Control and Tuning Methods in Switched Mode Power Converters
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Module - 02
Modulation Techniques in SMPCs
Lecture - 07
Power Stage Design of Basic SMPCs: Summary

Welcome. So, today is our lecture number 7. In this lecture, we are going to discuss about Power Stage Design of Basic Switch Mode Power Converters.

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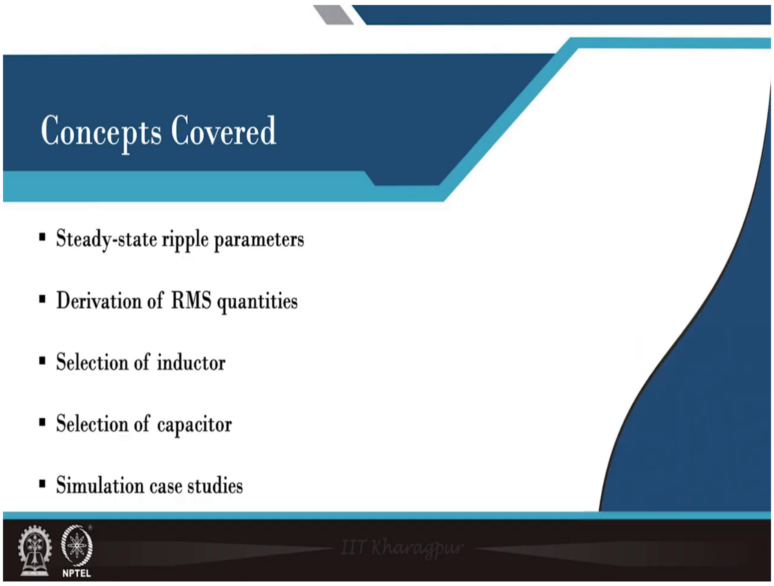

So, in this lecture, we are going to cover steady-state ripple parameters, derivation of RMS quantity. Then selection of inductor, selection of capacitor and then, few simulation case studies.

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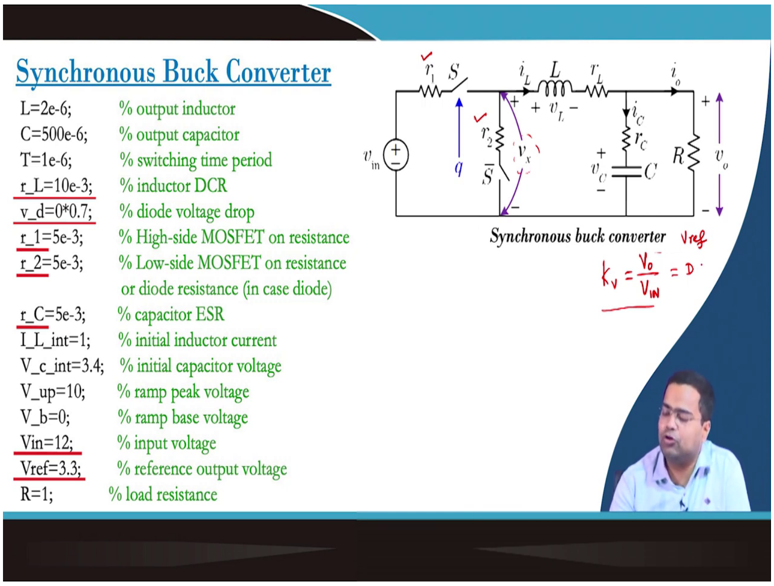

So, if we consider a synchronous buck converter which we have already discussed that means, if we take a realistic circuit which include the on-time of the just yeah. So, here which include the on-time of this switch S 1; S the main switch, then on time of S bar that is my the complementary switch and this v x is the switch node voltage terminal.

So, before I start actually, I am going to show a few simulation case studies in which I set the parameter like, L equal to 2 microhenry, C equal to 500 microfarad, T equal to 1 microsecond. Although, this L I have taken quite large because we are operating at 1 megahertz switching frequency. But this is just to show a case study. In fact, we will design we will want to see what is the effect of a larger inductor or a smaller inductor so, we will take a few case study.

We took the DCR of the inductor like you know r_L, then diode drop if you here we are not operating in conventional buck converter. So, there is no diode drop so, it is set to 0. The on-time of the MOSFET, the on-time of the complementary MOSFET, then ESR of the capacitor and we have also set some initial condition and input voltage we have taken as 12 volt and the desired output voltage is 3.3. Since we are setting duty ratio directly with no closed loop control, there are parasitic drops which I will show you.

In ideal condition, if we set the duty ratio that means on-time, what is the voltage gain of the ideal buck converter? K v is equal to V 0 by V IN that means, this is the voltage gain and if we operate under like a steady state condition, is a fixed switching frequency. Then we can take this as my duty ratio. And we will check whether we can get the desired voltage which is equal to V_ref can we get it or not if we take D equal to V ref by V IN.

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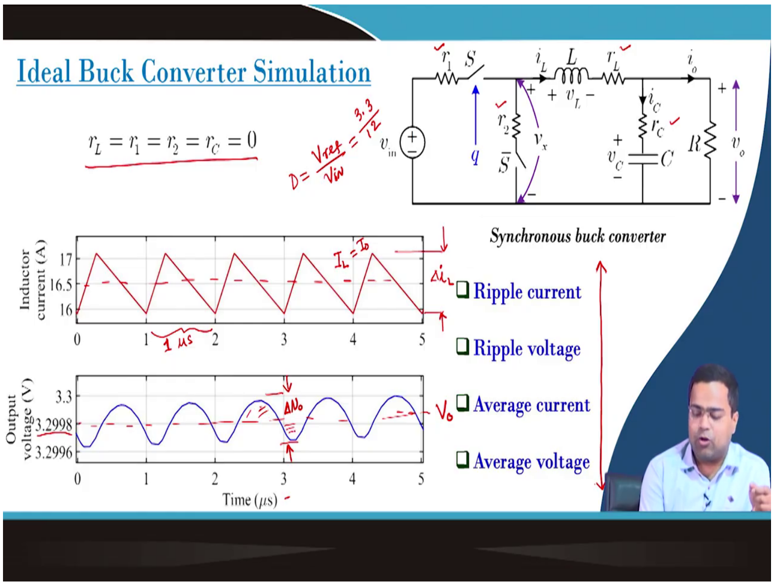

Next, now, in ideal simulation, we should I have replaced here all these parasitic r 1, r 2, r L which is the DCR of the inductor, then ESR of the capacitor all are set to 0, they are set to 0. And if we simulate this converter and this I have simulated using MATLAB. In fact, I have discussed how to building build blocks in MATLAB you know using a series parallels in a parallel RLC circuit series RLC circuit.

