Digital Control in Switched Mode Power Converters and FPGA-based Prototyping
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Module - 04
Modeling Techniques and Mode Validation using MATLAB
Lecture - 33
State-Space Modeling and Steps for Deriving Discrete–Time Models

Welcome back to this lecture. We want to first derive the State Space Modeling so that we can derive the discrete-time model of the complete you know digitally controlled DC-DC converter because, in the previous lecture, we have seen that if you only analyze the current loop stability, That is not sufficient and that is why we need to derive the full model.

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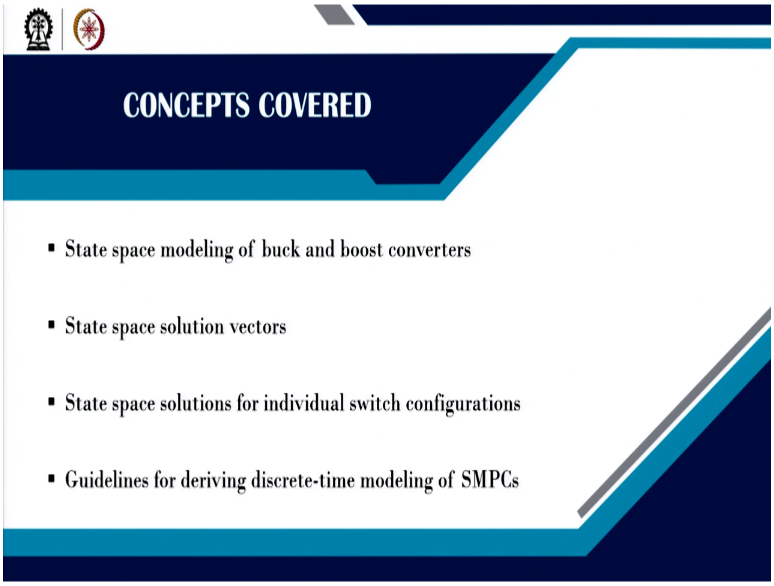

So, in this lecture we will talk about, we will recapitulate our state space modeling of the buck and boost converter. Then we will derive the state space solution vectors and then state space solution for individual switch configuration and then what is the what are the guideline to derive a complete discrete-time model.

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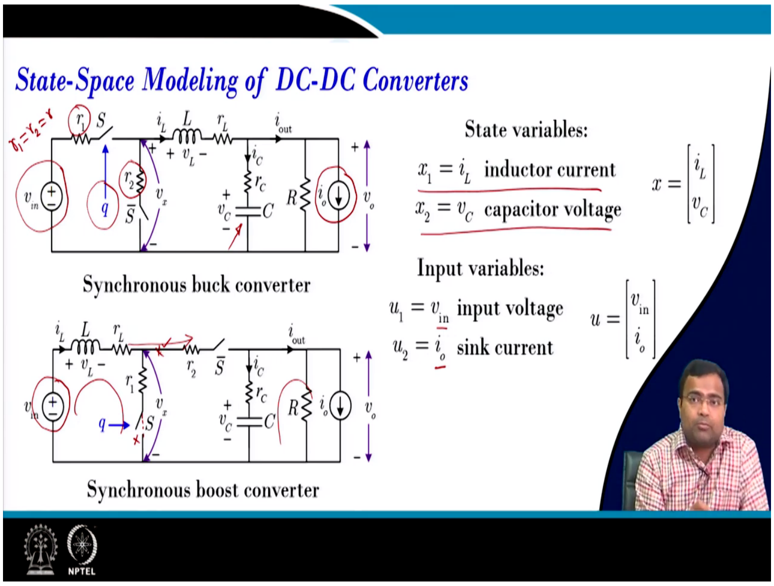

So, first state space modeling of the DC-DC converter. So, here I will talk about two like a synchronous buck, as well as a synchronous boost converter. And this you know this converter detail model we have already discussed, is a more or less practical converter with synchronous configuration.

Here we are considering two state variables, one state variable is the inductor current and another state variable is the capacitor voltage. And we are considering capacitor voltage at the state variable because we do not want the state to be discontinuous at the point of switching, that is why we are taking two state variables that are continuous because inductor current cannot change immediately neither capacitor voltage can change.

And we are considering the input variable which is the input voltage as well as the external load sink current. That means, if you consider another sink load current, then this variable will come as the input variable another input is the actual input voltage. And there will be one more input variable which is a timing parameter, there is a control like a duty ratio, and here since we are initially starting with an open loop system. So, we are not considering the control variable at this point.

