Digital Control in Switched Mode Power Converters and FPGA-based Prototyping

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Module - 04
Modeling Techniques and Mode Validation using MATLAB
Lecture - 40
Model Accuracy with MATLAB Case Studies - Comparative Study

Welcome back. So, in this lecture we are going to talk about Model Accuracy with MATLAB Case Study, we want to consider some Comparative case Studies.

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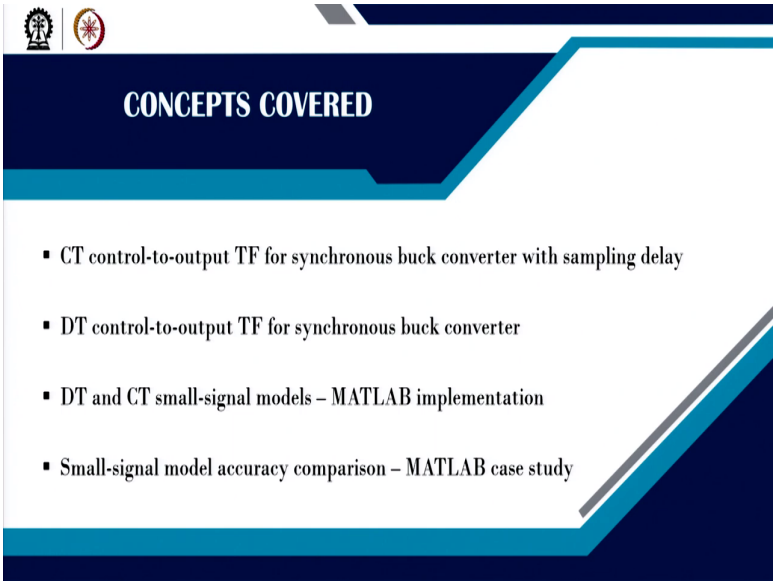

So, first, we want to recall or control continuous time control to the output transfer function, for a synchronous buck converter with sampling delay, we want to consider discrete time control to the output transfer function for a synchronous buck converter and then discrete time and continuous time small signal model MATLAB coding and finally, small signal model accuracy comparison using MATLAB case study.

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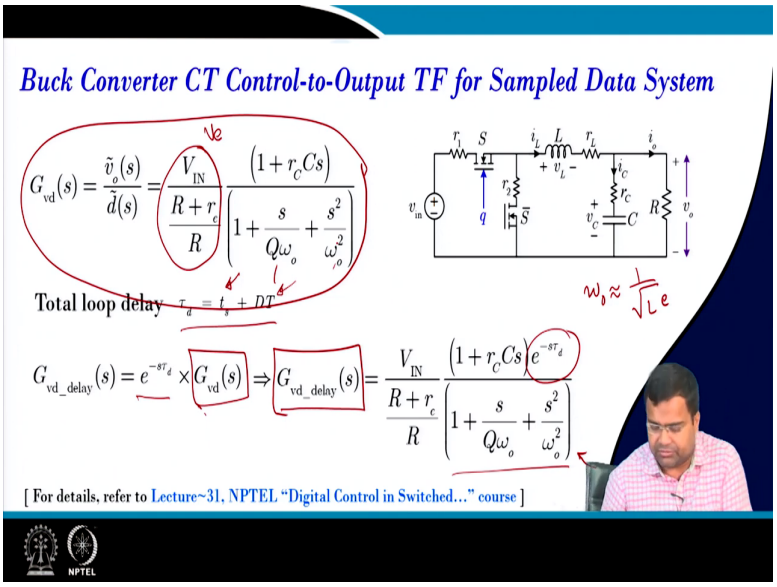

So, here if you remember our buck converter continuous time control to output transfer function; for the sample data system. So, we know G vd and this can be derived from our equivalent circuit model, this equivalent circuit. And we know in digital control there will be a total loop delay of t s plus DT. What is t s? That is the sampling plus you know computational time and DT is the DPWM delay.

Then the transfer function can be modified by considering the control to the output transfer function of the original converter into the delay amount which is e to the power minus s tau d, and then the control to output transfer function with delay can be considered the same thing multiplied by delay. And we have discussed this particular you know model in lecture number 31.