But in subsequent lecture, we will also build develop a Simulink model, the model for realistic buck and boost converter and we will also operate under different you know like a modulation and control technique. So, in this case, we have used a fixed switching frequency, which is 1 megahertz and it is like a 1 microsecond time scale. You can see from here to here it is 1 microsecond.

And here since we have consider an ideal case and we set the duty ratio D equal to we have considered V ref by V IN and for here, we consider V ref to be 3.3 volt and Vin to a 12 volt.

So, the duty ratio is set. Now, what we can see here that this is my inductor ripple current. We call it as an inductor ripple current this quantity and if we take the average here, this is my average inductor current. And for this synchronous buck converter or this buck converter, it is nothing but the load current. And if we take the average value of this output voltage, it is almost 3.3. Because we are talking about 3.29998 which is almost 3.3.

So, the average value and we are also discussing this part which is nothing, but delta. V 0 that is my ripple output voltage. So, we are concerned about; we are going to discuss what are the parameters like inductor ripple, output voltage ripple, average inductor current, average output voltage. All these we are going to discuss.

So, these are the four parameters that we are going to discuss, and we want to see what is the effect of this parameter on these parameters if we incorporate the real component like a parasitic.

So, in this case, we have seen that the waveform is show like waveform shows that the inductor current looks like a triangular waveform and the capacitor voltage profile is also consistent because we know that if the average if we take the capacitor current which is the subtraction of the inductor current minus load current.

So, that means, if we if this if you want to draw the capacitor current, this will be same as inductor current if we eliminate the DC value. And if the capacitor current is positive, the output voltage will rise and if the capacitor current is negative, output voltage will fall so, this is consistent.

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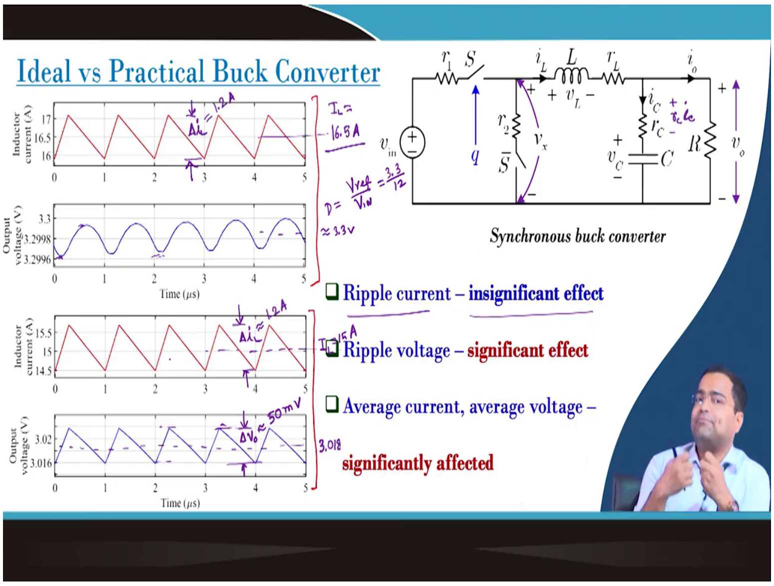

The next now, we incorporated the parasitic. Then, we now, we have consider r L, r C and what we discuss in the know the parameter set? These are the parameter set now, we have incorporated those parameters with ESR all these. So, this is my ideal simulation, which was shown earlier, and this is my practical simulation result of a practical synchronous buck converter where we have considered ESR, then DCR and on state resistance of the MOSFET.

What you can find here? If you compare like you know the four parameters that we discuss, what are the four parameters? One is the ripple inductor current. That means, if we consider this ripple current, this is my ripple current; this is my ripple current. If we consider the ripple current like here to here, this is my ripple inductor current. What is my ripple inductor current?

It is nearly 1, 1.2 ampere, output voltage that we discussed, and the ripple output voltage is almost negligible because if you see from this value to this value, it is like a 0.0004 that means 0.4 millivolt which is almost negligible. And, what is the average value that we saw in that simulation? It is around 16.5 ampere, which was our average inductor current value. The ripple we saw around 1.2 ampere something like that, close to that.

Now, what will happen and what is the case? In both the cases, we consider D equal to V ref by V IN which is 3.3 by 12 in both the cases. You see, the ripple current in this case that means, if we take this ripple current, this remains more or less 1.2 that means, this is almost same as 1.2 ampere that means, ripple current will not find much difference between these two 2 f form.

But, what about the average inductor current; the average inductor current, what is the value? It became 15 volt sorry 15 ampere; 15 ampere whereas, it was earlier 16.5 now it became 15 ampere so, there is a reduction is 1; 1.5 ampere. What about the output voltage average value?

So, it is almost 3.018 whereas, this was more or less it is nearly 3.3 volt. So, there is almost a reduction of how much is nearly 0.2 volt; nearly 0.2 volt difference. So, the 0.2 volt, the magnitude of the output voltage is reduced by almost 0.2 volt.

And what is the ripple parameter? If you take the ripple output voltage so, this is my ripple output voltage; this ripple output voltage is almost equal to how much? Around like a 50 millivolt whereas, this was around 0.4 millivolt. So, ripple current insignificant effect we can see from adding like if we add parasitic ripple voltage, this is significant effect suddenly, it has increased. The average current and average voltage both got affected significantly.

And another point, if you see the profile of the output voltage, this looks like almost in phase with the inductor current so, it clearly indicate that the contribution due to this ESR which is into i c, this is now prominent I mean this is the dominant effect. And since the capacitor current carries the ripple of the inductor current. Because we assume the load current to be constant.

So, the ripple current that means the ESR into ripple inductor current. That ripple will appear in the output voltage as a dominant effect and that is why it is in phase with the inductor current. So, what we can conclude from here that in a practical converter so, we will have some I mean we will have a significant effect of ESR so, you have to carefully choose the capacitor I mean we need to we should reduce the ESR magnitude.

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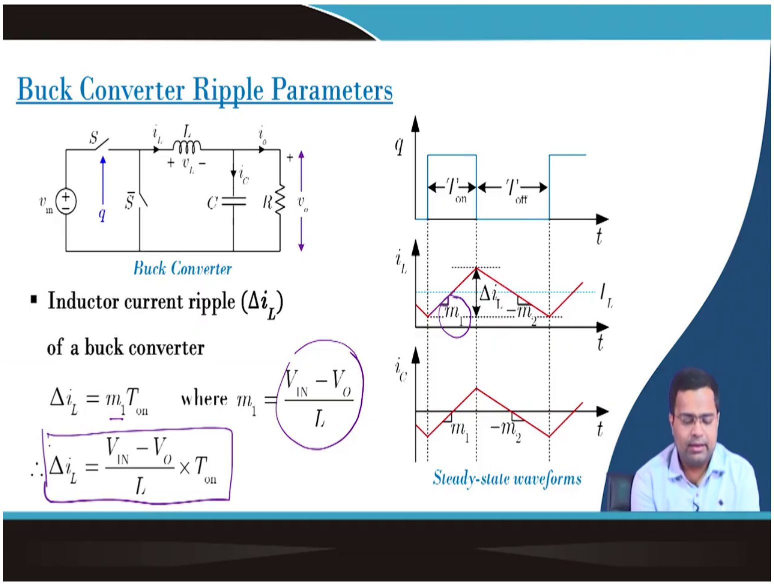

Now, if we take the buck converter with a ripple parameter, the inductor current ripple can be derived you know if we take m 1 to be the rising slope of the inductor current, this is a rising slope of the inductor current. So, I am talking about this quantity so, this is nothing, but V IN minus V 0 by L for a buck converter and from where we can find out my relationship like a find out the relationship of the ripple inductor current.