So, we want to first derive the state space model for different switch configurations and we want to get the complete solution.

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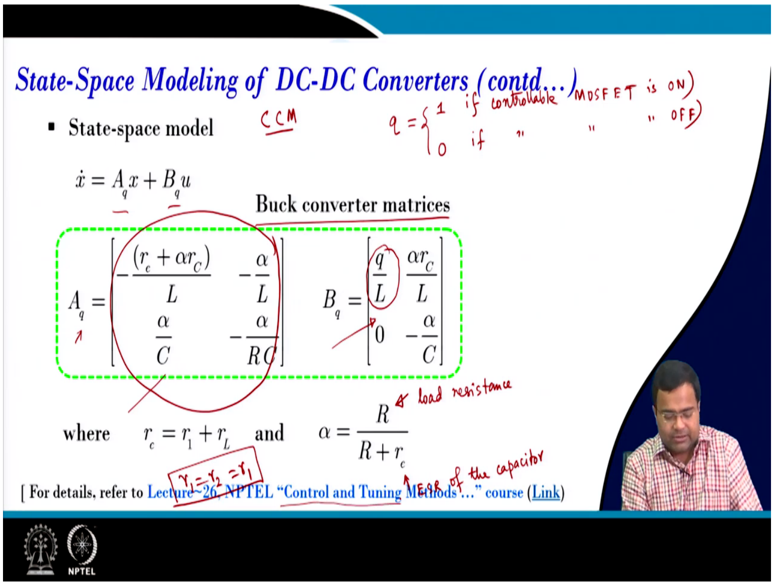

So, if you take the state space model of a DC-DC converter, from the previous diagram if this switch, is on then this path will be on and this path will be off. So, you will get one set of configurations. Similarly for this converter, if this switch is on it will be connected and this path will be disconnected.

So, you will get two separate systems, one is this inductor loop another is this loop, but when this switch is off and this switch is on then the current will flow in this path. So, depending upon the switch configuration, you will get two different state space equation, and this kind of equation is called a switch linear system; that means, each subsystem are linear, but there is a switching in between.

So that means, that is why we are writing A q and B q, for each of this switch configuration for q equal to. So, what value of q can take? q can either take 1 or 0 if the high side or I will say controllable MOSFET is on because the controllable MOSFET means if the controllable MOSFET is on ok. And it is if the controllable MOSFET is off and we are not talking about discontinuous conduction mode. So, it is under CCM, ok.

So, here it is a compact form for a buck converter, where this is for a buck converter, yes. That means, what is alpha? Alpha equal to R by R plus r c; R is the load resistance, this is the load resistance and this is the ESR of the capacitor ok. So, you will find for the buck converter a matrix is independent of q provided that the two-state resistance is identical. That means, i I take r 1 and r 2 identical; that means, if we take r 1 equal to r 2 equal to r, then this matrix will be the same. And what is r e? So, we are taking r 1 equal to r 2 equal to r the same thing ok.

So, here you can take r or r 1 whatever, but if they are different then there will be switching terms. Otherwise, A matrix and they are more or less closed. So, the A matrix is almost identical for both switching configurations, but you will get a drastic or fundamental difference in the B matrix.

What will happen to the B matrix? If the switch is on, this term will be non-zero, it is 1 by L and if is switch is off it will be 0. That means, in the buck converter if you see the input voltage either connected when the switch is on or it is disconnected and that will make a completely different B matrix for the buck converter.

Similarly, if you go to the boost converter you see the input voltage is always connected in CCM. So, you will see the B matrix will not have such a fundamentally different equation associated with the input voltage, but the B matrix for the buck converter is fundamentally different. The term associated with the input voltage. But in the boost converter, A matrix will be different; that means if you go to the boost converter and you will get the detailed derivation in lecture number 26.

