Advanced Microwave Guided Structures and Analysis
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Lecture – 9
Instantaneous Form of Maxwell's Equations (Contd.)

So, welcome to this next session of the lecture on Instantaneous form of Maxwell's equations. So, now we have seen that the equations the Maxwell's equations can be represented in the mixed circuit and the field quantities form. So, now the question arises as to why we do this exercise at all? What is the utility of this exercise?

If for instance, if Maxwell's equations so exact that they can analyze everything. Then why worry about the circuit quantities? In fact, we will see in many works after we solve a problem. We want to express or what we want to find equivalent circuit parameters, for a structure like a waveguide discontinuity or discontouring inside a waveguide; or a microstrip patch or any kind of radiator.

So, but why this exercise; because Maxwell's equation has enabled us to find out exactly the input impedance, the matching, the radiation pattern everything. So, then why is the, are the equivalent circuit model needed? In fact, in many cases it is difficult to find out the equivalent circuit model. But, in planer circuits for instance you can fit the model to softwares like ADS; and they will give you if you draw the circuit.

They will give you the equivalent parameters, which from which you can find the one to one correspondence between the design circuit and the equivalent circuit. So, they will essentially match the S11 in magnitude and phase essentially many of them just match the magnitude. But, ideally they should be matching both the magnitude and the phase of S_{11}, to give you the equivalent network parameters. But, why this, why doing this exercise?

So, this exercise is done because we are like we are Maxwell's equations is a very elegant; they are very they describe nature in a very-very accurate manner. They are integro-differential equations; but unfortunately sometimes they are too elegant for us or too mathematically rich for us to get a physical understanding of what is going on. And what do I understand by this physical understanding?

What is the meaning of this physical understanding? By this physical understanding, we mean that when we design a circuit we want to, for example find out its more detail about its functionality. What is the meaning of more details about this functionality? Let us suppose for instance, I have designed an antenna.

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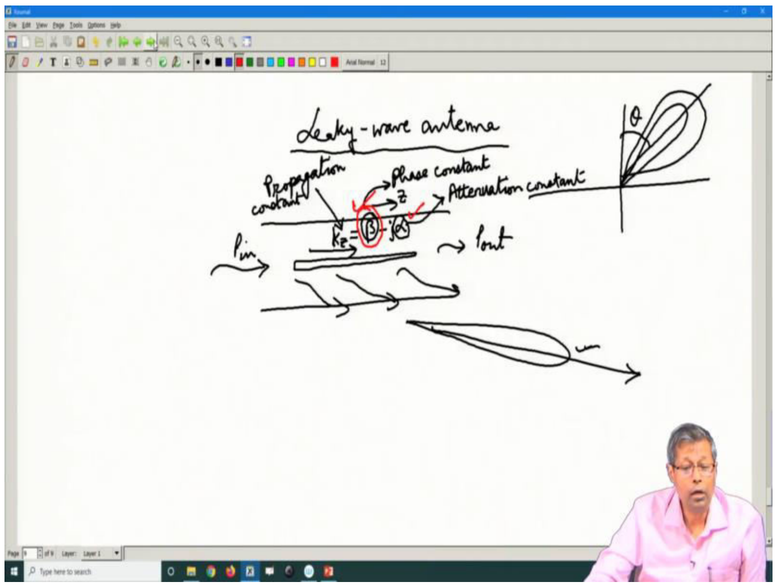

Let us suppose I will give you an example a Leaky-wave antenna; a Leaky-wave antenna. An example of a Leaky-wave antenna might be just a waveguide; so this is a waveguide, and on the narrow wall, I have cut a slot. This slot is not of uniform dimension; so the basic physics is as the waveguide is excited from the left hand side. So, this is the input port and this is the output port; energy radiates from this slot. So, we get we can get a pattern like this for instance.

Now, this is called a Leaky-wave antenna because as the wave propagates along the waveguide; the energy radiates from this structure. Now, the question is in a practical application of this Leaky-wave antenna; the one of the very important characteristics is which angle this beam is pointing. In the elevation plane, let us say what is this angle of the beam?

