Advanced Microwave Guided Structures and Analysis
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Lecture – 8
Instantaneous Form of Maxwell's Equations

Welcome to this session. Topic of this lecture is Instantaneous form of Maxwell's equations. So, in the last class we talked about the scattering matrix parameters. Now, the scattering matrix parameters are essentially used in, they deal with voltage waves and current waves as we saw. The voltage waves and current waves are linked by the characteristic impedance, we also saw that. We also saw that how the scattering matrix enables us to derive the properties of a network, and how certain parameters are independent on each other.

How the scattering matrix parameters are dependent on each other? Now, this scattering matrix language or this particular language of; or this particular mode of representation of scattering matrix enables us to find the, or characterize the phenomenon inside preliminary types of guided wave structures. Like for example, a two wire transmission line.

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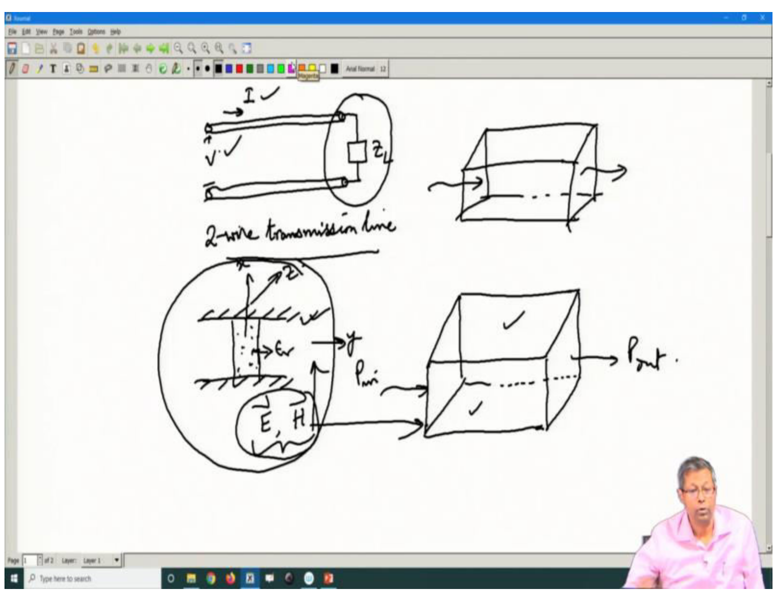

Let us draw a 2-wire transmission line. So, here in a 2-wire transmission line scenario; so, this is a 2-wire transmission line. So, there we define voltage waves and current waves; the voltage is between the two conductors, and the current flows in the conductor. So, we have the incident voltage wave, the reflected voltage wave, the incident current wave and the reflected current wave in such a scenario.

And we can characterize this kind of a transmission line; they characterize the reflection phenomenon from a load in the transmission line, by applying the the voltage equation at this load. But, if this transmission line is extended; if this concept is extended to let us say a rectangular waveguide. We all are aware of a rectangular waveguide something like this, a structure like this. So, this is an incident wave, this is a transmitted wave.

Let us draw, so this is a rectangular waveguide; so power is incident from this direction, and power goes out from this direction. So, in such a scenario, we have the boundary conditions on the top wall, on the bottom wall. The conducting boundary conditions on the top wall and on the bottom wall as well on the transverse walls; which is this wall and that wall.

So, in such a situation we cannot invoke the the transmission line concept; because of the imposition of boundary conditions on the two transverse walls of the waveguide. A more complex scenario may emerge, if I insert for example a dielectric rod in between the two parallel plates of the waveguide. So, if this is a pec, if this is a pec, and if this is a dielectric strip; and this is my z-direction, and this is my x, that is my y.

So, we cannot use the voltage and current concept in order to describe these kinds of guided wave structures because of the imposition of transverse boundary conditions on such a structure; so therefore we are compelled to use the more general classification, which is the concept of electric and magnetic fields, **E** and **H**. These **E** and **H** are linked through Maxwell's equations.

Now, there are two different forms of Maxwell's equations as we will soon discover. One is the instantaneous form of Maxwell's equation, where time plays like a distinct role; and the other is the time harmonic form of the Maxwell's equations, which we are going to cover later. So, let us go into the instantaneous form of Maxwell's equation, and how do we link up, how the **E** and **H** are linked together? So that is our prime concern here.

