Control and Tuning Methods in Switched Mode Power Converters
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Module - 11
Large Signal Controller Tuning
Lecture - 50
Large-Signal Controller Tuning in Buck Converter: Objectives and Derivations

Welcome this is lecture number 50, in this lecture we are going to talk about Large Signal Controller Tuning in Buck Converter, what are the objectives, and the steps for derivation.

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So, in this lecture, we are first going to recapitulate the PID controller tuning objectives, then large signal modeling of a buck converter, then large signal based tuning in a buck converter and the summary of tuning parameters.

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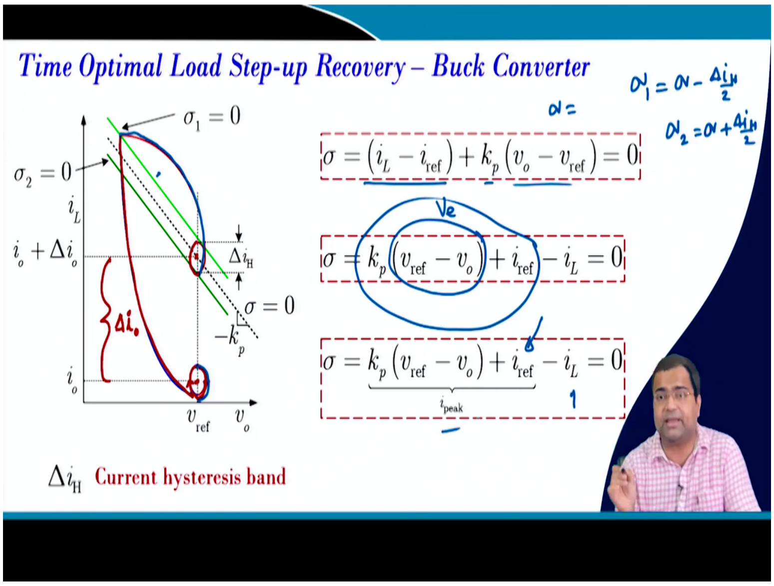

So, the time optimal load step recovery first let us try to understand because we have discussed in the last week like you know in the boundary control like lecture number 47. That as well as the 48 what is the concept of time optimal control in which you know if you start with one you know I think the load condition.

Suppose we are here, the load condition here, and now there is a load step transient. This is my load step size of delta i 0 step size and it has to go to the next load current right and during steady state it will have an on-off operation. This is like an on state, and this is like our off state. Then this is a load transient. It will go by this path, and it will hit this surface and in one switch action, it has to come back to this surface. It will hit and then it will again like continue like this like this.

So, that means, we will have you know this is our off state trajectory on state trajectory. So, this is the sigma. We know that sigma 1 sigma 2 which are used to generate the hysteresis band; this is only an offset hysteresis band. Otherwise, sigma 1, sigma 2, sigma 3 all have same slope. If we discuss about the sigma, this is a sigma, and we have a current hysteresis band.

So, thg sigma comprises error current gain into error voltage equal to 0 whereas, you know if we want to consider the band hysteresis band. So, sigma 1 will be sigma minus delta i H by 2 and sigma 2 will be sigma plus delta i H by 2 that is it. Now, if we rearrange sigma that means we are multiplied with minus 1.

So, it will look like a traditional error voltage like error voltage where V ref minus V 0. The reference current will look like a peak current reference consisting of the reference current, which can be load current plus the proportional gain into error voltage. So, it is like a proportional control with load current feed forward and which is compared to the inductor current; that means, this is a peak current.

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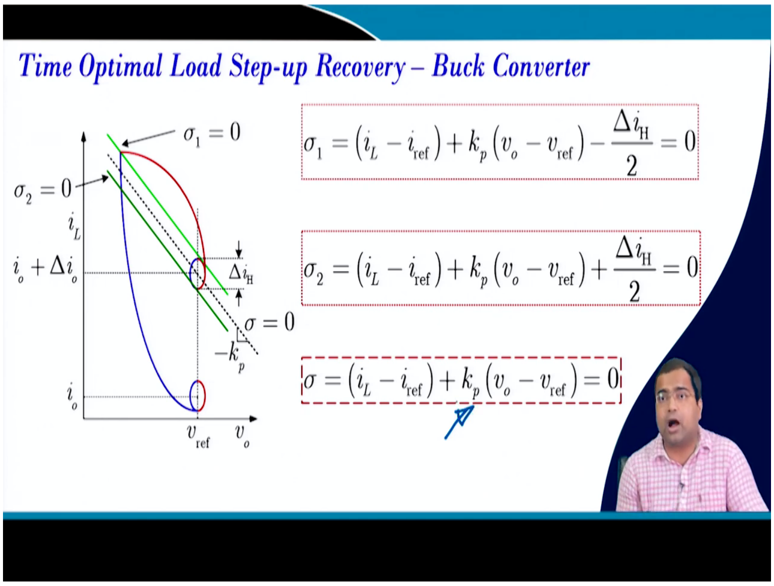

So, this can be translated and if you put a hysteresis band that we have discussed sigma 1 which is just to this band, is created delta H by 2 in either side. So, that the overall band is delta H and you have to choose delta H in such a way. So, our steady state switching frequency will meet our desired switching frequency requirement. That is it ok. Now, another problem you have to choose this proportional gain how to choose that.

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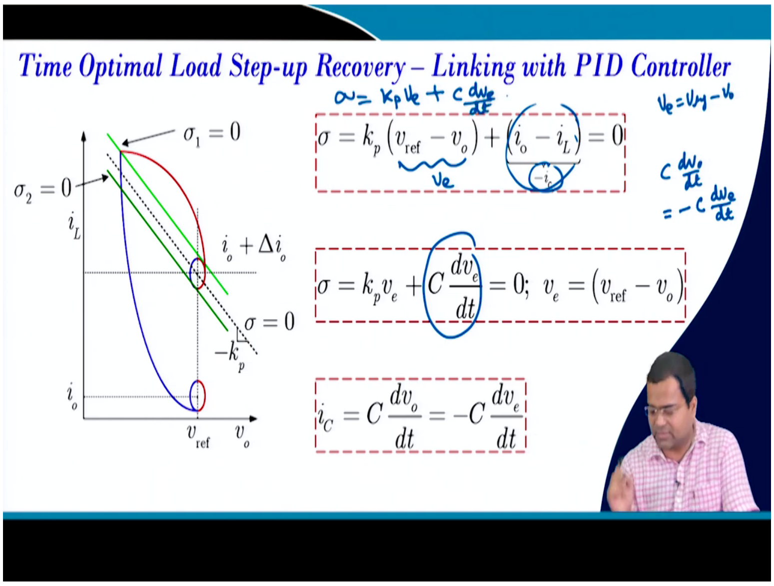

Now, if you rearrange if you link if you recall our earlier concept here this current if we replace i ref reference current is equal to load current then load current minus inductor current is minus capacitor current and the minus capacitor current means we have C dv 0 dt right.

Now, what is our v 0? We know that our error voltage is V ref minus V 0. So, this can be written as if V ref is constant it can be written as minus of that means, if we take this one minus of C dv e dt ok. So, this is exactly what minus here it is a minus sign.

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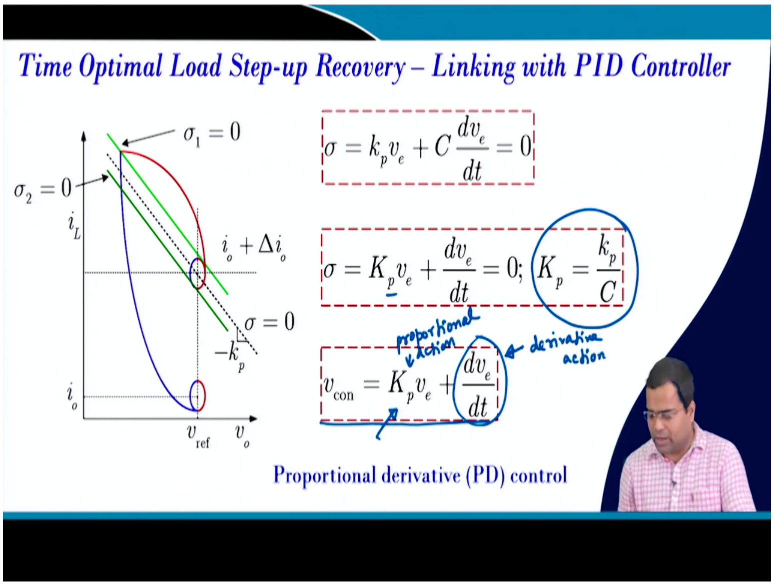

So, this I capacitor current expression we have given now the sigma will be replaced by. So, sigma we have this is our error voltage, so that means, the sigma here we can see k p into error voltage plus because this whole term is minus of C, C into dv e dt right C into dv e dt. Now, if we normalize if we divide by C then we will get small k p by C which we are taking as a capital K p.