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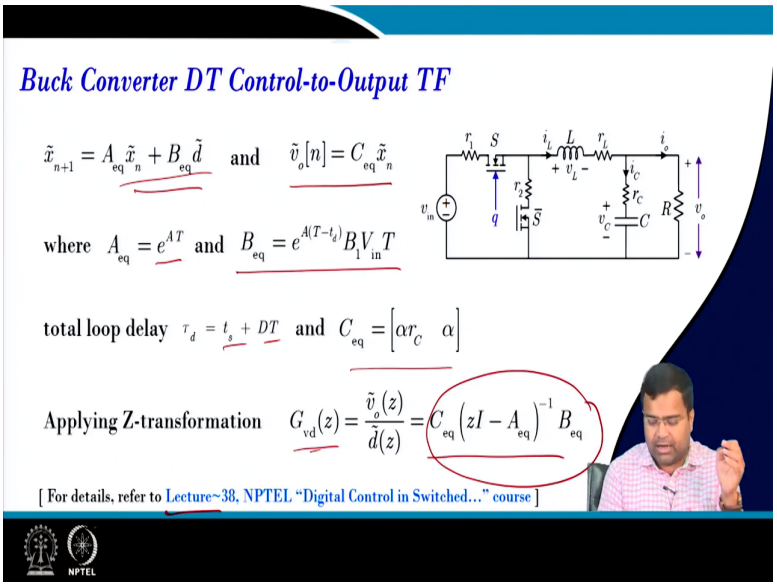

In NPTEL in this course; that means digital control in this particular course, in lecture number 31. Now, so buck converter discrete time control to output transfer function; that also we have discussed. Where we have obtained that the x n plus 1 tilde is equal to A equivalent to the x n tilde plus B equivalent to the d tilde and the output voltage is C equivalent to the x n tilde.

Where A equivalent for a buck converter, if we take the low side and the high side MOSFET Rds as identical, then their on and off matrices are identical. So, you will get e to the power at. Then B equivalent we also know e to the power A into T minus tau d that is the total delay B on V in into T. So, the total loop delay we have discussed that is the t s plus DT and this is the output matrix can be alpha r c in alpha.

Now, if we apply the Z transform here, we know that we can derive the discrete-time control to output transfer function; which is like this. Now, what is our objective? Our objective, first we want to draw the bode plot of G vd s without delay and then G vd with delay. Because in digital control we have the ADC delay plus DPWM delay and third on the same plot, we want to plot G vd z, but z is an irrational function of omega.

So, if you use MATLAB by default it will convert into a w domain which is a kind of bilinear transformation; that means, z will be replaced by w. So, which is a complex number in the frequency domain. So, we have discussed this like a discrete-time model, small signal modeling in lecture number 38. So, we are not going to repeat it.

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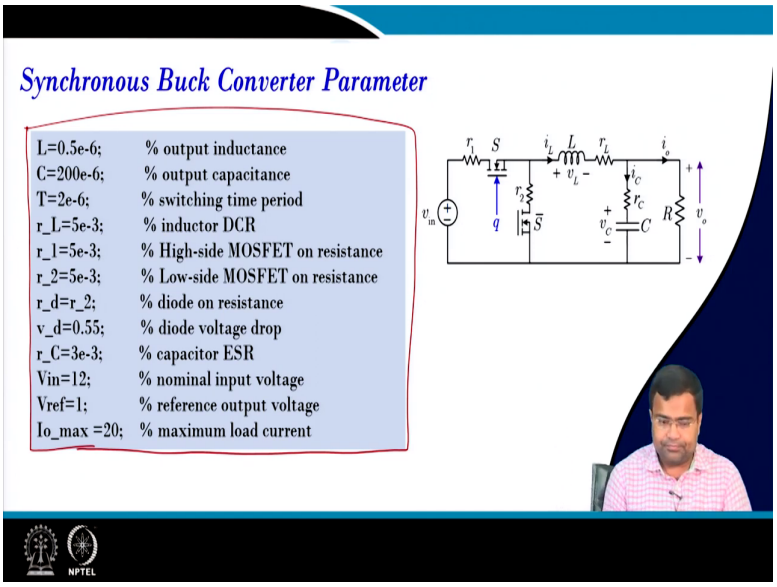

Now, we have considered the synchronous buck converter parameter, which is this and which we have considered for various other lectures. We have discussed also lecture number 35 and 36, where we have shown the large signal model validation. So, we have identified our discrete discrete-time accuracy, and it can exactly capture the cycle by cycle-by-cycle behavior with simulation.

