Advanced Microwave Guided – Structures and Analysis
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Lecture 3
Scattering Matrix Concepts (Contd.)

So, welcome to the next session of the Scattering Matrix Parameter Concepts, here we will investigate the scattering parameters of our two port junction.

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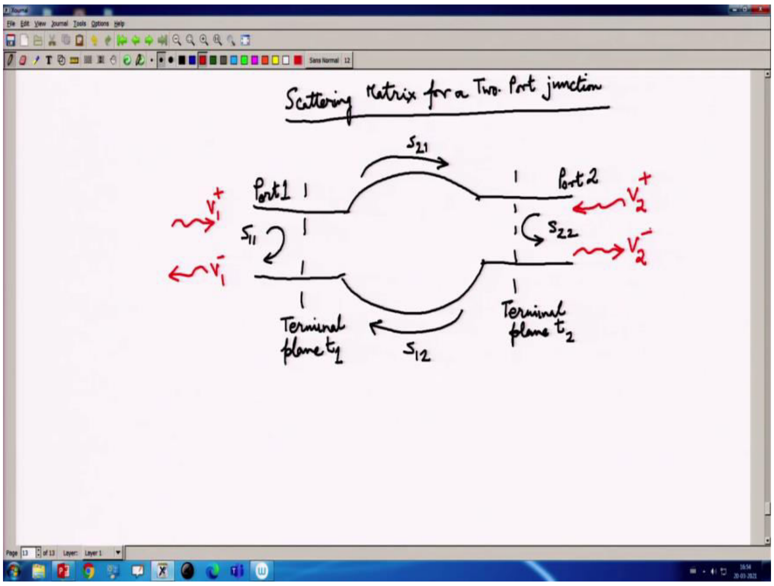

So, consider a two port network. We now specialize from the input network to the two port network. So, consider a two port network. Here it is S_{21}, S_{12}. So, this is the general depiction of the two port network. So, what do we have?

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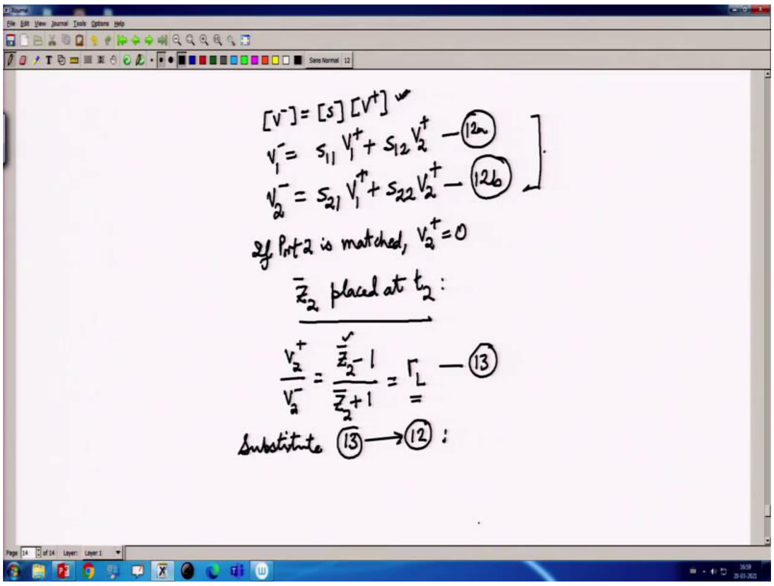

We simply can write down the equation involving the voltage waves, the incident and the scattered voltage waves as $[V^-] = [S]\,[V^+]$. The magnified or the explicit form of this equation for the two port network can be written as:

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

Now, if port 2 is terminated in a matched load, if port 2 is matched, V_2^+ will be equal to 0 straightforward.

So, S_{11} is the reflection coefficient at port 1 with port 2 matched because S_{11} will be V_1^- by V_1^+ with V_2^+ equal to 0. Similarly, S_{21} is the transmission from port 1 to port 2 and it will be V_2^- by V_1^+ with V_2^+ equal to 0.