What is the beam angle or the beam direction? And the other question might be how much is its directivity that is how much. Whether it is very directive or its directivity is lower; its beam width is broader. It is a larger beam width so lower directivity; so, the direction of the beam is controlled by the phase constant. So, if this is the z-direction and if kz is the complex propagation constant; so, that is given by beta minus j alpha.

So, this beta is the real part of the complex propagation constant; so this is the propagation constant, this is the phase constant, and this is the attenuation constant. So, what happens is that as the as the wave progresses down; because the energy leaks. This leakage of energy causes attenuation of the wave; and therefore, this attenuation constant.

So, the direction of this beam is controlled by this term beta, while the directivity how pointed the beam is; that is controlled by this term attenuation constant. The lower the attenuation constant, the slower the decay of the beam, the more directive of the beam will be. On the other hand, the larger the attenuation constant is, the faster the decay of the beam; and the wider the beam will be, so the directivity will be lower.

Now, typically you see that this is a complex environment. Because I would want if possible an independent control over the beam width, or the directivity and the direction of the beam. Ideally, I would want a different set of parameters which will control my beam width; and a different set of parameters which will control my directivity.

Always this is not for possible; this is an environment in which you know like I want to isolate certain parameters to perform which have to I mean to perform certain tasks. In the way I want to isolate parameters which have more control over the beam width; and certain parameters, which will have more control over that activity. One step towards achieving my goal is if I can identify network parameters.

If I can express the Leaky-wave antenna by means of like an equivalent circuit and from the equivalent circuit model if I can isolate certain circuit elements which are responsible for the directivity And certain circuit elements which can be responsible for the direction of the beam. If it can be found, so this is an example of a scenario, where I would like to find out a circuit model; corresponding to my very accurate analysis of the circuit using Maxwell's equations.

The same thing will happen in many many kinds of antennas; miniaturized antenna for suppose. So, I want to basically see that the coupling problem to the antenna; the reduction of the resonant frequency to the antenna. So, these two are distinct aspects of miniaturization. Because, when you miniaturize an antenna, when you reduce its resonance frequency, when you reduce its resonance frequency; so its coupling will degrade.

So, which part of the circuit is responsible for the coupling? If I can express the circuit, if I can express the physical geometry of the antenna by means of an equivalent circuit; so I can isolate, you can try to isolate certain elements of the circuit, which is responsible for the coupling to the antenna. Certain elements of the circuit which are responsible for generating particular resonances; let us say the first resonance this kind of circuit elements are responsible.

The second resonance this kind of circuit elements are responsible; so then I can get more control over designing or tailoring those resistances. or so those resonances. So, this might be examples of scenarios, where we want to gain much more knowledge about the circuit, than its only accurate analysis through Maxwell's equations. So, I will reiterate Maxwell's equations are very-very accurate; but they do not lead us to a physical understanding of the circuit.

Or antenna or guided structure discontinuity in many many cases; for which we would take like to take recourse to finding the equivalent circuit parameters if possible. And this is frequently done in electromagnetics; so therefore we do not lose sight of the circuit parameters at all; so, that must be very very understood. In fact, we try to achieve a one to one relationship between the field quantities and the circuit quantities.

So, the next part we will see how the equations can be expressed entirely in terms of circuit quantities; so, the representation entirely in terms of circuit quantities.

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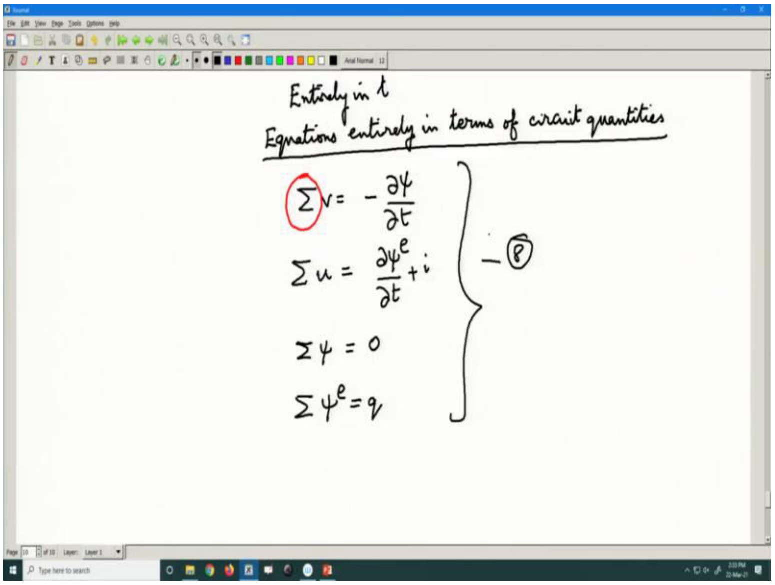