Now, you want to see, if we perturb the discrete-time model to obtain the small signal discrete-time model, it is considered to be very accurate, but it is mathematically somewhat complex because you have an exponential term.

So, if you see, we have this exponential term. So, even if you get this transfer function, you may not have sufficient insight in terms of poles 0s, but if you go back to the previous expression, you know the poles and 0, these are very well known, and there is a Q factor omega 0, which is a function of in a practical buck converter.

It is almost identical to 1 by Lc. So, that means, we know there is an Lc oscillation like natural undamped natural frequency due to the Lc, we also know there can be an there is e sr 0 and we will also know the Q former depends on the load resistance as well as you know this parasitic like a r 1 r l and so on.

So, that means, the previous model gives us insight; in terms of poles and 0s and we have considered in our earlier NPTEL course that means, the control and tuning method, which

course that how to design a controller. In digital control, before we design the controller, we want to first make an analogy. Can we obtain a somewhat simplified version of a digital controller? That means, can we use this continuous time model with delay and can we compare it with the actual discrete time model?

Because the actual discrete-time model is very nice and very accurate, but it has less insight in terms of poles and zeros and that will make us very difficult to design a compensator. So, we want to design first in this course particular lecture, we want to see, how closely we can predict the behavior of the digital controller using a continuous time model, by suitably get providing the delay.

So, here we have considered, this buck converter as the case study and I am again recollecting the concept; that means, if you take a digital control, I am first considering the controller a simple pi d or pi controller or type 2 whatever.

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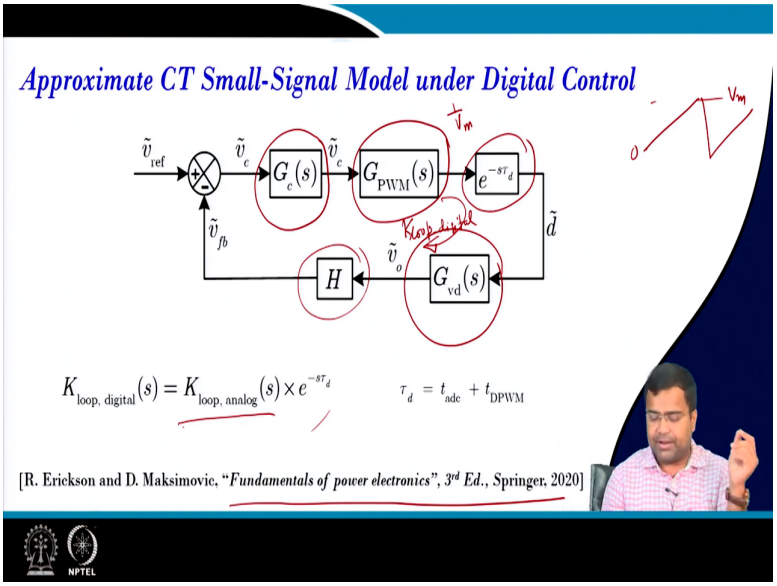

Then we have the PWM block and we know for analog control it is simply 1 by V m, that is 1 by peak voltage, where V m is the sawtooth voltage right this is our V m from 0 to V m, then this delay we know, this delay coming from ADC plus DPWM and G vd is the continuous time transfer function and H is the feedback gain and so total loop delay, that means if you consider this loop delay, K loop; under digital control, if you take it is under digital control, it is same as the analog control multiplied by the delay.

And the delay is adc plus DPWM, and you can get this you know basic concept; that means, how to model the digital controller, in this third edition of the fundamental of power electronics, where it introduces that, if we add delay, it can reasonably capture small signal behavior. So, we want to check using MATLAB.