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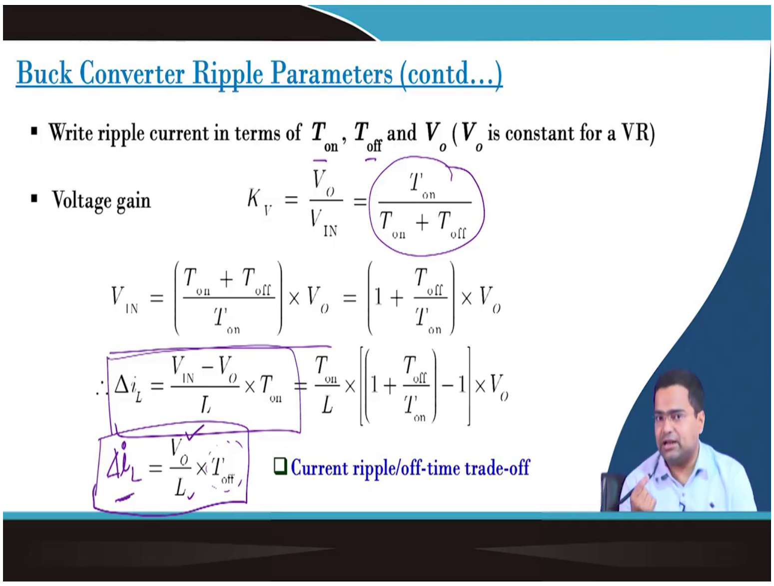

Now, if we continue, if we want to write so, what is known here in the previous equation? The output voltage we want to regulate so, this is a fixed value. Inductor that we have chosen at the design cycle at the beginning once you operate the converter like once you design, the inductor value is more or less get fixed. Although, it value can change you know depending upon the amount of current it is flowing through it because it will have some effect of; non-linear effect of the inductor, but in general, we can keep that inductance value to be more or less constant.

Now, we want to write the ripple parameter in terms of on time, off time and here, we want to write in V 0 rather than V IN so, where the voltage gain is V 0 minus V V 0 V IN and voltage gain can be written as this on time, off time. So, you can write the input voltage to be V 0 from where the inductor ripple can be written in terms of timing parameter which is finally, this. So, this is my delta i L so, this is my inductor ripple.

So, this can be derived that means, what it shows? The inductor current ripple depends on the output voltage which we in this case, we take this thing to be constant, inductance value is almost fixed we discuss, we have already discussed, but now this is the off time.

So, depending upon the off time the inductance, ripple inductor current can vary. If the off time increases, the ripple current increases. So, there is a tradeoff between ripple current and the off time, but off time is not in our hand because it depends on the voltage gain.

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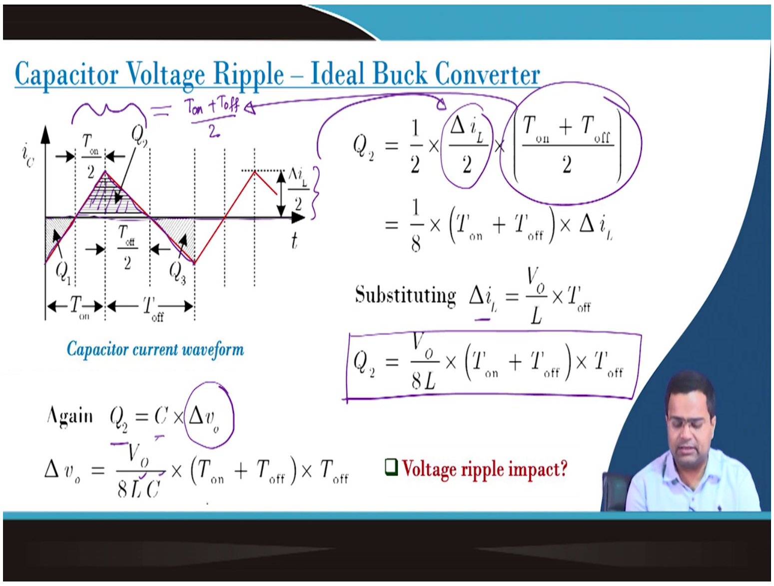

Now, if you take the capacitor voltage ripple again, you can draw the capacitor current and this profile is same as the inductor current ok. But here the difference is that the average capacitor current at steady state should be 0. Whereas, the average inductor current in steady state is equal to i 0 ok.

So, we can find out our intention to find out this area under the curve during the positive charge condition that when the capacitor current is positive and it can be shown that this time is nothing, but this time period is nothing, but T on plus T off by 2.

So, this term is here actually and this quantity is half of the ripple which is this quantity. So, we are talking about this quantity and area at the curve is half into height into the time, so you can find out. Now, we know that i L is V 0 by L into T off so, if you substitute, then you can get the expression of the positive charge of the capacitor. Now, this during this positive charge, output voltage will rise.

So, you can replace that capacitor positive charge is equal to C into the positive voltage rise that means, the ripple voltage. So, the ripple voltage can be expressed in terms of inductor, capacitor, output voltage and on-off time, the ripple impact.

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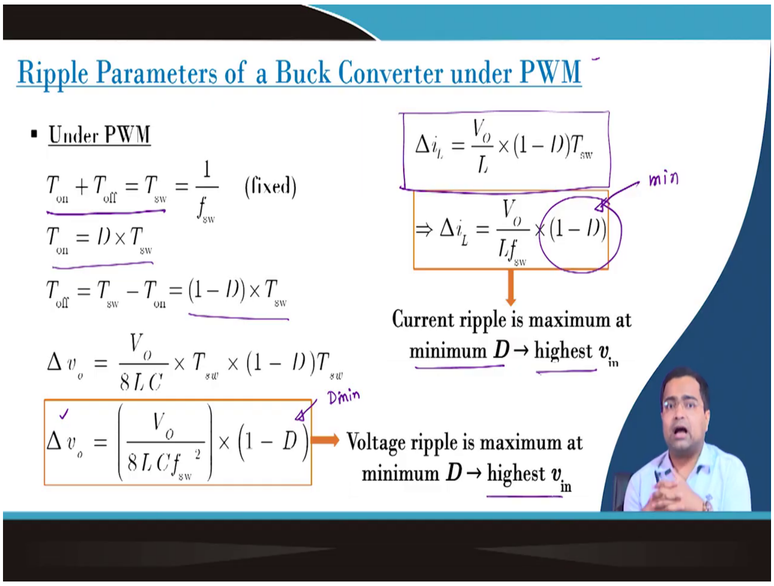

Now, we want to see the worst case, we want to analyze the worst case current ripple, voltage ripple and here, we have considered pulse width modulation, but we can take other modulation also. So, under pulse width modulation, the total time period is fixed, on time is replaced by D into T on sorry D into total time, off time is 1 minus D into total time and then, current ripple can be written in terms of duty ratio.