However, it is important for us to also recognize; that though we are using this concept of the electric and magnetic fields in this situation as well as this situation, this concept of voltage and current or the concept of circuit parameters; they are very very important to us. Why they are important to us? Because we want to, we would like to have more control in the design of certain structures by identifying certain network parameters, which can help us to perform the design in a much more better fashion.

Let us go ahead a little bit then, then we will understand it a bit more clearly.

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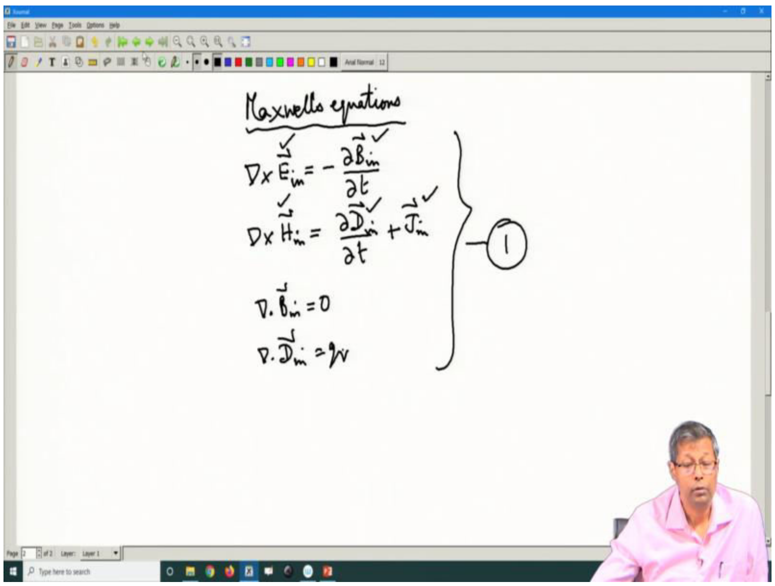

So, we will start with Maxwell's equations; they are equal to: So, $\mathbf{E_{in}}$ is the electric instantaneous electric field, $\mathbf{B_{in}}$ is the instantaneous magnetic flux density; $\mathbf{H_{in}}$ is the instantaneous magnetic field, $\mathbf{D_{in}}$ is the instantaneous electric flux density, and $\mathbf{J_{in}}$ is the impressed electric current. Other than that we have the two other equations.

So, the first two equations link the curl of the electric field to the time rate of change of magnetic flux density; that is the first equation. Let us actually call this as equation-1; these four sets of equations. So, the first equation out of these four sets of equations link up the curl of the electric field, with the time rate of change of the magnetic flux density.

The second equation links up the curl of the magnetic field to the time rate of change of the electric flux density, together with the impressed current. The third equation states that the magnetic flux forms closed loops; so there are no magnetic sources. And the fourth equation is Gauss's law, which tells that the divergence of the electric flux density is equal to the charge enclosed. So, these are the four fundamental forms of the point form of Maxwell's equations.

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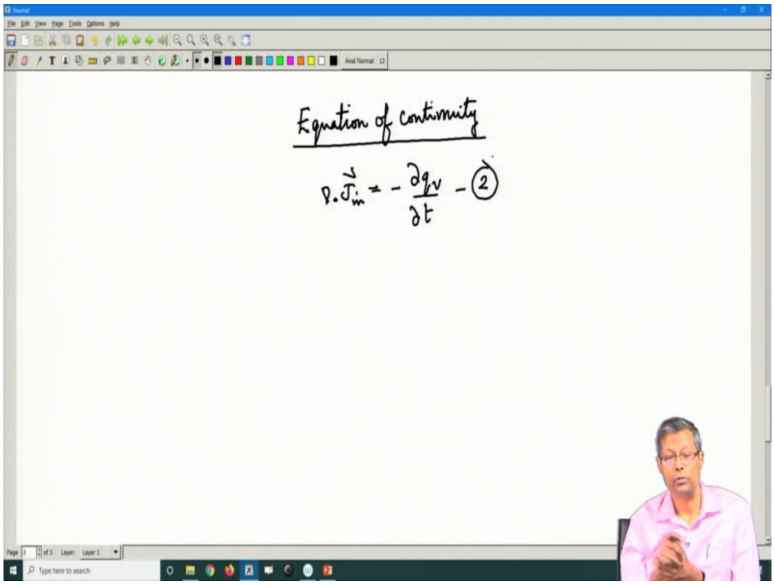