So, it will look like you know, like a control voltage, the switching surface. So, it is our control voltage. It is a PD controller, proportional controller gain, and the derivative gain is unity because we have normalized it. So, it is a PD control ok, proportional gain and derivative control proportional control. So, this part is our proportional. This is our proportional action this part, and this part is our derivative action ok.

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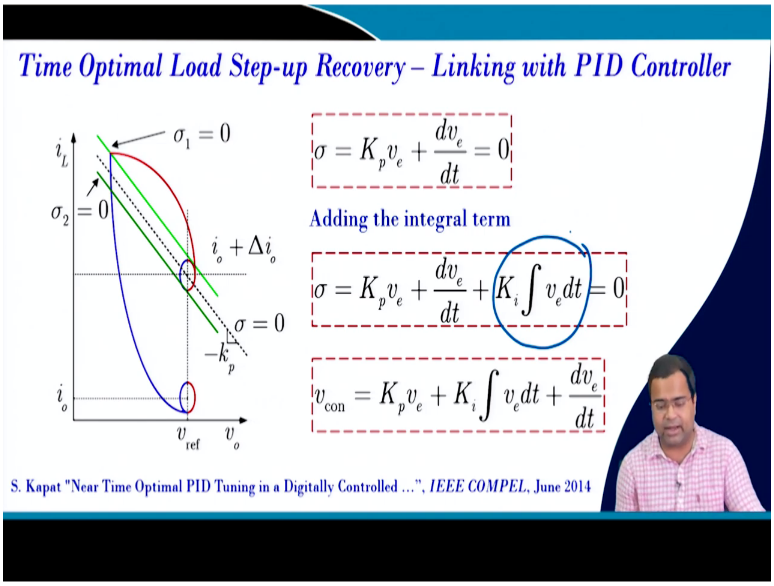

Now, this sigma K p into error voltage if we add an integral term that means we have an integral term into the integral of the error right. So, what we will get? Then the whole thing will look like a PID controller. ok see a PID controller formulation. So, we can in a buck converter we can show that this actually represents like a PID controller and this has been reported in this paper.

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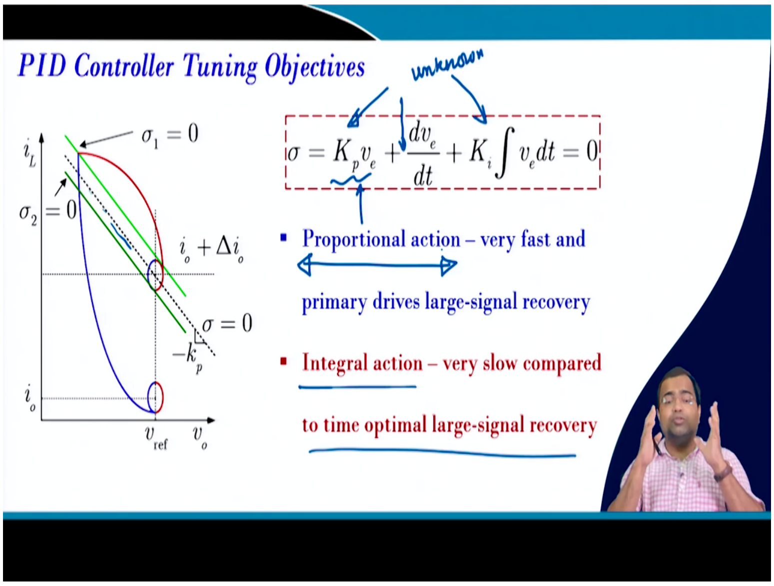

How to formulate this the next object. So, we have a PID controller. So, here, since our derivative gain it is normalized, so it is unity. So, we do not need to define or obtain any derivative gain. So, we have only 2 degree of like a two unknown variable this and this these are the two unknown. So, these are the unknown variable, that we have solved.

Now, out of these two, the proportional action is very fast, which primarily drives the switching action of the switching surface. This will act during a large signal transient right though it is. So, fast that during the large signal, this proportional gain is going to decide the shape of the trajectory.

But integral action is used to remove the residual error, so that the output voltage will approach towards the reference voltage. So, this action will come during steady state or close to steady state when the objective is to achieve almost 0 steady state error. But this has almost in negligible impact on this large signal recovery and where we want to achieve time optimal recovery at their our primary role is to choose a suitable proportional gain which will achieve the one switching action ok.

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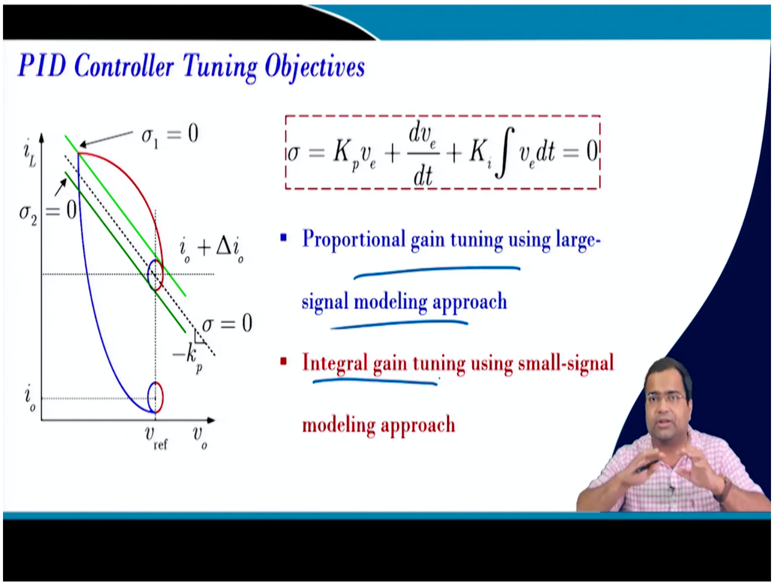

So, for the proportional gain here, we are using large-signal model. Thus is used to design our trajectory switching surface in such a way that we get one switching action, whereas the design of the integral gain is carried out using small-signal model. This will primarily contribute near steady state, and it has a very slow scale behavior near to steady state. So, that it will try to eliminate the steady state error.

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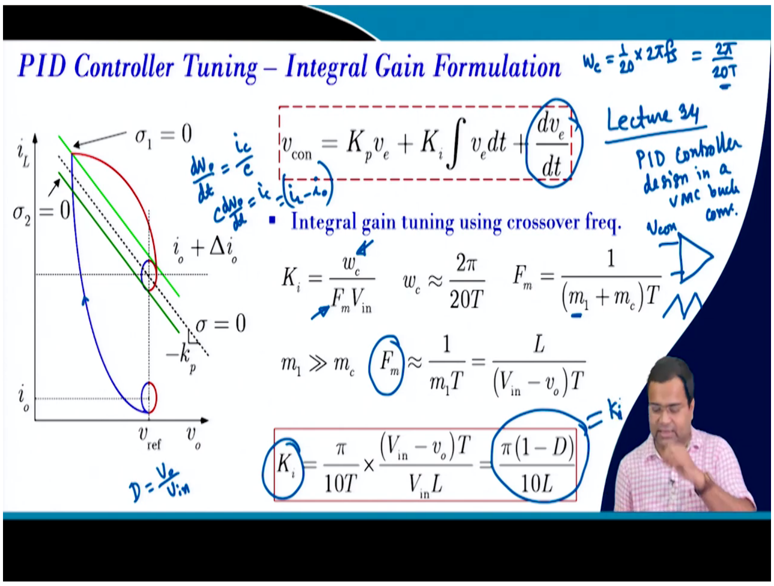

So, let us first design how to design integral gain. So, if you recall in lecture number 34 if you recall lecture 34 because we are talking about PID controller design in a VMC buck converter. So, this lecture will talk about PID controller design. And there we found if we use a PID controller we can actually by means of an analytical pole 0 cancellation stable pole 0 cancellation we can get first order closed loop equation. But unfortunately we could not get the suitable model matching up beyond like even up to one tenth of the switch frequency. Our model was limited.

So, here that is why we are considering this PID controller gain. We know the expression requires a crossover frequency. This is the modulator gain, and this is the input voltage. So, we need to get the crossover frequency one twentieth of the switching frequency because you know one twentieth means omega c will be one twentieth times 2 pi f s and f s is nothing but 2 pi by 20T 1 by T is a time period.

