Digital Control in Switched Mode Power Converters and FPGA-based Prototyping

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Module - 04
Modeling Techniques and Mode Validation using MATLAB
Lecture - 37
Derivation of Discrete - Time Small - Signal Models - I

Welcome back. So, in this lecture, we are going to talk about the Derivation of the Discrete-Time Small Signal Model and there are two parts. So, this is Part I and the next lecture will continue Part II.

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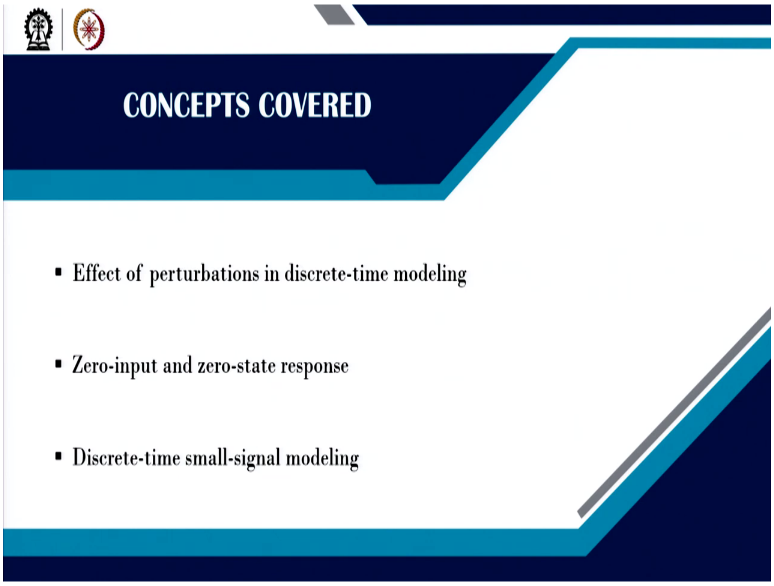

So, in this lecture we will first talk about the effect of perturbation in discrete time modeling then zero input and zero state response; that means, what is the effect of perturbation in the initial state and what is the effect of perturbation in both initial as well as the duty ratio there is a control input. And finally, we want to derive the discretized small signal model.

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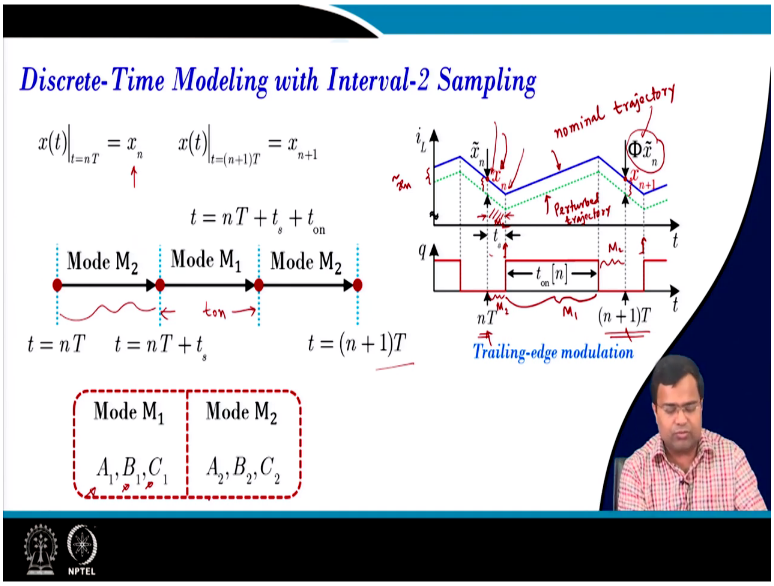

So, first, we will start with the interval 2 sampling discrete-time model. So, this waveform shows the waveform of the interval under interval 2 sampling under trailing edge modulation. So, you can see that at the rising edge of this clock, the switch turns on which means this is sorry this is not the rising edge. So, at this rising edge of this switching clock, the switch will turn on ok so, this is why it is trailing edge modulation.

And here this waveform; that means, this dotted one is the this is a nominal trajectory. So, this is a nominal trajectory is a current waveform, but you can think of trajectory because if we combine both current and voltage then it will look like a trajectory and the dotted line is a perturbed one. So, perturb trajectory in this case it is the inductor current waveform.