Now, if a normalized impedance $\bar{z}_2$ is placed at t_2, therefore there will be reflection from port 2. Therefore, we can write under this condition straightforward

$$\frac{V_2^+}{V_2^-} = \frac{\bar{z}_2 - 1}{\bar{z}_2 + 1} = \Gamma_L$$

where Γ_L is the reflection coefficient at the load. So, I can characterize the reflection at the port 2 by this reflection coefficient due to this load, normalized load impedance $\bar{z}_2$. So, substituting we will get $V_1^- - S_{11}V_1^+ = S_{12}V_2^+ = S_{12}\Gamma_L V_2^-$ (i)

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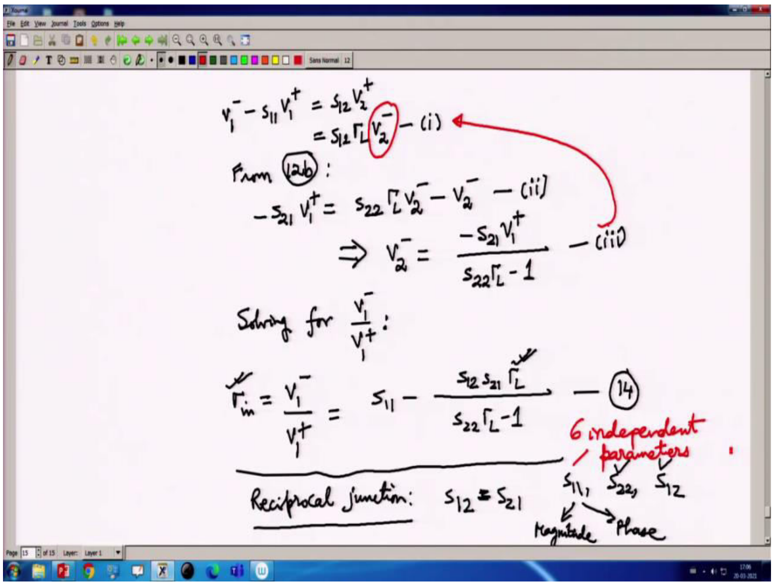

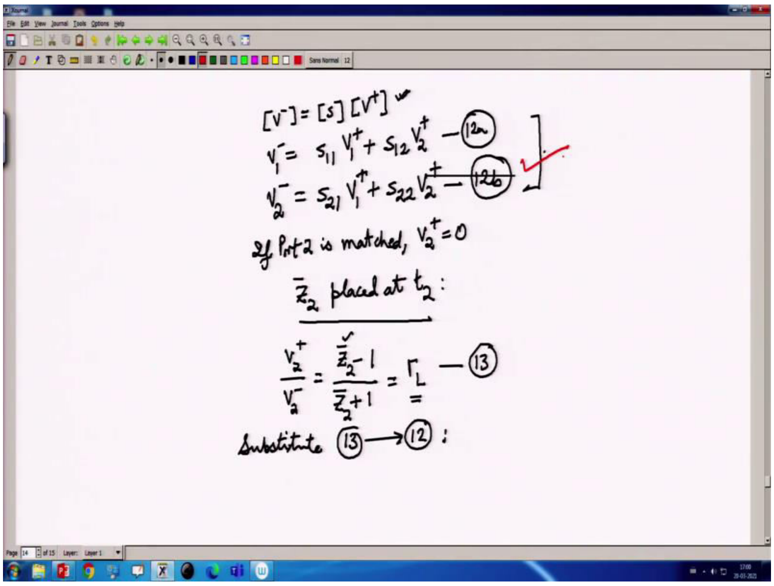

Also we can get, $-S_{21}V_1^+ = S_{22}\Gamma_L V_2^- - V_2^-$ (ii)

So, which will mean $V_2^- = \dfrac{-S_{21}V_1^+}{S_{22}\Gamma_L - 1}$ (iii)

So, now solving for V_1^- by V_1^+ we get, $\Gamma_{in} = \dfrac{V_1^-}{V_1^+} = S_{11} - \dfrac{S_{12}S_{21}\Gamma_L}{S_{22}\Gamma_L - 1}$

So, this gives the reflection coefficient at port 1, when port 2 is not match terminated that is we have a Γ_L at port 2. So, it gives the reflection coefficient at port 1 with port 2 not match terminated. Now, for a reciprocal junction or a reciprocal network, S_{12} equal to S_{21}. So, there are 6 independent parameters in the network and what are they? S_{11}, S_{22}, S_{12}, both their magnitude and phase and therefore, there are 6 independent parameters.