So, we are choosing one twentieth why because we have seen that this PID controller tuning for small signal the model validity somewhat comes somewhat reasonably well below one twentieth or even less. So, we are choosing one twentieth switching frequency.

Now, what is the modulator gain? Here we are considering the derivative of the output voltage, and in a buck converter, the derivative of the output voltage is nothing but the know capacitor current into c. If you take c dv 0 dt then it will be i c which is nothing but i L minus i 0.

Where i L has a slope which is a rising slope of m 1 because we are talking about step up transient. This is particularly the trailing edge modulator, where we want to decide the turn on time based on the switching logic. Basically, it will come from the closed loop system, but the off time will be decided from the type of modulator that we are using. For the trailing edge modulation, the off time is constant.

So, that means, rising slope the modulator gain here since we are using a derivative action, it is injecting the in inductor current information, so even though you can use a separate ramp. That means, if you have a sawtooth waveform, separate sawtooth waveform, you can use, but this is like a control voltage.

But this sawtooth we are taking a combination of the derivative of the output voltage, which is nothing but which carries the ripple information of the current and the sawtooth waveform. So, our combination of modulator gain will be 1 by m 1 slope that is the rising slope plus the compensating ramp into T.

But since m 1 we are taking a smaller slope and we will show our simulation case study, the m 1 is very large and what is m one for a buck converter? It is m 1 is nothing but V in minus v 0 by L. So, if you substitute f m can be approximated by this and if you write the integral gain can be finally obtained pi 1 minus d by 10 L where what is D? because in buck converter you know v 0 by V in is equal to D. So, if you substitute, so this is our k i that we are going to consider and we will see simulation case study.

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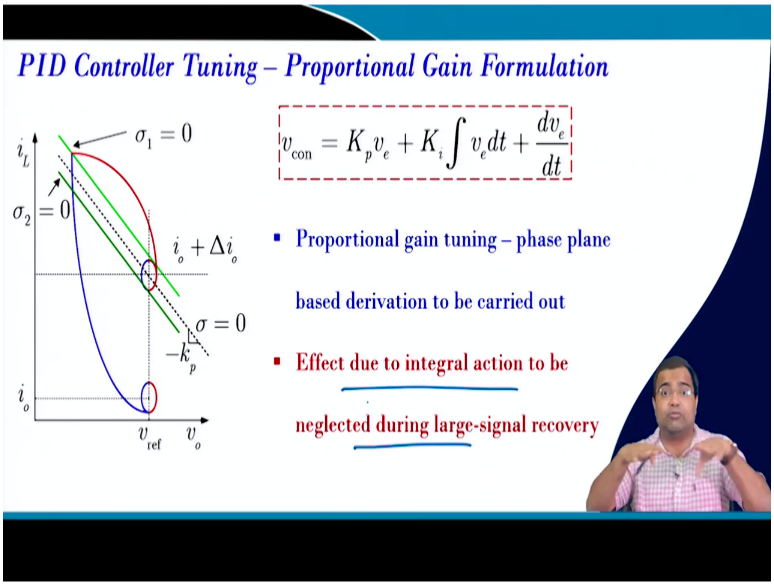

Now, we have to find the proportional gain and I told you the proportional gain needs to be found using large signal model where you need to incorporate the ripple information and that we are using phase plane geometry and that. And during this, when we carry out the phase plane analysis we will take, we will not take the effect due to integral because we told this effect primarily comes near steady state.

So, in large signal recovery, it is a proportional gain which shapes the trajectory. So, we will ignore the effect, but it will create an offset because it will ensure that steady state output voltage is regulated ok.

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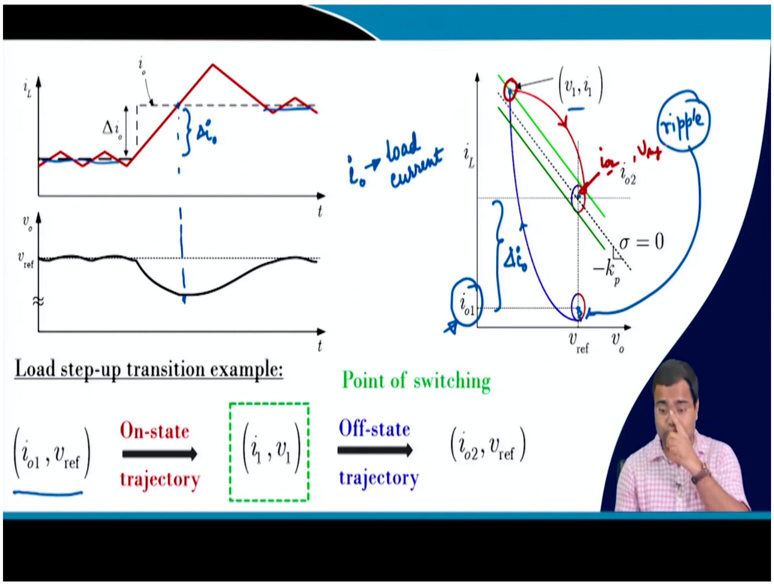

So, now coming back to our time domain waveform in time optimal control where the inductor current average was here and you need to achieve the next average current here and there is a ripple current right. And you know as long as the inductor current is smaller than the load current, your capacitor voltage capacitor current is negative and as a result the capacitor voltage will dip. But once it crosses, then your output voltage will or capacitor voltage will slowly increase ok.

And this is the corresponding phase plane diagram you can think of now our load transition state. So, we are starting from the initial load current of i 01. So, please remember we are using i 0 as the load current. Generic symbol load current which was i 01 at the beginning of the transient and at the end at steady state it will be i 02. The difference here we will find that i 0 s a load step size because it is coming from here, so delta i 0 ok.

So, a starting point it is starting from and we are ignoring here the ripple a ripple effect. This ripple effect here the starting point we are ignoring the ripple effect, so that we can take i 01. So, this will not change very drastic transient respond because we are using a modulator. So, we may not keep using the closed loop control perfect time optimal, but we will get close to time optimal.

So, that is why this assumption is ok. Otherwise you can incorporate this ripple parameter here this particular chain, but it will simply increase the complexity of the derivation. So, you want to make the derivation simple and it will be, of course, approximation, but that approximation is ok. So, we will see simulation case study.

So, it will start from i 01 and V ref this point, then on state trajectory it will follow this trajectory it will reach up to here. So, final value will be i 1, v 1. So, since we are plotting v 0 versus i L v1, i 1. Next it will follow the next trajectory which is this off state trajectory and for that this is my initial condition and the final condition will be i 02 is my load current and also V ref is my reference voltage i 02 V ref.

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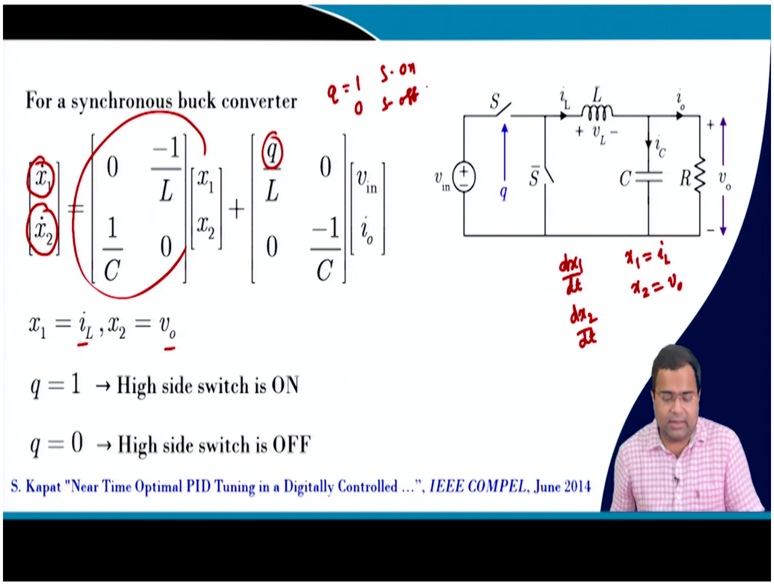

Now, for a synchronous buck converter, we can write the state space equation dx 1 dt where x 1 is my inductor current and x 2 is my output voltage for ideal case. It is also the capacitor voltage and we can also obtain dx 2 dt. So, since we have derived this state space model multiple time I am not going to derive.

But if you write dx 1 dt and dx 2 dt you can write in the state space form and the states are inductor current and output voltage and this q is a gate signal. So, when q equal to one for S on and it is 0 when S off that we have discussed high side low it is a high and low and this is also given the derivation in this paper you can get it detail.

