Digital Control in Switched Mode Power Converters and FPGA-based Prototyping
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 05
Frequency and Time Domain Digital Control Design Approaches
Lecture - 45
Design Case Study and MATLAB Simulation of Digital Voltage Mode Control

Welcome. In this lecture, we are going to talk about Design Case Studies. We are going to consider a Design Case Study and MATLAB Simulation of Digital Voltage Mode Control.

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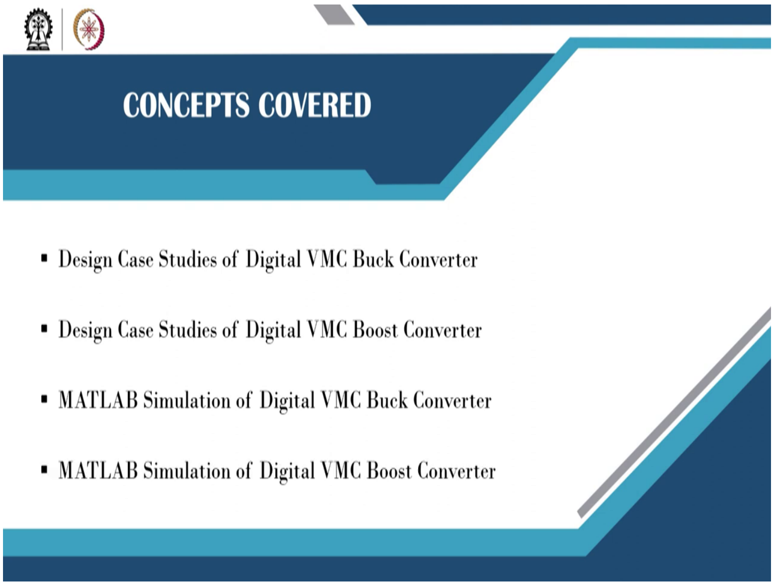

So, here we will first consider some design case studies of digital voltage mode control buck converters, then we will consider the design case study of digital voltage mode control boost converters along with their simulation MATLAB simulation for both as well as buck and boost converters.

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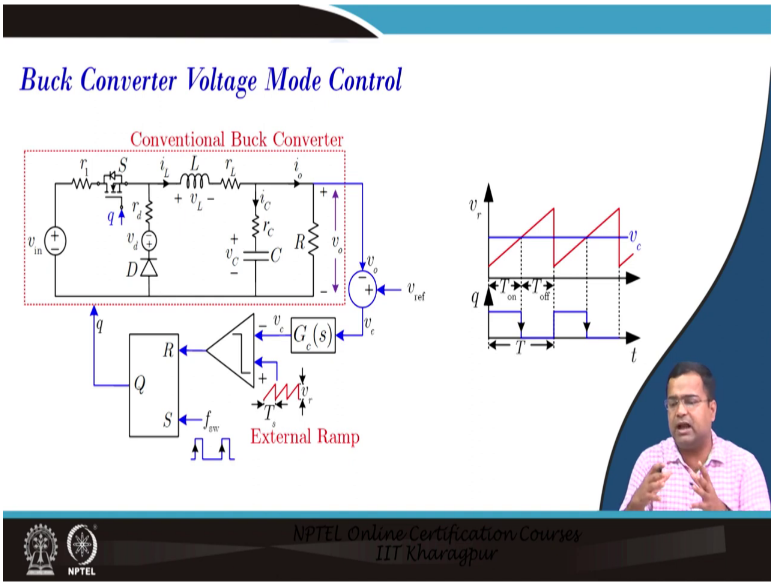

So, here this is now the continuation of I think lecture number 43, where we have discussed the design methodology for digitally controlled voltage mode control buck as well as a boost converter. So, we are taking those results. So, this is the typical diagram of analog voltage mode control.

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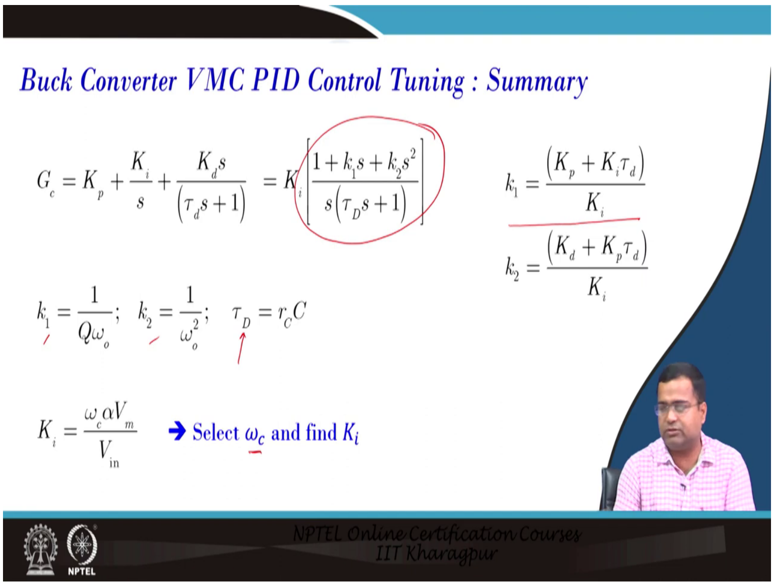

And if we are going for PID controller tuning then we have discussed that we can go for stable pole-zero cancellation which is method 1, where we can set this parameter k 1, k 2. These are the simplified version of the practical PID controller. And then we know the method in lecture number 43 we have discussed where k 1, and k 2 are functions of K p, K I, K d, and the tau d, which is the time constant of the practical derivative controller. Then we have to select crossover frequency so that we can find out the controller integral gain.

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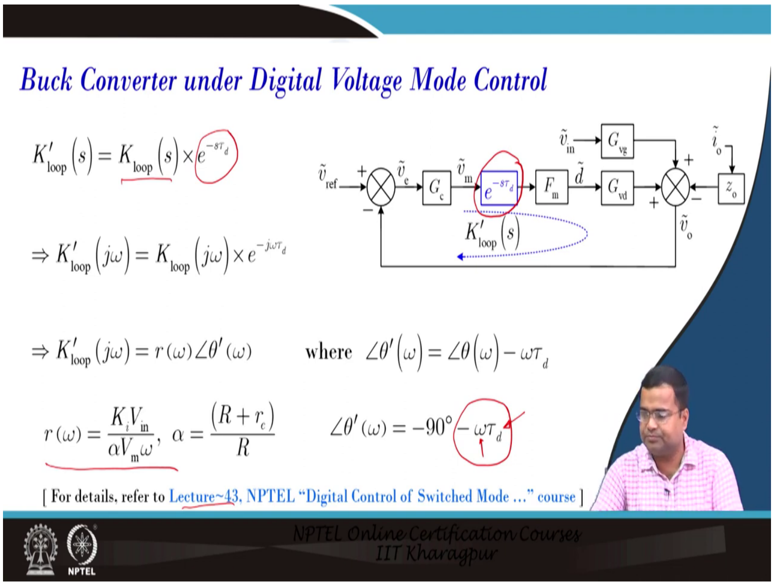

And this was referred to in lecture number 43 in our this course that how can we take this design for digital voltage mode control. So, in the digital voltage mode control we have considered the small signal model of the analog voltage mode control along with this additional delay. As we have discussed in the small signal performance as long as you operate close to one-tenth of the switching frequency if you just incorporate this delay, they are very accurate I mean they match very nicely with the accurate discrete-time model.

But we have not come across a situation where you know when you go to the harder demonstration that it may lead to subharmonic instability which cannot be predicted by this model. But this model is good enough to characterize a small signal behavior; so, we have discussed this in lecture number 43 in this course.

So, we know that if the K loop is the loop transfer function of the analog voltage mode control, then the digital voltage mode control we can just simply multiply it by this delay transfer function. And this delay accommodates the controller computation, ADC conversion time as well as DPWM delay. Then what we have discussed?

This delay will not change the gain plot, but it will change the phase plot; that means, the gain plot will remain unaffected by this delay. But this delay will introduce an additional phase lag which is minus omega tau d, where omega is the instantaneous frequency and tau d is the amount of delay that is introduced due to the digitization process.

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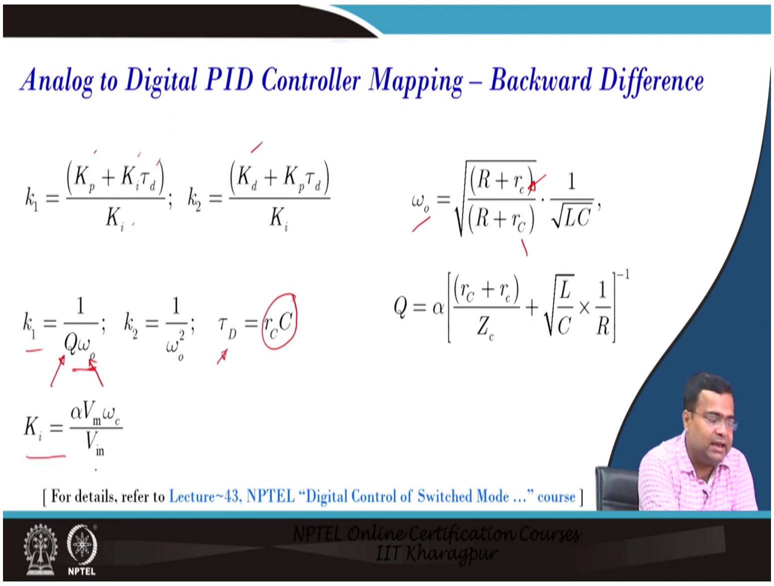

Now, we want to do analog to digital PID controller mapping using the backward difference formula. So, we have already discussed that k 1, and k 2 are the polynomial numerator polynomial of the controller practical PID controller and their coefficient can be written in terms of K p, K i, tau d, and K d. Then what we will do?

In the case of the pole-zero cancellation method in the voltage mode control design which we have discussed in lecture number 43 k 1 will be 1 by Q omega 0, where Q is the Q factor and omega 0 is the undamped natural frequency. And the tau dot tau d which is the derivative time constant field time constant, we can simply take this to be the ESR of the capacitor; that means r c into C, the time constant due to the ESR of the capacitor.

Then we already know that omega 0 for a practical buck converter will be R plus r e and this r e is the equivalent resistance which consists of DCR RDS on of the switch and DCR of the inductor and this is an ESR and R is a load resistor.

Then Q format this thing we have discussed; then finally, we will get the K i by suitably considering the crossover frequency that we have discussed.

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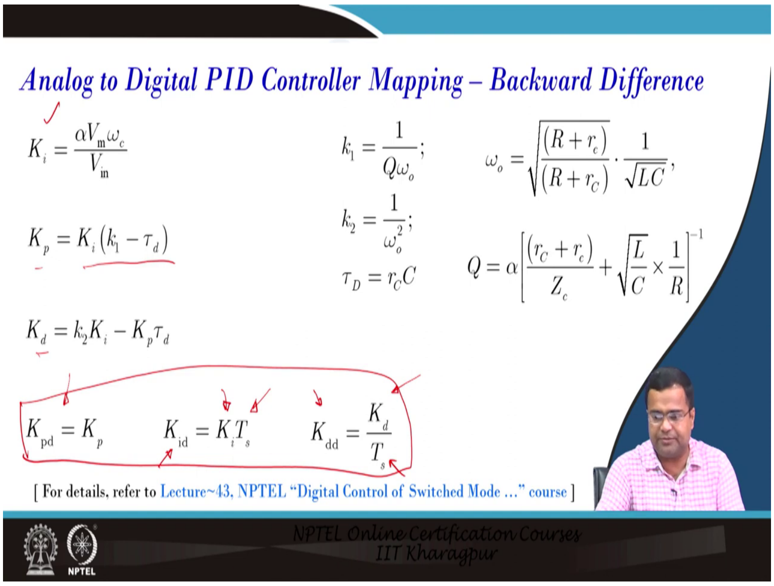

Now, if we can find out these k 1, k 2 values then we need we can find out; that means, we got k 1, k 2, tau d and k i and from here we can find out K p, K i, K d, and tau d; so, four equation four unknowns. So, you can find out K i is already obtained from setting the suitable crossover frequency, then the K p which is the proportional gain for the continuous time case, is K i into k 1 minus tau d.