So, now, if we look at this expression, the current ripple will be maximum when this duty ratio is minimum and this is here minimum duty ratio. And in a buck converter, what happen under minimum duty ratio that means, what is the for a given output voltage, minimum duty ratio corresponds to the maximum input voltage so, highest input voltage.

And if we take the output voltage ripple expression, it will also be maximum the output voltage when D is equal to minimum, we are talking about minimum duty ratio and that will be minimum when the input voltage is maximum or the highest input voltage. So, the worst-case voltage and current ripple will actually we have to consider the worst case voltage and current ripple corresponds to the highest input voltage condition.

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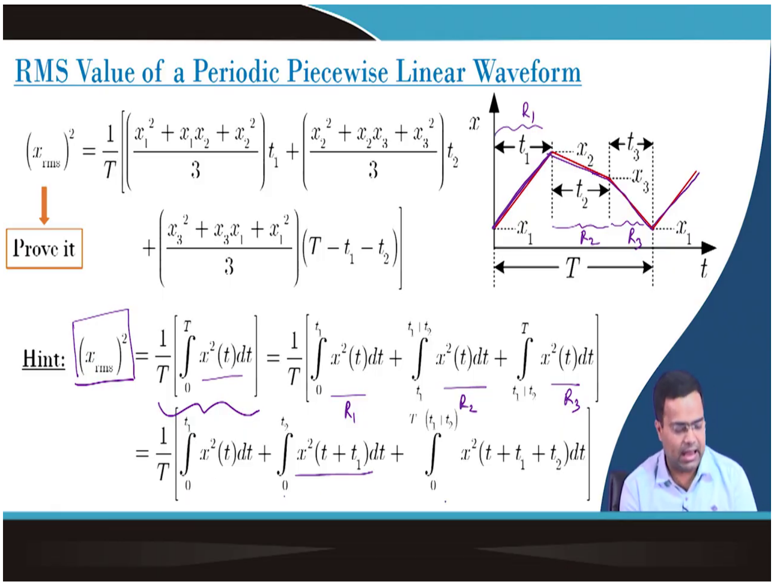

Now, we want to find out the RMS quantity. So, RMS quantity of a periodic piecewise linear waveform. So, here we are talking about a piecewise linear waveform that means, in general, it can it is the like inductor current even this piecewise approximation is also valid for capacitor current, it is also valid for switch current even if you consider diode current so, these are valid ok.

Now, even if the current is 0, this piecewise approximation also valid that means, under DCM that means, as long as the current waveform looks like a with a linear slope. The slope can be 0, slope can be positive, slope can be negative it does not matter as long as it is a; it has a linear slope any periodic waveform with linear slopes you know it can be represented.

Thus RMS value of this waveform can be represented by 1 by T particularly for this waveform x 1 is the value of this waveform at T equal to 0 and it is x 2 when T equal to t 1 after the interval of t 1. Then, after second interval it is x 2 here t 2 that means, this is my duration t 2, this is my duration t 1 and this is my duration t 3 which is nothing, but capital T minus t 1 minus t 2. So, it can be represented by this.

How can you prove that? Ok, I am just giving a hint; although, I am going to prove it in the next slide. So, if you take the RMS current of this you know periodic waveform and here for a special case, we are talking about a periodic waveform with piecewise linear approximation, it is shown it looks like a piecewise linear waveform.

In fact, this piecewise linear current is common in many dc-dc converter you know whether it is isolated, non-isolated in various other converters also as long as we can get a piecewise linear current approximation, we can apply this formula.

There can be n number of interval more number of interval does not matter. So, as long as is a periodic waveform and piecewise linear, this is valid. Now, if you want to get the rms square, this is a basic equation 1 by total time, then integral 0 to T, the time period x square t dt ok. So, you can break this the whole integral into three pieces, because there are three region the region 1, region 2 and region 3.

So, this corresponds to region 1, region 2 and region 3. Now, this is a definite integral so, we can change the in the definite integral, we can change the interval limit that means, the integral limit and we have to update this time. So, the same thing we have just change the integral limit.

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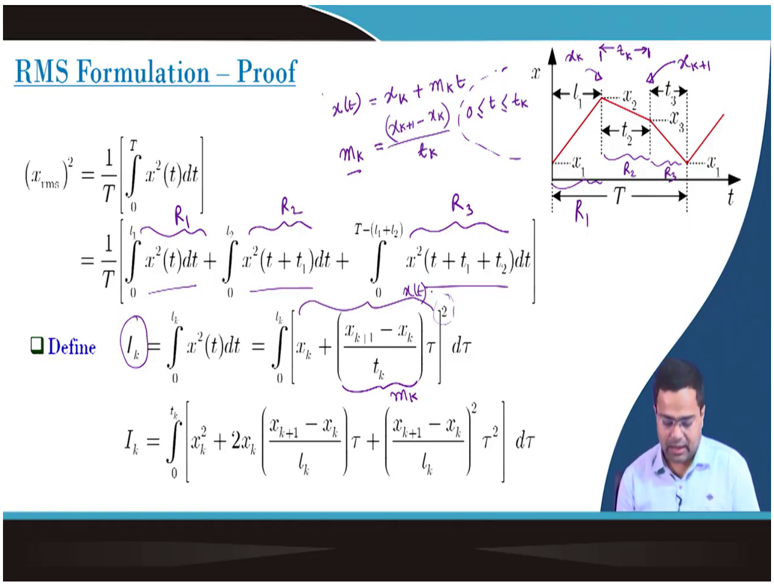

And in the next slide, we are continuing with that RMS waveform with the integral limit that we have already discussed. So, now, each region like R 1, R 2, R 3 that means, this equation corresponds to R 2, and this corresponds to R 3 region that means, we are talking about this region to be R 1, this region to be R 2 and this region to be R 3. Here, if we take a generic expression that means, if we take a generic region where the region of the integral is 0 to t k x square to dt.