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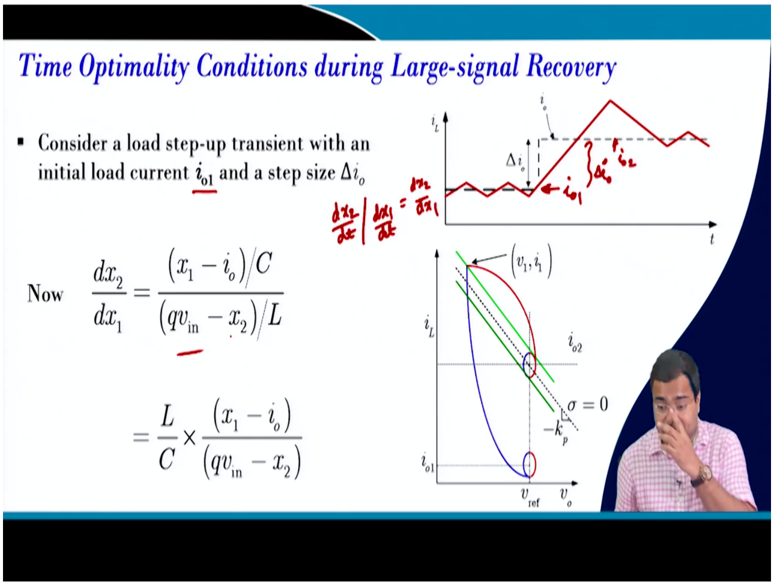

Next, the time optimal recovery it start with i 01 load step that we have discussed. This is our i 01 and then it goes to i 02 the next value i 02 and that difference is delta i 0 step. So, now, we can write dx 2 dt divided by dx 1 dt and that will give you dx 2 by dx 1 here.

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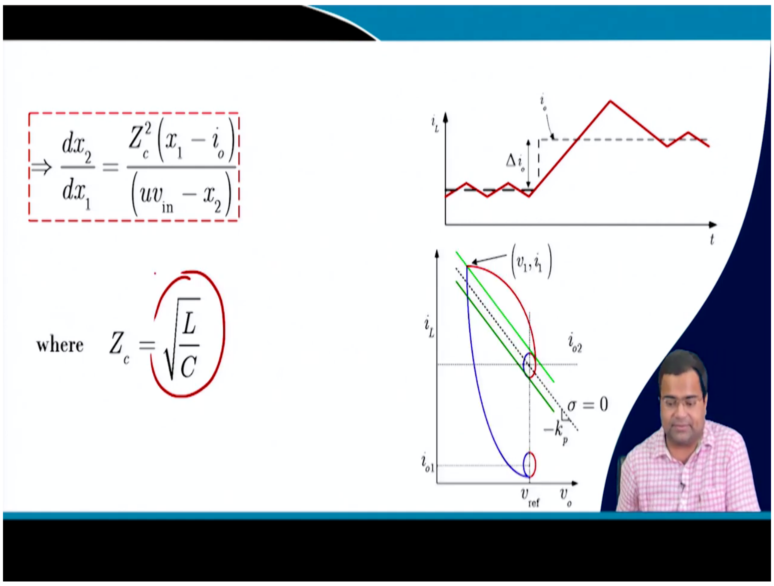

Then you can write and this next this Z c is nothing but the characteristic impedance which is square root of L by C.

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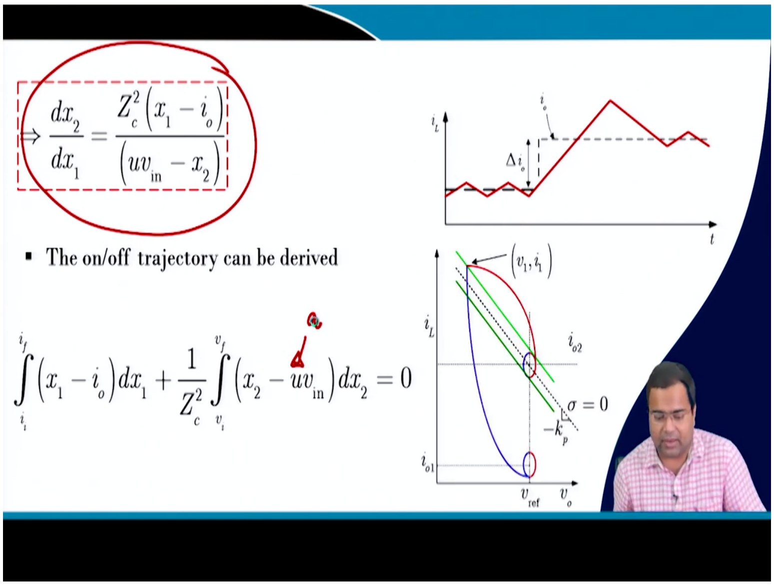

Next part we have this equation and we need to solve it and if you solve, it will give the trajectory equation. So, the trajectory of the common solution is this where q will determine or here it is u in this case u. So, here it is nothing but q, q equal to 1 or 0. So, the common solution of this trajectory can be solved by putting different integral.

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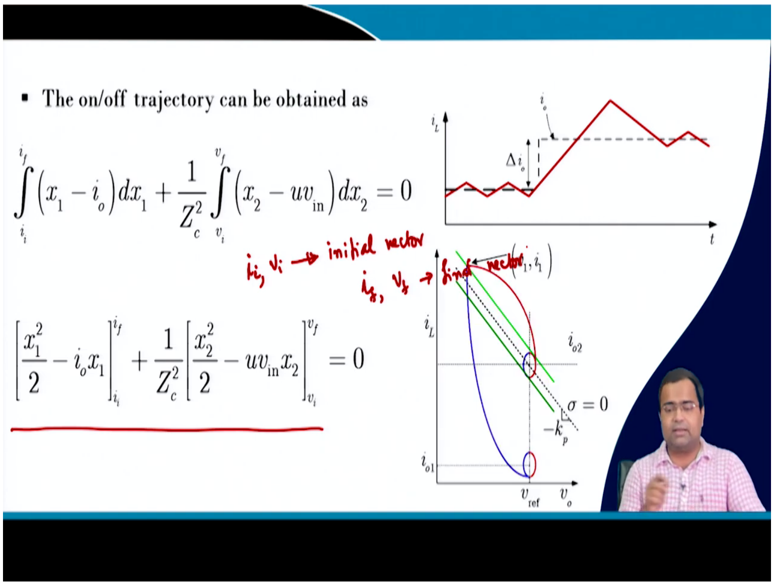

And if you solve it then you will get a generic form like this where i, i v i are the initial vector and i f and v f are the final vector. So, you can choose initial and final depending upon whether you are considering on state or off state right. So, you can accordingly select this trajectory.

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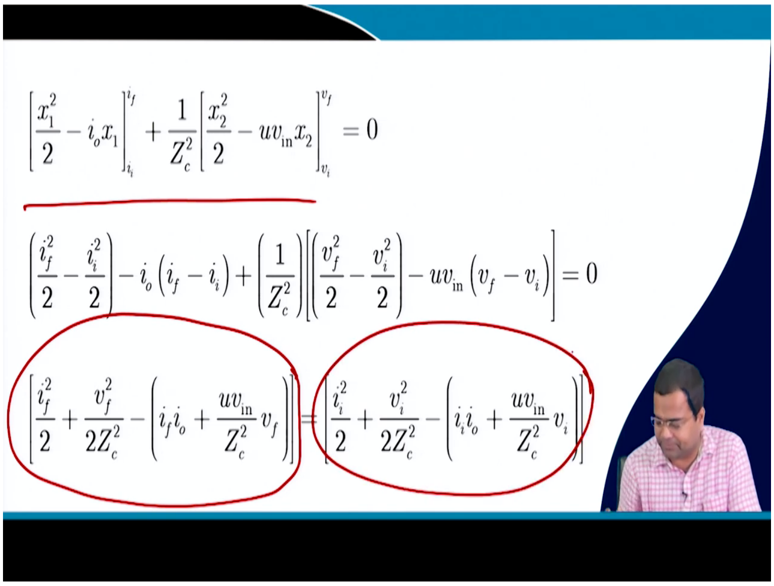

So, that means, if you solve this by putting the definite integral. So, this is the equation and if you rearrange, I am taking all the final quantity left side and the initial quantity right side.