Then K d is the derivative controller game and then once we get this we want to convert it into digital because this is a backward difference formula where the proportional gain will remain the same whether it is digital or analog. But the integral digital discrete-time integral gain is nothing but the continuous time integral gain multiplied by the sampling time and in this case, it is a switching period.

And that discrete time derivative gain will be simply continuous time discrete that gain like a derivative gain divided by the sampling time which is a time switching period.

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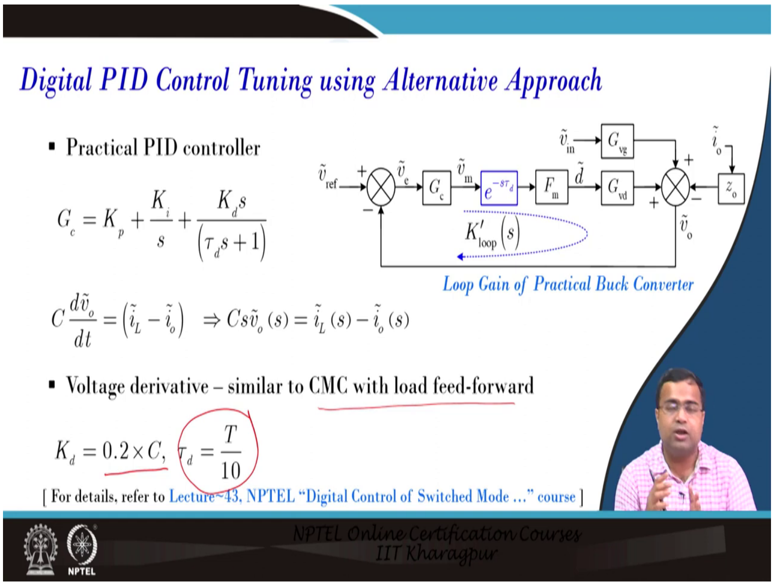

In that way, we can get the digital PID controller gain and we can design the controller.

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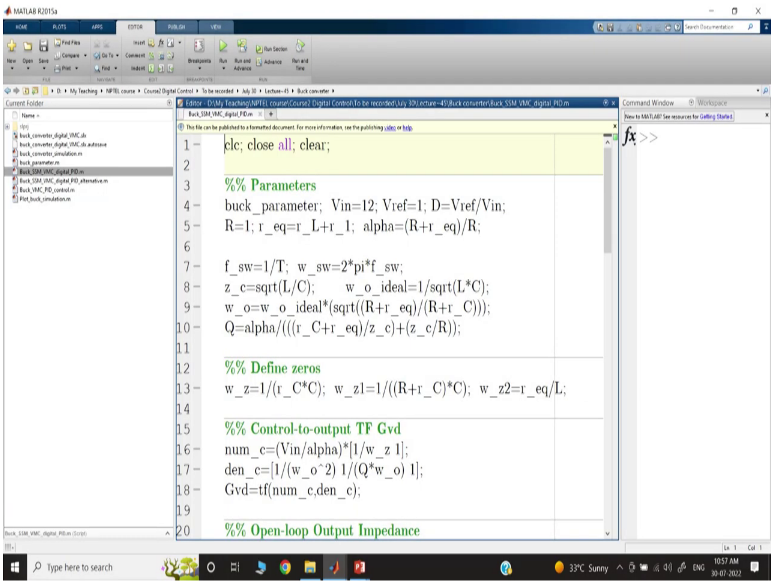

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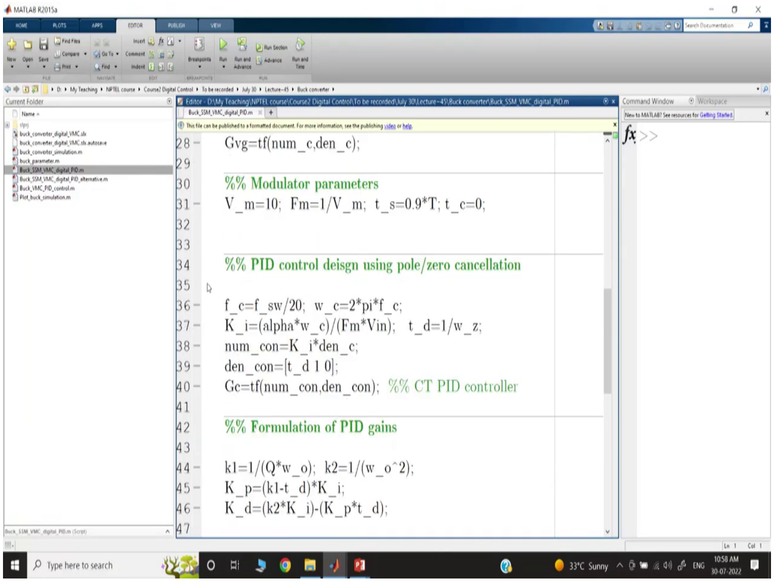

So, let us go to MATLAB's case study where we want to show that we want to design the PID controller and this diagram shows the voltage mode control buck converter.

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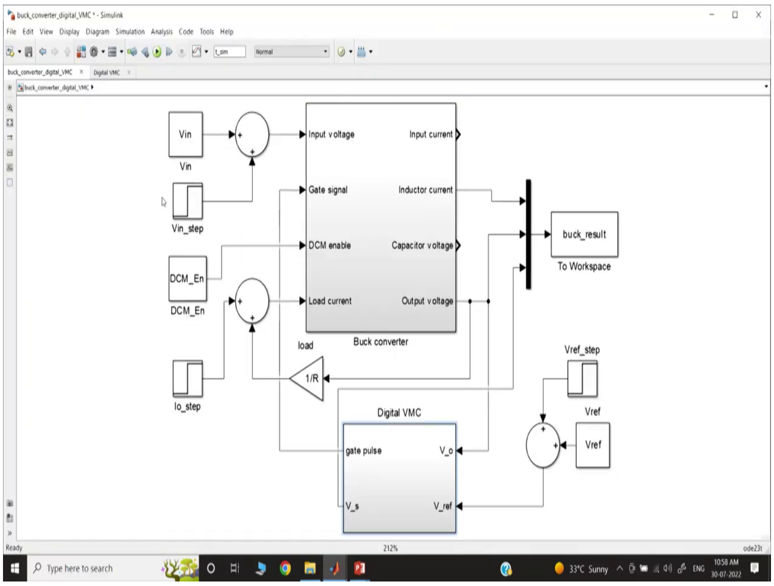

And we already know the buck converter power stage and in week three we discussed in detail how to develop this model MATLAB model, step by step we have discussed sufficient detail.

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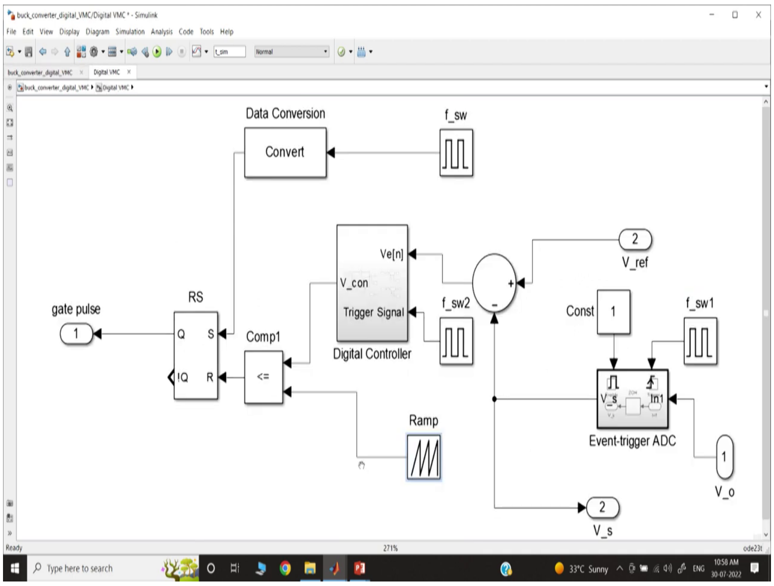

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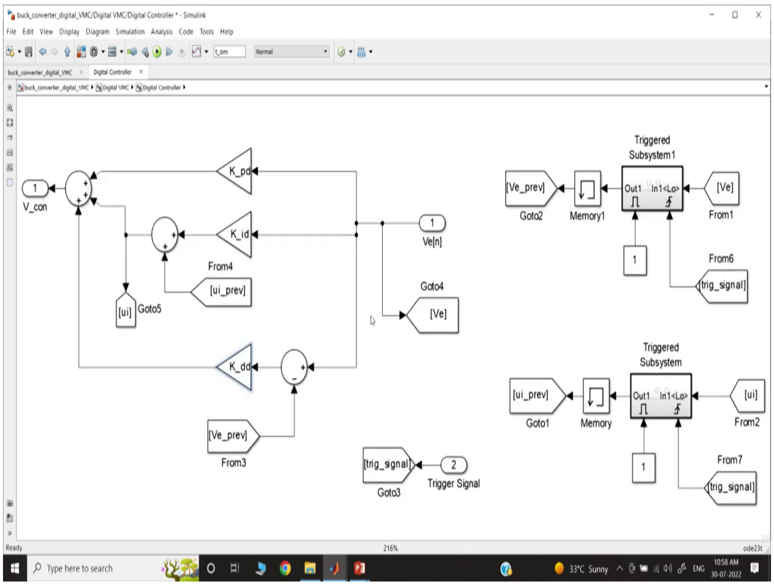

So, in this digital voltage controller, again we have discussed in lectures numbers 22, and 23 as well as we have also discussed yeah other lectures. So, in week 3 we discussed how to implement that custom PID controller block using a clock I mean it is a clock-synchronized PID controller that also we have discussed.

So, this is a proportional gain of the discrete-time PID controller, this is the integral gain discrete-time integral gain, and the discrete-time derivative gain; so, this is a thing we have discussed. And here the output voltage is sampled by using a clock; so, it is flexible how do you decide the clock sampling edge that clock, we can decide I mean it's the user's choice.

Then this sample goes and is subtracted from the reference voltage and then it goes to the controller the controller generates the control output voltage and it is compared to the sawtooth waveform. And the actual digital realization of this sawtooth can be generated by using either a counter or it there are other architectures that we have discussed in week 3.

Once it is generated, then this is a trailing edge modulator then the gate signal will be generated. Now, we want to go for the design of the PID controller. So, we will follow the traditional continuous-time PID controller design. And this we have discussed in our earlier NPTEL course; that means, the control and tuning method which we have discussed in lectures numbers 34 and 35 in detail, how to design this PID controller.

34 lecture is dedicated to you know we have discussed in detail the PID controller design using pole stable pole-zero cancellation. Here we are setting the control voltage to 10, it is again the user's choice. The sampling time as it we are sampling 0.1 times before; that means, I will show you first we are sampling then you are switching. Now, for the PID control design we want to set a crossover frequency and we are choosing the crossover frequency one-twentieth of the switching frequency.

Then all other controller parameters are decided because we have discussed that we are using stable pole-zero cancellation. So, first, we need to know what our desired crossover frequency and accordingly it will generate the integral gain and then we can find out. And tau d is a practical PID controller of the derivative time constant and that is used to cancel; that means, that particular pole is used to cancel the ESR 0; so, we have discussed it in detail.

Once we get all this K i and the new continuous time controller, then we know this. We are using a stable pole-zero cancellation. That means the controller numerator is K i times we know that 1 plus k 1 s plus k 2 s square. So, that term is used to cancel the denominator of the converter polwhichhat is why from there we can write k1 equal to 1 by Q that we have discussed; Q into omega 0. k2 equal to 1 by omega 0 square, e, and K p can be obtained from this k1, k2, K i and tau d; so, these things are written.