Let us say this is my, this duration is my t k; t k duration where the starting value here is my let us say it is x k and the final value of this duration is x k plus 1 ok. So, that means, this waveform, the starting value that means, what is the equation of this waveform during this interval? It will be x k plus the slope of this waveform m k t, and this is during the interval t equal to 0 to t k. What is the slope? The slope m k is nothing, but its final value x k plus 1 minus x k divided by t k so, this is my time period.

Now, this slope can be positive if x k plus 1 is larger than x k, the slope is negative when x k plus 1 is smaller than x k and it can be 0 when both are equal. So, it does not matter whether it is positive, negative or 0, but it should be constant. Now, if you take this slope then and then, this x k plus 1 so, this corresponds to my m k that we have represent. So, we can write, and this corresponds to my x of t during the interval 0 to t k.

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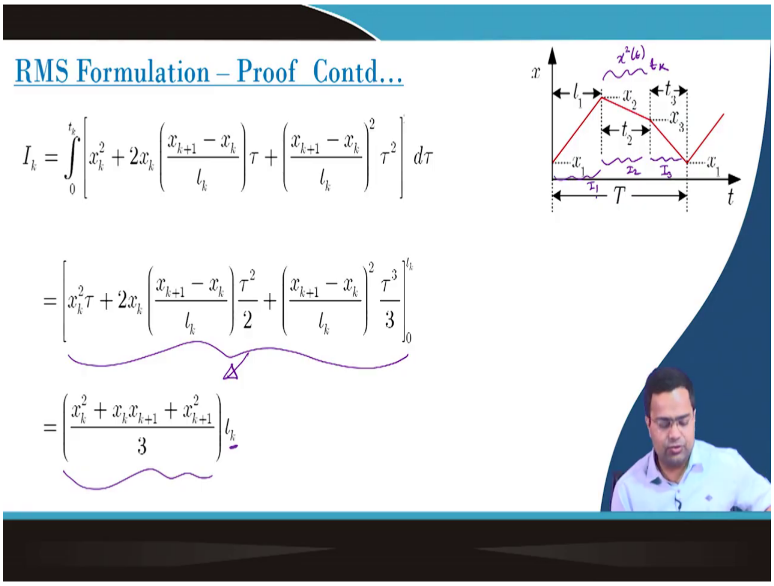

Then, if we write the full expression of this square term because there is a square term here and then, if we integrate after computing the integration, we will get this expression and after simplification, we can get this last expression. So, that means, if you take this x square t during this interval t k interval, we will get this value. And this is nothing, but x; x k square x k into x k plus 1 then plus x k plus 1 square by 3 into t k that is the direction of the interval and this we can compute for this interval as well.

So, I can take that I 3 I can calculate, I can calculate I 2, I can calculate this is my I 1 so, this integral I can compute for different region ok.

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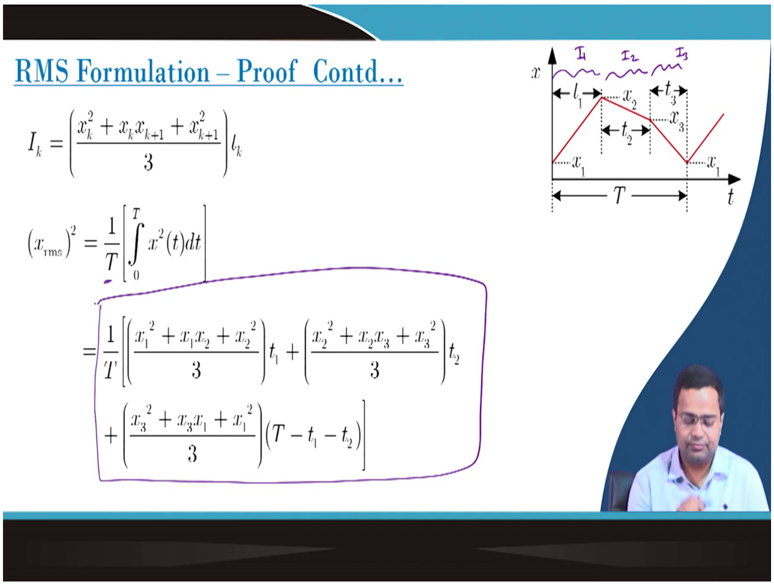

So, then the RMS current which was our original expression that can be written as the sum of this integral that means, I am talking about this is my I k sorry I 1, this is my I 1; this is my I 1, this is my I 2, and this is my I 3. And if I add them and divided by total time, then we will get x RMS square so, x RMS square and this is what we have actually written at the beginning.

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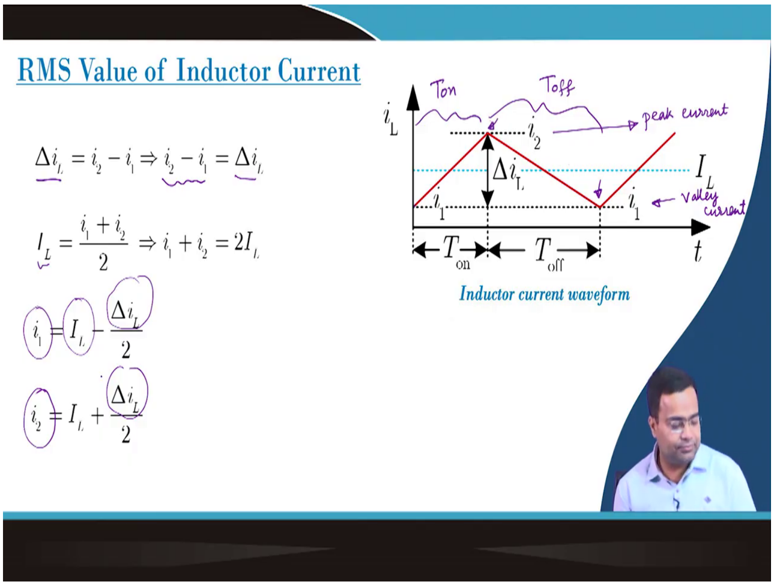

Now, we want to compute this for the inductor current. For the inductor current, you know it has only two interval like a this interval and this interval. What is this interval? This interval is on time and this interval is off time. In the beginning, the value is i 1 at the beginning and the final value is i 2. For this interval, i 2 is the initial value and i 1 is the final value.

So, then what is delta i L? That is the ripple current, that is i 2 minus i 1. So, i 1 is the peak, this corresponds to peak current, and this curve corresponds to this i 1 corresponds to the valley current; the valley current, this corresponds to the valley current. So, we can find out the ripple current in terms of peak and valley and we can get the peak and valley expression in terms of delta i L.

We can also get the average inductor current in terms of this quantity. Why it is important? We have to find i 1 and i 2 in terms of average inductor current and the ripple current how they are related.