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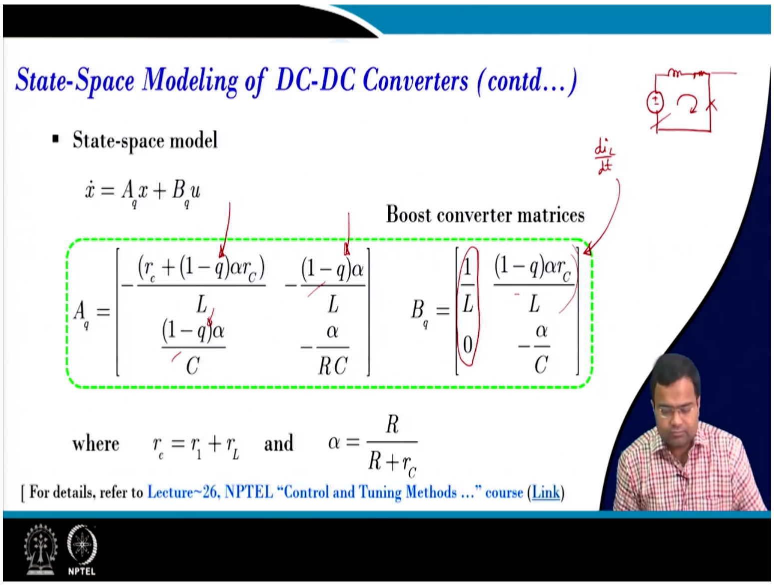

If you take the boost converter you see a matrix is heavily dependent on q; that means, if q equal to 1, these terms are 0 this term is 0 and if the q equal to 0 these terms are is coming into the picture. But this is the term associated with the input voltage, which is always common, but the term associated with the load current will also have a difference because the load will be connected or disconnected for which configuration you have to see. Load is always connected, but I am saying that this first term will have a di L d t term right?

So, whether di L d t will be linked to the load current, that will not happen in mode 1 because the mode 1 in a boost converter you will see the input voltage inductor resistance, you know there will be a resistance it will be; it will be like this. So, the inductor is disconnected from the capacitor side. So, this term will be 0, but the second time second mode di L d t will flow in this path. So, then it will be connected.

And this will make the boost converter because we know this connecting and disconnecting of this mode 1 and mode 2 and in you know kind of a because when the inductor can rise then capacitor voltage will fall. And that behavior will make the boost converter have a unique property for the non-minimum phase and that will lead to right half plane 0. And that is why this is an indirect power converter because it first takes the energy and then gives it back to the source and that will make the boost converter control very difficult.

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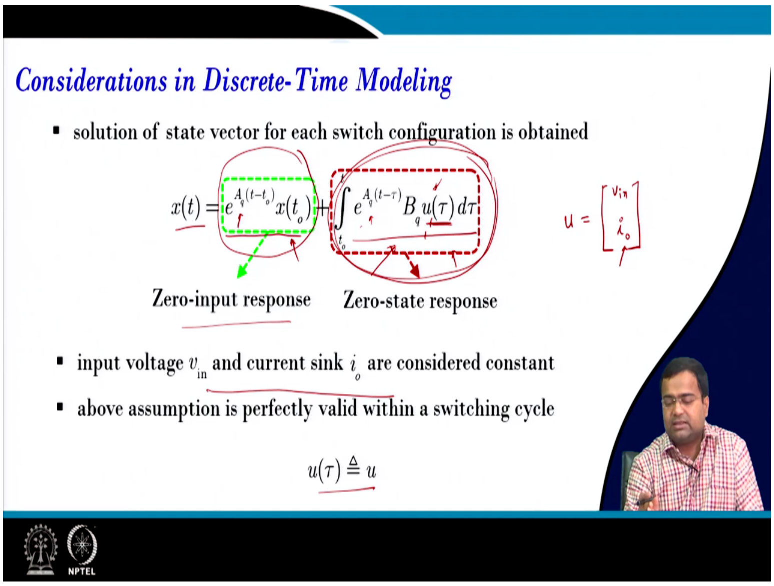

And this is also what you can get in lecture number 26. Now, how to get the solution of the state vector? So, any state vector solution in a general form can be written like this and this is a very standard textbook you know, if you go to any control system state space analysis this is a very standard technique and this state solution will depend on the A q.

So, if A q and B q changes, then this equation solution will be different. And the first term is called 0 input response because if you do not apply any input, then the response is due to the initial condition being 0 input response.

In the second term if you take the initial condition to be 0, then it will be the response due to the; that means, if you apply an input signal then 0 state response ok. Sometimes this is called a homogeneous solution, this is called particular integral and this is also sometimes known as an unforced response and this is a force response. And this can be also shown that this is a convolution of this input with you know this particular matrix.