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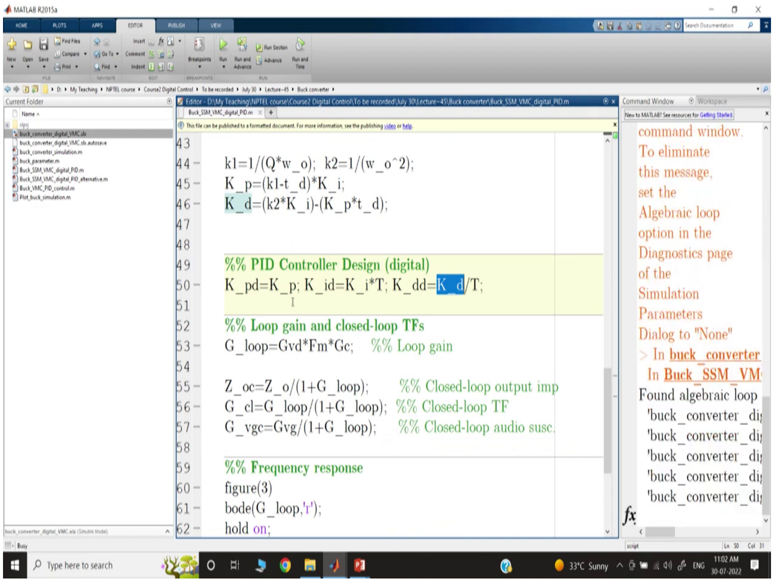

These are all analog controller gains, then we have to use a backward difference formula that means, the proportional gain will remain the same. Integral discrete time gain is the continuous time integral gain into T and discrete time derivative gain is nothing but continuous time derivative gain divided by T.

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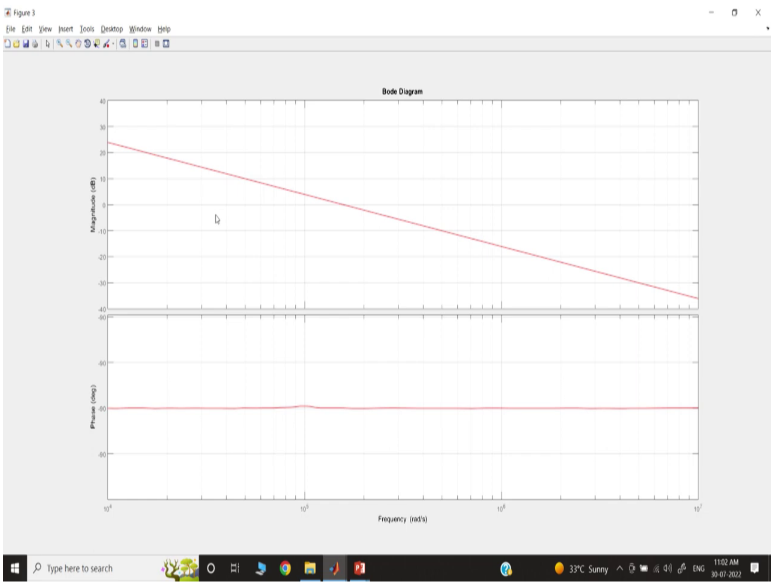

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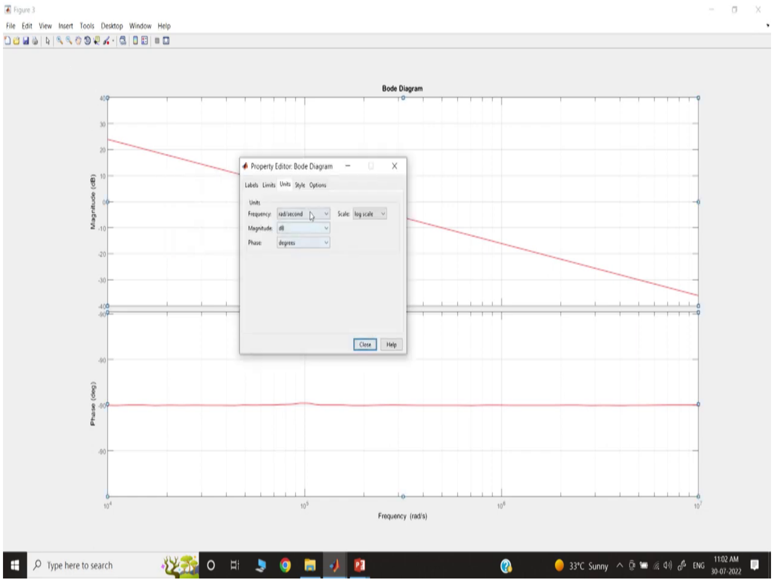

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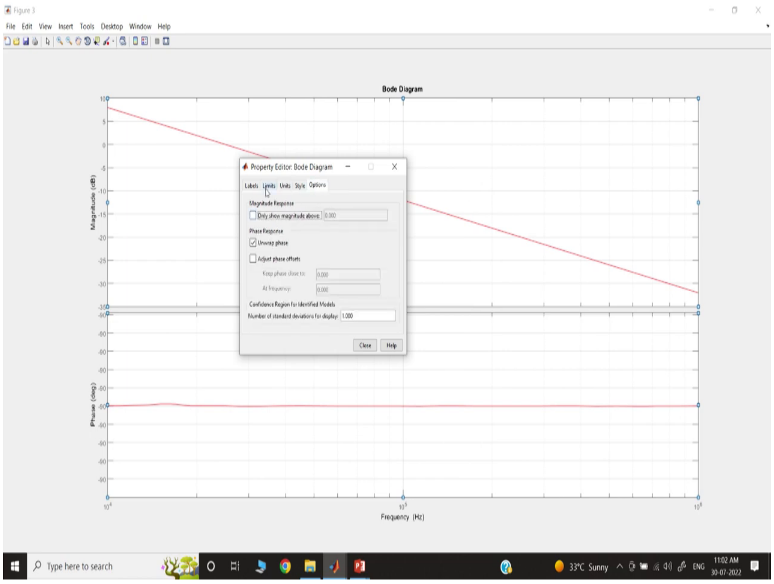

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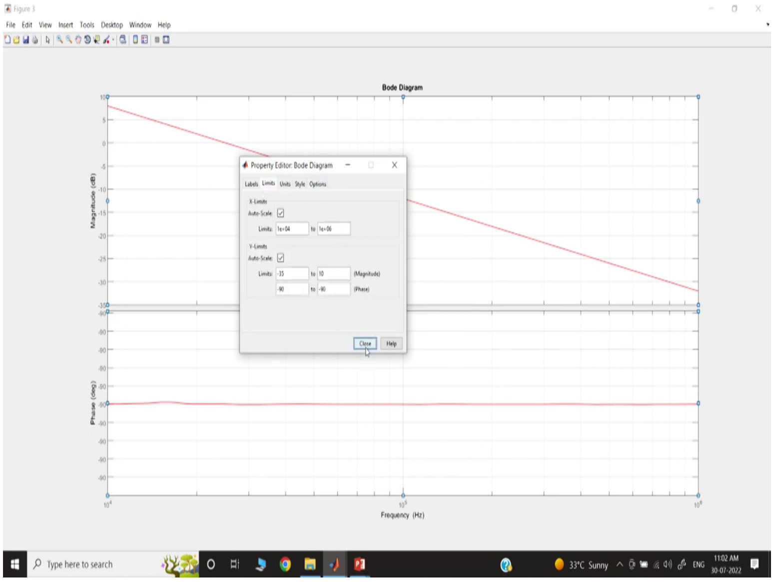

So, now, we want to first run this simulation, and first, we want to see that if you go to the bode plot this is a continuous time transfer function. And I want to show that if we go for you know let us say we want to check what are the stability margins.

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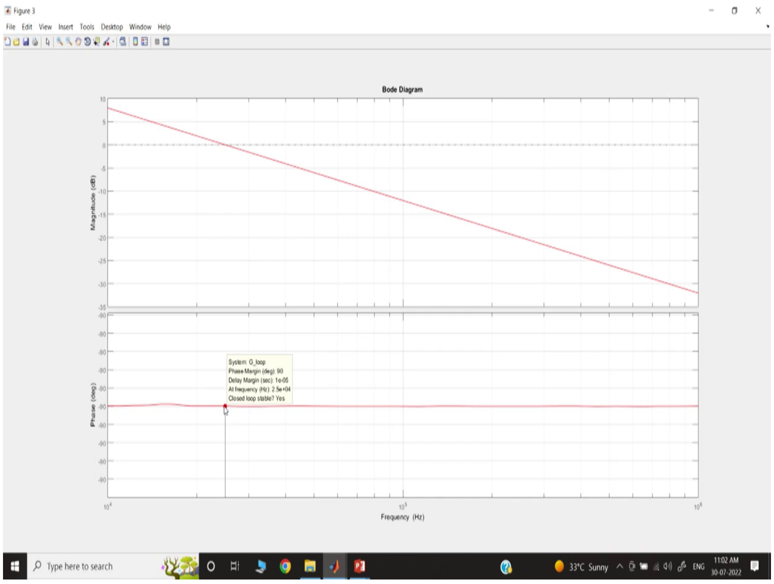

So, you can see we are operating using a 500-kilohertz switching frequency and this is you can see the frequency is 25 kilohertz; that means, we are using one-twentieth of the switching frequency. And phase margin is 90 degree because the phase is 90; so, the phase margin is 90 degree.

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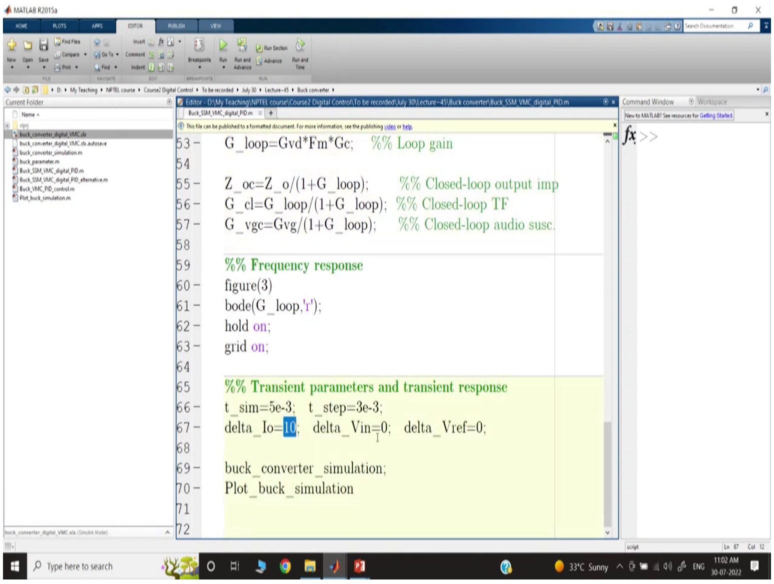

Now, this results in a transient response, we have applied a, what kind of transient? Here we have applied a 10 ampere load current transient, and we have used 12 volt input and 1 volt output.