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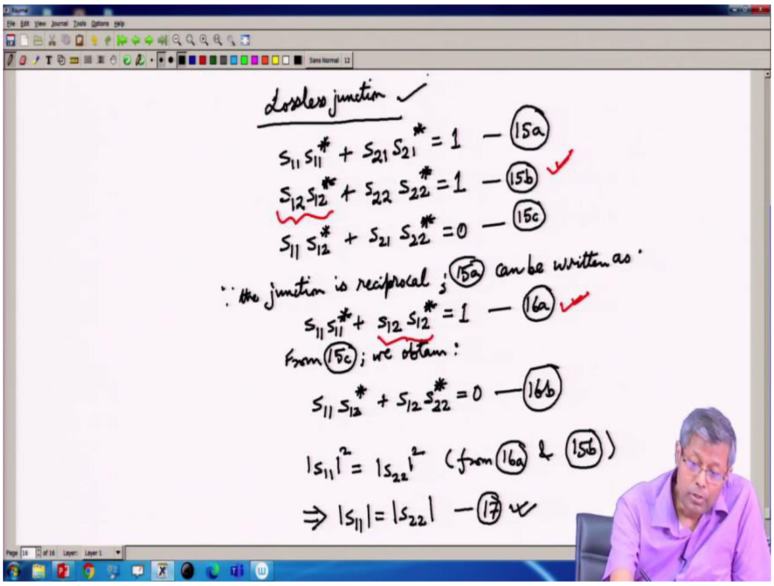
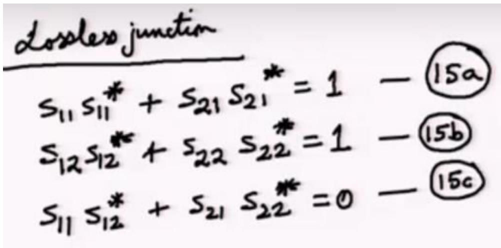
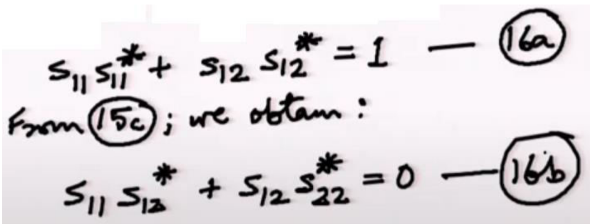


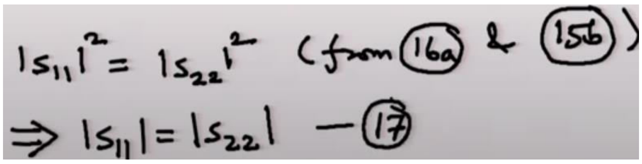
If in addition the junction is lossless, we will have three independent parameters. How? Let us see. So, for the lossless junction we can invoke the conditions and we can apply them for the 2 port network and let us see where we come to after we apply those conditions.



So, these are the three conditions. So, since the junction is reciprocal, we can just replace S_{21} by S_{12}, we do not need S_{21}. Because the junction is reciprocal 15c can be written as:



We just replace S_{21} by S_{12} because of the reciprocity the junction is reciprocal. So now, if you compare 16a and 15b, what do we get? We get



And therefore that implies, mod S_{11} equal to mod S_{22}. So, what does it mean? The reflection coefficients at port 1 and port 2 are the same for the lossless junction but what about their phases?

