideal assumption; that means the slopes are the same, it remains unaffected, then you will get the peak current in terms of the square root of duty ratio into load step size.

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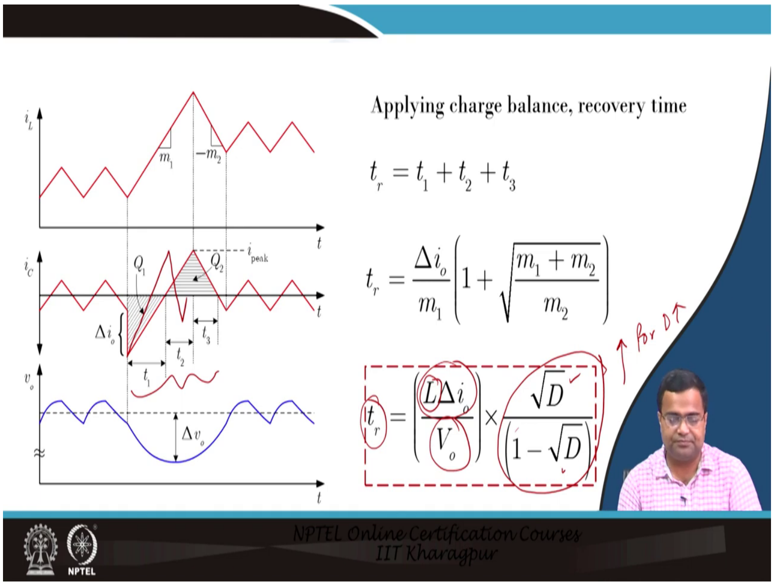

Then if we want to find out the total recovery time that we have discussed; that will be the sum of t 1, t 2, t 3 and it turns out to be the recovery time depending on L into delta i 0 divided by the like steady state output voltage into root D by 1 minus root D.

And we will discuss whether it will increase or decrease. So, from here it can it is clear; that means for the given duty ratio, given output voltage, and given load step size, the recovery time can be decreased by reducing the inductor value. That means, if we reduce the inductor value, it will recover faster, and as a result, you will get a faster recovery, but a smaller inductor will lead to a larger current ripple and that may increase the conduction loss, so that is the penalty, ok.

Then for the given inductor if the inductor is fixed, load step is load step size is fixed output, and voltage is fixed; if the input voltage decreases, that means the duty ratio increases, then it can be shown that this term will go up for increasing duty ratio because this will go up and this will go up. So, 1 minus D will be smaller and it is in the denominator. So, as a result, the overall term will increase.

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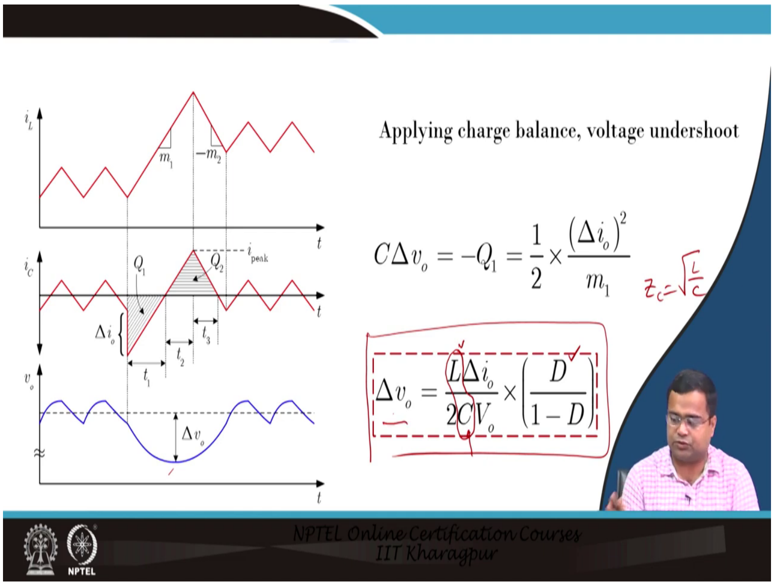

So, that means the worst case you know recovery time will be at the lowest input voltage or the highest duty ratio, ok. Similarly, it depends on the load step size. Now, if we apply the charge balance, we can find the undershoot expression; that means this undershoot and it turns out to be the undershoot is a strong function of the characteristic impedance, which means we know the characteristic impedance is the square root of L by C.