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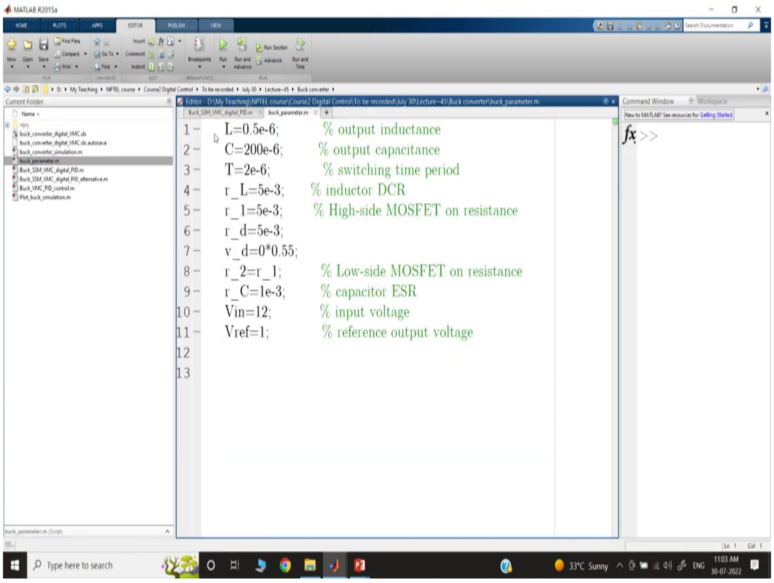

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And all the parameters we have taken the earlier parameters which like L equal to 500 nano henry, C equal to 200 microfarad then T is equal to 2 microseconds 12 volt input 1 volt output. So, once we get you will see the transient response using stable pole-zero cancellation is not good.

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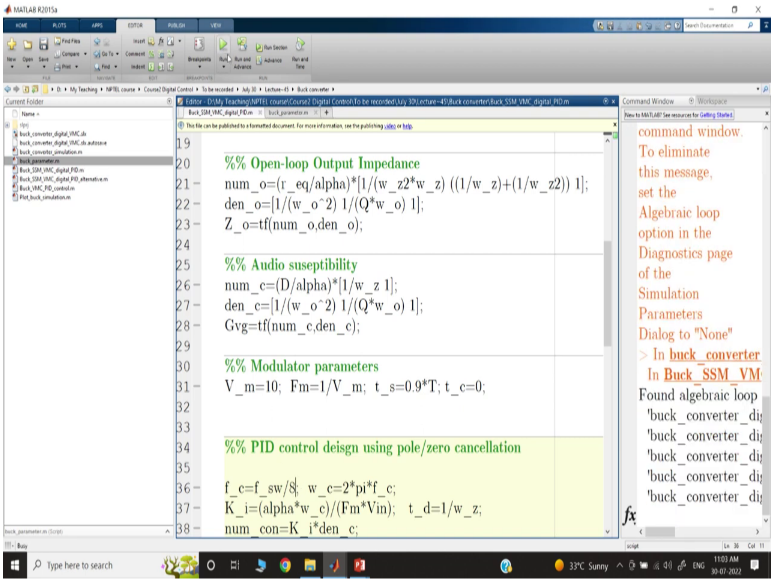

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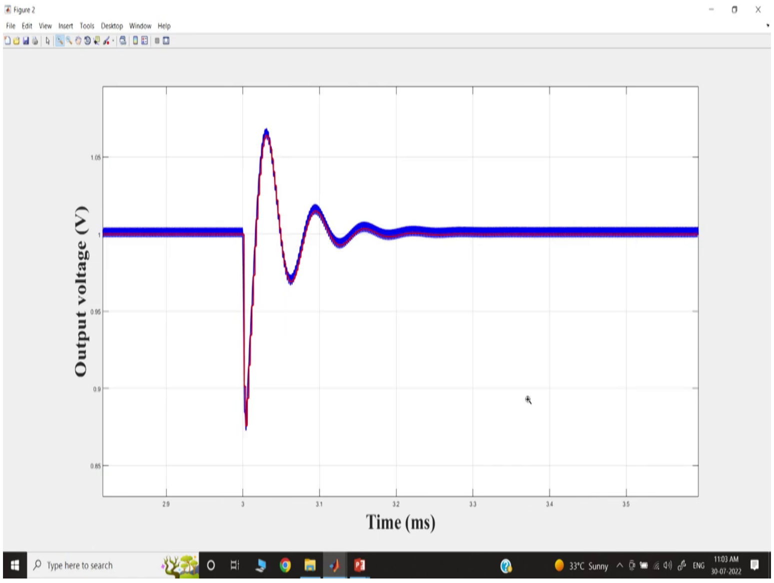

So, if we want to get a better response one way you can increase the cross-over frequency, instead of one-twentieth let us say one-eighth of this. And if we again run this simulation, we will see that it is somewhat better, because we want to reduce the overshoot undershoot. But we have discussed beyond that one-eighth or even one-tenth will be a good choice, the model validation comes into the picture that we have discussed in our earlier course.

So, we will not we do not want to design using a small signal base beyond this bandwidth.

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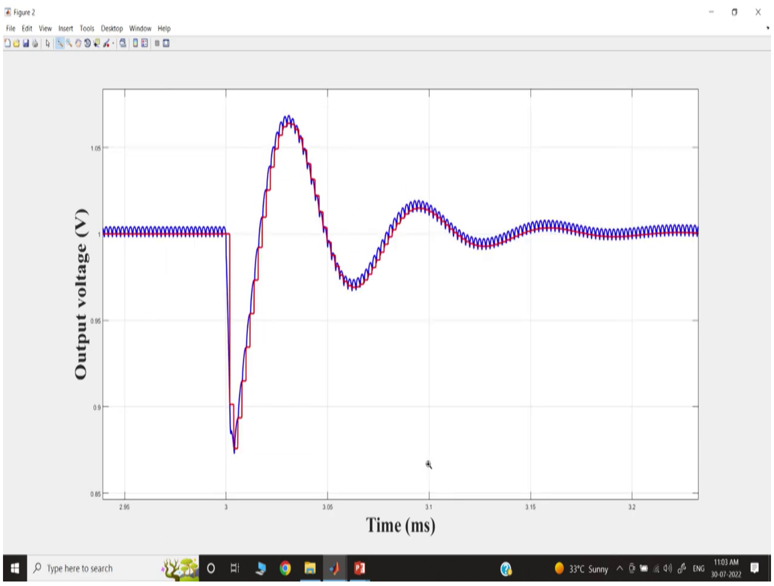

So, this is the response that is coming from the output which gives rise you know the settling time is here you can see we have applied a 3 millisecond we have applied transient. And if we want to zoom out, we will see it takes around 0.2 millisecond settling time with an overshoot of around more than 60 milli volt and the undershoot is somewhat around 120 milli volt which may not be acceptable.

I mean that is not a good sign by the means of this stable pole-zero cancellation. Then we want to discuss the alternative method and this method is discussed in lecture 43. In the alternative approach the same thing same PID controller, but right now we want to decide if it is not any pole-zero cancellation. So, we will take K d which is the derivative gain to be 0.2 times C and the tau d is T by 10 earlier we canceled the tau d we have taken to cancel the ESR 0, here we are not doing anything.

We are just taking the derivative gain 0.2 times the capacitor value that is output capacitor tau d. But this design we have not discussed sufficient detail the methodology, but I want to bring out that the derivative action gives rise to some kind of voltage derivative that carries some information of inductor current along with the load current feed-forward; so, you want to utilize that information.

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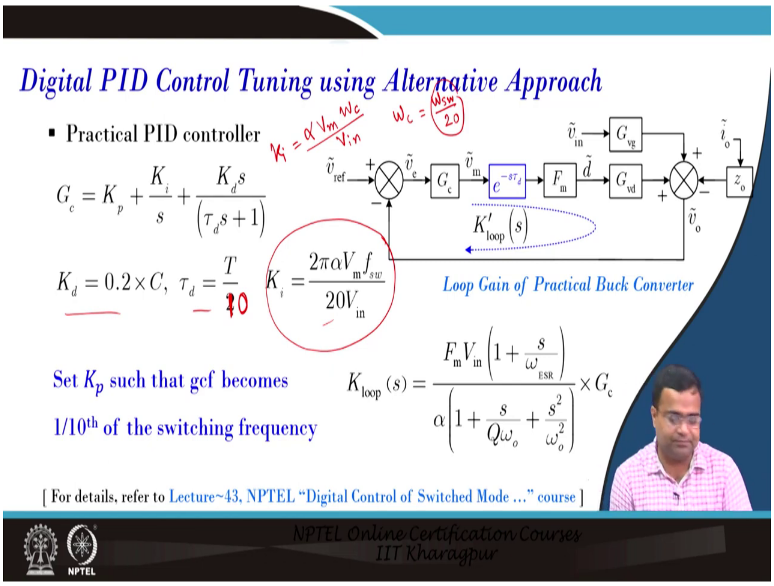

Then loop transfer function. So, we set K d and tau d sorry tau d we have taken T by 10; so, you can continue to take T by 10 here; so, it should be T by 10; T by 10. Then K i you can take this value which is nothing but from here you can write K i equal to alpha times V m omega c divided by you know V in where for choice of K i we have taken as if omega c equal to your switching frequency by 20 that is why we have chosen.

But again, we can play with the integral game, but this is a typical choice. So, we are not deciding on omega c yet, but we are choosing the K i by taking one omega that switches frequency by 20 times which is a K i gain. Then rest we have to decide on K p and the K p can be decided to achieve a gain crossover frequency somewhat higher than one-tenth of the switching frequency.

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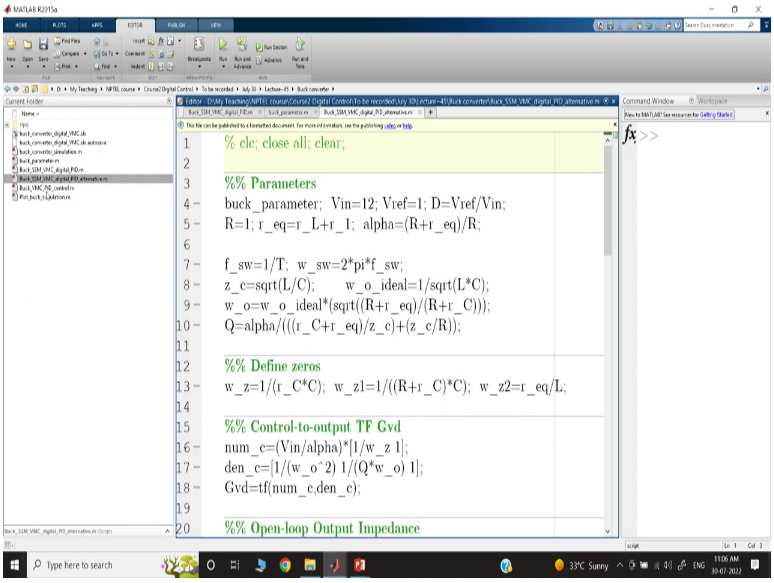

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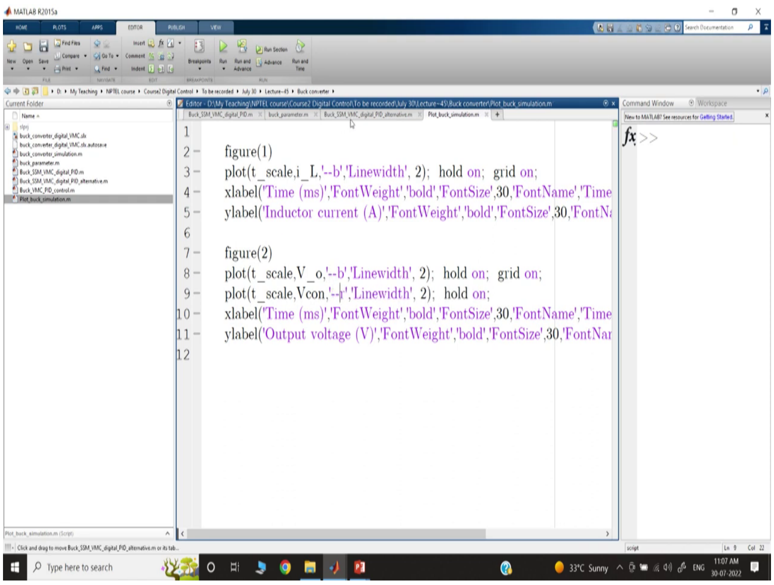

So, let us go to the alternative method in that and we want to compare the result. So, we are going to an alternative method and we want to plot on top of that to see what is the response means, we have to compare the response, and let us go we will take you to know this dotted dash line to see the response; that means, we want to compare.

