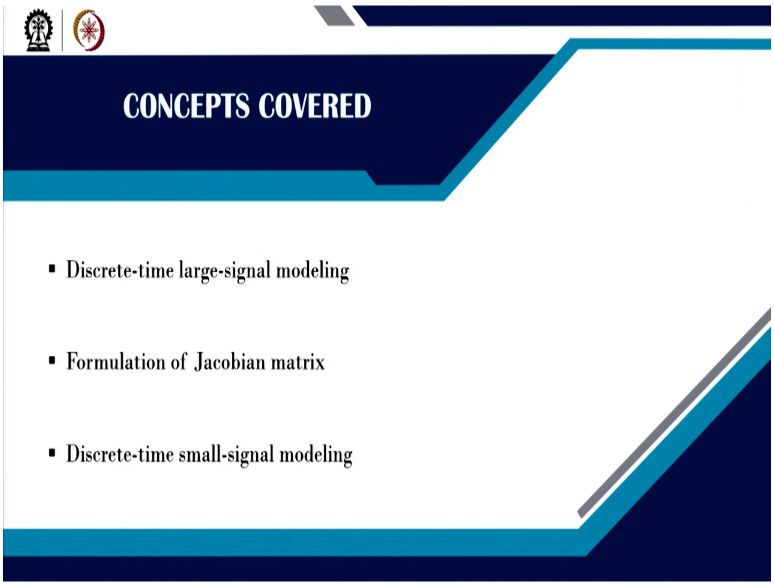
Digital Control in Switched Mode Power Converters and FPGA-based Prototyping

Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 04
Modeling Techniques and Mode Validation using MATLAB
Lecture - 38
Derivation of Discrete - Time Small - Signal Models - II

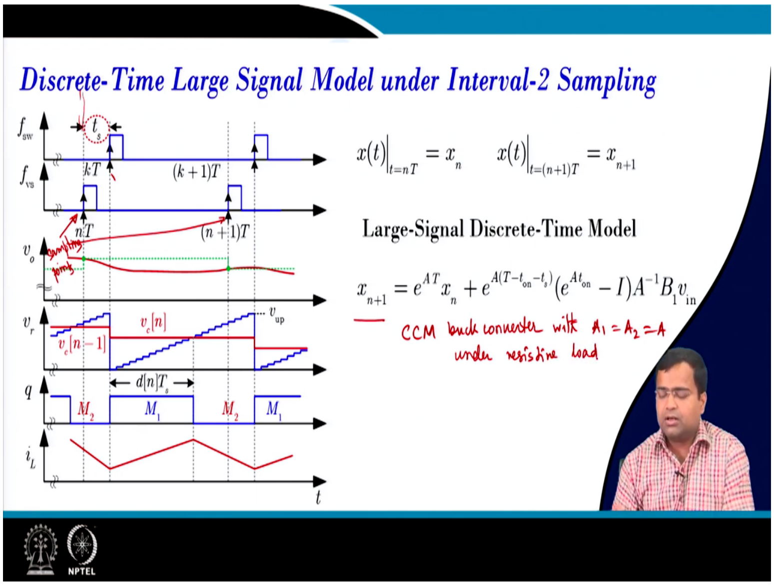
Welcome. So, in this lecture, we are going to continue I mean this is a continuation of the previous lecture where we want to Derive the Discrete Time model straight away from the discrete-time large signal model.

(Refer Slide Time: 00:37)



So, you want to derive the discrete-time small signal model from the discrete-time large signal model. So, first, we will again recapitulate our discrete-time large signal model, then we need I will show how to calculate the Jacobian matrix and then how to discrete-time small signal model.