So, if we can make the characteristic impedance 0; because in traditionally majority of the power supply we generally take small inductor large gap and then the this is small, by that way we can reduce the output you know undershoot. But I will show you in this lecture, it can be counterproductive for the reference voltage transient. So, for load transient, either we can reduce the inductor to reduce the undershoot, which will have an effect in terms of the current ripple will increase.

We can increase the capacitor, but that will penalize the power density because the size of know capacitor will increase and this will also you know decrease the reliability; because the larger gap is more unreliable, I mean reliability will decrease. And also the undershoot will be affected at a high duty ratio and will get a higher undershoot, larger undershoot; that means under the smallest input voltage condition or lowest input voltage condition, you will get the maximum to undershoot.

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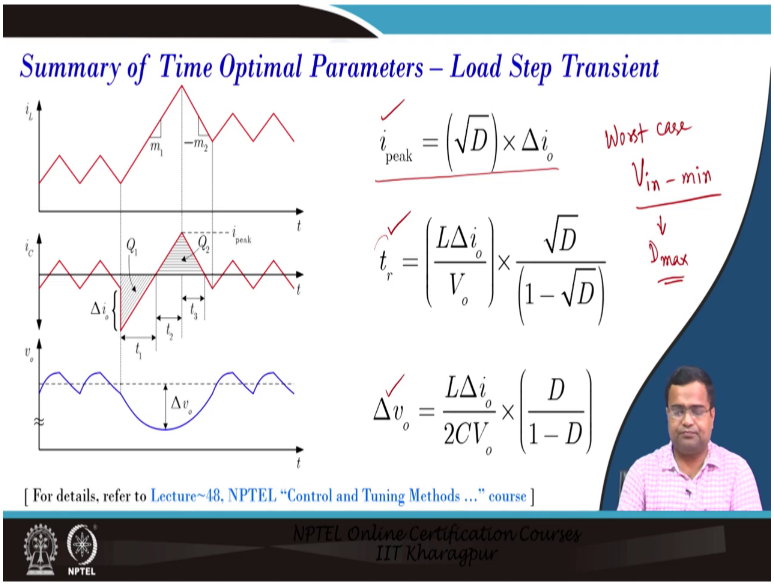

So, now if you summarize the peak current, again the peak current will be maximum for the largest duty ratio; the recovery time will be maximum for the larger duty ratio and the output voltage. So, the worst case will be V in min minimum value of V for a given output voltage ok, for the given load step size. You will get everything highest; that means peak current highest, recover time largest, and undershoot largest for the minimum value of input voltage, where this will lead to D max, the maximum duty ratio, ok.

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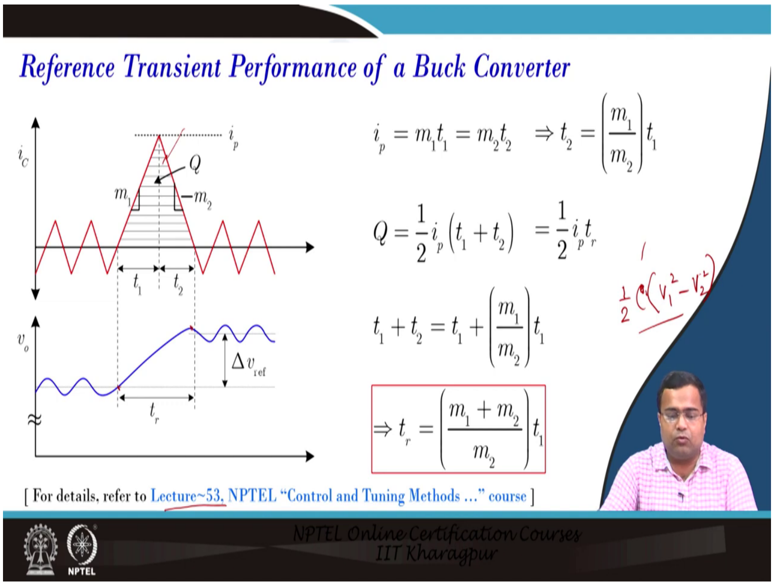