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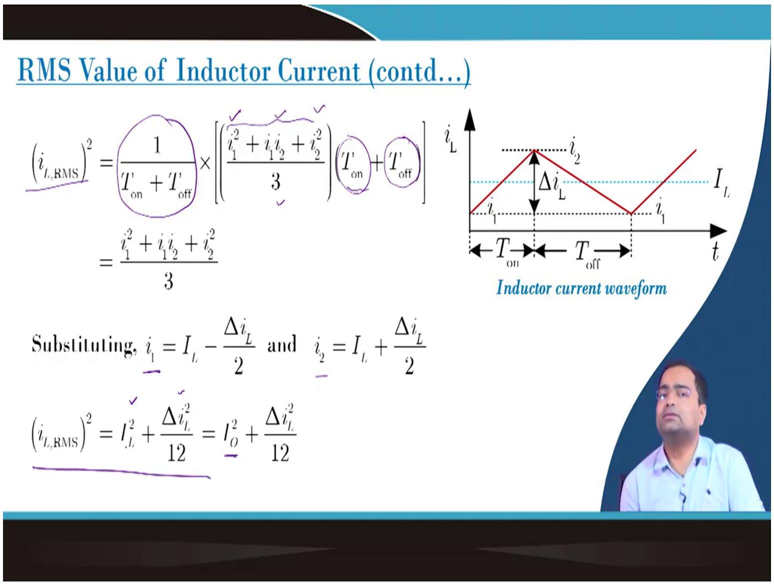

Because we want to find out the RMS current, the RMS current is what? We know the formula 1 by total time period, then in this case i 1 square i 1 i 2 i 2 square by 3 and since both i 1 and i 2 are common for both the interval only they are whether in one case it is peak valley.

So, like beginning value, but it does not matter as long as this formula is concerned, then the total time is T on, it is common T on, the same expression, it is also common for T off. So, if we add them together, then we will get the i L, RMS current is i 1 square plus i 1 i 2 plus i 2 square by 3.

If you substitute the peak and valley current in terms of their average current and the ripple current, then we will get i RMS square is I L square by the average inductor current square plus the ripple current square by 12. So, in buck converter, the average inductor current is same as the average and load current.

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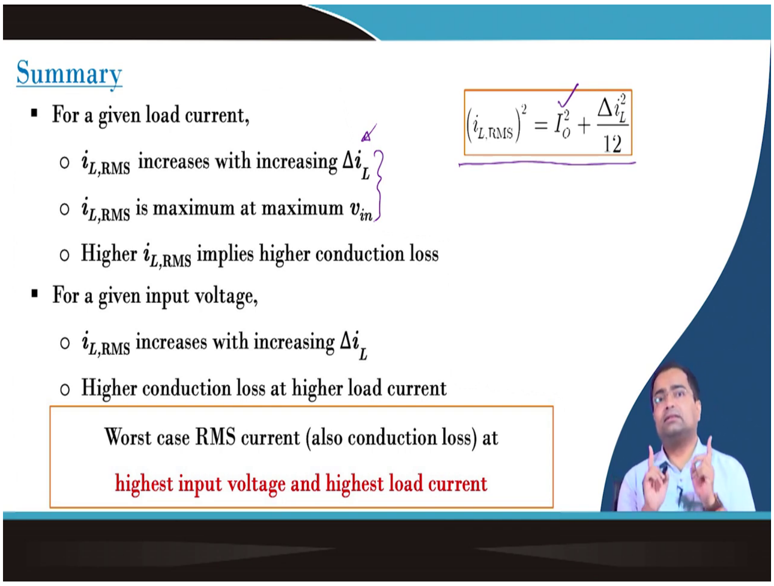

So, we can summarize that RMS current for a buck converter is the load current square plus the ripple current square by 12. For a given load current that means, if this is constant, i L, RMS that means, RMS inductor current increases with the ripple current and it is maximum when the ripple current is maximum and that is maximum at the input maximum input voltage that we have discussed.

This is particularly important for estimating the conduction loss because if the RMS current is more, then the conduction loss will be more. So, higher RMS current will lead to higher conduction loss. Now, for a given input voltage, if input voltage is given, now the load current is varying so, the RMS current increases with delta i L.

RMS current actually sorry it should increase with the load current because for given input voltage, given output voltage, the RMS only load current changes, the ripple current remains more or less same. So, if the load current changes, it increases with load current so, higher conduction loss at higher load current.

And the worst-case RMS current will be at the highest input voltage and the highest load current condition. So, that is the worst-case condition for the RMS current and this is also important that means, in this condition, at this condition, our conduction loss will be maximum.

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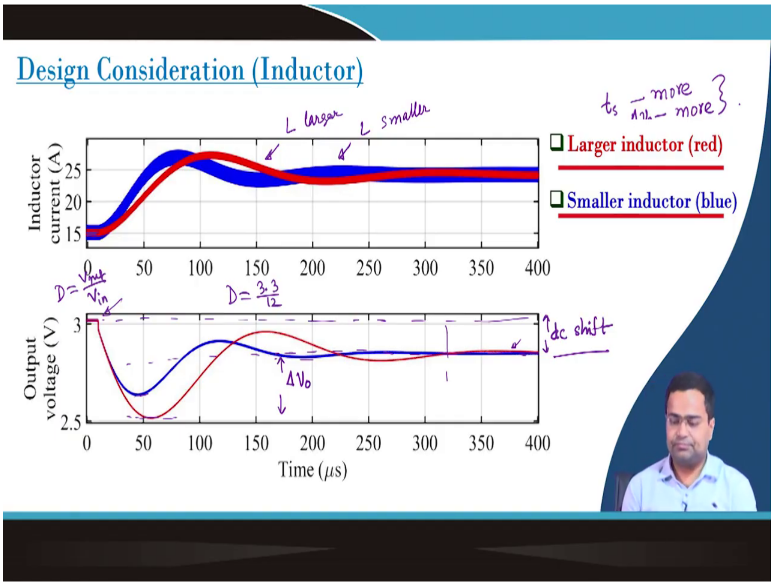

So, the design consideration, we understood the RMS quantities now, we want to show, we want to discuss that should we choose a larger inductor or a smaller inductor. So, it is clear from the previous expression if we take a larger inductor, then the ripple current will get reduce because and if the ripple current gets reduced, then the RMS current get reduce. So, we can save some conduction loss that means, we can reduce conduction loss, but the inductor size is increasing.

But we also want to see that was the steady state effect, we want to see what is the effect in the dynamic performance. So, here, I have simulated a case study. Again, this is the case study of a practical synchronous buck converter where we have set the duty ratio D equals to V_ref by V_in and which we have taken D equal to 3.3 by 12. Since there are parasitic drops so, the output voltage is not maintained at 3.3 volt because there is no feedback loop. So, it will be less than 3.3 because of drop.

And you will see this drop will increase at higher load current when the output voltage average value further decreases that means, if you take this point and this point, there is a reduction in the there is a shift in the dc shift, there is a dc shift. And this dc shift lead the more drop across that means, inductor DCR you know the on-time resistance of the

MOSFET, this can be expected by close loop control by adjusting the duty ratio, but here we are working or here, we are operating at a fixed duty ratio.