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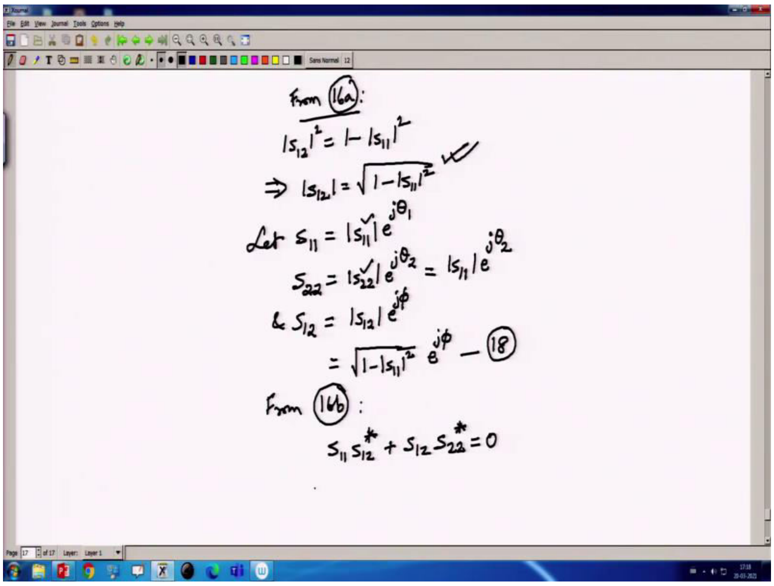

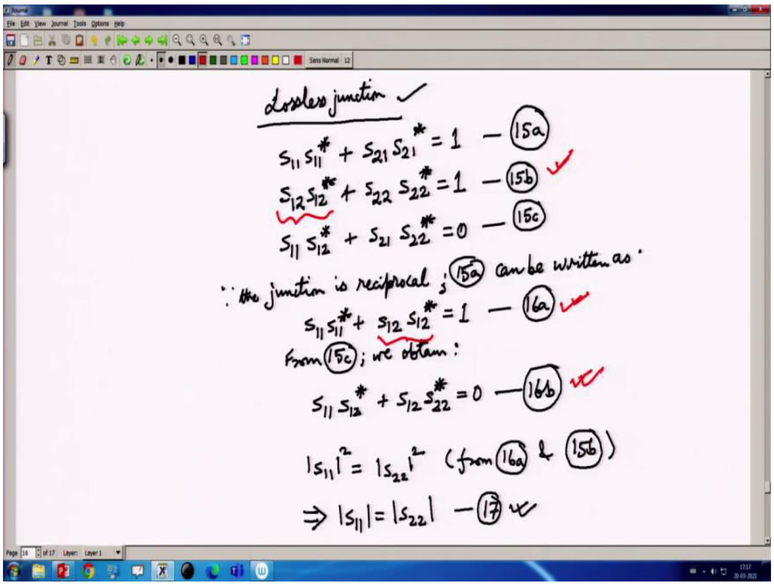

To investigate that, let us go to the next part. But before that, from 16a we can write, the relationship between S_{12} and S_{11} and therefore we can restate this as

From (16a):

$$|S_{12}|^2 = 1 - |S_{11}|^2$$

$$\Rightarrow |S_{12}| = \sqrt{1-|S_{11}|^2}$$

Now, as we were saying that how are their phases related? So, for that let us assume

$$\text{Let } S_{11} = |S_{11}| e^{j\theta_1}$$

$$S_{22} = |S_{22}| e^{j\theta_2} = |S_{11}| e^{j\theta_2}$$

Regarding S_{12}, we can write

$$S_{12} = |S_{12}| e^{j\phi}$$

$$= \sqrt{1-|S_{11}|^2}\, e^{j\phi} \quad \text{— (18)}$$

From (16b):

$$S_{11} S_{12}^* + S_{12} S_{22}^* = 0$$

Let us call this equation number 18. Now, we do substitutions, we just substitute their magnitude and phases.