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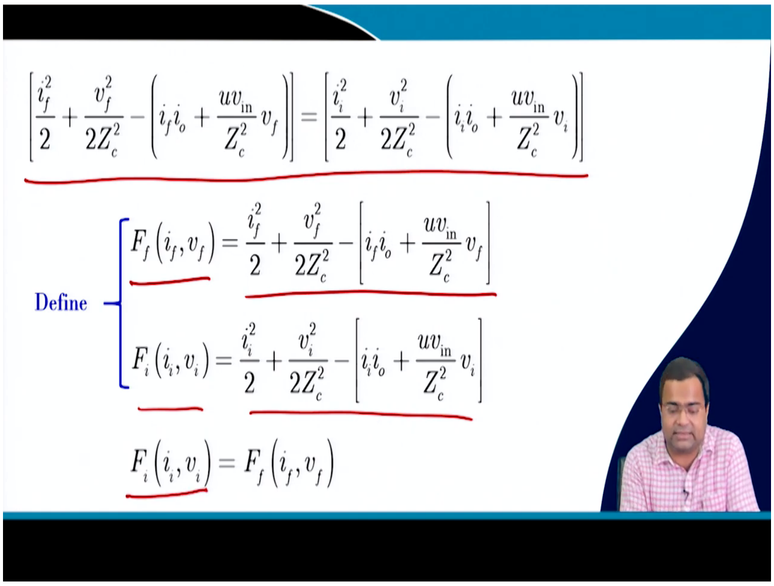

And that I am denoting as a function which consistent with the final quantity and this function to the initial quantity. So, this is I am defining and you see a form here you are getting final initial quantity is equal to final quantity ok.

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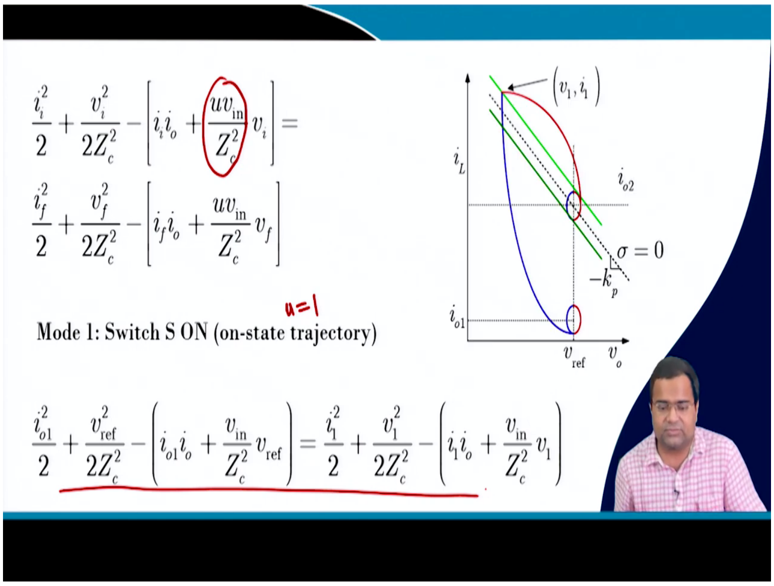

Now, for on-state trajectory we have to substitute u equal to 1 right and if you do that this u equal to 1 this term, so you can rewrite this equation right. So, this equation we have already derived from the integration.

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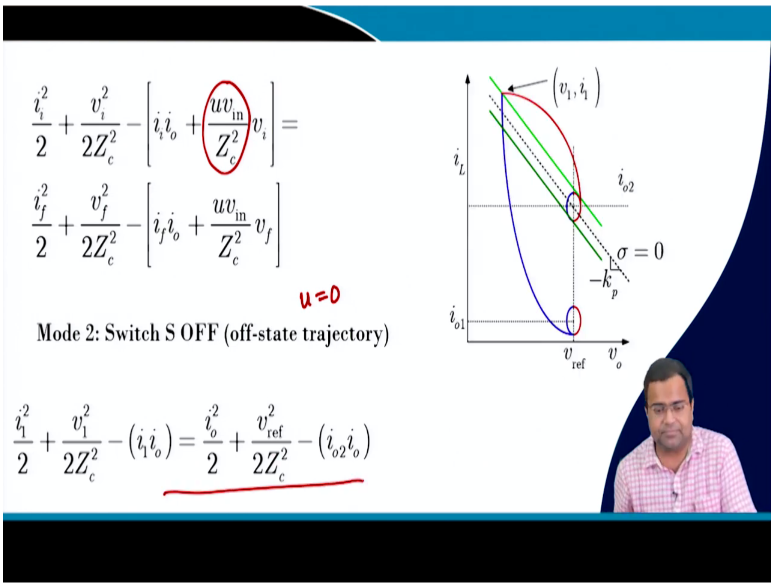

Next, if we rewrite the equation for off state we need to put simply u equal to 0. So, that means, this term will get vanish and the input voltage is not there. So, we will get the off state trajectory.

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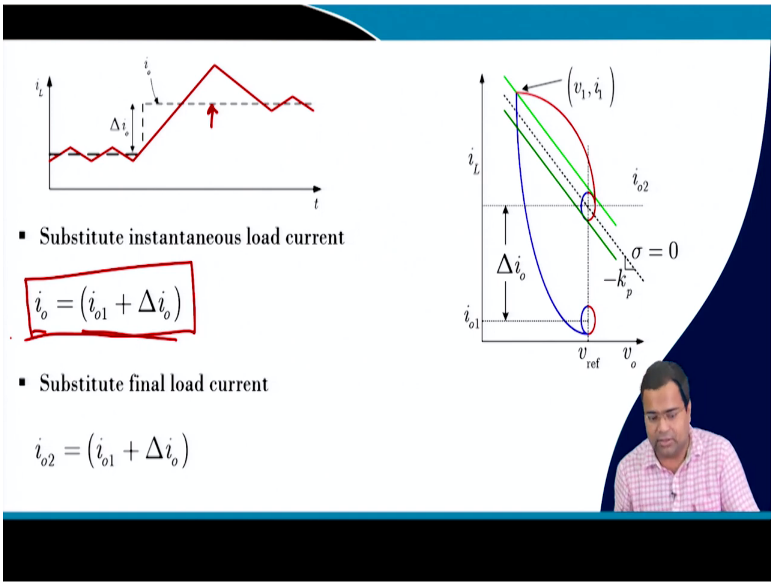

Then you have to substitute the instantaneous load current for i 0 the final load current; that means, the final load current, which is the instantaneous value, is i 01 plus i delta i 0. And if you substitute the final load current. So, sorry, the initial instantaneous load current because when you are talking about instantaneous means derivation suddenly there is a load step. So, new load current is nothing but our i 01 plus delta i 0 and that is also my final current.

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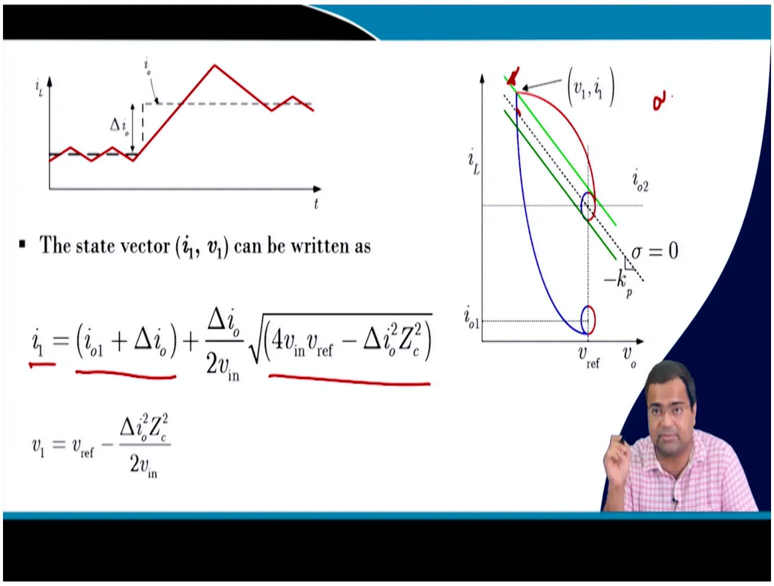

So, in our expression, if you substitute, you know if you go back. So, here you have this i 0 i 0. So, this i 0 is the instantaneous load current, ok. So, this i 0 is the instantaneous load current. So, this instantaneous load current we are putting i 01 plus delta i 0 and i 02 it is the final current, which is also i 01 plus delta 0, but i 01 is the initial current.

If you substitute from that previous equation, you can find out i 1 in the form of the final current into this v 1 in this format. So, this you can check it out because I am not going to derive all steps, but this can be obtained from the earlier equation that we have written from this you know off and on state.