But what is our interest here? Our primary interest to see the average inductor current and the ripple inductor current. So, we can see the rate trace corresponds to the value when L large that mean larger inductor and it is consistent that we are getting smaller ripple, but average is independent of the inductance value. An average voltage also remains same for two unique value of the inductor and the blue trace corresponds to L small, smaller, it is larger.

But what is more important? If you see this undershoot, that means if we compute from this value to this value delta V 0 so, this delta V 0 is larger, that means I am talking about this steady state point is much larger for larger inductor. And it is smaller for smaller inductor because this value is much smaller. And also, if you see the settling time, that means, how long it takes to come to a steady state that also increases.

So, for larger inductor, our settling time is more and delta V 0 undershoot is also more and this can be reduced using smaller inductor that means the choice of inductor is also important in terms of dynamic performance ok.

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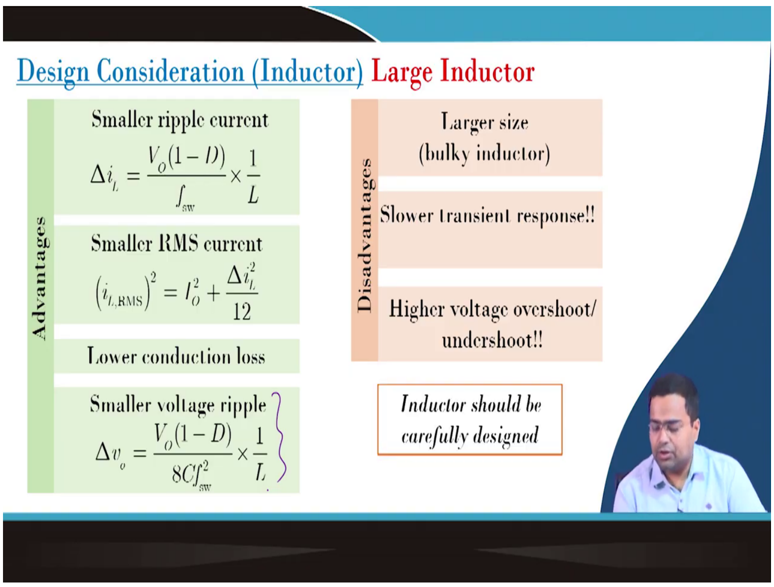

So, the design consideration of the inductor. If you take a large inductor, the advantage will be we can reduce the ripple parameter, we can reduce the RMS current so, you can reduce the conduction loss and we can also reduce the output voltage ripple. What are the disadvantage? The inductor size will increase naturally. Because you are taking a larger inductor, I mean large, the value of the inductance is high. And we saw that load transient response is slower, there is a natural response of the converter.

So, if the natural response is slower, that means, it indicates that when we design controller also does small series analysis, this will be reflected in the control to output transfer function.

And it also leads to higher overshoot and undershoot, and those things cannot be avoided if we use a larger inductor even with a closed loop control. The relative undershoot with for a larger and smaller inductor that relative you know reduction using a smaller inductor that concept is remains same even under closed loop condition.

That means, using a closed loop, you can reduce the inductor, the output voltage undershoot, by using a smaller inductor. So, inductor should be carefully selected.

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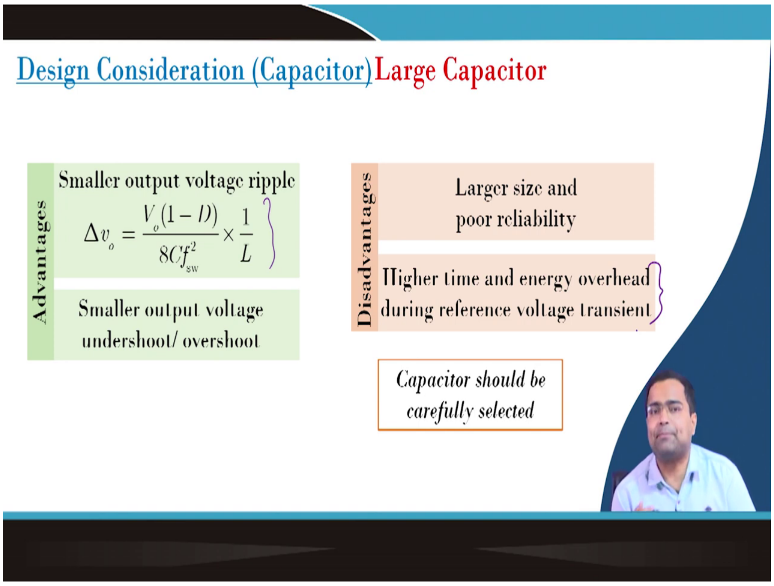

For a capacitor, if we use a larger capacitor, naturally the output voltage ripple will get reduced. This is obvious and we can also reduce the output voltage undershoot by choosing a larger capacitor. But, the larger capacitor size will increase and if the size increases in capacitor, the reliability issue also becomes important because it is more prone to failure the larger capacitor.

And also, we have not discussed here in the future, we will discuss. A larger capacitor requires more energy when you want to change the voltage of the output voltage of the dc-dc converter.

This is important for dynamic voltage scaling even it is also important for dc-link voltage, shifting of dc-link voltage of a dc-dc converter. So, larger capacitor requires more energy for shifting from one voltage level to the other voltage level. So, it will increase energy overhead and time over head. So, we have to select capacitor carefully.

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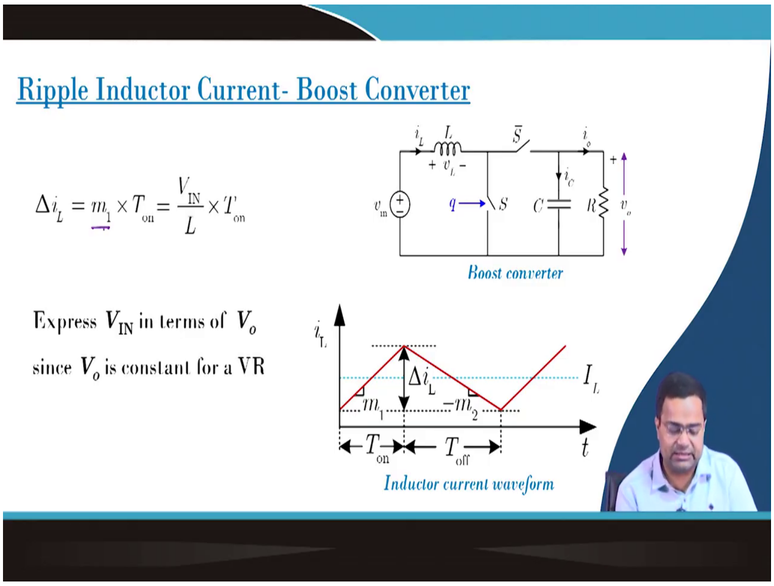

Ripple inductor current of a boost converter can be derived similar to a buck converter by finding the rising slope, then and expressing in terms of input output voltage.