Now, if you go for a reference transient, this is again discussed in lecture 53 in our earlier course; we can show that you know lecture 53 we have discussed, we can show that the recovery time can be, it counts intuitive that for large reference transient for changing one voltage to the other voltage, you have to make changes half C, that means half C output cap V 1 square minus V 2 square.

That means the energy requirement will increase if the capacitor size increases; because we want to change from one V 1 to another V 1 to V 2, that value is fixed. So, if the capacitor is large, larger energy is required and that will come through the inductor. So, you will have a larger current overshoot as well as a larger recovery time.

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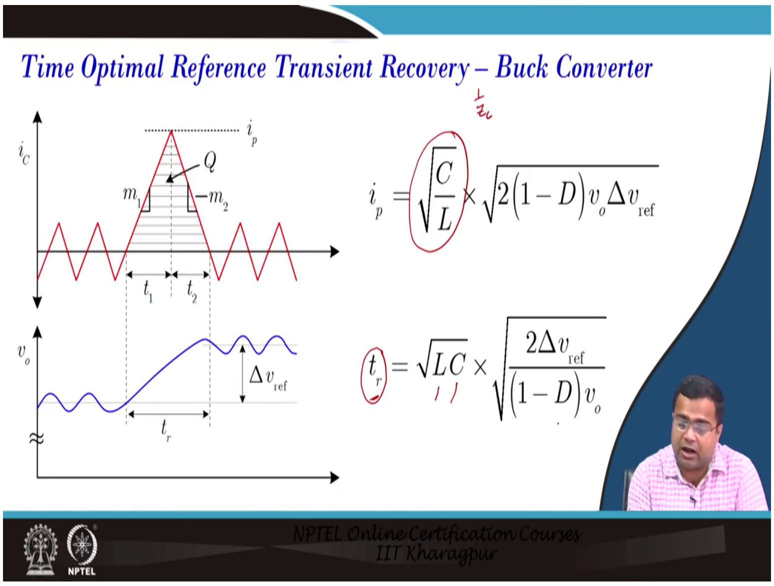

So, we want to find that, if we calculate the peak current; it will be just opposite to the characteristic impedance.

So, this is nothing, but 1 by z c right into the square root of 2 into 1 minus D into v 0 into delta v ref. And if we consider you know the recovery time, it is the square root of L C into the square root of 2 delta v r divided by 1 minus D into v 0. So, what you want to mean; the recovery time will increase for the larger value of the inductor as well as the larger value of the capacitor in the case of reference transient.

And it can be shown even for a larger duty ratio also it will increase.

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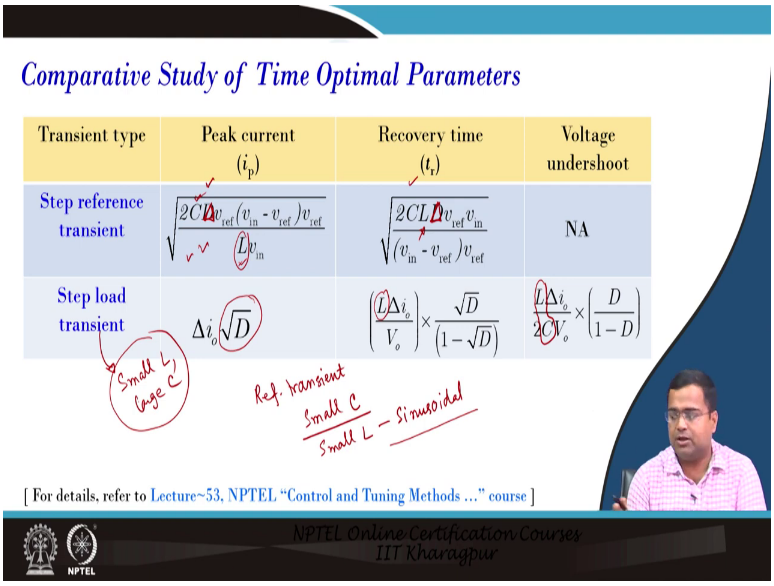