And this amount of perturbation that we have given here is the x n pert. And this x n perturbation I would say this is different; that means, this is the same difference where x n is the initial state at the beginning of the cycle; that means if we consider the blue waveform even if you consider whether nominal or perturb. So, any of this waveform at the point of this is n T which is a sampling point, and n plus 1 T, which is also a sampling point.

So, if we take this sampling point whether we take nominal or the perturbed trajectory at the sampling point we are talking about x of n which is the value of the state at nth clock instant at the beginning of nth sampling instant. And the end of the nth sampling instance there will be another sampling. So, we are talking about x n plus 1 and if there is a perturbation in the initial state we want to find how this perturbation is getting propagated at the end of the cycle or end of this sampling instant that mean interval and that we call about phi of x n tilde.

So, we need to find phi. How to get it? So, let x m is that you know the state variable that is the representation at the beginning of the nth sampling instant and this will start with mode 2 because we are talking about this particular duration the switch is off and we call it as a mid mode 2. So, this is the duration when it is mode 2 and this is the on-state duration which will be mode 1 again this interval will be mode 2. Mode 2 means the off state and mode 1 is the state.

So, here this duration interval is a t s and that is the delay. So, we are inserting; that means, we are capturing the sample here, but the actual switching is happening here. So, this delay is provided to accommodate ADC conversion time as well as the controller computational time. So, this t s is the duration of this mode 1 that is the delay. Then mode 1 starts and mode 1 duration the on time that is the on-time in mode 1 that is the duration of this interval ok so, duration of this mode 1.

Now then next the rest of the duration the final time is the n plus 1 t where we are taking another sample. So, we are talking about the mathematical model between the two subsequent samplings where the starting value is x n and the final value is x n plus 1. We want to see if the x n we perturb the initial condition what will happen at the end of this sampling; that means, the cycle and we are talking cycle means it is synchronized with the sampling time.

So, this is for mode 1 the matrix is because we already know the state space model for mode 1 the matrix is A 1, B 1, and C 1 what is A 1? A 1 is the in the state matrix. B 1 is the input matrix and C 1 is the output matrix. Similarly, for mode 2, there are A 2, B 2, and C 2.

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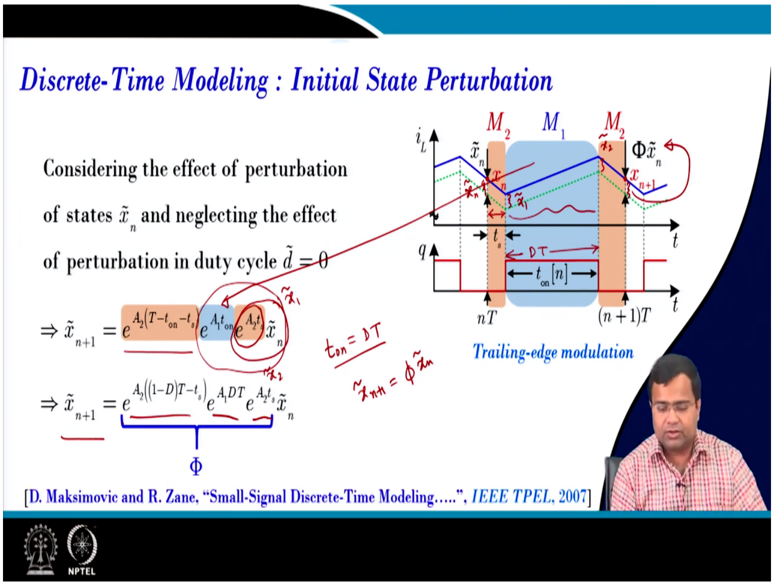

So, how do you? So, now, we are considering the initial state perturbation; which means, this is my x n perturbation this duration and we are trying to find this one; that means, this 1 and this is nothing but our phi of x n. So, here we are not disturbing the duty ratio; that means, we are not changing. So, this is like an, in this case, a constant on time or if it is under pulse width modulation the constant duty ratio then x n plus 1 can be obtained by this.

So, first x n propagate through this and this is my mode 2 and the mode 2 matrix is A 2 then it propagates through the next mode which is my A 1. So, it goes to this; that means, this one is now x 1 tilde; that means, after m we can consider this is my x 1 tilde. And it will be propagating through this step which is A 1 t 1. And this 1 we can consider the x 2 tilde so; that means, we are talking about this is the x 2 tilde, and then end of this it will propagate through this matrix which is nothing but this one.