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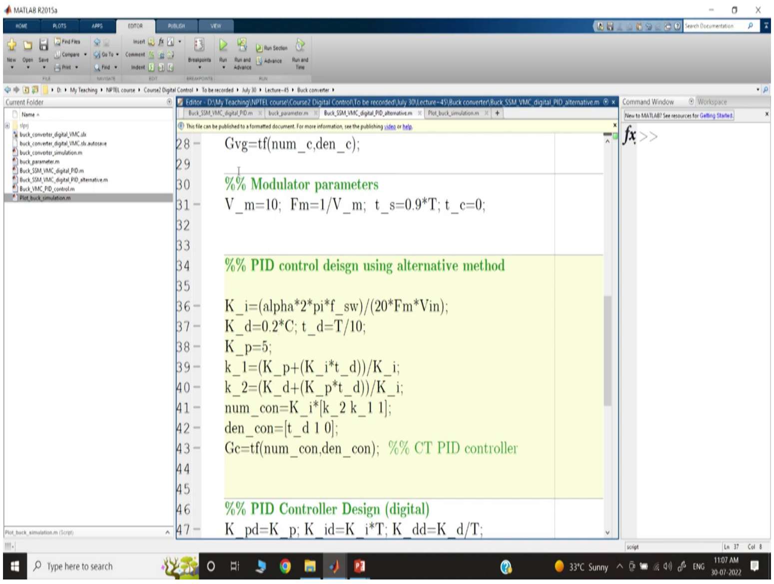

So, in the alternative method we have considered K i to be 20 times that we have discussed, K d we can take 0.2 with no issue, and K p we have selected. And we will see what is my crossover frequency. Let us first run it.

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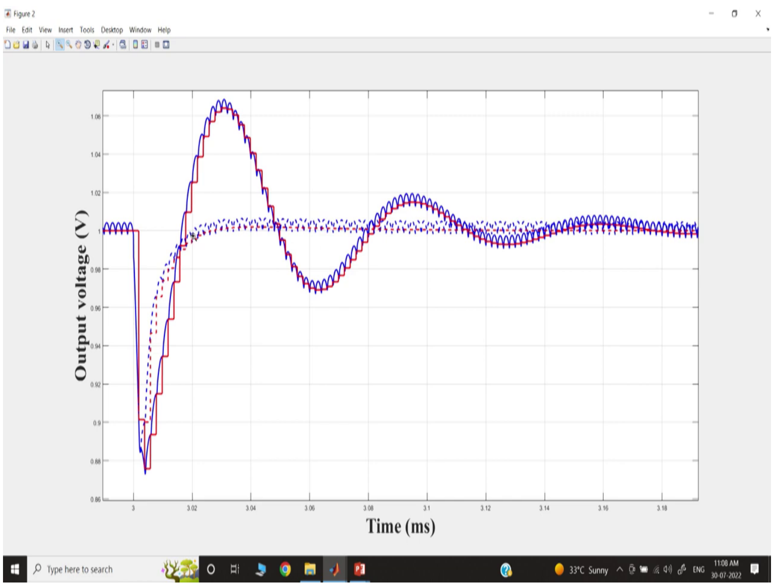

So, we are running it and we want to check the response.

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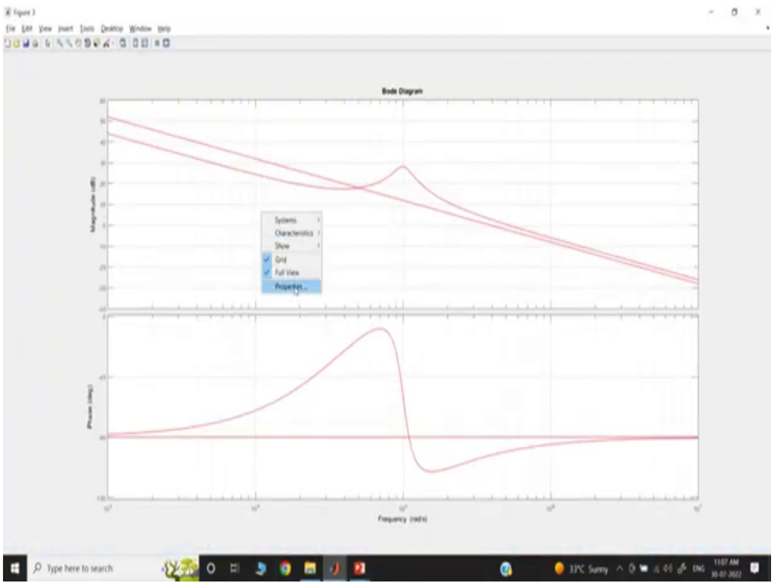

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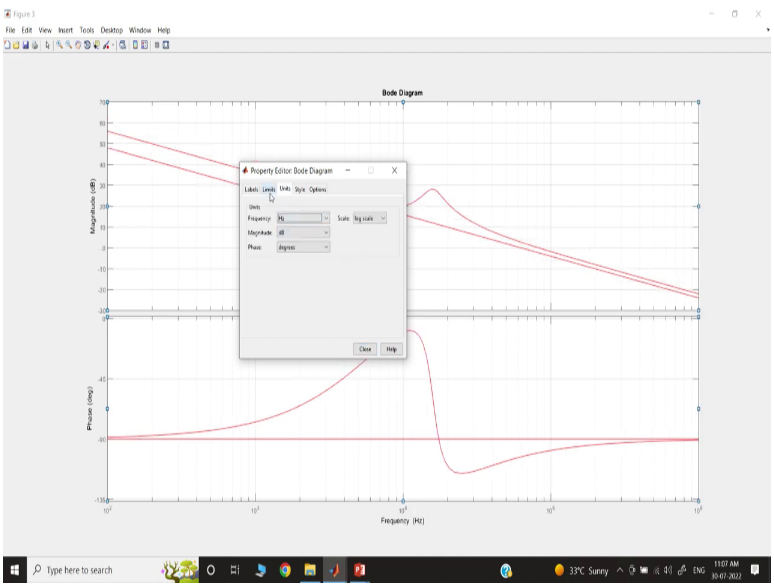

So, before we run let us go to the MATLAB Bode plot; so, here we want to convert it into unit hertz then we want to check the stability margin.

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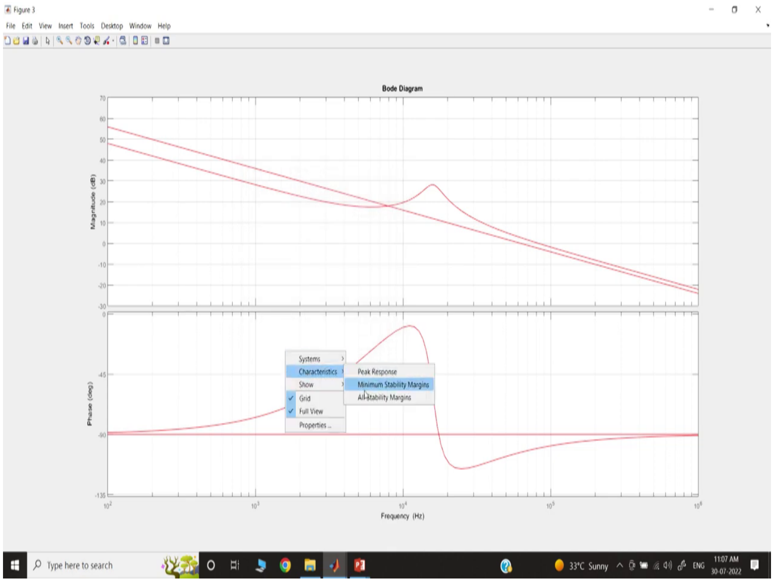

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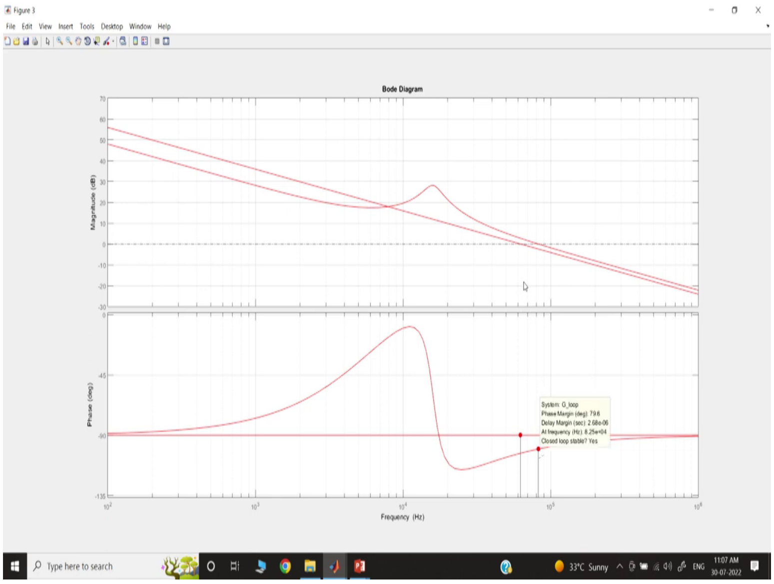

So, in the second method, we are achieving little higher crossover frequency, but here it shows the phase margin is around 80 degree; whereas, the stable pole-zero cancellation was 90 degree and the crossover frequency is a little less.

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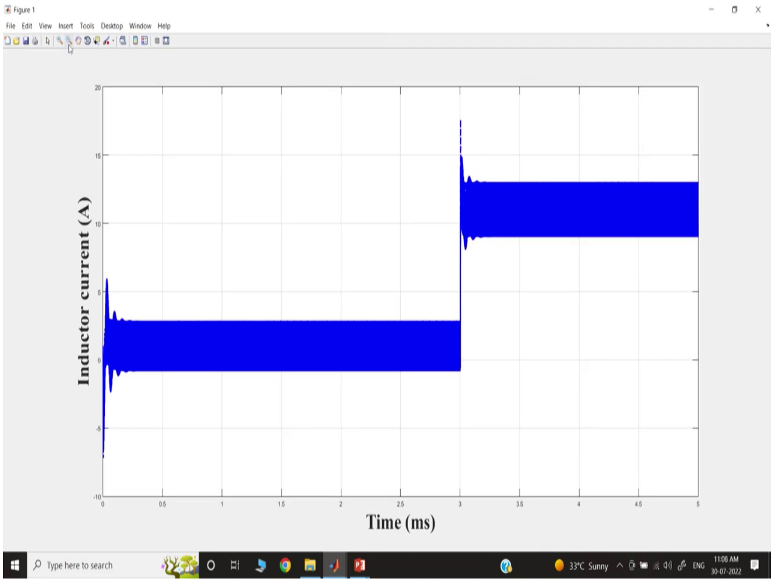

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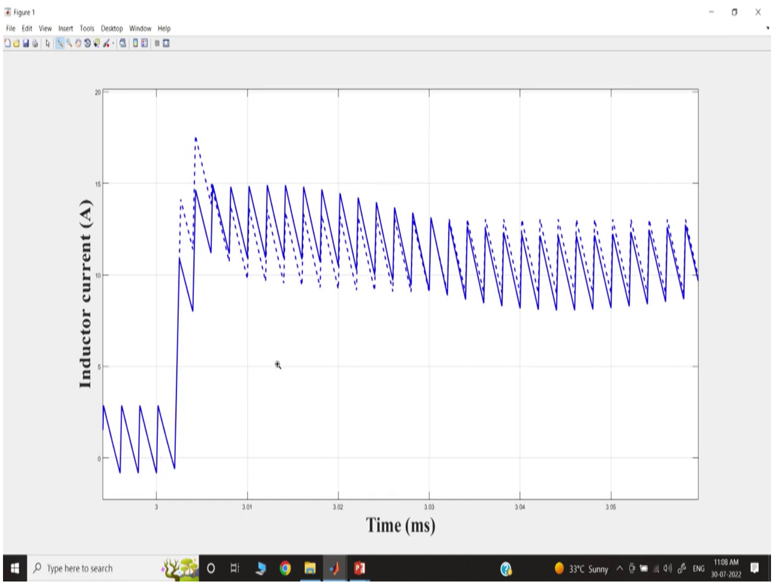

But if you see the response transient response in both cases, I want to show that the dashed line is coming from an alternative design method and under steady-state, they must be identified that we know ok.