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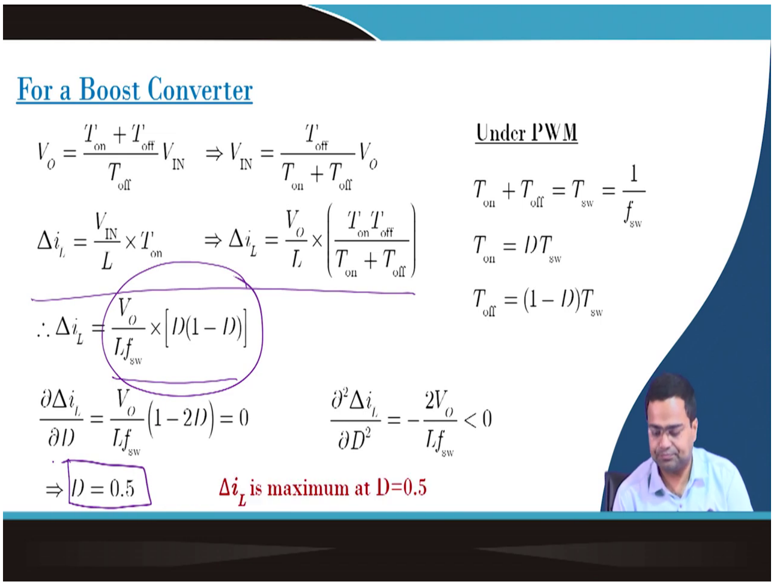

So, for a boost converter, the ripple inductor current under pulse width modulation I mean the process is same as earlier, only you have to formulate the ripple current in terms of the output voltage, inductance and the duty ratio. And it is interesting to see in this case that inductor current ripple when the output voltage is constant is maximum if you differentiate with respect to duty ratio, it is maximum at D equal to 0.5.

So, in a buck converter, we got the current ripple is maximum at minimum duty ratio, but for a boost converter, it is maximum for 50 percent duty ratio. On either side of the duty ratio, you can reduce the inductor current ripple, it is maximum.

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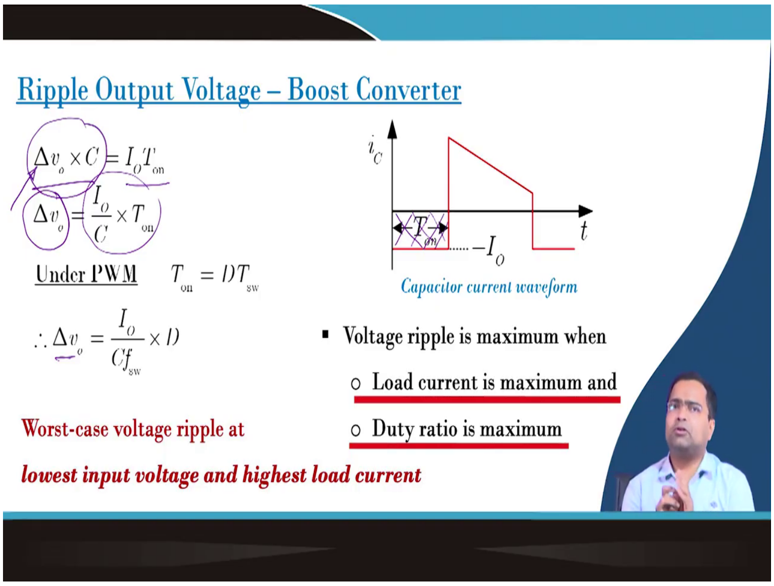

If you look at the output voltage ripple, then the capacitor current can be replaced I mean it is actually the minus of load current when the switch is on and switch is off, it is i L minus I 0. So, you can find out if you take this negative interval that means, you are talking about this area under the curve, then what is the area under the curve? I 0 into T on is equal to delta v 0 into C.

So, this is my undershoot or basically negative ripple that means how much it is deviate. So, output voltage ripple can be expressed in terms of load current and that on time.

So, you can find out under pulse width modulation, the output voltage ripple is maximum at maximum duty ratio; it is also maximum at maximum load current. So, the worst-case output voltage ripple will happen at highest load current condition and highest duty ratio that means the lowest input voltage condition. So, this is different from the buck converter example.

So, we found the ripple voltage ripple current for buck and boost converter; we found the RMS current, in the similar manner, we can find the RMS current for a boost converter and we can analyze that under what condition the ripple current is maximum. So, here we have taken only pulse width modulation, but we can also consider other modulation technique.

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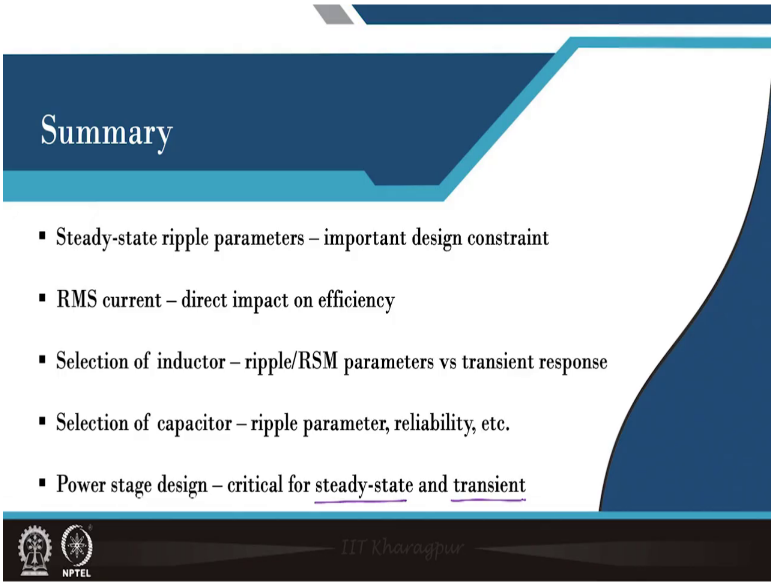

So, in summary, we have discussed steady state ripple parameter; we have considered the important design constraint. RMS current formulation also I have shown like you know and how it is going to affect the like a in the conduction loss and as a result, consequently the effect will appear in the efficiency as well. The selection of inductor is discussed that means, it is a tradeoff between the steady state RMS, ripple parameter as well as the transient response.

The selection of capacitor is important that in terms of ripple parameter as well as is reliability so, we have to carefully design inductor and capacitor. So, the power stage design is very critical, and it has to be designed not only from steady state standpoint, we should also keep in mind the transient point of view. With this, we want to finish today's lecture.

Thank you very much.