And we will get this n result you know here; that means, we are getting the final. That means, the perturbation at the end of the cycle will be propagating through first mode 2, then mode 1 then mode 2, and here we are talking about t 1 equal to D into T, because we are talking about pulse width modulation ok. And this whole matrix will be like phi because we got x n plus 1 tilde is the phi times x n tilde and this is presented in this paper. So, you can refer to the details.

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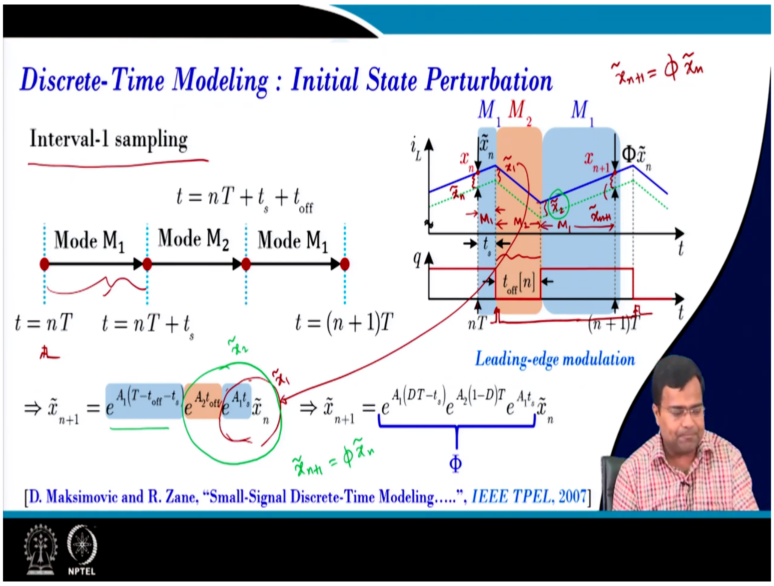

Next, this initial perturbation if you consider leading edge modulation where the leading edge modulation at the rising edge of the switching clock switch isswitchessorry the rising edge of the switching clock. If we consider the rising edge of the switching clock this is the rising edge of the switching clock the switch turns off and the switch turns on again.

So, at this rising edge switch turns off. And the switch turns off whenever a certain condition is made. Here since we are using off time. So, under open loop conditions, we generally fix the on-time and off-time. So, this duration off time is constant. But if we use any control method; that means, valley current mode control, then this off-time switch will be turned on again when the inductor current hit the valley current ok.

And we have discussed in our previous lectures about various architectures of valley current mode and then peak current mode control. Now under the leading edge of the switch, the sample is generally captured during the on state and that is why it is called interval 1 sampling.

That means the sample is captured at the on the state. Again we are talking about the perturbation of the initial state which is the x n tilde and we want to figure out what is the perturbation at the end of this cycle. So, here we can again consider the x 1 tilde, and we are considering the x 2 tilde, and finally, this is our x n plus 1 tilde and we want to obtain the x n plus tilde phi times of x n tilde.

So, that is our objective and again we can follow it starting with which is my sampling time the edge of the sampling then in the next mode goes to the duration of this mode is the delay. And again this delay is nothing but the delay due to ADC conversion and computational time and this mode is my mode 1; that means, the on state and that is exactly shown. This state is my mode 2 when it is off and again the rest of this interval is mode 1.

So, then if you propagate again the same the perturbation first propagates. So, this is my this one is my now x 1 tilde, which I have shown here then if I change the color this one will be my x 2 tilde which is shown here. And then it is propagating to the remaining interval which is A 1 into t minus T off t s and then you can get the final. So, this is like it is propagating stage by stage and this is the overall x n plus 1 equal to this and this is nothing but your phi.

So; that means, we are getting x n plus 1 tilde to be phi times x n tilde. And this is presented in this paper. So, in this paper, you know initial perturbation looking at the waveform you can get this perturb matrix by looking at how they are propagating.