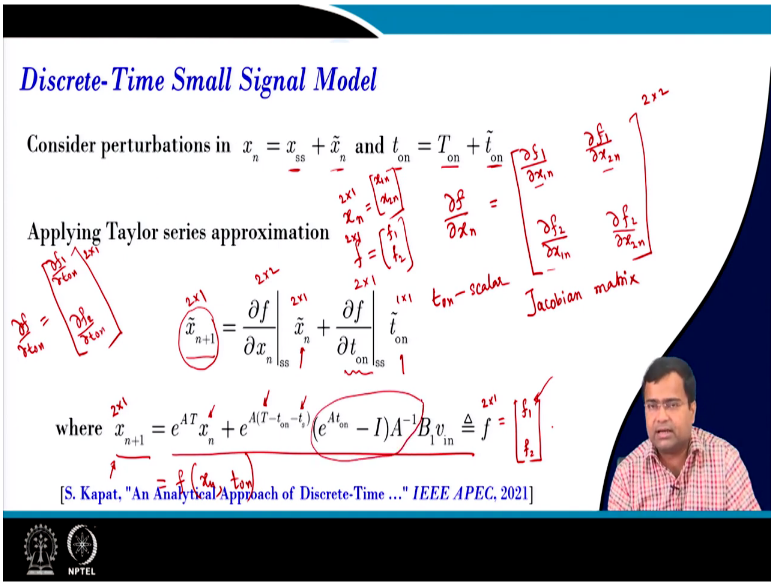
(Refer Slide Time: 00:53)



So, here discrete-time large signal model under interval to sampling. So, this waveform we already know; that means, this is our sampling point, these are the sampling point sampling points and these are the switching point this is the switching point and there is a delay and that is to accommodate a to d converter conversion time and computational time.

Here let us say the state variable here is x n at t equal to n T that is the beginning of the nth sampling interval and then at the end of the sampling interval it is x n plus 1 and we know the large signal model discrete-time from the previous to previous lecture x n plus 1 can be written as this is for a buck converter. So, this is for a CCM buck converter with A 1 equal to A 2 equal to A under resistive load ok. So, we know this expression we already know how to derive a discrete-time small signal model for this particular case.

(Refer Slide Time: 02:13)



So, if you understand this model then we can derive a small signal model for other cases also. So, first, if we talk about this perturbation x n it is a steady state value around that steady state value we are considering the perturbation and similarly, the on-time can perturb these are perturbations and this is a steady state value.

Now, we know the large signal model x n plus 1 this expression and this is a vector because we are talking about two states one is the inductor current and the capacitor voltage and these are the values of the state at n plus 1 clock sampling instead and this; that means, this will also be 2 crosses 1 and let us say it is f 1 and f 2 ok. Now we can obtain it straight away; that means if you write this you f. So, this is a function of; that means, here it is a function of it is a vector function of x n and t on.

On the other parameters are constant; that means, the time period is constant sampling delay is constant, but only the initial condition can change and the on-time can change because on time is a control variable. So, how to obtain this part of the small signal model? That means, what is the perturbation in the final state at the sampling instant in terms of the perturbation of the initial state and terms of the perturbation on time?

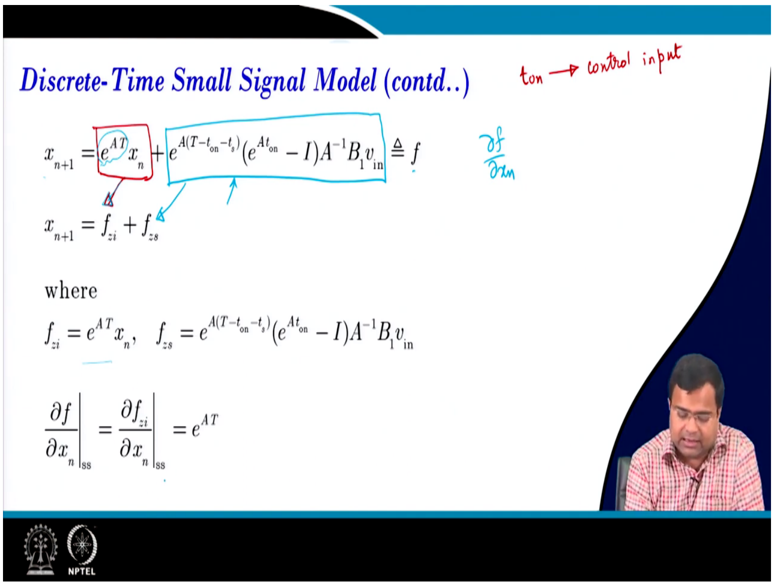
And in the previous lecture, we saw we considered individually the perturbation of the initial state and we saw the effect. We have considered individually the on-time perturbation or the duty ratio perturbation and we saw the effect and then we combined here we can obtain mathematically. So, first of all, if you take it; that means, it is a function of two. So, what is dou f dou x n? Because x n is a 2 cross 1 vector and f is also 2 cross 1 and this is nothing but f 1 f 2. So, this can be obtained.