In addition to that we have the equation of continuity; so this is the equation of continuity. It links the divergence of the current to the time rate of change of the electric charge; so, this is the continuity equation. And now corresponding to these sets of equations, we can form the integral form of these equations which are valid for a region. So, while equations 1 and 2 refer to a single point they are point forms of equations. The next form we migrate to is the region form, where we express these as integral equations; which can be applied in a region of space.

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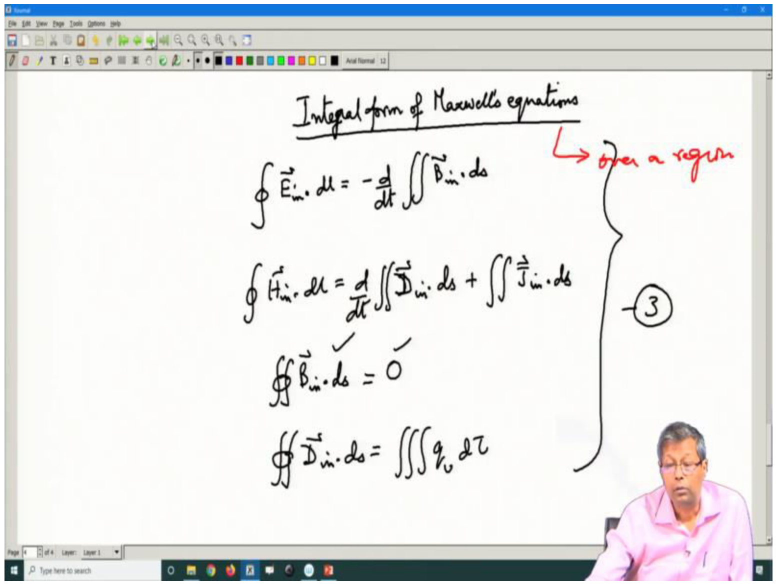

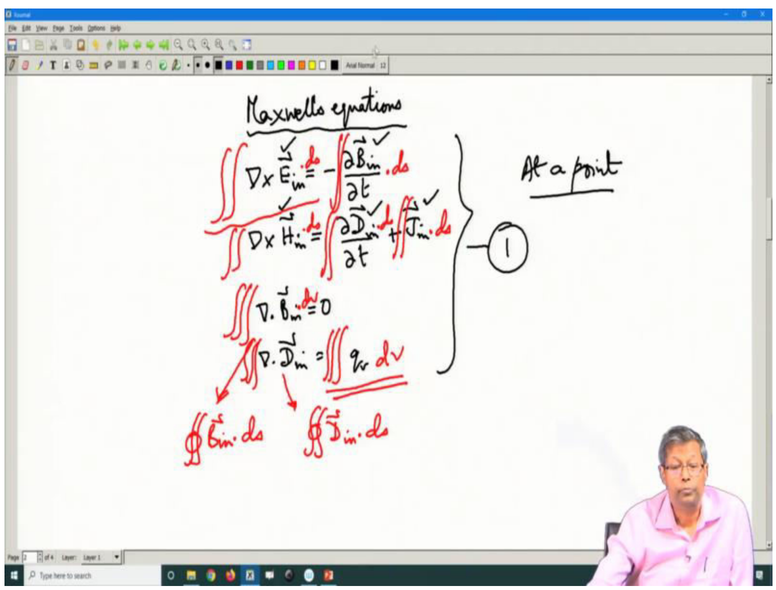

So, that we call the integral form of Maxwell's equations; and what are they? Let us write that down. So, I think we all understand that when we take the double integral of the first equation on the left hand side. So, we take a double integral here dot ds; and we take a double integral here dot ds. We can express this as closed integral E dot dl using stokes theorem; and this remains as we can take the differentiation out, and we it becomes B in dot ds.