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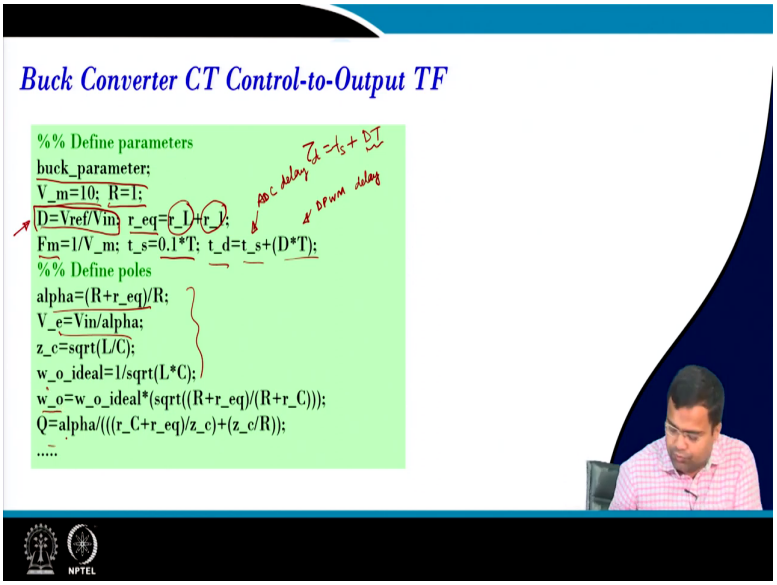

So, first, we will run will create a DOTM file, and we will again take the buck parameter, V m, we are keeping load resistance to be 1, this duty ratio may not be needed, because if you are using a closed loop control. The r equivalent is the r dcr, this is my dcr plus this is the on-state resistance, this is the modulator gain 1 by V m and we can introduce a delay 0.1. And this D is needed because we have an additional delay tau d, which is my t s plus D into T. The D T is a DPWM delay.

So, you can see the total delay is at ts plus D T. So, this is my ADC delay, delay and this is my DPWM delay, ok. Then we can define pole alpha, then all these things we have already discussed how to plug in; that means, this is the that means, if you go back to our expression of the continuous time, this I am taking as the V e which is V in by r plus re by r ok. Then there are the q w 0.

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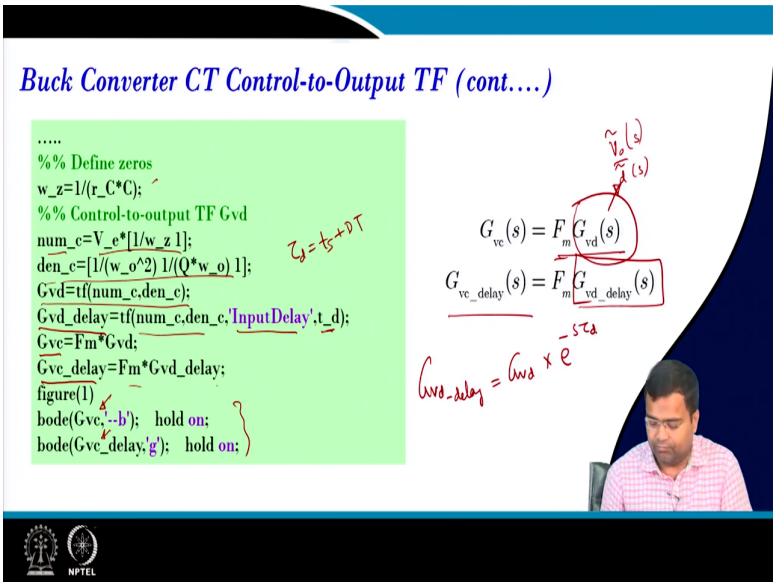

So, all these transfer functions we have plugged into the MATLAB model. W 0 practical then Q then after that e s r 0. So, the numerator, if we follow the transfer function, will be this denoted will be this G v d will be this, this is the continuous time, control to output transfer function without delay. And the delayed version of that will be the same transfer function, which means the polynomial numerator and denominator with an input delay which is t d.

And what is tau d? Tau d is t s plus d t. Then our control to output transfer function with modulator gain will be Fm into G vd and the delayed version will be F m into G vd delay. Now we are plotting, first, the Bode plot, without delay control to output transfer function with delay control to output transfer function and these are all continuous time.