And suppose x n has two states x 1 n x 2 n then how do I get it? So, you can get you f 1 dou x 1 n, dou f 1 dou x 2 n dou f 2 dou x 1 n, and dou f 2 dou x 2 n this matrix is known as the Jacobian matrix and these are 2 crosses 2 matrices. Here that means, we are partially differentiating with respect to the first element with sorry we are partially differentiating the first element of the function vector function with respect to two states separately.

And then we are again taking the second element of the function vector function and separately differentiating with respect to this and this is a standard thing. So, this is a Jacobian matrix for you f dou x n. Now, what about t on? t on is scalar it is a scalar quantity than what will be dou f dou t on? What does it look like? So, it will look like you f 1 dou t on and dou f 2 dou t on. So, it will be simply a 2 cross 1 vector.

Because here f is 2 cross 1 and t 1 is a scalar. So, we can get another matrix this will be 2 crosses 1 and this is scalar, this is 2 cross 1 this is 2 cross 2 and overall we will get 2 crosses 1 and you know the direct thing they are already well known, but in this paper, it also presents what will happen if the matrix is noninvertible then how to deal with this? So, all these things are presented in this paper.

(Refer Slide Time: 06:50)



Now, in the descriptive small signal model since we have a vector function and you can see there are two parts the first term is here, and we call as a 0 input because this term there is no input what is the mean input? So, here we are talking about t on to be the control input because, in any switching converter, the only control variable is the on-time and off-time adjustment of the gate signal.

So, under pulse width modulation we generally control the duty ratio which is equivalent to the on-time. So, the duty ratio in the time period is on time which is why we consider t to be a control variable control input. So, we call this term to be input so; that means, if you take this part in this part, we call it a zeroth state; that means, here zero state means the first thing effect due to the initial state that combination.

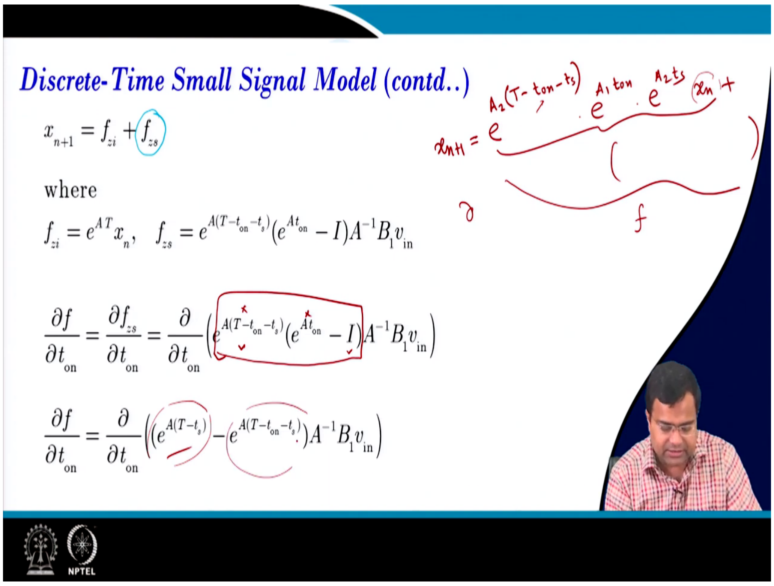
That means, if you set if you talk about the perturb term and if you do not consider any perturbation on time. So, the perturbing effect due to this will be 0 and if there is any perturbation in the initial condition it will be captured by the 0 input function. So, we are separating the x m plus 1 of the total vector into two components; one zero input component, and there is a zero state component.