So, that is exactly what we are doing; so, that is this. Similarly, the next equation can be expressed. That is equal to closed line integral of $\mathbf{H}_{\mathbf{in}}$ dot dl is double integral $\mathbf{D}_{\mathbf{in}}$ dot ds, plus double integral $\mathbf{J}_{\mathbf{in}}$ dot ds. So, what we essentially do is exactly the same thing; so we do the double integral of this side dot ds, double integral this dot ds; and double integral this dot ds. So, we apply we come to the integral form of the first two equations. The next equation will entail the the integral of the divergence.

So, let us do that; that will be equal to 0. Just before we do that let us call this equation; yes, we call that equation-2. So, let us, in fact, club all these equations and the next equation will be $\mathbf{D}_{\mathbf{in}}$ dot ds; so, this will be the third bunch. So, how do we obtain this by using divergence theorem? So, we perform the triple integral of this; we perform the triple integral over this that is equal to 0. And then that becomes $\mathbf{B}_{\mathbf{in}}$ dot ds.

And similarly we perform the triple integral here that becomes D in dot ds, and we perform the triple integral here. So, this becomes the total charge enclosed because qv is the charge density. So, that is how we graduate to the integral forms of Maxwell's equations, which are valid over a region of space. So, these are applied over a region and these are the equations which are valid at a point. So, now we need to find out the integral form of equation number-2.

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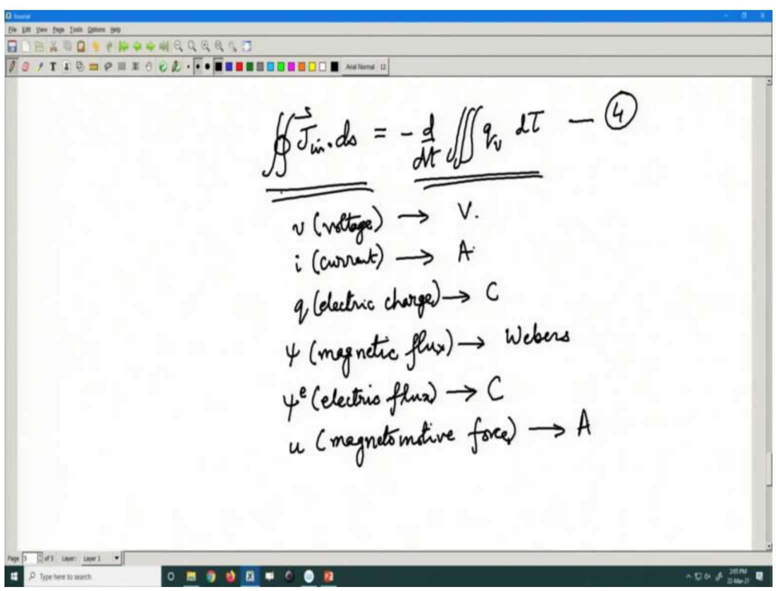

So, this is the integral form of equation number-2, where we link the left hand side; we perform the triple integral of the left hand side. And use divergence theorem to convert into J in dot ds, the close surface integral J dot ds; that is equal to the time rate of change of the total charge, so qv d tau is the total charge. So, the total current flowing out, it says the total current flowing out must be equal to the time rate of change of the total charge in a closed region.

So, we call this equation-4. So, associated with every field quantity, there is a circuit quantity. So, these circuit quantities are let us write down the circuit quantities. V the voltage measured in volts, i the current measured in amperes, q the electric charge measured in coulombs, ψ the magnetic flux measured in Webers; ψ_e, the electric flux measured in coulomb.

And u, the magneto motive force again measured in ampere. So, these are the circuit quantities corresponding to the field quantities. Now, we need to find out the relationship, we need to find out the relationship between the circuit quantities and the field quantities; so they are related. So, how they are related? Let us see.