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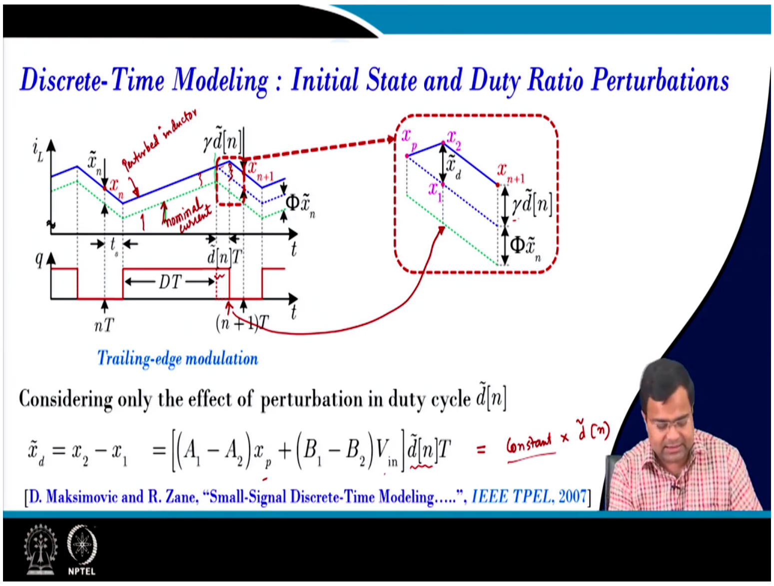

Next, we have only considered the initial perturbation. But now we also want to consider the duty ratio perturbation. So, here there are two things first you know you can see the blue one first one the blue one this dotted line up to this point is my nominal trajectory ok. That means this is my nominal trajectory. So, in this case, this is my nominal trajectory nominal current here it is inductor current.

And the first thing is if we apply an initial condition perturbation; so, this solid line indicates the perturbed current inductor current is after perturbation. And here we are applying a duty ratio perturbation. So, it will also get perturbed and this will introduce an additional state perturbation this additional test state perturbation, which will propagate through the rest of the duration.

So, this duration perturbation is due to the perturbation of the duty ratio. So, how do you compute the overall perturbation? So, if we zoom this portion then it turns out that at this time; that means, we are talking about this time after the perturbation this goes here. So, at this time your state perturbation occurring is due to the perturbation of the duty ratio and that is captured here that difference. And this difference is for you know gamma times D tilde and this difference, which was due to the initial condition perturbation is due to phi into x n tilde.

And we want to get, so if you consider only duty ratio perturbation then this x d tilde can be obtained by subtracting x 2 minus x 1 and it can be shown using averaging A 1 minus A 2 into x p where x p is the state under the steady state; that means, for the nominal part and then v in is the also steady-state value and you can get this d in tilde that is the difference in the duty ratio.

So, effectively this is something like you know some function; that means, some constant term into d tilde n and we will see how to represent this. And this concept term depends on what? A 1 A 2 matrix B 1 B 2 matrix input voltage time period and so on. And these things have been presented in this paper.

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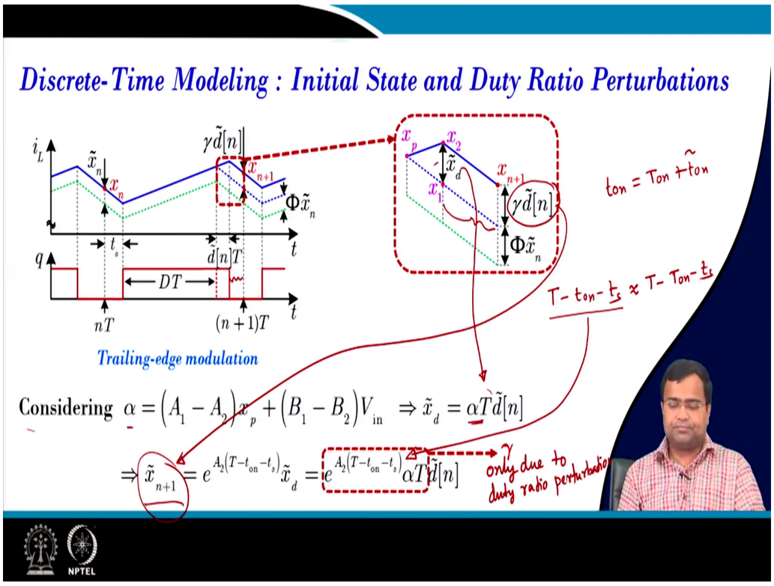

So, our objective so, is like an alpha; that means when you are talking about alpha. So, you will get alpha T which means, this perturbation is represented by because this is the cause of this perturbation is the duty ratio cause of this deviation of the state is due to the duty ratio perturbation.