Because in the 0 input component, there is no on the term, and in the zero states in component, there is no x n term, but as we move forward it may so, happen that e to the power A this matrix there is you know the cascaded chain of this exponential matrix, there can be t on the term, but when you consider the variation the perturbation the first term we are only talking about the variation in the initial state.

So, for those of you who know the case of that case, we will consider the on-time to be constant. For the second case, we will consider what we will consider. We will consider x n to be constant and we are only talking about the effect due to you know; that means, here the effect is due to on-time variation. So, we are just separating now where f z 1 equal to this term and it is visible from the.

So, what do we want to obtain first? We want to obtain you f dou x n since f has two parts and the first part has x n as a function of x n. So, we will state I take you f z I by x n that will be e to the power A T.

(Refer Slide Time: 09:40)



If you take the dou f dou t on particularly for this example only this term has t on dependency, but it may so, happen I told you that there can be t on dependency in general. For example, in if you take interval two DC-DC converter what was the original expression? We know that x n plus 1 we started with what interval two sampling at least the first term will be t to the power A 2.

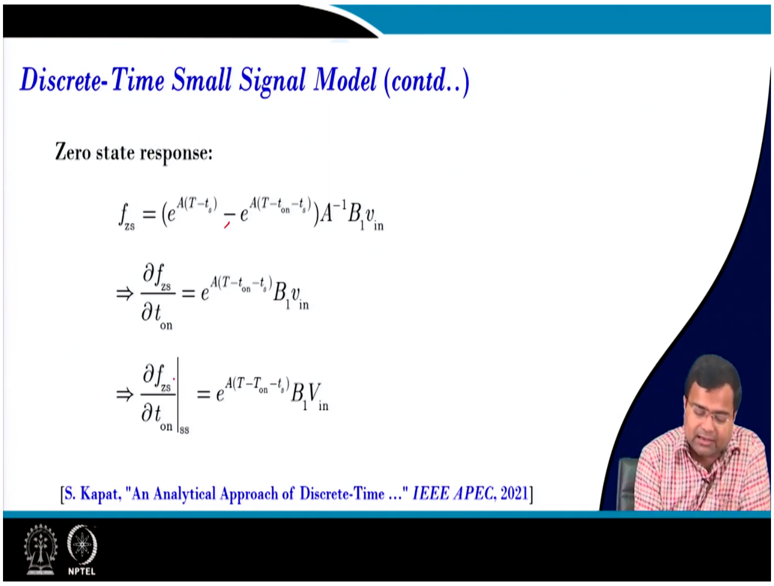
So, if you go back to the large signal model, it will start from right to left. So, the left will be T minus t on minus t s then e to the power A into t on then e to the power A 2 into t s into x n. So, this is a first-term plus the other term. So, I am not writing. So, in this case, you can see this function is a function of both x n as well as t if A 1 A 2 are not equal then you cannot simplify like this.

So; that means, in that case, if you obtain dou x n plus dou f; that means, the whole thing is f dou f dou x n then you need to also consider you know x n this will be straight this will be replaced by their steady-state value, but if you consider dou f dou t on then we have to consider x n as a constant, but we need to also take out this matrix component when we take differentiate respect to t on so; that means, in that case, it will not be so, straight forward.

But it is straightforward mathematically we can always obtain this expression so; that means, if we t on if we differentiate in this case first we will do we simplify this expression; that means, we multiply the first term if we multiply this term and this term what we will get? It will be A T minus t s and that is exactly this and if you multiply this term with this term it will be simply this term only.