So, this is our G vc, this is our standard buck converter V 0 2 d this one and then, we are adding multiplying with the modulator delay; modulator gain. This is our delayed version, everything is the same only this is coming with an additional delay, which means our G vd delay is nothing but G v d into e to the power minus s tau d.

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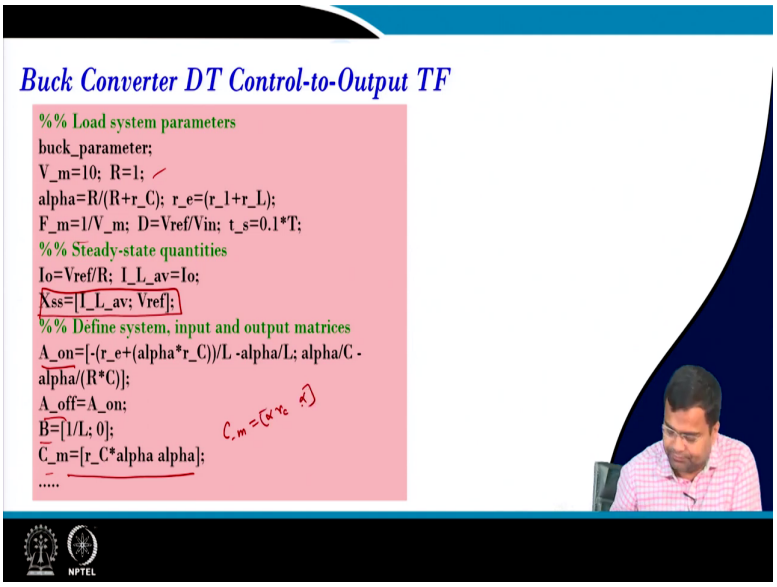

Next, where buck converter discrete time out control to output transfer function. Say again, the same parameter everything same V m, now we are using the discrete-time small signal model. So, for that, this will be derived around the steady state operating condition, and we are taking the average inductor current and the reference voltage. Next, we are taking on time matrix, off time matrix and, since r on rds, one's are identical for both the low side and high side.

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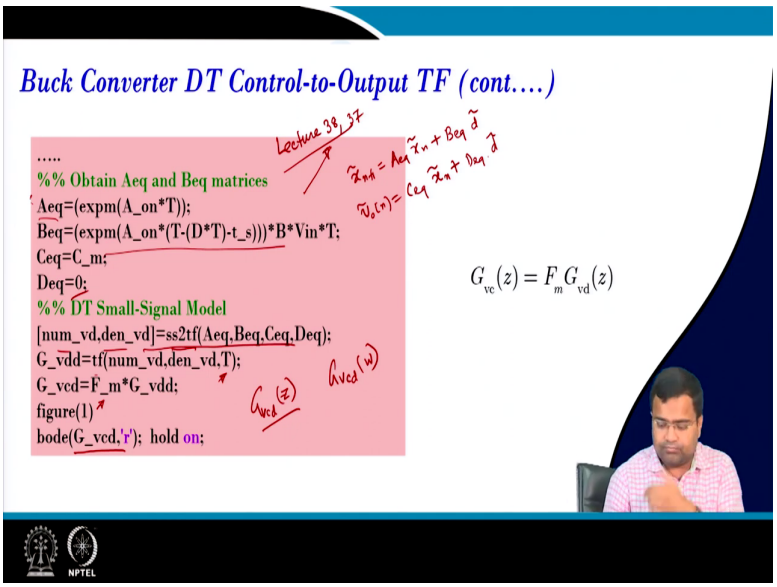

So, if on and off-state matrices are identical, then we will get a B matrix and a C matrix. Next, we can compute an equivalent, these are discussed in lecture number, if you follow the lecture I think 38 lecture 38 as well as 37. So, you can get a discrete-time small signal model. So, b equivalent also you can get from there, in this course, and then the C equivalent is the C m, what is C m? It is nothing but this.