So, there are different interpretations, but the bottom line is this 0 input response and 0 state response, but this term is easy to handle, but this term is a somewhat difficult one, because of how to get the solution. So, generally, if we consider a DC-DC converter; this u term what is what? So, we took the u term to be 2 terms, one is input voltage another is the load current right?

So, in general, the input voltage is constant for all the cycles. If you do not consider any small variation of the input voltage and we also consider the sink current constant, because it is a constant current sink. So; that means, these two variables are considered to be constant. So, this term is constant for this whole duration. So, you can take u. So, you can simplify this term by taking out u of tau, it is a constant and just take the integral for the rest.

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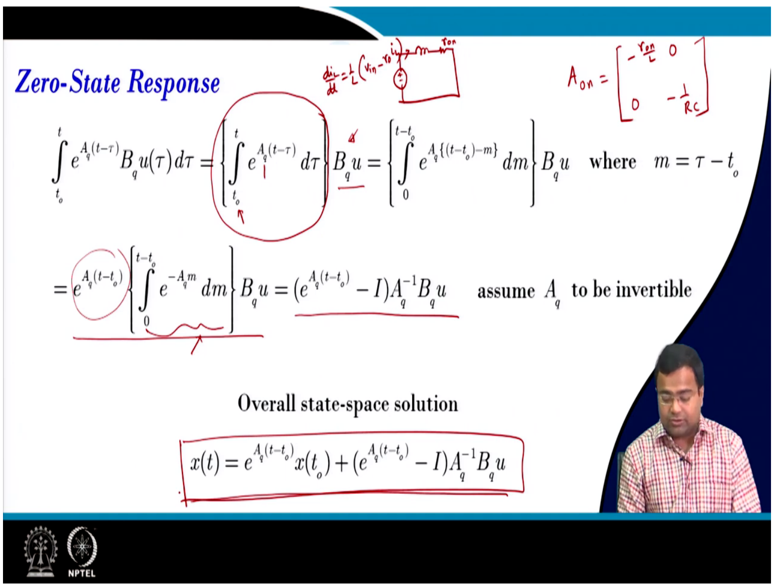

That means you can take this out, but the B q will vary depending upon whether q is equal to 1 or 0. So, you are only dealing with the integral of this term ok. So, now this will also depend on the e q matrix. Now, if you change the integral limit; that means, we are taking any arbitrary initial condition to t.

So, we want to convert into 0 to t minus t 0 and this expression will come to the same thing; that means if you take 0 to t any. So, e to the power A; that means, 0 to t minus t 0 it will be A q t minus t 0, this is a common term and now this is integral. If the A q matrix is invertible, then this solution can be written in the analytical form in this way, if this solution is invertible.

But if this solution is not invertible, then in general if you take a practical boost you will face this kind of difficulty in a boost converter. If you take an ideal boost converter during mode 1, the boost converter is a matrix it can be shown as an ideal boost converter. On stage, the matrix will be 0, 0, 0, minus 1 by R c. So, that will make the m A 1 matrix noninvertible and that is not very straightforward to get.

But if you take a practical boost converter, then you may find that there will be some amount of ESR ok; that means, you will get to know there will be some ESR; that means, minus of ESR or some, like some offset non-zero term will be there. Something will be there or you can, not ESR I would say it will be DCR. Because if you take the boost converter circuit like this, inductor resistance, and then this then what will be L di d t? So, di L d t will be 1 by L.

So, it will be V in. So, I am taking this term minus. So, it will be if this term is r on it will be r on into I L. So, you will get minus r on by L and because of this term, this will be also invertible ok. So, overall state space solution will look like this ok so; that means if it is; if it is noninvertible there is a way to handle that, but we are not discussing this in this course.