So, in that sense we can write the equations as sigma v or summation over v is. So, this summation denotes summation over a closed contour for a line integral quantity. It denotes the summation over a closed contour for a line integral quantity; and summation over a closed surface for a surface integral quantity. So, therefore this comes directly from equation number-6; what we wrote so summation of v is equal to -$\frac{d\psi}{dt}$.

So, summation of v is line integral E dot dl; so the summation of all voltages. And then we have summation u as $\frac{d\psi_e}{dt}$, plus i. So, if you look at the second of equation-6 line integral $\mathbf{H_{in}}$ dot dl; that is summation over the magneto motive force. And that is equal to $\frac{d\psi}{dt}$ plus i which is unchanged. Then, summation over ψ is 0, where ψ is closed integral **Bin** dot ds. And then the summation over ψ_e is q, where ψ_e is integral $\mathbf{D_{in}}$ dot ds.

So, this bunches of equation part into equation number-8; they are the equations you see here there is no field quantity, there is no field quantity. We have expressed it purely in terms of circuit quantities.

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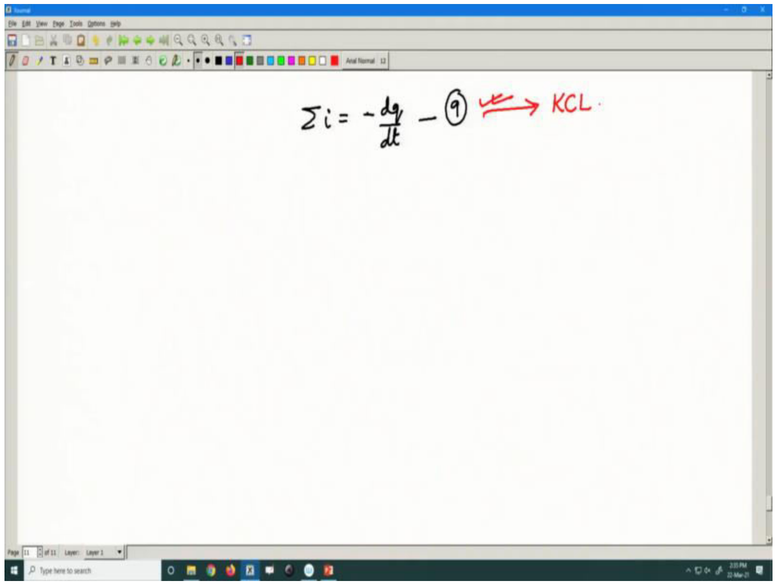