Because for off state i 1 is the initial state, and i 02 is the final state and v 1 is the initial state and V ref is the final state. If you go to on state i 01 is the initial state and V ref is the initial voltage, i 1 is the final volt current and v one is the final current. So, by that way, if you will get two equation two unknowns you can solve it i 1 i 1 v 1 is the unknown.

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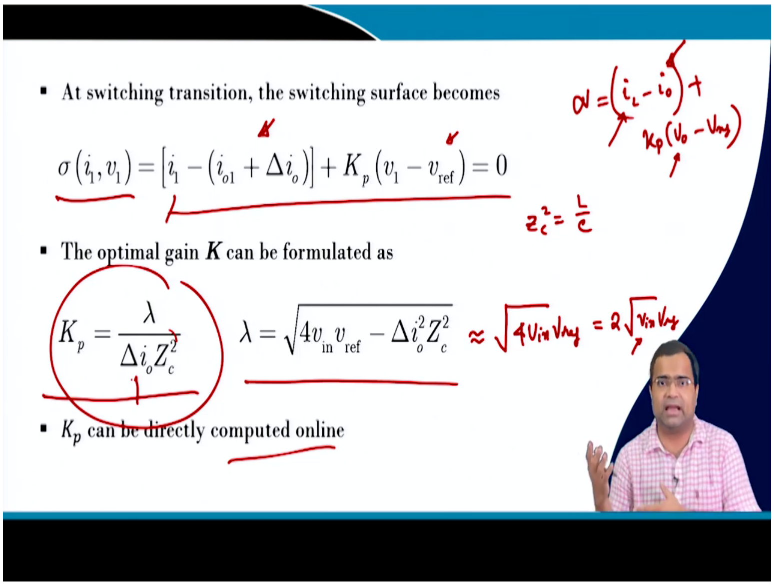

Now, at the switching surface where it will intersect right if you draw the switching surface. So, at this point sigma it is basically intersecting with you know v 1 and i 1 and we are again approximating that we can instead of sigma 1 we can take simply sigma there is small approximate because we are ignoring that variation due to the ripple.

Then sigma x i 1 v 1 if you substitute again, I told you i 1 is the initial current and this is a new load current or the instantaneous load current and this is my output voltage reference because that is there. Because what is our sigma, it was i L minus i 0 plus K p into V 0 minus V ref.

So, this i 0 is nothing but our i i 01 plus delta 0 i L is the point where it intersect. It is i 1 v 0 is a point where it intersect V 1 and V ref. So, then if you solve you can get K p because earlier we got i 1 v 1. So, that is known everything is known except for K p. So, you find out K p and that will take this form. And you can do so that means, K p can be now this can be approximated to square root of four V in V ref which is nothing but twice V in V ref. Since V ref is more or less constant for a voltage regulator. So, we only need to compute the square root of V in.

Another thing about this Z c, Z c square is generally you know that is our L by C right. So, Z c square is nothing but L by C sorry L by C and which again L and C are more or less known at the design stage ok and their variation is not very significant. So, you can compute online only we need to know the load step size.

Even if you do not know the exact value fine if you can know somewhat close its fine, but you can get much better response than the linear control. But if you can perfectly if you can estimate the load step or load current right or if you can sense in indirectly in case of LED driving or other application, but this is not a problem.

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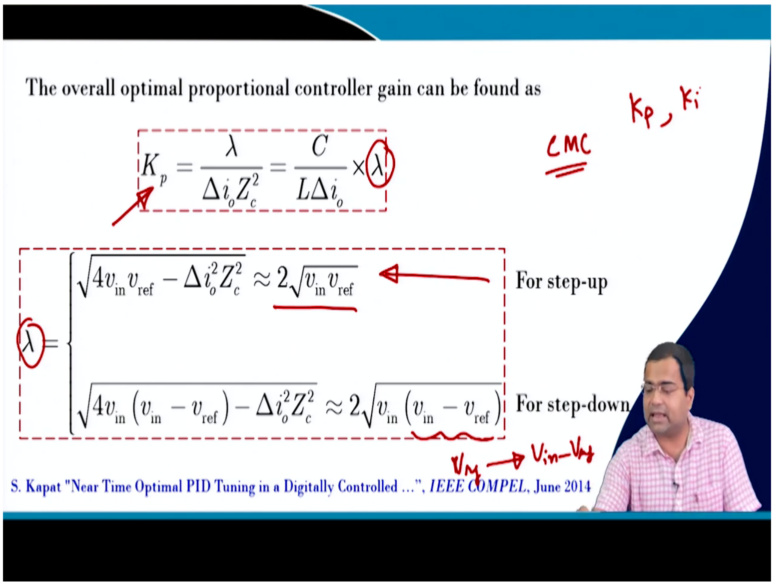

So, the overall time optimal control that the optimal gain will K p where this lambda will be equal to approximately equal to twice square root of V in V ref, but this is during step up transient. So, the method that we have followed to derive step up gain you can do the same method for the step down and you will get everything else is same except the V ref will be replaced by.

So, here you will find the V ref to be replaced by V in minus V ref. For 50 percent duty ratio both are same, but for higher gain input voltage high input voltage the falling slope will be higher than the rising slope and that is also already there in this paper.

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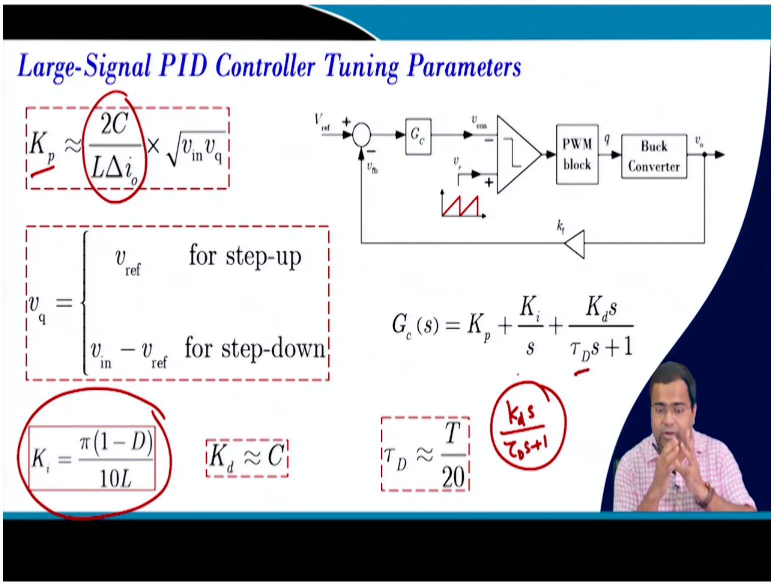

So, the using large signal, so this is what we discuss for current mode control we know how to find K p, we have discussed how to find K i because K i was coming from small signal model and if it is a voltage mode control like a converter, then K p is this. This is the same that we have discussed. Only some modification will be here and K i we have discussed the same K i will be used and K d derivative is equal to C.

And we have to since we are using a derivative control, we need to use a band limited derivative; that means, K d into s plus tau D s plus 1 and this tau D to be much smaller than time period because we are using fixed frequency control. So, that tau D should be small, so that you can get close to the derivative action, because that will carry the ripple information.

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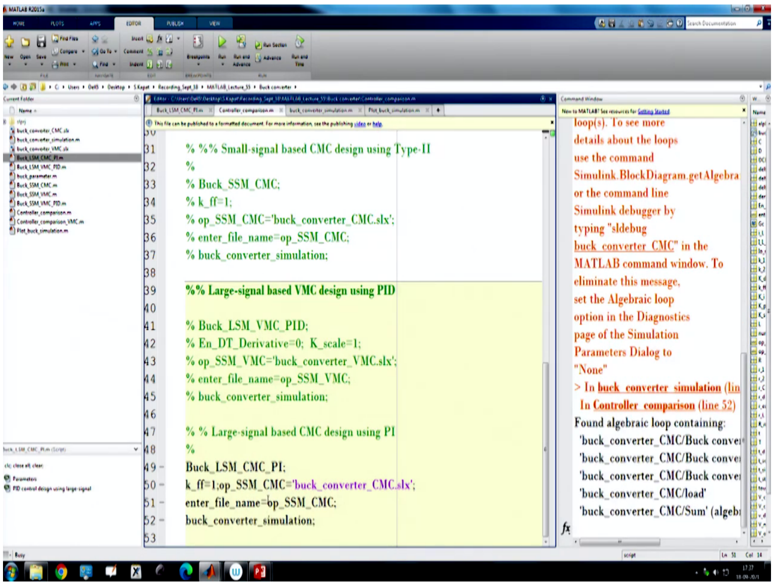

So, let us go to a simulation, say you know case study and check. So, I am [FL] first showing you know if we use a current mode control.