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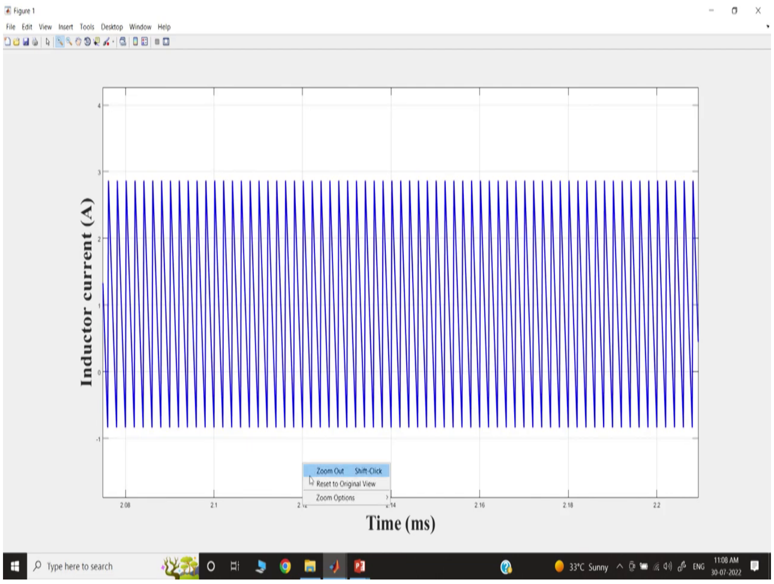

So, now if you go to the voltage recovery; so, you can see the dash response is much better because it is settling time is 0.2 millisecond which means, 20 microseconds, it just takes 10 cycles; whereas, the other method pole-zero cancellation is a non-robust method it takes longer cycle. And also there is no overshoot at such for the second case; that means we are talking about this particular response there is no overshoot.

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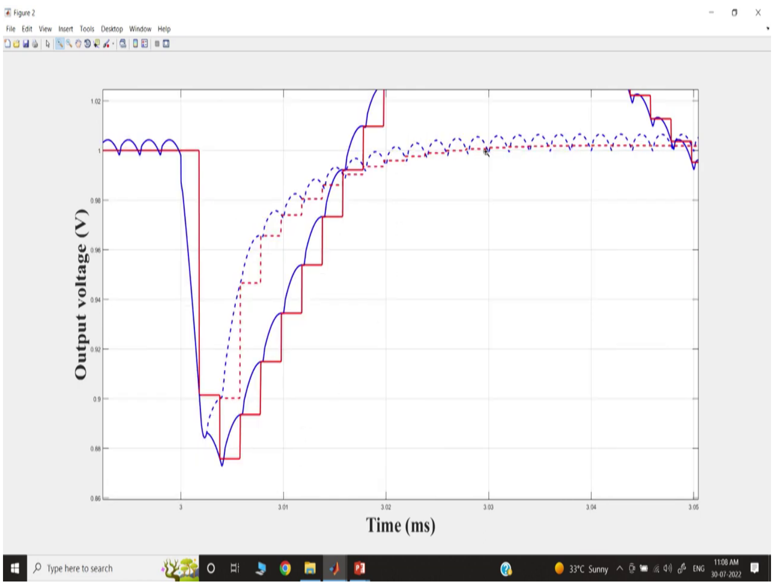

So, which means we can get a better design by considering the derivative action as the load, it is carrying some load current information inductor current plus load feed forward; so, viewing from that angle we can better design. So, what is the problem with the output impedance? Because in stable pole-zero cancellation, we are trying to shape the loop. But if you recall the output impedance expression, it is the open loop output impedance divided by 1 plus loop transfer function.

So, whatever loop you will save in the numerator you still have the open loop output impedance and that creates a problem. So, there is an alternative way to you know to shape the output impedance, that we have discussed in lecture number 36 in our earlier NPTEL course, the control and tuning method.

That means it turns out to be a simple you know lead compensator will be good enough, but in the digital control method or the alternative design method we can shape the PID controller, and accordingly, we can get the discrete-time PID controller gain. So that means, we have learned that this alternative method can better handle the load transient performance.

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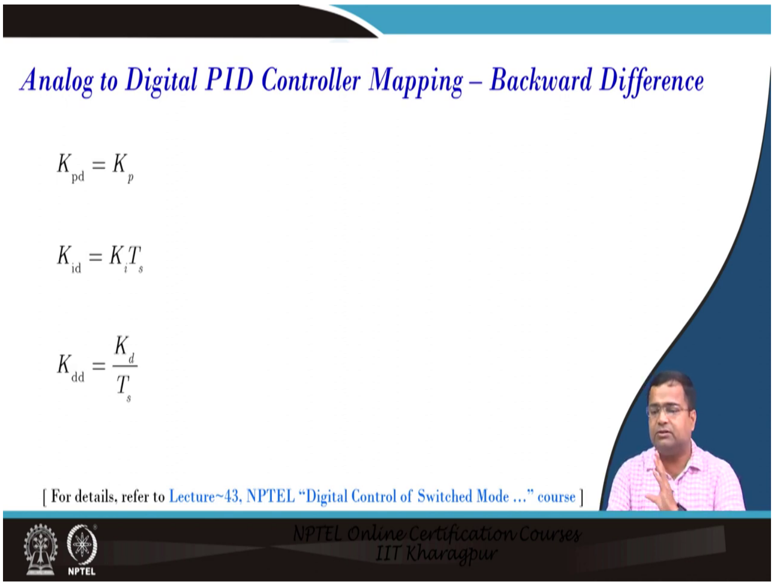

And we can convert this PID controller gain K p, K I, and K d by suitably scaling those from continuous time to discrete time.

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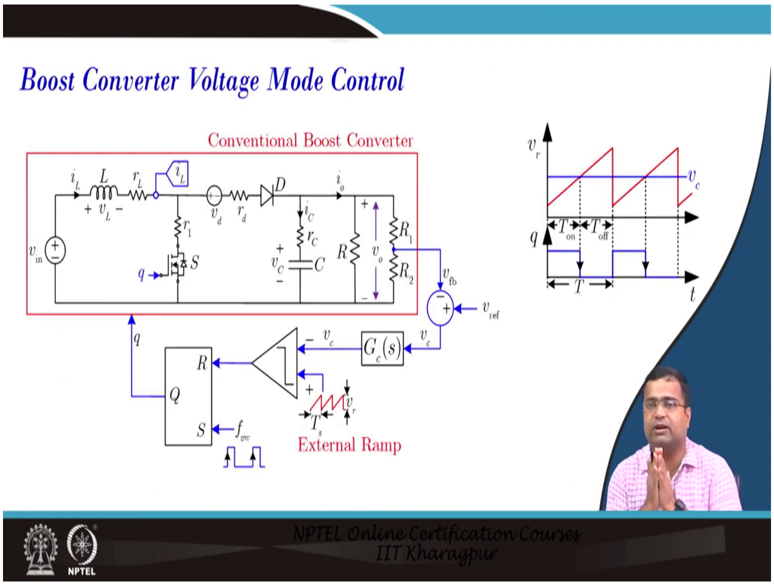

Now, we are going to the boost converter voltage mode control, because this class is mainly to show MATLAB simulation. So, in this converter, it is analog voltage mode control and then in this control analog voltage mode control, we have this all controller in analog continuous time.

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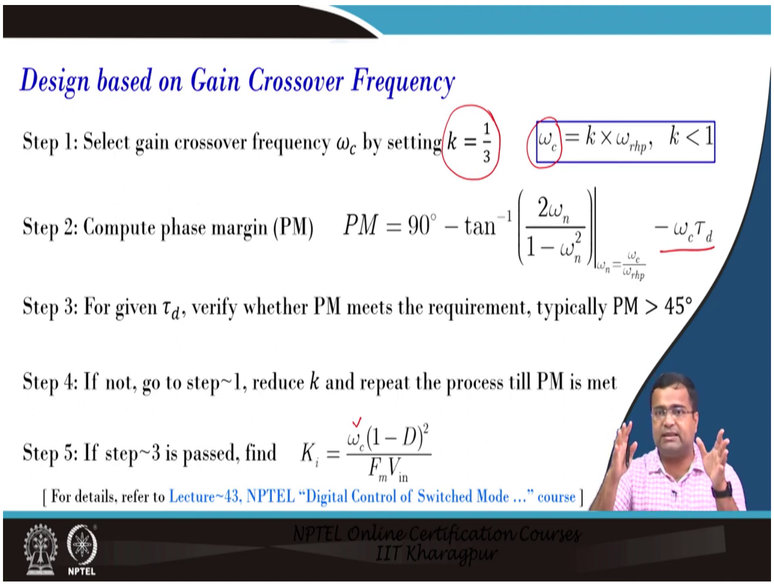

And what is the methodology to design? We have discussed in lecture number 43 we discussed the summary of the steps. Number 1, we select the crossover frequency by setting some fraction of the IP zero ok, then we will compute the phase margin that how much phase margin you want. So, for 0.3 times what is the phase margin that we are getting? Is it?

Now, we know in the case of continuous time voltage mode control boost converter design. We discussed this in lecture number 36 in our earlier NPTEL course. I think 36 or 37 where is the voltage mode control design? So, there using stable pole-zero cancellation we can shape we can by setting the suitable omega c, and we can find out whatever my phase margin is. Similarly, we can also design by setting the phase margin and it will accordingly set the crossover frequency.

But in the case of digital control, we have an additional term that is coming due to the that is additional phase lag due to the delay in the ADC conversion and controller computational time. So, we have to accommodate and then we need to check whether for the given time delay are you able to meet the required phase margin of at least 45 degrees or above. If not then you have to again reduce the cross-over frequency by scaling down this factor.

And then we have to iterate the same method step and check whether the phase margin is met or not. Once it is there then we will set the K i as the integral gain which is omega c that is coming from this value into 1 minus d whole square by F m into V. So, if that is set then we got K I, once we get K i then since we have made stable pole-zero cancellation; so, we will get all the other parameters.

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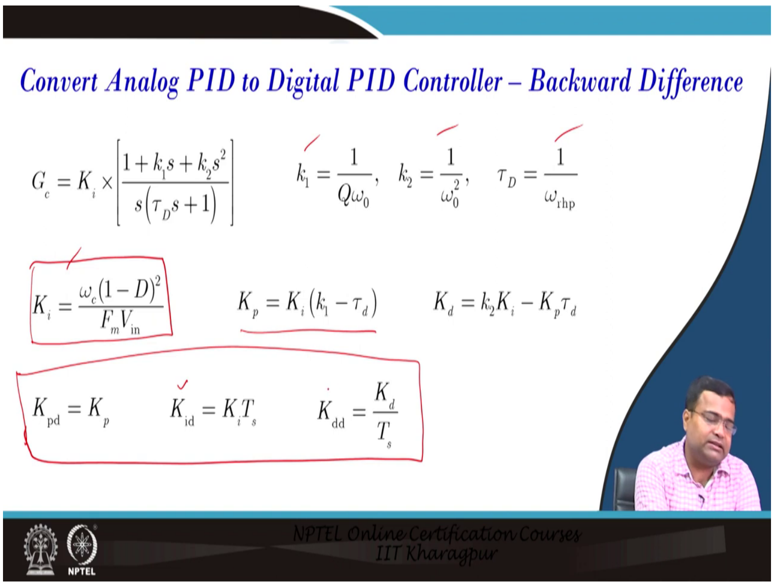

So, how do we get it? That means this is the PID controller. We already got this expression of K i by setting our crossover frequency to meet a certain desired phase margin. Then we can get the K p from this parameter, this all four parameters are four equations you can find out individually the proportional gain derivative gain in continuous time.