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$$|S_{11}| e^{j\theta_1} |S_{12}| e^{-j\phi} + |S_{12}| e^{j\phi} |S_{11}| e^{-j\theta_2} = 0$$

$$|S_{11}|\,|S_{12}| \left(e^{j\theta_1 - j\phi} + e^{j\phi - j\theta_2}\right) = 0$$

$$|S_{11}| \sqrt{1-|S_{11}|^2}\, \boxed{\left(e^{j\theta_1 - j\phi} + e^{j\phi - j\theta_2}\right)} = 0$$

$$\Rightarrow e^{j\theta_1 - j\phi} + e^{j\phi - j\theta_2} = 0$$

$$e^{j\theta_1 - j\phi} = - e^{j\phi - j\theta_2}$$

$$e^{j\theta_1 - j\phi - j\phi + j\theta_2} = -1$$

$$|S_{11}|e^{j\theta_1}|S_{12}|e^{-j\phi} + |S_{12}|e^{j\phi}|S_{11}|e^{-j\theta_2} = 0$$

$$|S_{11}||S_{12}|\left(e^{j\theta_1 - j\phi} + e^{j\phi - j\theta_2}\right) = 0$$

$$|S_{11}|\sqrt{1-|S_{11}|^2}\left(e^{j\theta_1 - j\phi} + e^{j\phi - j\theta_2}\right) = 0$$

Then because S_{11} is the same as mod S_{11} is the same as mod S_{22}. I can write S_{22} star as S_{11} e to the power minus j theta 2. That will be equal to 0 and then we can take these two terms common. So, we get the above equation.

Now, this implies

$$e^{j\theta_1 - j\phi} + e^{j\phi - j\theta_2} = 0$$

$$e^{j\theta_1 - j\phi} = -e^{j\phi - j\theta_2}$$

$$e^{j\theta_1 - j\phi - j\phi + j\theta_2} = -1$$

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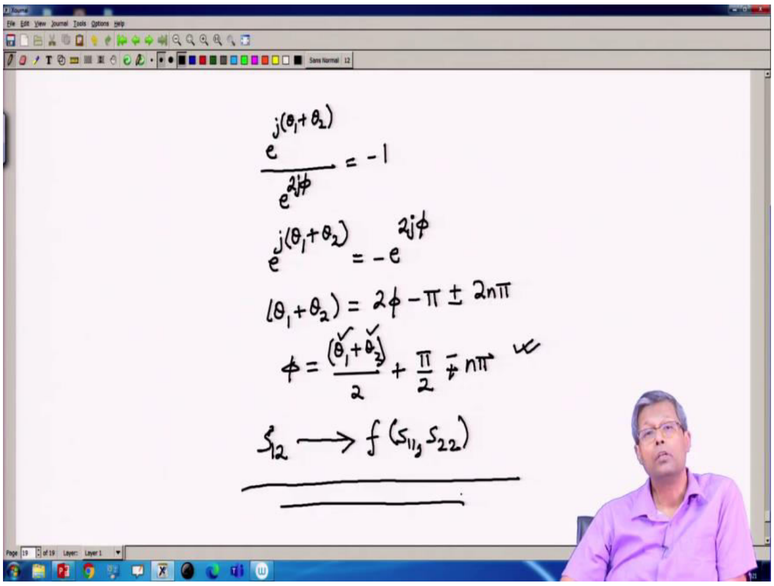

And then I can express this thing by a division, and therefore, I can write

$$\frac{e^{j(\theta_1+\theta_2)}}{e^{2j\phi}} = -1$$

$$e^{j(\theta_1+\theta_2)} = -e^{2j\phi}$$

$$(\theta_1+\theta_2) = 2\phi - \pi \pm 2n\pi$$

$$\phi = \frac{(\theta_1+\theta_3)}{2} + \frac{\pi}{2} \mp n\pi$$

So, we see that S_{12} is completely specified, it is a function of S_{11} and S_{22}, it is completely specified in terms of S_{11} and S_{22}. So, this simplifies the description of the circuit, because the magnitude and phases of S11, S22 can be easily measured. So, this completes this section we will next continue with the next part. Thank you.