So, that means if we compare the reference transient versus load transient; for load transient, we know the peak current depends on the duty ratio for a given load step size, but for reference transient, that means for a higher duty ratio the peak current is higher. But if you take a reference transient, the peak current depends on the capacitor.

And this should be delta v ref, not D v ref; delta v ref, this is delta v ref, that is the difference in the voltage, that means what voltage you want to change. So, the peak current increases with the larger capacitor and smaller inductor; that means it is just the opposite. If you take the peak current, peak current does not depend too much on load transient response with the L and C.

But if we go to recovery time in the case of step load transient, we know the smaller inductor will reduce the recovery time and the smaller inductor will also reduce the recovery time for the reference transient; but it will increase the peak current because it is in the denominator. If you take the output undershoot for the load transient, we will need L by C; that means a smaller inductor and larger capacitor is desirable, but it is just the opposite for the peak current.

If you take a large gap, you will get a very high peak current for the reference transient and if you take a small L, it will also increase. If you take the recovery time, the large capacitor will increase the recovery time; so that means it gives rise to a design compensation, which means for load step transient, we need a small L and large C, but this is conflicting for the reference transient.

So, for ref reference transient, we need a small C; because if you take a small C, it will reduce the peak current as well as the recovery time. If you go for an envelope tracking power supply, if you use a buck converter for envelope tracking; the capacity is very very small almost negligibly small, otherwise it will simply limit the bandwidth of the converter, ok. And so, the general requirement for reference transient is smaller C, that is conflicting with the load transient response.

But if we go for reference transient, the choice of the inductor is something you have to be very careful; about because if you take a small L, the recovery time can be reduced or the bandwidth can be increased, but the peak current can be high. But if you are trying to track a sinusoidal current difference, then it is desirable; you can have a small L also, this is for sinusoidal tracking, it is for sinusoidal tracking which is better, ok.

For sine wave tracking, but if you are going for reference transient tracking, reference step; then L can lead to large smaller, L can lead to higher peak current. So, you have to be careful.

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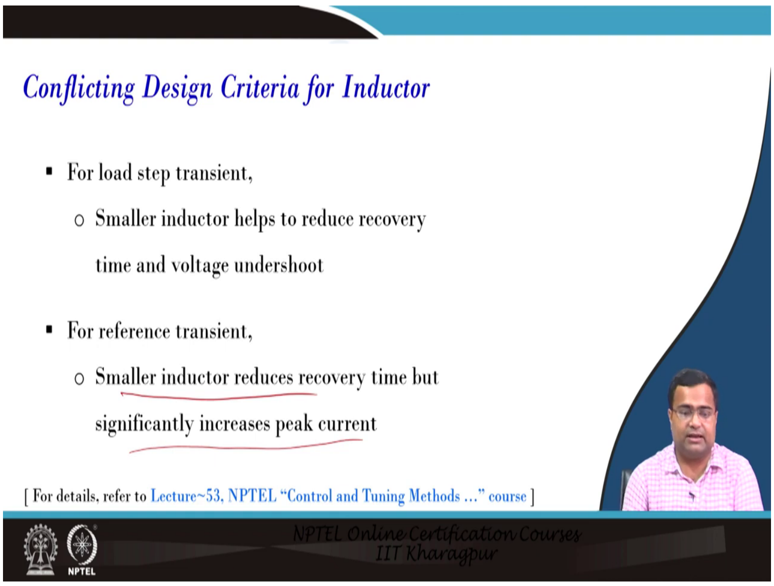

So, the conflicting current criteria; load step transients, and smaller inductor help to reduce recovery time and voltage undershoot. But for a reference transient, a smaller inductor significantly increases the peak current. So, that is a conflict.