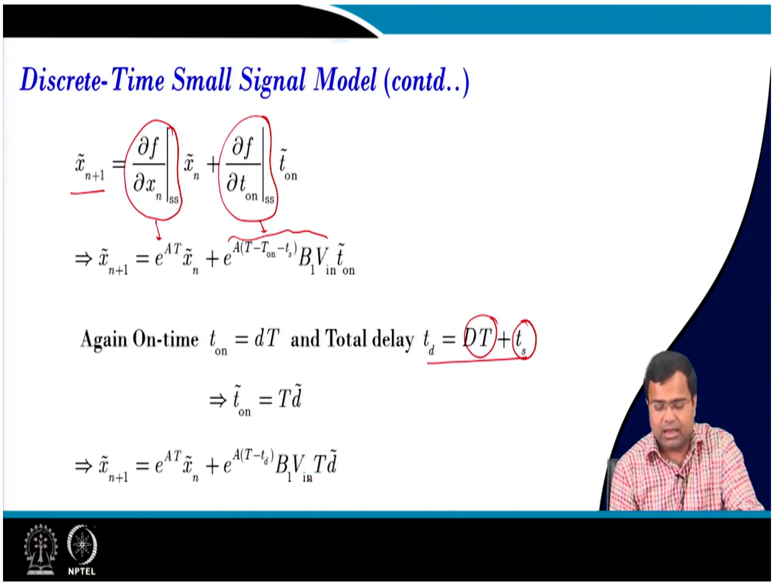
So, it is there and if you differentiate the first term there is not in dependency it will be 0 and in the second term, there is a t on dependency.

(Refer Slide Time: 11:59)



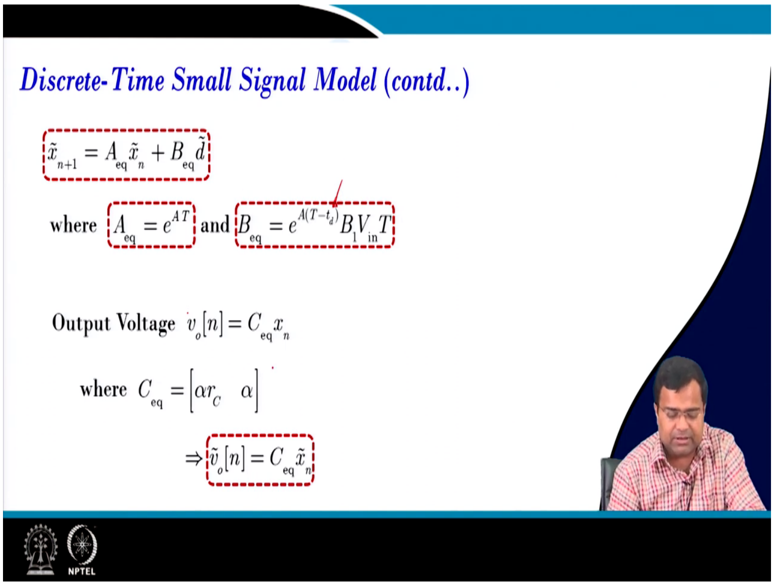
So, now we are continuing. So, zero state response, and if we differentiate with respect to t on so; that means, this negative sign is there and one negative will come out with respect to t on. So, you will get positive. So, overall this function will be. So, it will be like this. And The details of these steps are discussed in this paper. So, here you will get dou z s f z s by t on will be this term.

(Refer Slide Time: 12:30)