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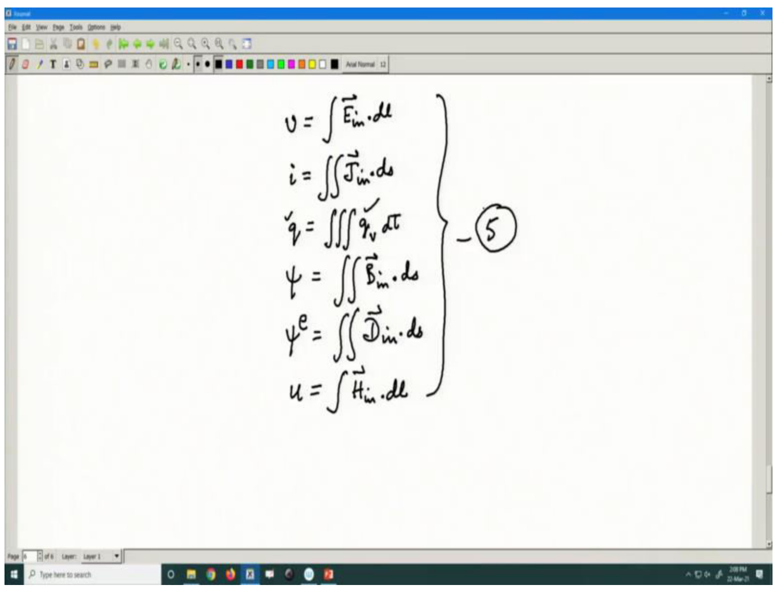

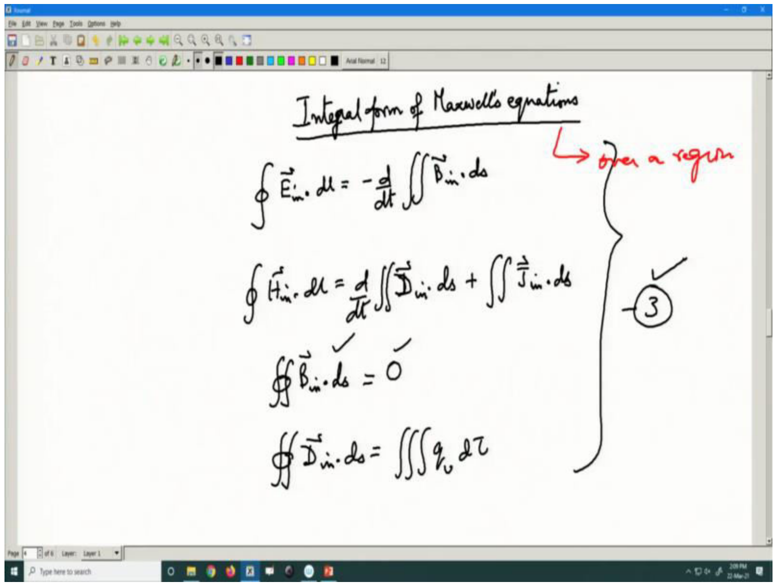

The voltage is given by $\mathbf{E_{in}}$ dot dl integral of $\mathbf{E_{in}}$ by dot dl. The current is given by double integral $\mathbf{J_{in}}$ dot ds. The q is given by the total charge is given by the triple integral of the charge density times $d\tau$, the elemental volume. So, this is the total charge that is the charge density. The magnetic flux is given by the magnetic flux density dot ds, double integral of that.

ψ_e which is the electric flux is given by the electric flux density dot ds; and u the magneto motive force is given by $\mathbf{H_{in}}$ dot dl. So, you see how therefore, so we call this set of equations equation-5. So, how the field quantities on the right hand sides are related to the circuit quantities to the left? So, the one to one correspondence between the field quantities and the circuit quantities. So, equation 1 to 4 are called the field equations, because they only involve the field quantities. So, equation 1 to 4 are called the field equations, because they only involve the field quantities.

So, corresponding equations for the circuit quantities are called circuit equations. Now, corresponding to equation number-3, all these equations corresponding to this equation. We can write the equation in the mixed field and circuit form. How do we do that? We just simply use these notations and we can write the equation number-3 in mixed field and circuit form.