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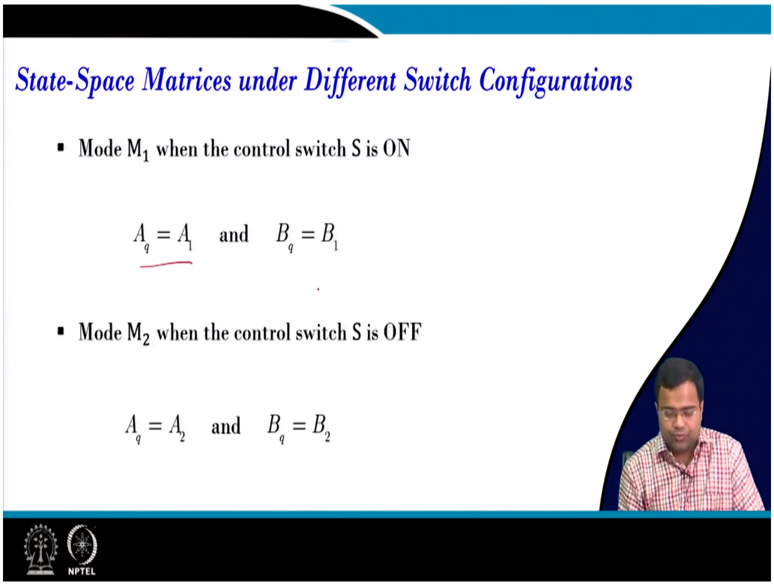

So, overall discrete time model looks like this. Now, we want to get this solution vector for different switch configurations for various converters. So, mode 1 will be A 1, and mode 2 B 2 switch off.

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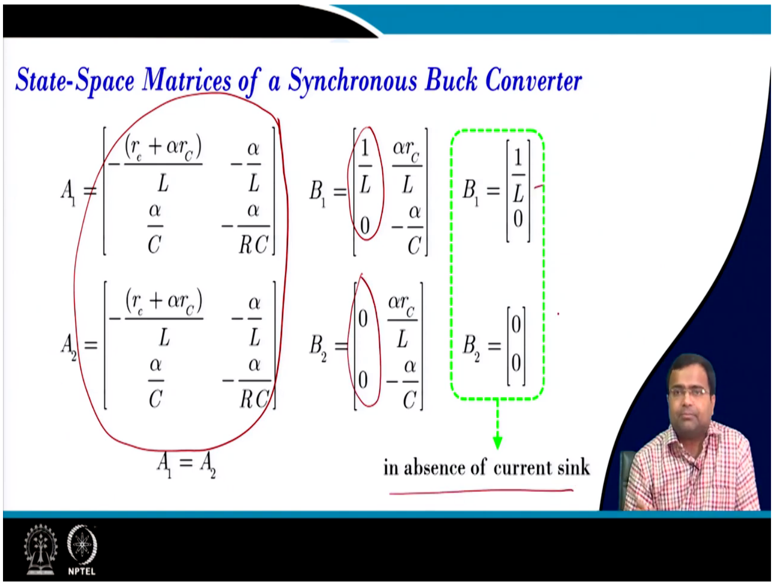

And for a buck converter mode 1 equation I told you. So, they are identical if r 1 r 2 are the same. That means, on the state resistance of the two switches is identical, then you will get the same, but you will get a different B matrix that we also discussed. And if we do not take any current sink, it is simply this term and this is a well-known method.

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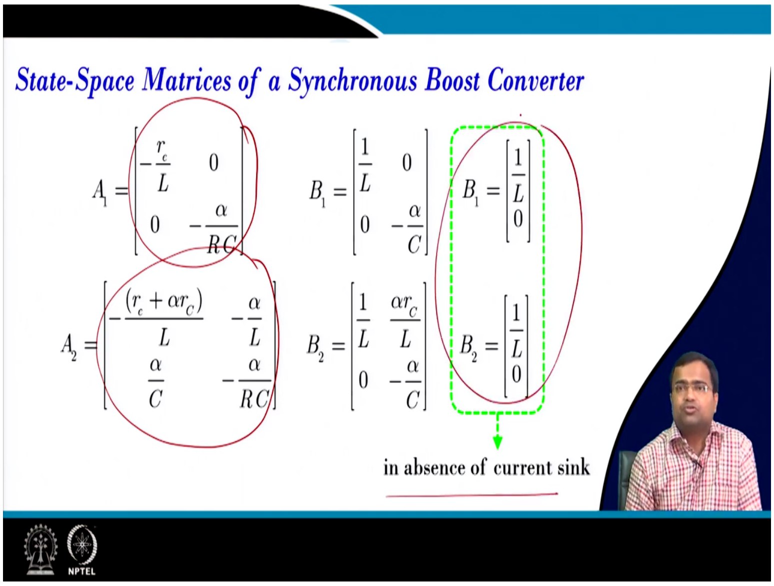

For a boost converter A 1, and A 2 matrices are fundamentally different ok. And B 1 B 2 matrix is different in case you consider load sink; external sink load, but if you ignore the external sink load then the B 1 B 2 matrix is identical for a boost converter in continuous conduction mode. So, then how to get the discrete-time model? So, you can get the solution vector that we discussed earlier; that means, you know whatever we discuss here.