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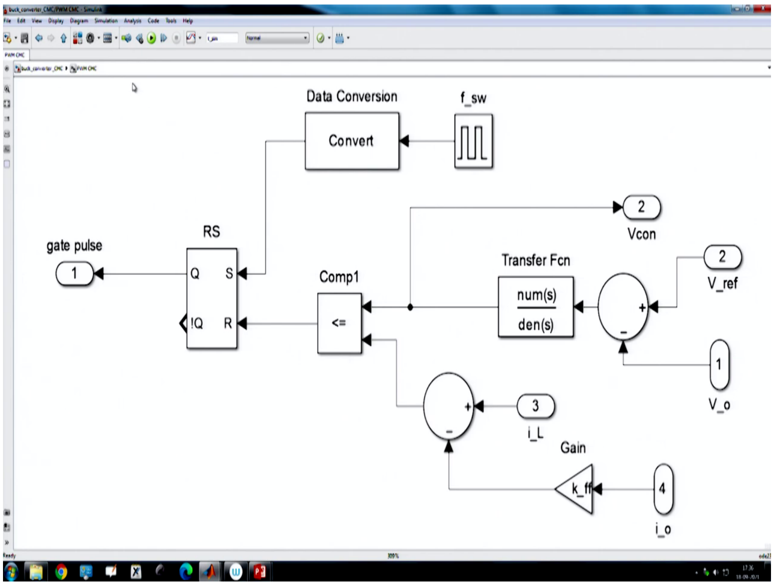

Suppose if we go back to our current mode control.

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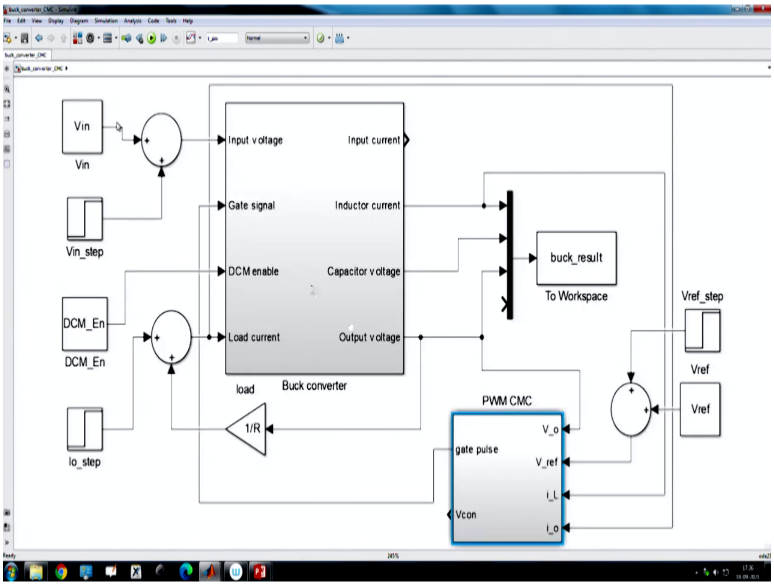

And we all know about this structure current mode control I have discussed multiple time, but here we are taking load current feed forward.

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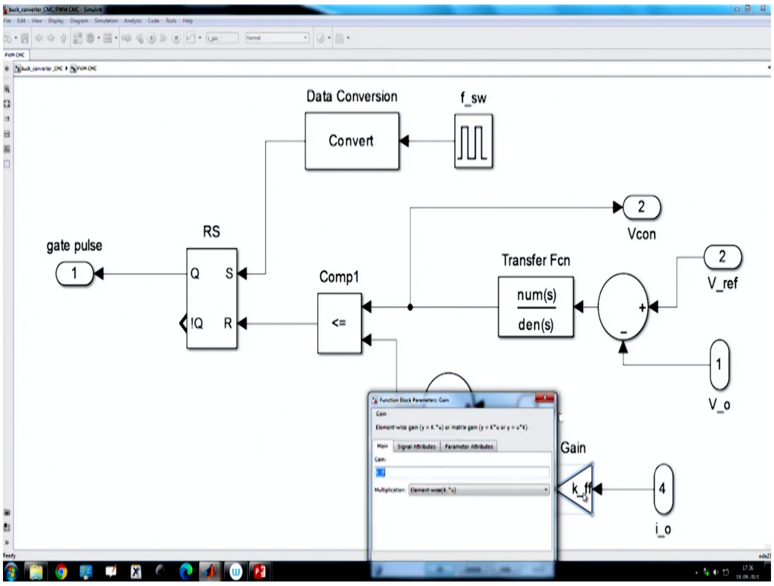

And here this feed forward gain I am setting from internally either if you set 1 for buck or if you set 0 there will be no load current feed forward.

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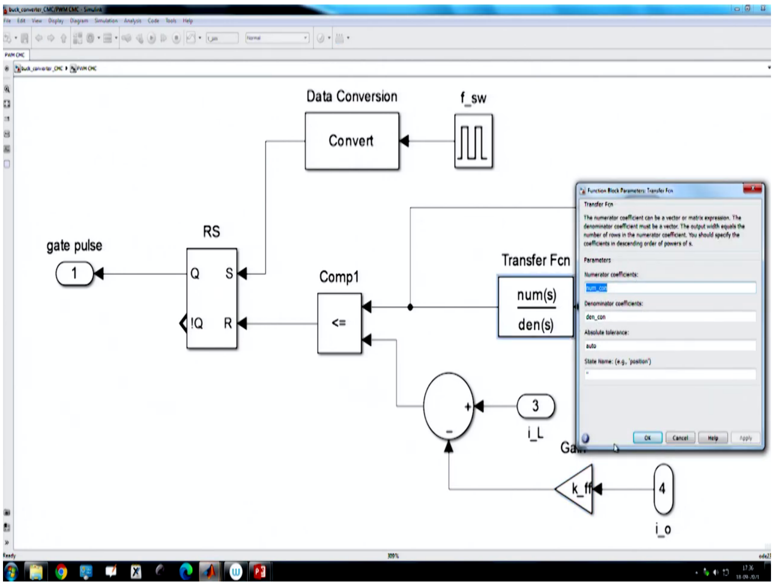

And here we are setting the gain from the MATLAB code. So, we can use simply proportional gain even we can use a PI controller because in our tuning parameter we have to choose this PI controller ok.

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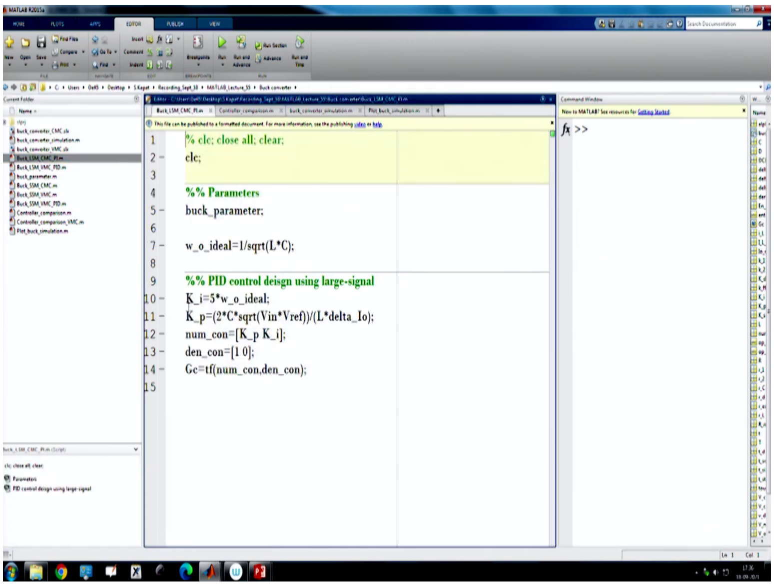

Now, if we go back to current mode control using large signal based PI controller. So, this derivation I think this should be a different derivation. So, now we want to run this buck converter using current mode control PI controller and if we run it you know we are giving all the values from here. Because I will discuss in detail.