Once we get them we have to convert them into discrete time using the backward difference formula where the proportional gain will remain the same, and the discrete-time integral gain is the continuous time integral gain into sampling time. Then a discrete time derivative gain is a continuous time derivative gain by sampling time.

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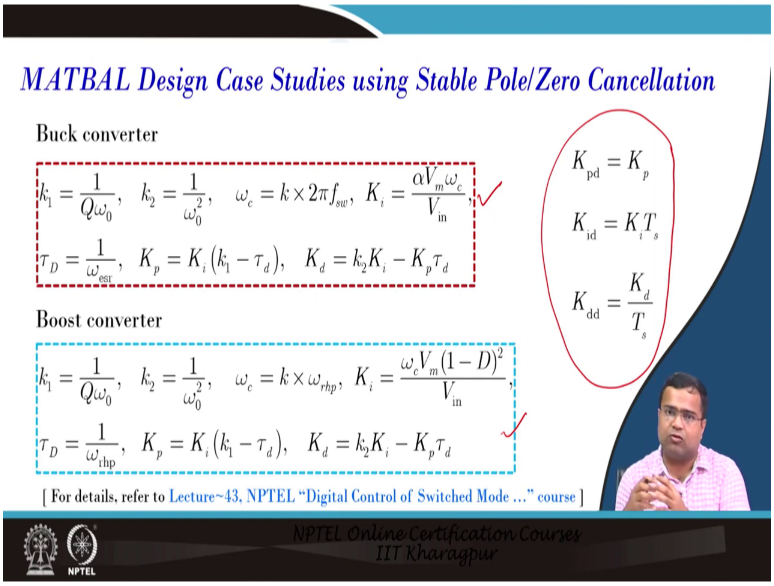

And this we have discussed in lecture number 43, and we want to summarize that if you want to go for the pole-zero cancellation method stable pole-zero cancellation for the buck converter, we have already discussed the case study of buck. And for the boost what will be the final expression of the K p, K I, K d as well as the tau d values; so, that also we have discussed in lecture 43?

Then we need to convert this continuous time controller gain into discrete time controller gain and that way we can set the controller parameter and we can implement whatever digital control platform we are using.

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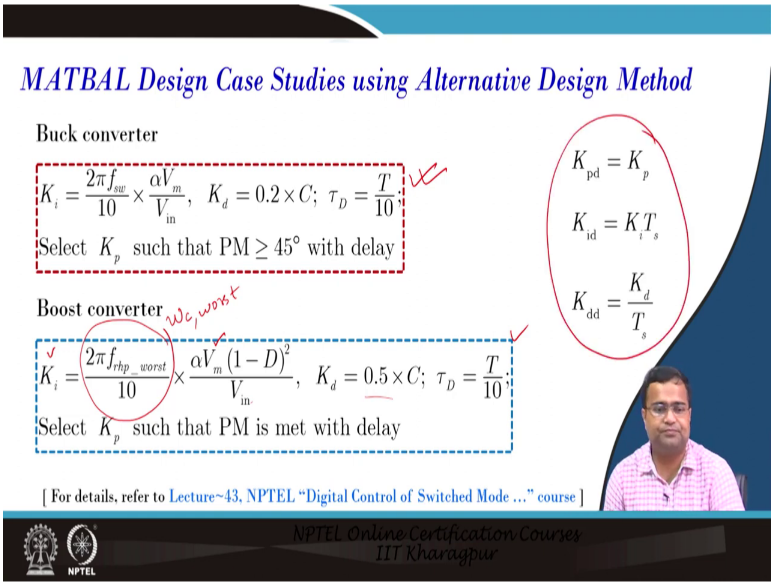

But we have also discussed the alternative method. In the alternative method, we have already you know shown the simulation case study for the buck converter. And we show we have observed that pole transient response is much better using this alternative method compared to this traditional method.

Now, you want to see a case study for the boost converter using the traditional pole-zero cancellation method that we have discussed. And then this method of this you know how to set this K i, K d because here we will take the K I to be worst case rhp 0. Because this sets something like omega c, omega c worst case; is the worst case omega c.

Then alpha V m, V m by V m this is our voltage mode controller peak voltage V n is the input voltage 1 minus D whole square. And tau d T by 10 and then K d 0.5-time output capacitor and we have to take the K p such that some phase margin required phase margin is met.

So, let us go to MATLAB we want to show; so, these are the earlier voltage mode control case study; so, we want to go to MATLAB and we want to open. So, this was for the buck converter, now we will go for the boost converter. So, in the boost converter, we want to first open the boost converter PID controller case study.

And I can show you that this is the diagram of the boost converter. This digital voltage controller is exactly structurally the same as buck only the parameter will be different. But for the boost power stage, we have already discussed how to implement it in MATLAB custom simulation; so, we are not going to discuss it again. Here, we want to design the boost converter using stable pole-zero cancellation.

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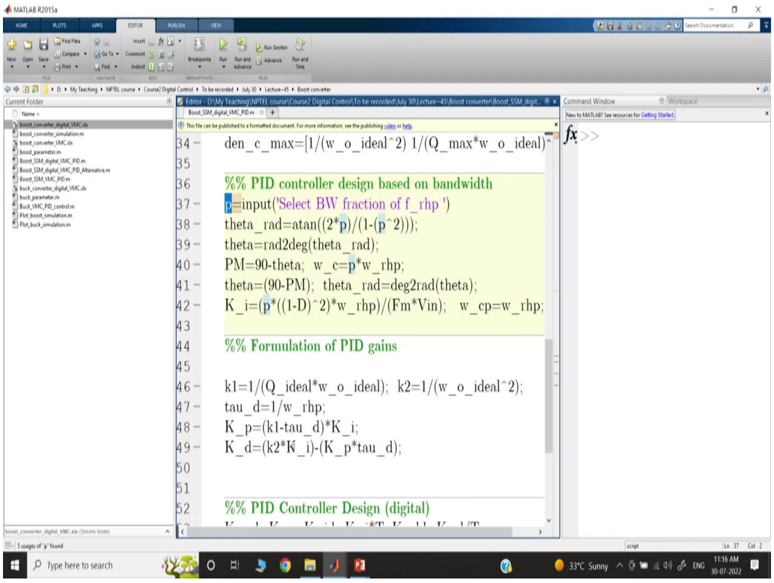

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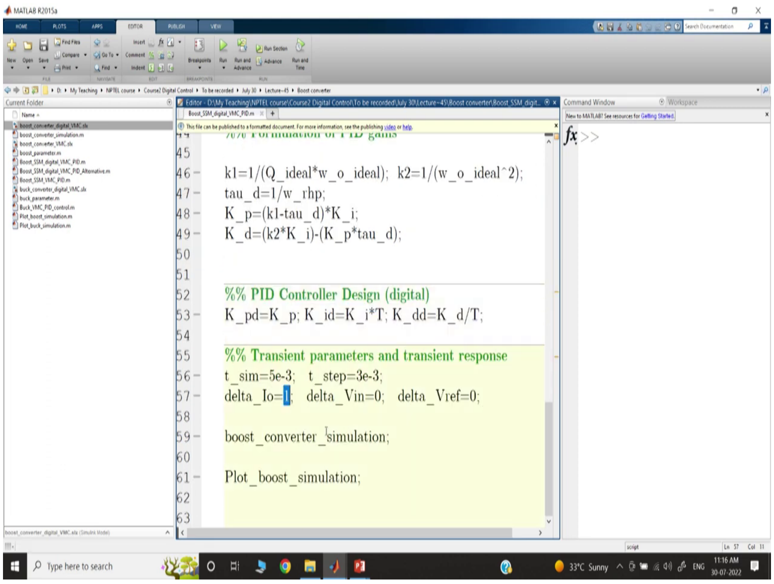

So, that is why design based on the bandwidth and we know that we can also design based on the phase margin. So, we will select p means what is the fraction of rhp 0 that you want to create the bandwidth; accordingly, it will create phase margin. Then you will get k1, k2 all these values and finally, it will show the digital PID controller value that we have discussed.

Here we are considering a load step of 1-ampere load step for a given initial load current of you know starting value R equal to 2 ohm let us run this simulation; so, maybe R equal to 1 ohm we can take.

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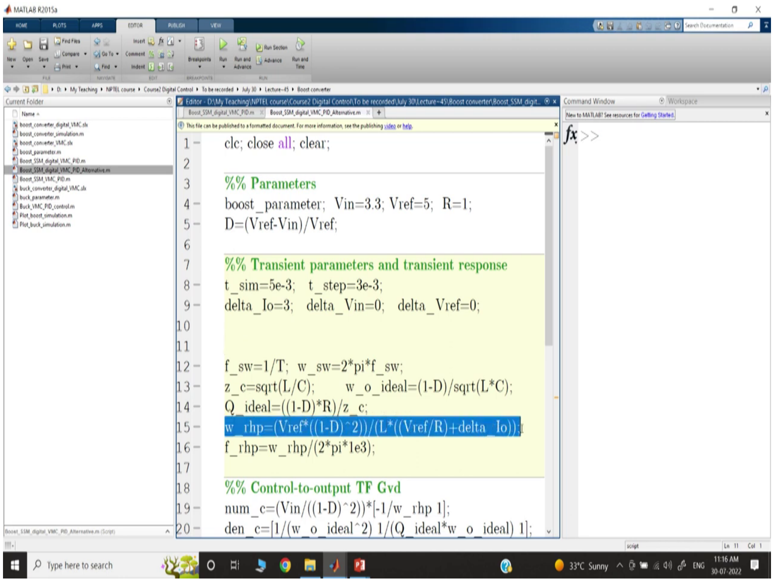

An alternative method is R equal to 1 ohm and we want to make a simulation case study let us say we want to make a 3-ampere case study. So, here we will consider the rhp 0 according to this expression; so, let us go back. rhp 0 will be according to this expression and we want to because we want to make identical simulation case studies; so, let us put it here ok.

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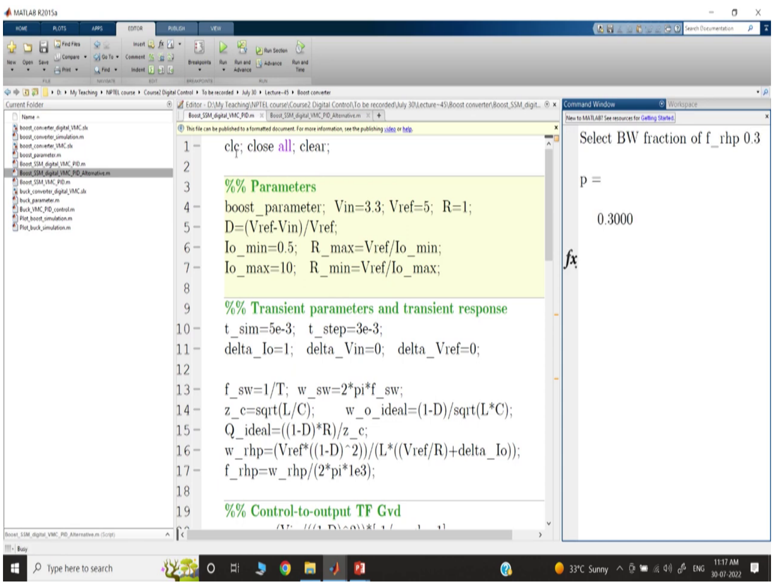

So, first I want to show R equal to 1; so, it is the pole-zero cancellation method ok; so, it is the pole-zero cancellation method.