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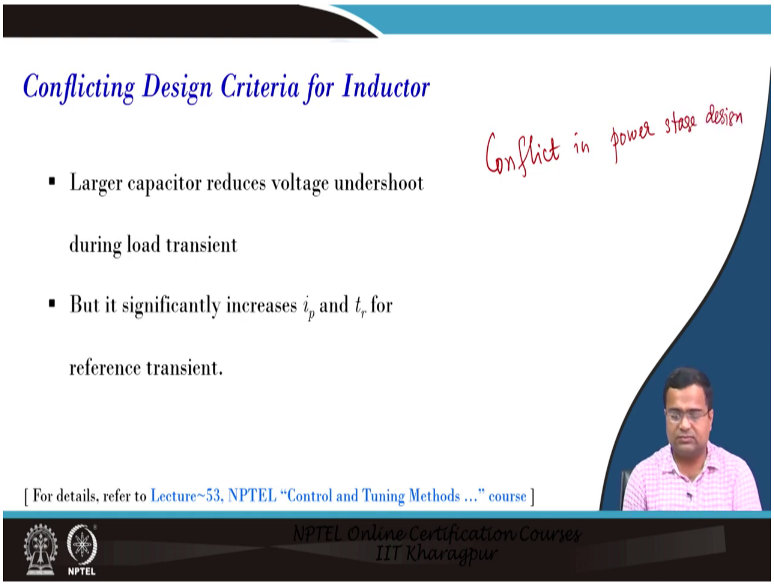

Then if we take the capacitor, a large capacitor reduces the undershoot under load transient; but it significantly increases the peak current as well the recovery time for reference transient. So, this leads to a conflicting power stage design, conflicting; so this gives rise to a conflict in power stage design, ok.

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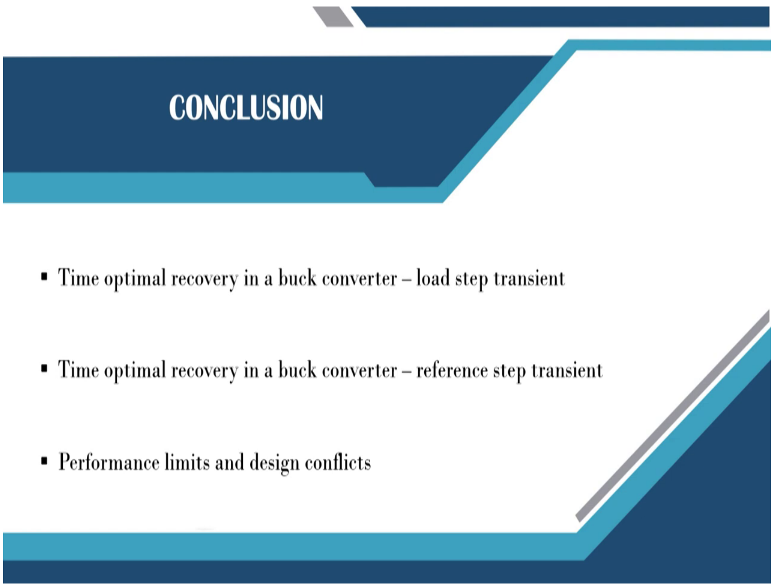

In summary, we have discussed time optimal control in a buck converter for load transient, we have discussed reference transient and we have also discussed what are the performance limit and what is the fastest possible performance that can be achieved and with we have discussed what are the design conflict. So, now, with digital control in the subsequent lecture, we want to discuss how can we implement such near-time optimal or time optimal algorithm using digital control.

And that cannot be achieved, that performance can be almost the fastest performance that can be achieved; but that can never be achieved using a power signal with design. So, this is where the beauty of digital control comes that, how can we achieve this time optimal recovery using digital control, and then only we can claim that we can significantly improve performance and efficiency compared to you know analog control, that is it for today.

Thank you very much.