So, this is nothing but C m is that alpha r c alpha. Then, we will get the numerator; now since we are this is in the that means, this expression is in the state space form; in the perturbed state space form, b equivalent into d perturbation, then we have V 0 n which is C equivalent into x n tilde plus d equivalent into d tilde.

So, the d equivalent is 0. So, we can simply use state space to transfer function and G vdd transfer function, we have to specify; which means, we got the numerator and denominator, by converting state space to transfer function. Since it is in discrete time, so we have to specify the sampling time; that means, it will consider a discrete-time transfer function.

Then we have to multiply with our modulator gain and we will simply bode plot. This way, we are getting G vcd in the z domain. So, when you do a bode plot, it will plot G vcd in the w domain, where the z and w domains, are more or less closed for lower frequency approximation, so they use some bode in the bilinear transformation.

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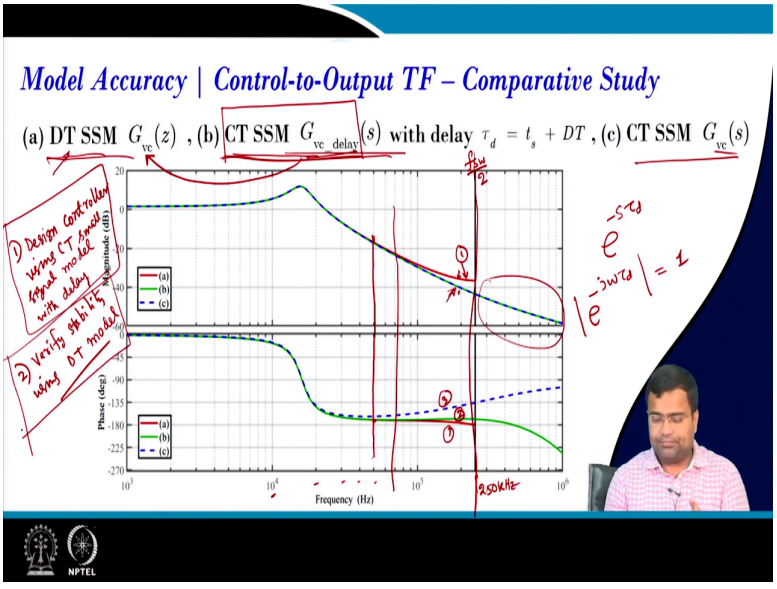

And this is the plot, in this plot, the first one is a which is this one, which is the discrete time control to the output transfer function, and since it is in discrete time, you cannot plot beyond half of the switching frequency. So, this is half of the because it is a Nyquist frequency range. After that it is invalid. Even for a continuous time also it is invalid because, we are applying an averaging technique followed by linearization, so it requires a minimum of 2 cycles to get the effect.

So, that is why all the small signal models are standard averaging like conventional state-space averaging, the small signal model is valid up to half of the switching frequency. If we consider this line, up to this half, you will find the first one, is the accurate gain plot and the blue and the green; that means, there green and the blue, gain plot is identical.

So, where the blue one is the continuous time transfer function with delay. And the c which is the dotted blue line is a continuous time without delay. Since, e to the power s tau d, if we write in terms of j omega; that means, e to the power minus j omega tau d, if you take the magnitude it is 1. So, that means, with and without delay, this transfer function and this transfer function gain should be identical because a delay will not alter the gain; that is why they are overlapping.

But, you can see the effect in the phase. You can see without delay, the phase is higher whereas, due to this delay, the blue and sorry, the green and the red one is the exact discrete-time model, small signal; that means, it is derived, it is a discrete-time small signal model, which is derived from the discrete-time large signal model and 2 is the continuous time model with delay and 3rd one with the continuous time model without delay.

So, in terms of phase response, the continuous time model, with delay more or less captures that discrete time model; which means, they are more or less close, very close. Only there is a slight difference in gain, but that is fine because that is even close to half of the switching frequency, but typically we design compensators with nearly one-tenth switching frequency; that means if you know this is our 250 kilohertz because our 500 kilohertz is our switching frequency.