So, this can be captured by alpha T times of this duty ratio perturbation this is an approximate version and you can write the overall state perturbation due to. So, if you only consider the duty ratio perturbation only then x n plus 1 perturbation. So, this is only due to duty ratio perturbation. So, if you only part of the duty ratio then it will cause a perturbation deviation in the state x d and this state will propagate through the rest of the interval. And what is the interval time?

So, you see it is the effect coming here. So, you are talking about this. So, this duration is nothing but T minus t on minus t s, and t on is you know it like a capital T on plus ton perturbation, which is a duty ratio perturbation and in this case, it is approximately taken as T minus Ton minus t s; that means, it is assumed that perturbation is very small.

So, you can get this time here you can get this time because you need to have a steady state value and this is A 2 time. So that means, this will be reflected as this function. So, this is the reflection, which is coming due to the perturbation here it is only due to the duty ratio perturbation ok.

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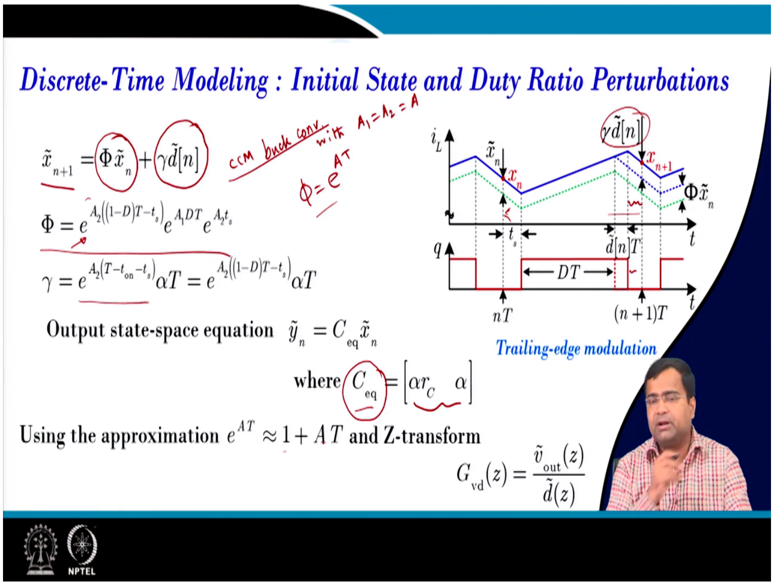

So, it is given here now the overall perturbation can be; that means, here since we are only talking about the perturbed trajectory and we are applying some sense of averaging to get this expression. So, here the perturbed model becomes a linear model. So, we can apply the supervision theorem. Why? You can break the final perturbation into two parts. The final perturbation is due to only the initial perturbation that we have seen.

The final perturbation is due to only the duty ratio perturbation we have seen and the total perturbation will be the sum of these two perturbations and this comes straight away from the superposition principle. So, you can get the phi that you have discussed and you can get gamma from this perturbation gamma is propagated through only this duration whereas, the initial perturbation happens throughout this whole interval.

So, that is why for a buck converter if you consider this A 1 A 2 matrix is common. So, this is all matrix for a buck converter if you consider a CCM buck converter or a synchronous buck converter where A 1 A 2 with A 1 equal to A 2 equal to A. So, you will get phi equal to simply e to the power AT; that means, this propagation is passed through the entire duration, but this only passes through this duration.

So, for the overall output equation, you can write the C equivalent into x n perturbation and this C matrix for this case since the C matrix is common for the buck because there is no discontinuity in the output voltage. But if you take a boost converter then you have to be careful to select this C equal matrix depending upon where you are taking the sample is it the interval 1 sampling or interval 2 sampling and you will get a structurally different C equivalent matrix for a boost converter and that also there are research papers.

So, that C equivalent matrix may lead to some at least marginal effect in the right of 10 0 effects depending upon where you are taking the sampling. So, use approximation if you approximate because these functions all are exponential matrix functions, but you cannot get a rational transfer function using this. So, you have to approximate and if you approximate then you will get a rational z transform.