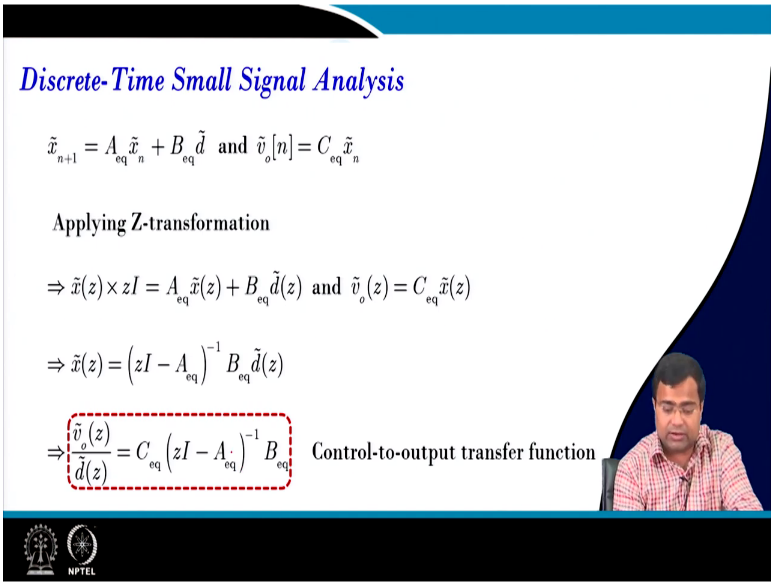
So, in summary, if you want to get total perturbation at the output, it will be this matrix into x n perturbation and this matrix into t on perturbation and this is nothing but a to the power A T and this is nothing but this term and again if you take a PWM pulse with modulation if you replace t on by dT and the total delay if you consider the delay due to DPWM as well as the ADC delay. So, it will be t on perturbation will be d. So, you can replace here this term and this will be B 1 V in.

(Refer Slide Time: 13:07)



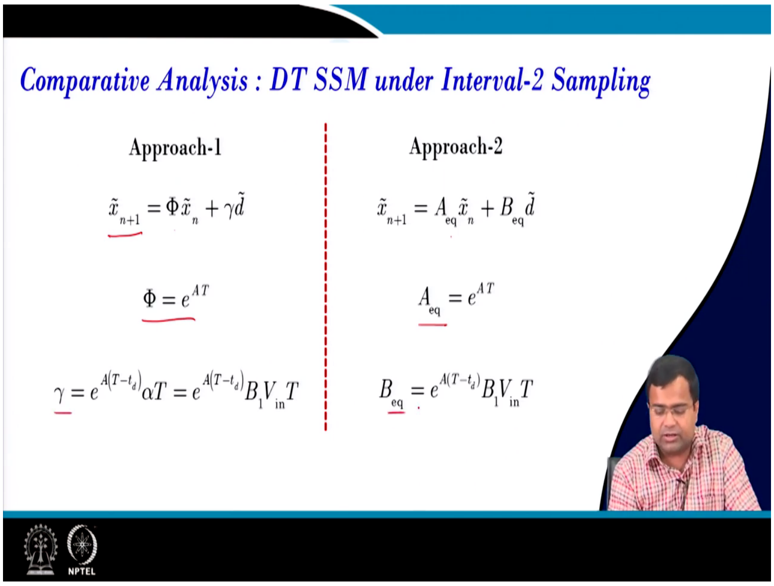
So, overall x n plus 1 perturbation is A equivalent to x n perturbation B equal to d perturbation that A equivalent equal to e to the power A T and B equivalent to e to the power A T minus t d, and this t d includes the delay due to DPWM plus ADC conversion time and this is an output function perturb model and if you take perturbation here then you will get simply C equivalent to x n perturbation.

(Refer Slide Time: 13:33)