So, one-tenth will come around 50 kilohertz, which means this is our 10 kilohertz, 20, 30, 40, 50. So that means, if we consider this particular one, so the gain is identical; that means, there is no difference and the green and red, are also phase are identical. So, even if you slightly increase this; that means, if you go slightly beyond, gain and phase are a perfect match for the

discrete-time small signal model and continuous time small signal model with delay; that means, it is reasonable that, this transfer function with delay can be used to design digital controller.

But, the question is if that is so, then what is the need for a discrete-time model? And we will show using a case study that, even though we match small signal model behavior, very accurately, and closely, the averaging technique loses the information of ripple information. So, I will show you due to this sampling, there will be subharmonic instability if you try to push the gain when this model cannot capture the subharmonic instability, but the discrete-time model can perfectly capture it.

So, we will show that case study, in the design case study, during the design. But, here we want to make sure that, as far as long as they are stable; there is no subharmonic, then you can you know you can design the controller, low bandwidth controller; I would say one-tenth or one-fifth of the switching frequency that kind of control, small signal based controller you can easily design using continuous time model with delay ok.

So, actual stability has to be verified with a small signal discrete-time model. So that means, in summary, we can design a controller using a continuous time small signal model, continuous time small signal model, and continuous time small signal model with delay. And verify stability using a discrete-time model; that is the final thing. Because the discrete-time model, since it retained the ripple information, so you can predict the first scale or the subharmonic instability, due to sampling.

Because in this course, I will be showing, even for you know for low duty ratio or even the condition where the analog control is perfectly stable, even if you introduce a sampling delay, that not only causes some deviation in the phase; that means, it will degrade slightly phase, but more predominantly, it may introduce sub harmonic, which cannot be captured by this model.

So, that is why this model is good to design the controller, but finally, we have to check the stability of this discrete-time model.

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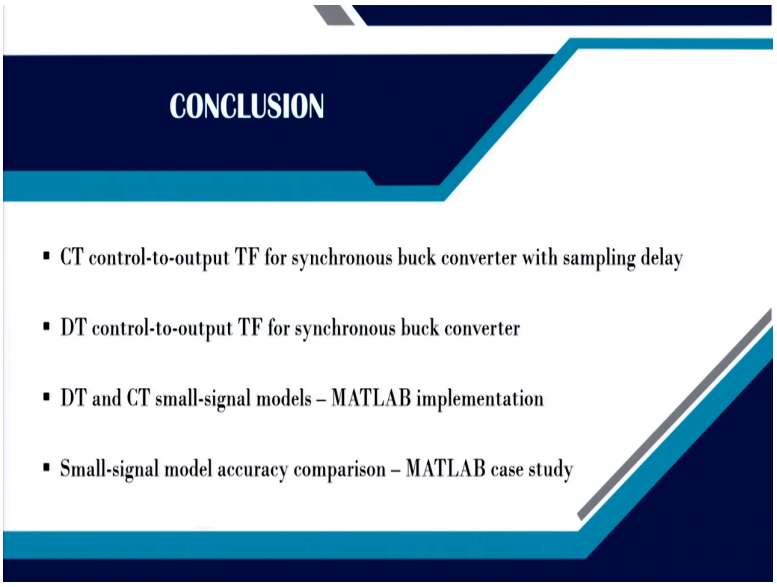

So, in summary, the continuous time control to the output transfer function of the synchronous buck converter we have discussed, with and without delay, we have discussed discrete time control to the output transfer function, and we have compared, we have implemented I have shown you the step for implementing discrete time as well as the continuous-time small signal model, and how to obtain the Bode plot.

And finally, I have shown our MATLAB case study to show that you know up to a small signal bandwidth that means even up to one-fifth of the switching frequency, the gain and phase can very accurately match with both discrete-time small signal model and continuous time small signal model with delay.

So, that is why in summary, we can say the continuous time small signal model with delay, will be a very good solution to start designing the digital controller; because we can get physical insight; in terms of poles and 0s. But, we will say in the subsequent design class, that we need a discrete-time model to predict stability, even though we can predict various non-linear phenomena, or even if you want to do some kind of spectral spreading the discrete-time model is very effective.

So, in summary, I think we have understood all the modeling aspects, we have validated using MATLAB and in the subsequent lecture we will be talking about, the design process that is it for today.

Thank you very much.