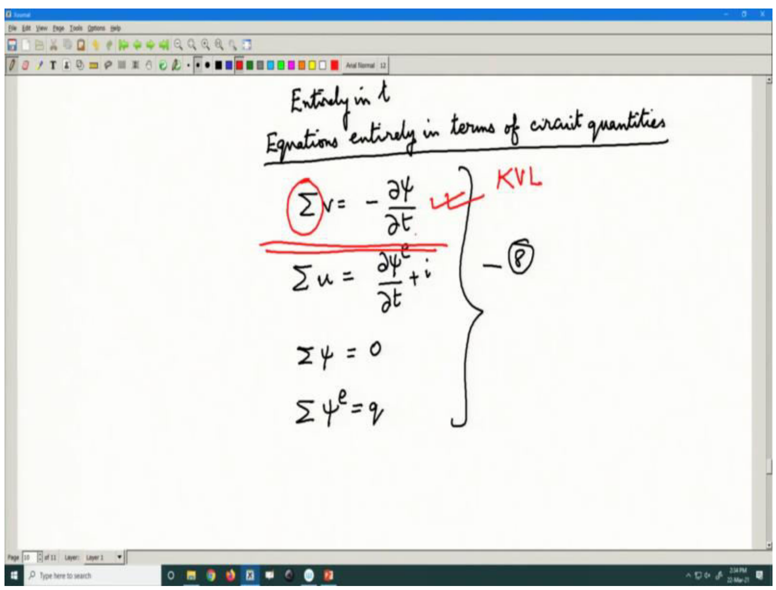

So, furthermore, we have equation from equation-7 summation of i as minus $\frac{dq}{dt}$. So, i is equal to $\mathbf{J_{in}}$ dot in double integral $\mathbf{J_{in}}$ dot ds; or summation over i is double integral $\mathbf{J_{in}}$ dot ds. Now, if you go back to the first of equation-8. It is a generalized statement of Kirchhoff's voltage law or KVL; because it is linking the summation over emf to the time rate of change of the magnetic flux. So, it is a general statement of Kirchhoff's voltage law.

Similarly, this equation is a generalized statement of Kirchhoff's current law. So, these are generalized Kirchhoff's voltage law and Kirchhoff's current law. From here we next go to constitutive relationships.

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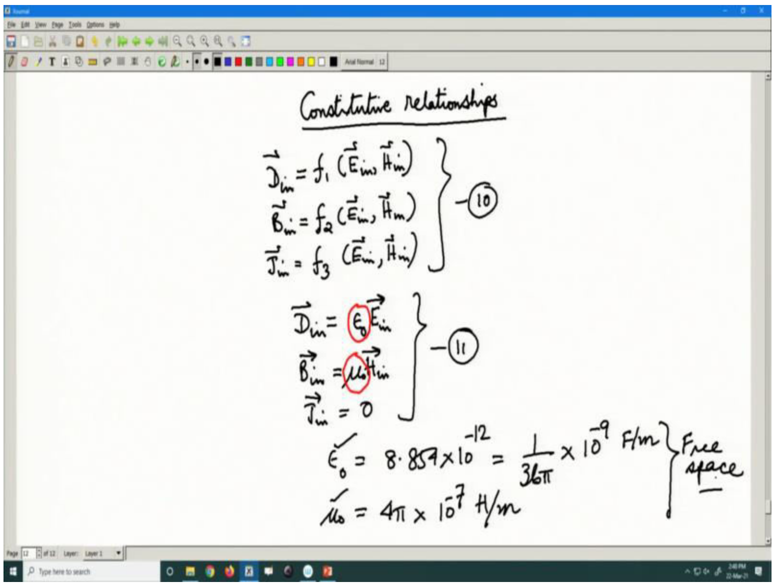

We next go to constitutive relationships. Electromagnetics links the electric and magnetic field quantities; and in addition to that we have material parameters. And those material parameters are epsilon and mu, the permittivity of the medium and the permeability of the medium. So, what is the linkage? How do they enter into electromagnetics form part of constitutive relationships, so, the effects of the material parameters on the field equations?

And we will also discuss their particular significance; practically, in issues like there are many issues. But, we can just take an example to elucidate our point, which is again antenna miniaturization. But, first let us write down those constitutive relationships. And we have J in equal to f3($\mathbf{E}_{in}$, $\mathbf{H}_{in}$). So, these bunches of equations $\mathbf{D}_{in}$ equal to f1 of the instantaneous electric and magnetic fields.

The magnetic flux density is again a function of the instantaneous electric and magnetic fields; and the impressed current a function of electric and magnetic fields. Now, what are these functions? So, $\mathbf{D}_{in}$ it is epsilon zero $\mathbf{E}_{in}$; it is mu zero $\mathbf{H}_{in}$, and $\mathbf{J}_{in}$ equal to 0. So, you see the parameters mu zero epsilon zero, and mu zero creeping in inside the field quantities.