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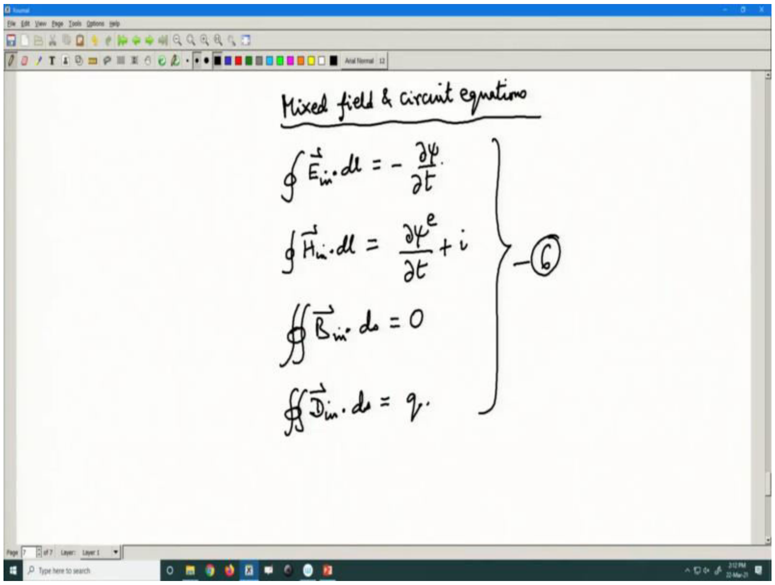

These are we will write the mixed field and circuit equations, very simple. So, if you look at the first equation of equation number-3; so integral closed integral $\mathbf{E_{in}}$ dot dl. That is equal to $\frac{d\psi}{dt}$ replacing double integral $\mathbf{B_{in}}$ dot ds by the circuit quantity ψ. Then we have $\mathbf{H_{in}}$ dot dl, $\frac{d\psi}{dt}$ plus i. So, how do we find?

So, if you go to the second of equation-3; you will find integral $\mathbf{H_{in}}$ dot dl on the left side, and replacing $\mathbf{D_{in}}$ dot ds by ψ_e and then replacing integral double integral $\mathbf{J_{in}}$ dot ds by i. So, therefore the closed line integral of the magnetic field is related to the electric flux densities, time variation plus the current.

Next very simple, closed integral $\mathbf{B_{in}}$ dot ds is 0; and closed integral $\mathbf{D_{in}}$ dot ds is q. We call this bunches of equation equation number-6. In addition equation number-2 can be expressed the continuity of current can be expressed also in the mixed field and circuit form.

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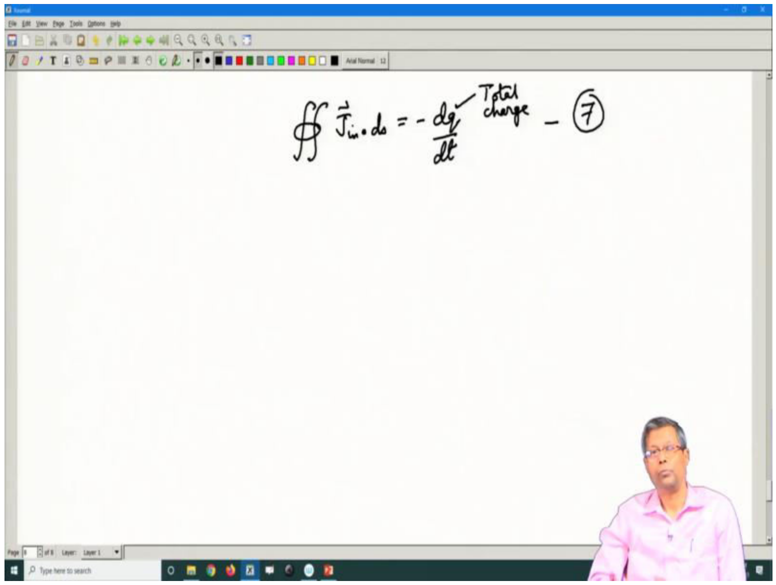

So, expressing equation number-2 in the mixed field and circuit form, we get go back to equation number-2. It is reading $\mathbf{J_{in}}$ dot ds by using divergence theorem, double integral J in dot ds; and if we take the triple integral of the right hand side, it becomes the total charge. So, this is the total charge and the as we said by equation-7; we link the the current J to the total charge. So, this is equation-7, where we link the current J to the total charge; so, this is equation-7.

So, let us stop here; this will continue this session and next we will see why this formalism is taken. Why this interrelationship between the field quantities and circuit quantities, the mixed field circuit form; or, even as we discovered later the total circuit form; where we express the same equations using total circuit quantities. Why is this important? So let us wait till the next session for that. Thank you.