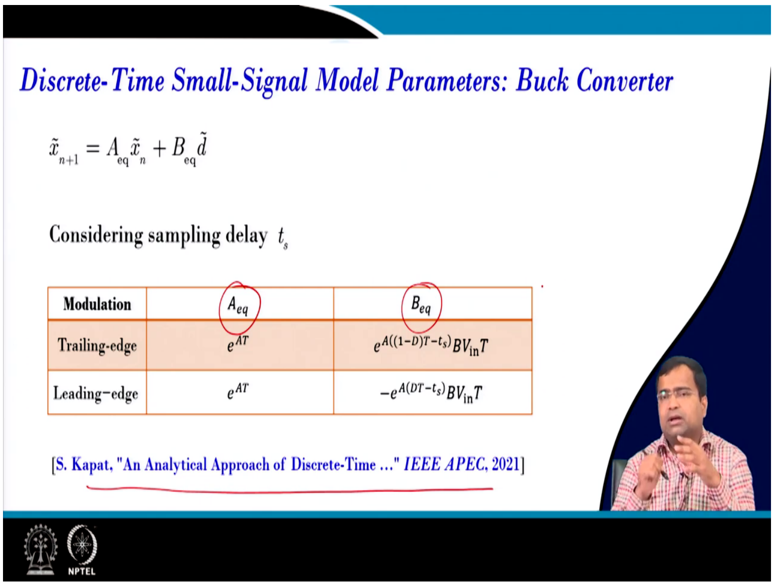
So, the discrete-time you can obtain apply z transform then you will x of z z I A equivalent. So, you can get z transform and then x of z can be obtained by z I minus equivalent inverse B equivalent dz and if you want to get control to output transfer function v 0 z by d z is simply C equivalent z I minus A equivalent to the power that whole inverse into B cube.

(Refer Slide Time: 13:59)



So, if you compare the previous lecture and this lecture you know there we represented x n plus 1 perturbation as a phi into x n perturbation gamma into d perturbation. Here x n plus 1 perturbation A equivalent to x n perturbation plus B equivalent to d perturbation and phi and A equivalent are same then gamma and this B equivalent are also same so; that means, we can state I get the small signal model from the large signal model itself.

(Refer Slide Time: 14:31)

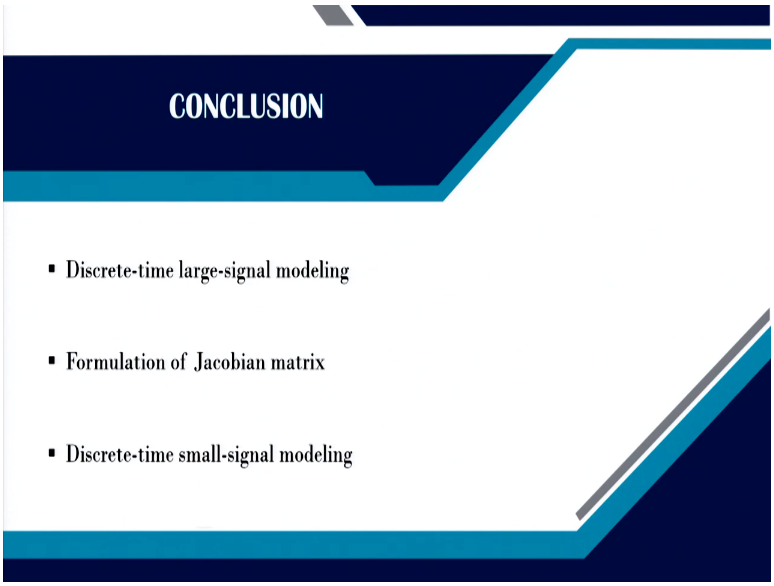


And you know if you take the overall this discrete-time model considering the sampling delay then you can get the trailing edge and leading edge modulation various expressions of A equivalent and B equivalent and the detail of the steps summary of and this technique actually where we obtain small signal model directly from the large signal model using Jacobian, this model can be extended for variable frequency; that means, constant on-time constant off-time control trailing edge leading edge various modulation.

And one by only considering the large signal model and also since we are also considering large signal model. So, we can validate the large signal behavior of the system, and also large signal model can be used for non-linear analysis. So, in summary, all these aspects are detailed and discussed in this paper. So, if you want to get the non-linear phenomena the large signal behavior at the same time you want to get the small signal for the controller design.

So, all this can be combined or unified and for different techniques, the details of the small signal models are discussed.

(Refer Slide Time: 15:41)



So, in summary, we have discussed discrete-time large signal modeling, we have shown how to compute the Jacobian matrix and we have also discussed how to derive the discrete-time small signal model straight away from the discrete-time large signal model; that means, we can develop the digital controller based on our small signal analysis. So, we want to finish it here for today.

Thank you very much.