Or the inter relationship between the field quantities; so, you see now so we call this equation as equation-11. So, epsilon zero is given by we all know its value 8.854×10^{-12}; or $\frac{1}{36\pi}10^{-9}$ farad per meter. And mu naught is given by $4\pi\times10^{-7}$ Henry per meter.

So, this is the permittivity and permeability of free space. If the matter is different from free space for any other kind of matter; then we will have the relationship.

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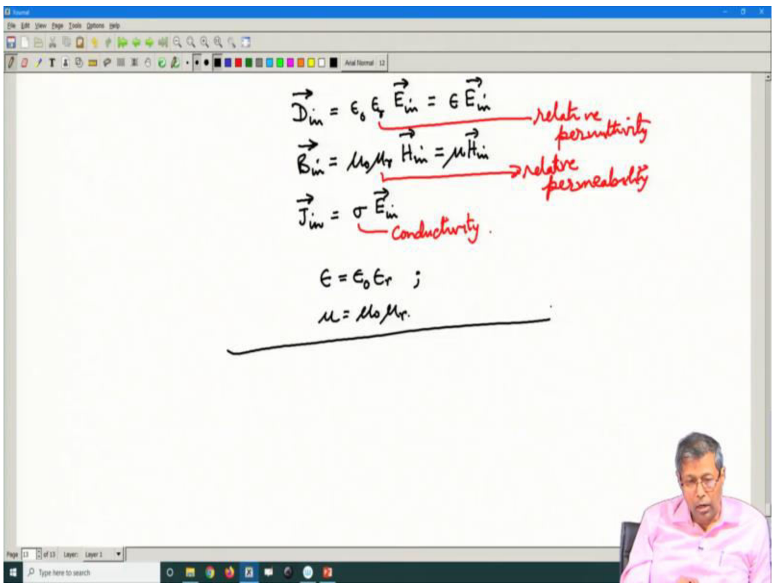

D$_{in}$ equal to $\varepsilon_o\varepsilon_r$ **E**$_{in}$ that is equal to epsilon **E**$_{in}$; where ε_r is the relative permittivity and epsilon is the permittivity. Similarly, **B**$_{in}$ equal to $\mu_o\mu_r$ **H**$_{in}$; that is equal to mu **H**$_{in}$, where μ_r the relative permeability and mu is the permeability. And **J**$_{in}$ is sigma **E**$_{in}$, where sigma is the conductivity; so this is the conductivity. This is the relative permittivity and this is the relative permeability.

And therefore we have epsilon equal to $\varepsilon_o\varepsilon_r$; and mu equal to $\mu_o\mu_r$. Having said that what is the real significance of this? Why do we how does this matter to us? In what way are the medium characteristics of the medium parameters important in electromagnetic design? To again see this we can cite as we talked about an example.

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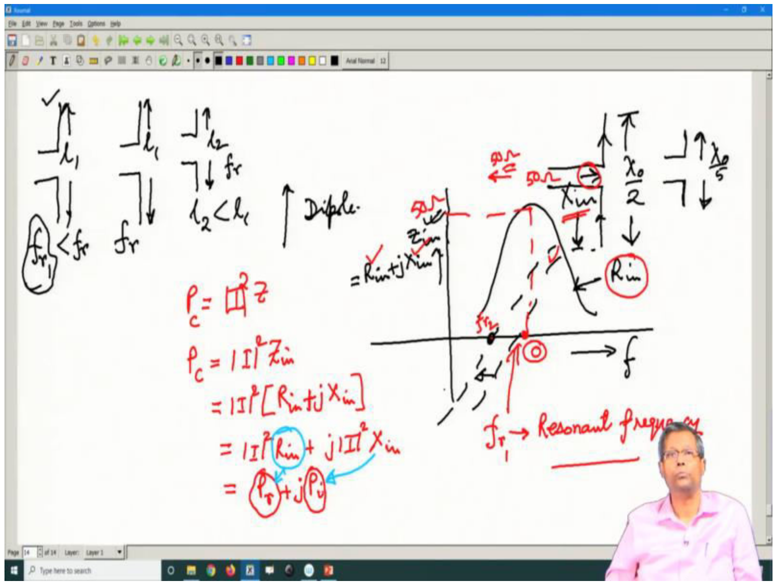

Let us say that we are designing a small antenna; this dipole is radiating in free space. So, when this dipole is radiating in free space, we all know that the resonance length of a dipole this is the symbol, so the resonant length of a dipole is lambda zero by 2. So, what we understand by resonant length? We understand that if this is the frequency; and this is the input impedance.