So, this is the solution vector I can get, but this is this solution is for individual mode. So, how to get the complete solution? So, if you look at this diagram, now we want to derive the complete discrete-time model. How to do that?

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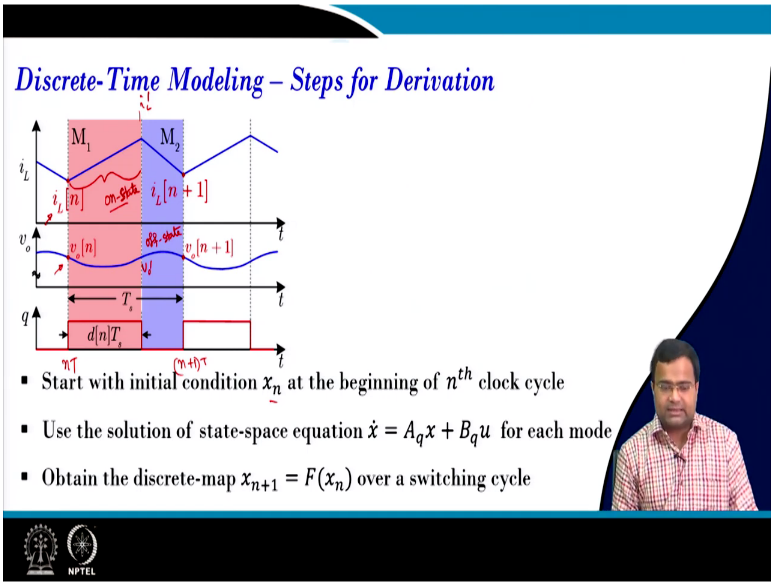

So, we already have derived for individual subsystems; that means, if you take on the state, this is my on the state you know then we also know off state ok. So, on-state and off-state. So, the on-state solution will start with the initial condition; that means, this is my nth cycle and this is my n plus 1 cycle.

So, during this nth cycle, we will start with the initial condition which consists of this inductor current and output voltage. Similarly, you can get the capacitor voltage initial current. And we want to; do the intermediate variable condition and then use the solution. So, the on-state solution you can get is if this time you will get I L 1, if this time you will get v 0 1, then next place you can go.

Then i L n plus 1 will be a function of i L dash and then v 0 dashes, then you can get the complete solution x n plus 1 over a switching cycle. So, these are the steps that will continue in the next lecture.

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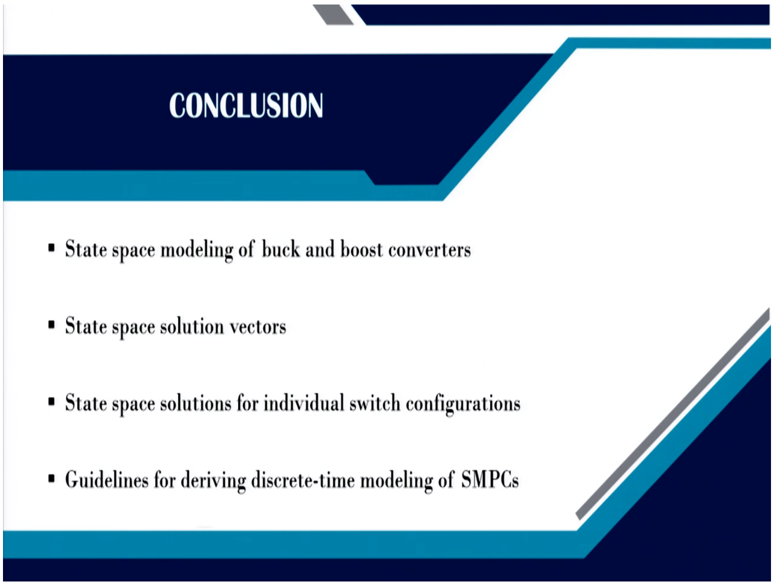

So, in summary, we have discussed state space modeling of buck and boost converters, state space solution vectors, state space solution for individual switch configuration, and guidelines for deriving discrete-time modeling of switch mode power converters. So, in the next lecture, we will derive the complete discrete-time model of the DC-DC converter under digital control. I want to finish it here.

Thank you very much.