

Advanced Microwave Guided-Structure and Analysis
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Lecture 42

The Reciprocity Theorem, Computation of Amplitudes of Forward and Backward Propagating Waves for Electric and Magnetic Current Sources in the Waveguide (Contd.)

Welcome to this session which is a continuation of the application of the reciprocity theorem in the computation of the n^{th} forward going and n^{th} backward going wave amplitudes radiated by electric and magnetic current sources located inside a rectangular waveguide. In the previous lecture we had considered the computation of the n^{th} forward moving amplitude using the reciprocity theorem radiated by a current source or radiated by an electric current source \mathbf{J} inside the waveguide.

In this part of the lecture, we are going to write down the expressions of the n^{th} backward moving amplitude of the fields radiated by the electric current source inside a waveguide as well as the n^{th} forward and backward going amplitudes of fields radiated by a magnetic current source inside a waveguide.

So, we saw previously that in order to find out the amplitude of the n^{th} forward moving wave inside a rectangular waveguide, we invoke the test mode the second set of fields which involves the n^{th} reverse going wave. In order to do the reverse thing, which is to find the amplitude of the n^{th} reverse going wave we considered the test mode to be the n^{th} forward going wave.

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$$\begin{aligned}\vec{E}_2 &= \vec{E}_n^+ \checkmark \\ \vec{H}_2 &= \vec{H}_n^+ \checkmark \\ A_n^- &= -\frac{1}{P_n} \int_V (\vec{E}_n^+ \cdot \vec{J}) dv \\ &= -\frac{1}{P_n} \int_V (\vec{E}_n^+ \cdot \hat{u}_z \hat{e}_z) \cdot \left(\frac{j}{\omega} \right) e^{-j\beta_n z} dv - (46)\end{aligned}$$

So, we will now consider in order to find out the amplitude of the n^{th} reverse going wave the electric field **E2** to be **En** plus which is the forward moving n^{th} harmonic and **H2** to be **Hn** plus which is the forward moving n^{th} harmonic for the magnetic field. So, this is the forward moving n^{th} harmonic for the electric field. This is the forward moving n^{th} harmonic for the magnetic field the set of fields **E1**, **H1** corresponding to the scattered modes or scattered fields or the total scattered field inside the waveguide consisting of the summation over all Eigen modes, they stay the same as previously.

So, now, if we do the same exercise once over again we will find the amplitude of the n^{th} moving backward wave as minus 1 by P_n the volume integral **En** plus dot **J** dv and that will be the same as 1 by P_n , we expand **En** plus and that is **En** plus **uz ezn** dot **J** $e^{-j\beta_n z}$ dv. Let us, call this equation 46. So, this is the amplitude of the n^{th} reverse going wave radiated by the electric current source **J**. So, we have the electric current source **J**.

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The image shows a handwritten derivation on a digital whiteboard. At the top, it states $\vec{J}_1 = \vec{J}_2 = 0$ and \vec{H} with a checkmark. Below this, equation (47) is derived for the positive-going wave amplitude A_n^+ :
$$A_n^+ = \frac{1}{P_n} \int_V (\vec{H}_n^+ \cdot \vec{H}) dv$$

$$= \frac{1}{P_n} \int_V (-\vec{h}_n + \hat{u}_z h_{zn}) \cdot \vec{H} e^{j\beta_n z} dv \quad (47)$$
Then, equation (48) is derived for the negative-going wave amplitude A_n^- :
$$A_n^- = \frac{1}{P_n} \int_V (\vec{H}_n^- \cdot \vec{H}) dv$$

$$= \frac{1}{P_n} \int_V (\vec{h}_n + \hat{u}_z h_{zn}) \cdot \vec{H} e^{-j\beta_n z} dv \quad (48)$$
A note at the bottom states "P_n is given by (45)".

In order to find out the n^{th} negative going wave, in order to find out the n^{th} positive going wave radiated by the magnetic current source \mathbf{J} we set \mathbf{J}_1 equal to \mathbf{J}_2 equal to 0 and excitation with only a magnetic current source \mathbf{M} and there we will find. Please do these on your own in exactly the steps we have outlined for the electric current source. So, there we will find A_n plus is 1 by P_n volume integral \mathbf{H}_n minus dot \mathbf{M} dv that is 1 by P_n volume integral minus \mathbf{H}_n plus $\mathbf{u}_z h_{zn}$ dot $\mathbf{M} e^{-j\beta_n z}$ dv.

So, we call this equation 47. So, this is the n^{th} forward moving wave radiated by the magnetic current source \mathbf{M} placed inside the waveguide. So, we have the magnetic current source \mathbf{M} sitting here and then considering the other problem which is the n^{th} reverse going wave radiated by the magnetic current source \mathbf{M} inside the waveguide that can be obtained similarly so, considering the other problem which is the n^{th} negative going wave amplitude radiated by the magnetic current source \mathbf{M} .

So, that can be found as A_n minus equal to A_n minus equal to 1 by P_n volume integral \mathbf{H}_n plus dot \mathbf{M} dv and then expanding \mathbf{H}_n plus it will be 1 by P_n volume integral \mathbf{H}_n plus $\mathbf{u}_z h_{zn}$ dot $\mathbf{M} e^{-j\beta_n z}$ dv. So, we call this equation 48. So, P_n in all this is given by 45.

So, this completes the expression or rather the expressions for the forward going wave amplitude radiated by the magnetic current source and the reverse going wave amplitudes radiated by the magnetic current source placed inside the waveguide. So, they can be exactly

derived as I said following the similar steps for the electric current source. This concludes the section on the application of the reciprocity theorem to compute the forward and the backward moving waves inside the waveguide. Thank you.