So, this input impedance is defined as Rin plus Xin; so this is Rin and this is suppose Xin. So, what is the meaning of this Rin and Xin. It means that this is the input impedance to the dipole; that is the impedance seen looking at this point. So, if this is the feed to the dipole so impedance seen looking at this point; so the dipole current is here. So, it consists of two parts; one is the resistive part, another is the reactive part.

The resistive part results in radiation; it is the real part it corresponds to the real part of the radiated power. So, if I express power that is equal to I^2 Z, this is the complex power; or $|I|^2$ Z. The complex power is $|I|^2$ Z; so I can express this as $|I|^2$(Rin + j X in). So that will be equal to; so $|I|^2$Rin, plus j $|I|^2$Xin.

So, that is equal to Pr + j Pi, so this corresponds to the real part of radiated power; and who contributes to that? This is the parameter which is contributing to the real part of the radiated power Rin. And this corresponds to the imaginary part of the radiated power; and who contributes to that? It is contributed by this parameter Xin.

So, we will go to a bit more details, when we go to the conservation of power. But, then the issue remains that from this side when we are matching this transmission line. What do we need? We need that we have to see a 50 ohm here. The input impedance look seen here must be 50 ohm; because there we have a 50 ohm line. So, this input impedance ideally should be 50 ohm; how can it be 50 ohm?

It can be 50 ohm, if this input resistance is. If this is 50 ohm, and this is 0. Only then, this transmission line will see a 0 reactive load here; it will see a 0 reactive load here. And it will see an impedance of 50 ohm there; so this is called matching. So, we say that, for matching the reactance curve or the X in curve, this X in; which is this curve. This curve should be crossing the 0 axis at the resonant frequency; this we call this the resonant frequency.

So, this is the resonant frequency; we call this the resonant frequency. So, at this point, we have the resonance and that length of the dipole is lambda 0 by 2. Now, suppose we want to reduce the size of this antenna; suppose we want to make the antenna small. Let us say we want to have an antenna which is lambda 0 by 5. So, an antenna which is lambda zero by 5 in length should be giving me a decent radiated power or should be resonant.

So, when will that happen? You see that means the resonant frequency should be shifting to the left. So, for a smaller size antenna, the antenna size is small. The resonant frequency should be shifting to the left side; so if I move, if I have a situation like this, where this resonant frequency shifts to the left. So, this is my new resonance frequency; so this is let us say fr1 and this is fr2.

So, we can say that this antenna is a smaller; because it has a reduced resonance frequency. You can see this more clearly, if I draw an antenna, which is this length; I can make this tell that this antenna is smaller. Then antenna is smaller than this antenna, if the length of my second antenna is lower than this antenna. Let us say this is l1, if my and this antenna is lower than this antenna; the size of l2 is less than 1l.

And the resonant frequency of this one and the resonant frequency of this one is the same. I have achieved miniaturization of this antenna. You will say that because this antenna is smaller and the resonant frequency is the same. Or, I can make the size of this antenna the same; and if the resonant frequency of this antenna which is fr1, if it is less than fr.

Then also I will say miniaturization is achieved; because for an antenna which is having a lower resonant frequency; its size will be typically larger, its size is typically larger. So, if its size can be maintained the same, if its size can be maintained the same as l1. But, its resonance frequency is fr1, then I also can say miniaturization is achieved. So, therefore if I move my frequency to the left hand side that means I am I am achieving miniaturization.

So, how can this be achieved? How can this be achieved? So, at least to understand the problem, the miniaturization problem; next we will attack how this is achieved. And how the material parameters tailoring parameters how does that fit in this picture; so, there will understand the significance of material parameters to the miniaturization process. So, let us stop here; we will continue from here. Thank you.