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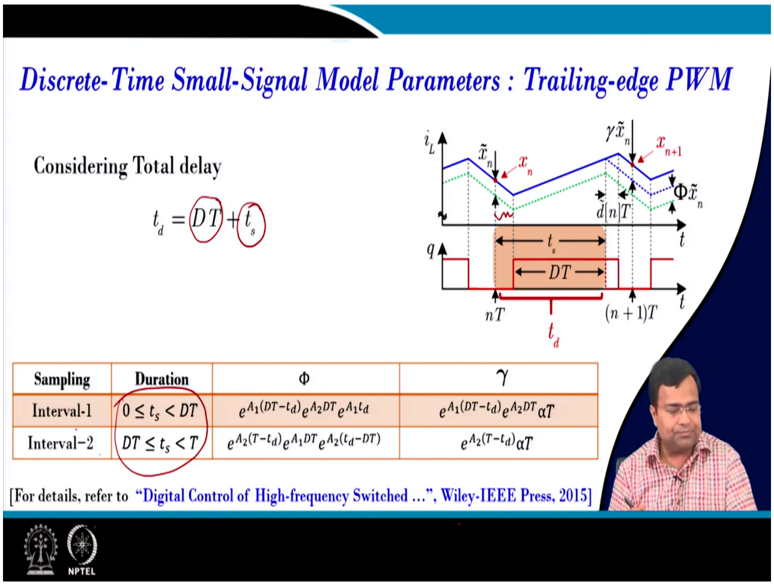

So, if you consider the total delay because we have discussed that tau d t d the 1 delay is due to this which this is the delay due to controller computation ADC conversion, but there is also a delay due to DPWM why? Because we will compare that if the analog PWM if you consider and the digital PWM consider. So, digital PWM if you just sample it and will hold for the whole cycle.

So that means, from the start to that the switching time the signal is not allowed to change which is why it is introducing a delay of DT. So, if you consider this total delay then you can get the phi n gamma for interval 1 and interval 2 sampling and whether the duty ratio of this sampling delay is greater than DT or less than DT based on that you will come up with two different forms and what will be phi and gamma they are presented in this book you can get detail.

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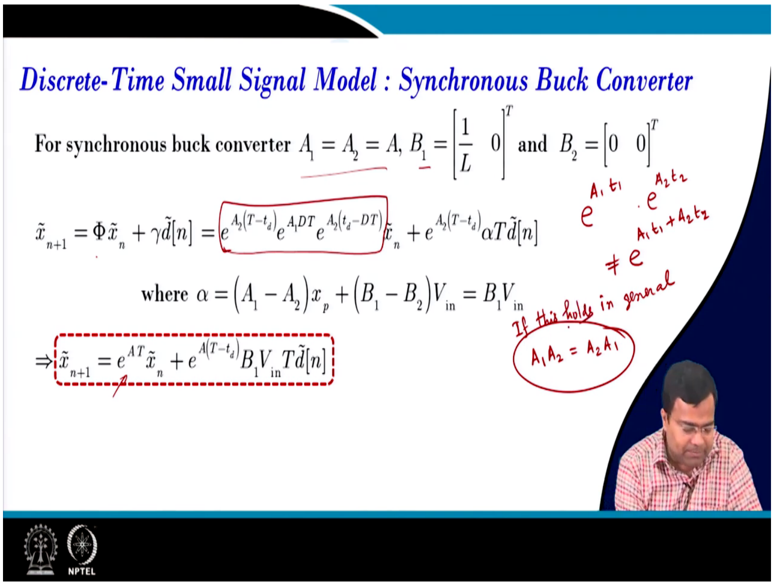

So, if you talk about a synchronous buck converter A 1 and A 2, are the same and B 1 is nonzero, but B 2 is 0 if there is no load because we are not talking load sink then this phi will simply become alpha. So, this will be e of AT because this phi this all A 1 A 2 matrix are common. So, you can write e to the power; that means, A 1 let us say some t 1 into e to the power A 2 t 2. In general, they are so; in general, it is A 1 t 1 plus A 2 t 2 it is not in general.

When it will happen? If the matrix satisfies this property if is valid this hold?