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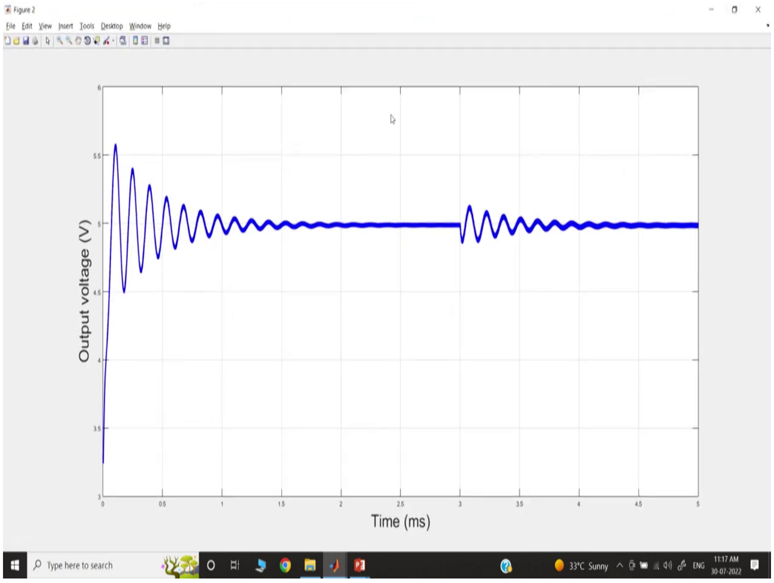

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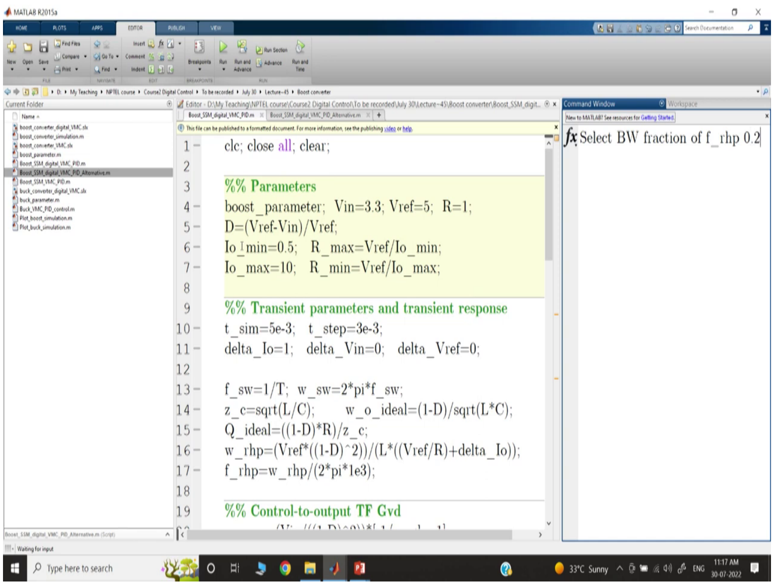

So, I have taken 0.3 times of this; that means, this is for you know ok; let us run again because we will take 0.2 times. Now, we are going to show a stable pole-zero cancellation, the PID controller using stable pole-zero cancellation. So, we are taking initial load resistance of 1 ohm and we want to apply a 3-ampere load step. So, let us see we want to design the crossover; that means, the controller PID controller by setting the crossover frequency 0.3 times the rhp 0.

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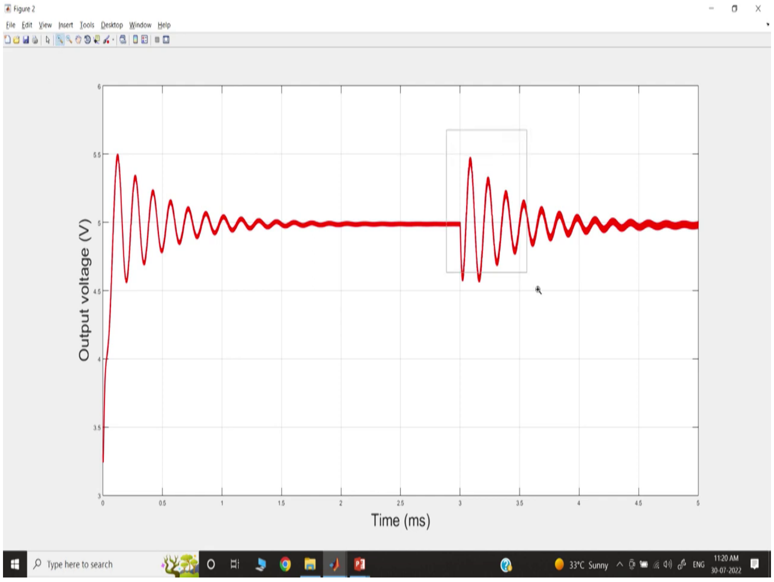

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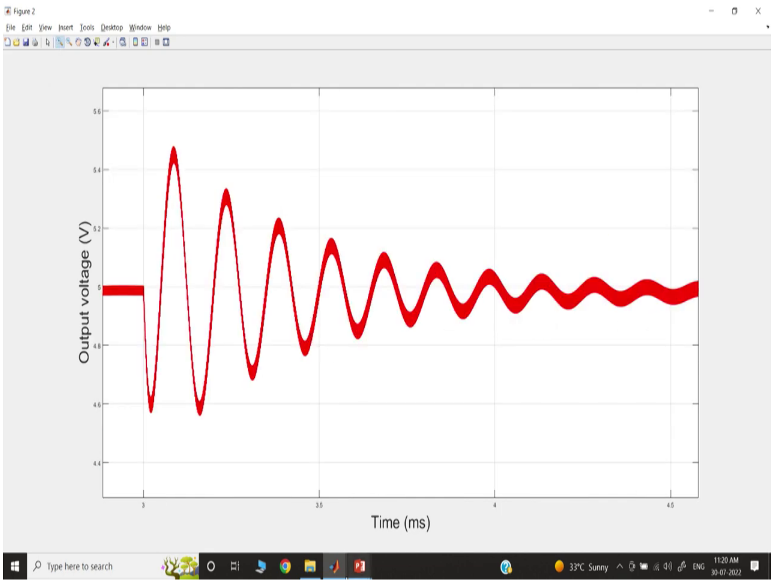

So, this is the response of the boost converter ok; so, this is under voltage mode control. And now we want to go for the alternative method that we have discussed.

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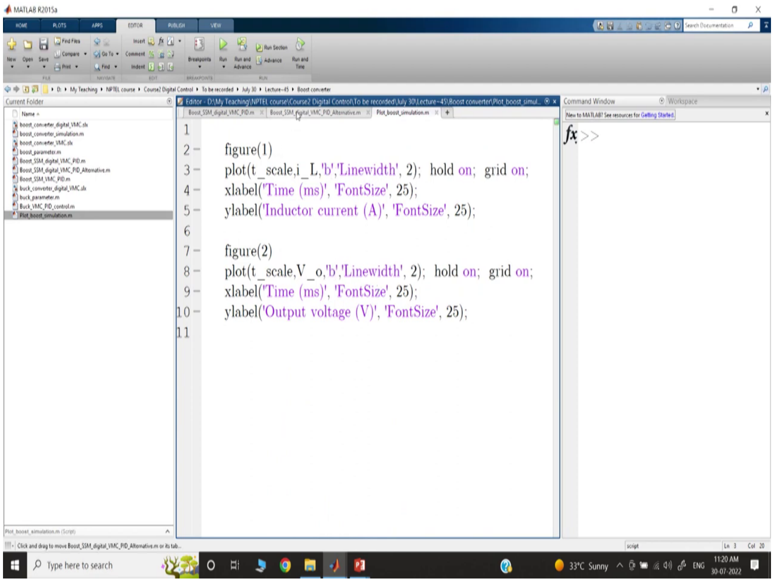

So, in the alternative method what we will do? We will design the PID controller by setting K i to be the same as we discuss and K d which means, the fraction of omega C times 1 minus d whole square into rhp 0 by F m into V m. F m is nothing but 1 by V m, K d equal to 0.5 times C, and K p for a small gain 1. So, let us run it and see what happens.

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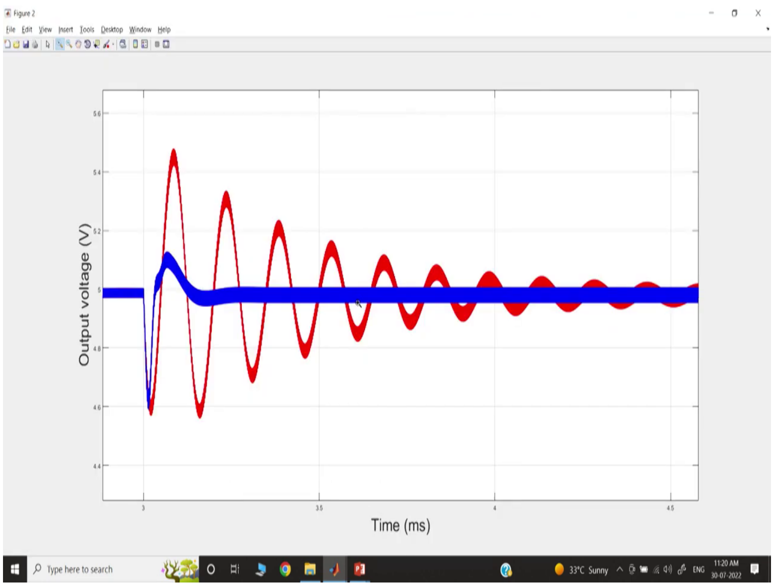

Again, we are setting 0.3 times. As you can see the blue color is the one alternative design method that is much better than the stable pole-zero cancellation. You can get a better responsive stable pole-zero by you know by shaping the proper output impedance of the boost converter.

But in this particular case just by loop cancellation; that means if you take the red color that was the response of the boost converter using stable pole-zero cancellation where we are only shaping the output like a loop transfer function. But when you take the loop transient, we have to consider the closed-loop output impedance.

So, in that point of view it is somewhat difficult to handle, but if you go for the alternative method since you are getting a fraction of the derivative gain fraction you know that C; that means, 0.5 C and K I we are setting same as the earlier formulation. But K d sorry K p we have taken a smaller value and we get very much a faster response as well as a very well damped response for the boost converter using the alternative design method.

So, in summary, I would say the second method the ordinary design method may be more useful when you deal with a low transient response. And then you can accordingly convert all these K p, K i, K d in the digital domain.

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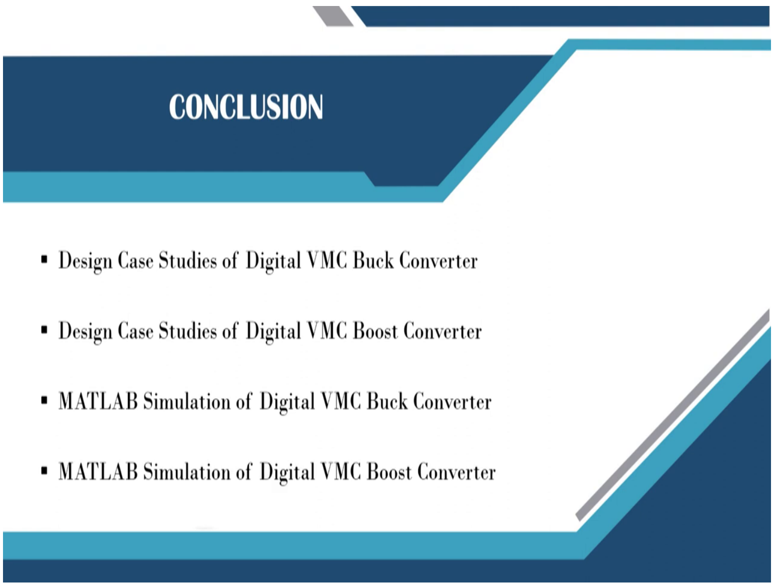

So, in summary, we have discussed the design case study of the digital voltage mode control buck converter. Then we have also discussed the design case study of a digital voltage mode control boost converter. We have shown MATLAB simulation case study for both buck as well as the boost converter digital voltage mode control. So, we want to finish it here for today's lecture.

Thank you very much.