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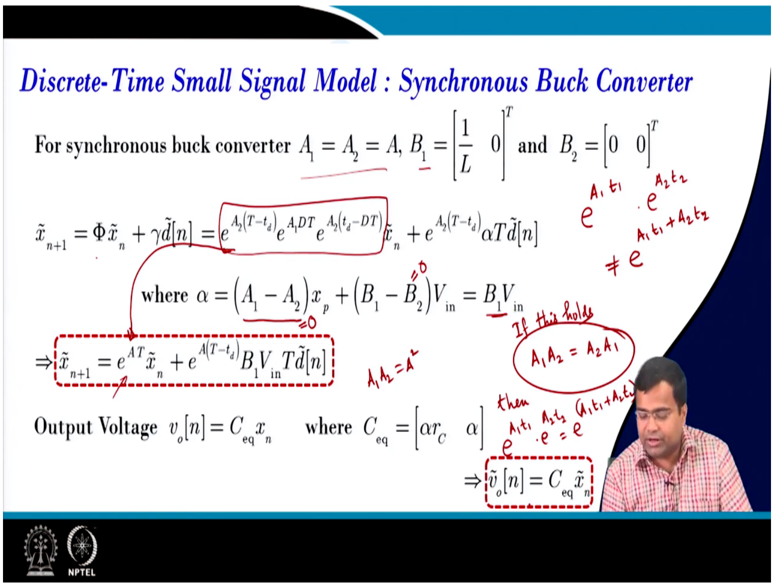

If this holds then we can write A 1 t 1 into A 2 t 2 is equal to A 1 t 1 plus A 2 t 2. So, we can write only when these commutative properties hold is valid. But in the case of buck converter if all A 1 A 2 of the matrix are identical. So, if you take A 1 A 2 which is nothing but A square. So, it is already satisfied. So, in this bug converter yo,u add up all matrices and since they are M same, you will end up with this result.

Similarly, if you talk about the last one all A 1 and A 2 are the same. So, you will get it and since buck converter A 1 A 2 matrix are same, this term will be 0, 0 and this B 2 is already 0, null matrix. So, will get only B 1 V in ok

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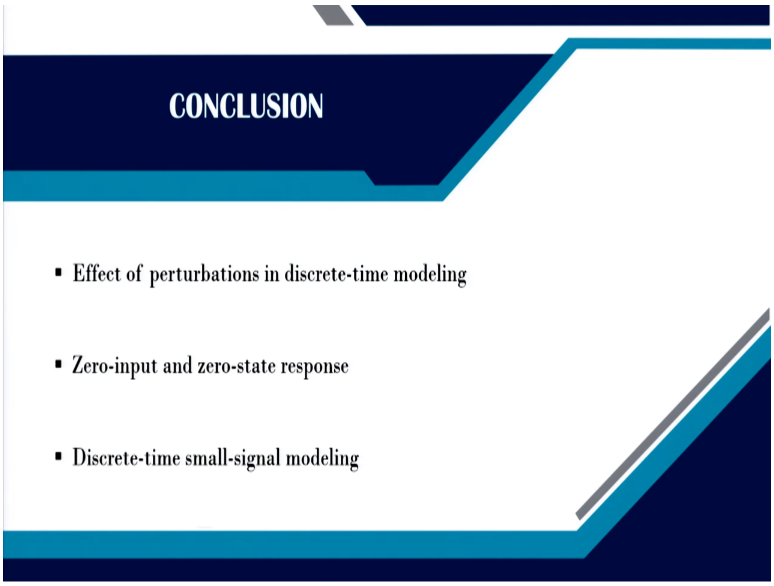

So, you will get the overall C matrix you already know so; which means, you can obtain the small signal transfer function. So, in summary, we have considered the effect of perturbation in the discrete-time model we have also shown zero input at zero state; that means, zero input means the perturbation due to the initial condition only. And zero state means the perturbation due to the duty ratio perturbation only.

And we have also discussed discrete-time small signal modeling. So, in the next lecture, we are going to talk about you know another alternative approach; that means, this approach was simple from drawing the waveform you can capture the perturbation effect. But in the zero state response, you need to apply a little bit of averaging technique to get it other this technique is much more straightforward.

And the next lecture we will be talking about by taking starting with our large signal model and how to directly you can derive the discrete-time small signal model. So, I want to finish it here for today.

Thank you very much.