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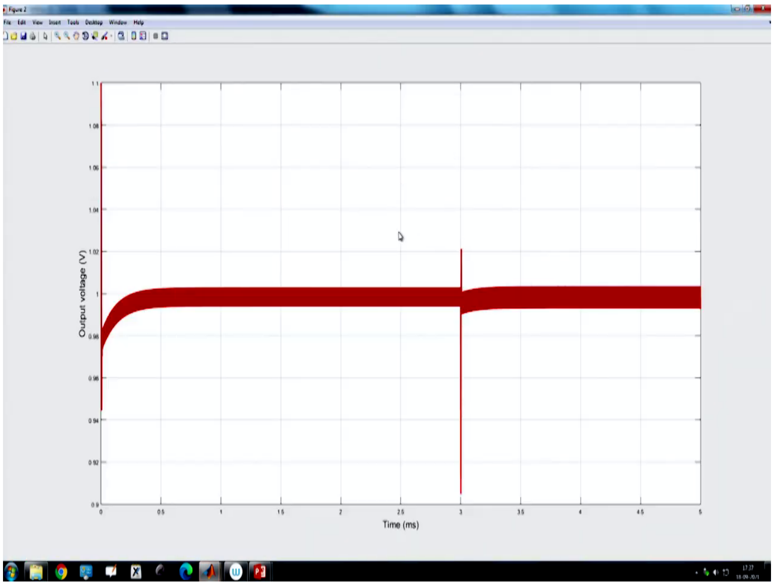

This is under current mode control

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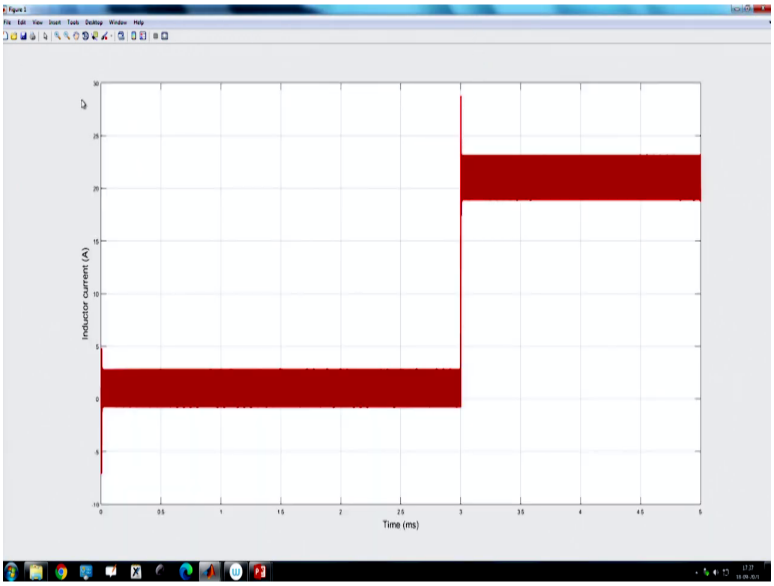

Where it undergoes a 20 ampere load step from 1 ampere to 20 ampere.

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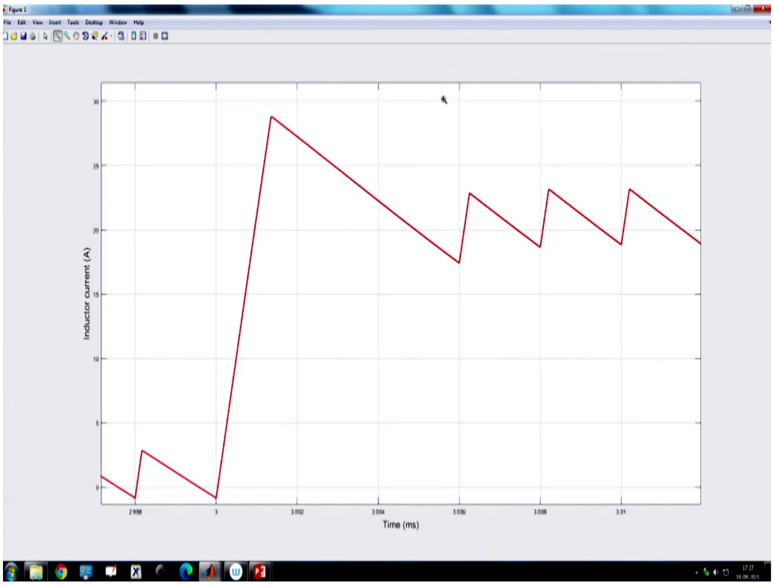

And you can see it is almost like a close to or time optimal.

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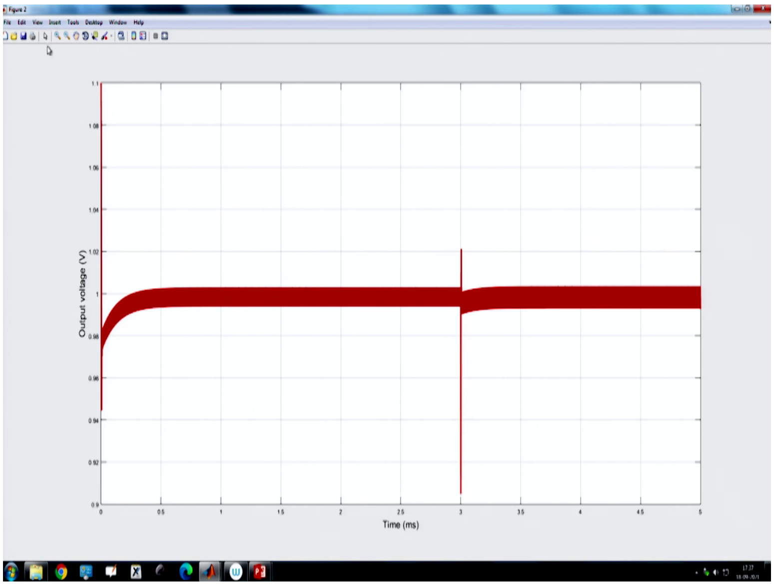

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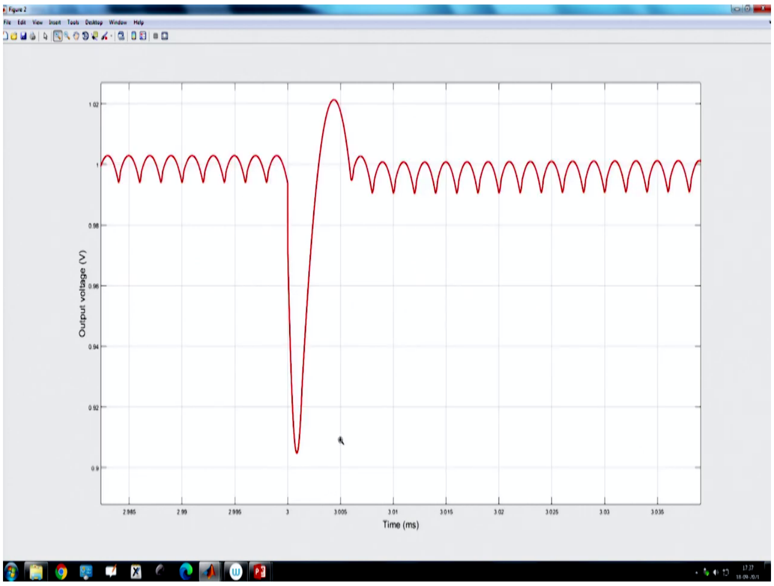

Where output voltage is not exactly time optimal because there are some effect, but it is much faster.

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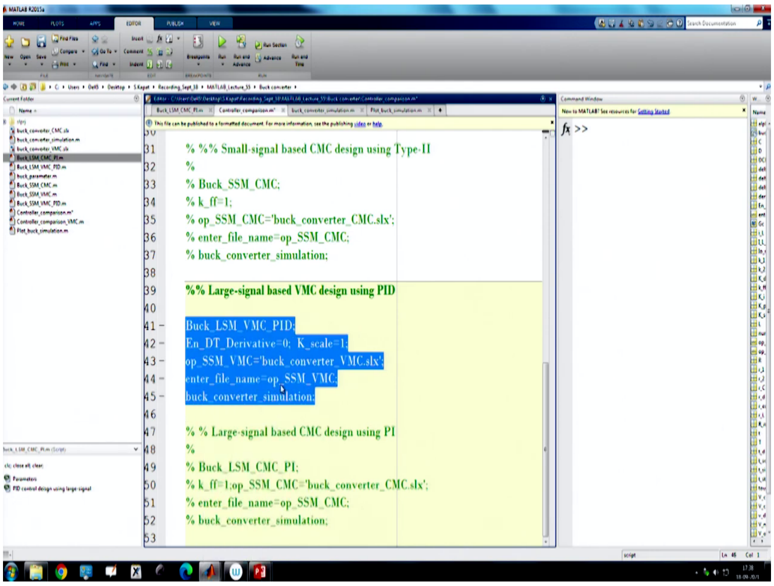

Now, if we go for voltage mode control. So, if we disable this on top of that, if we you know comment it and if we go for voltage mode control design voltage mode control design again, it is a large signal based design and we will discuss this in the subsequent lecture.

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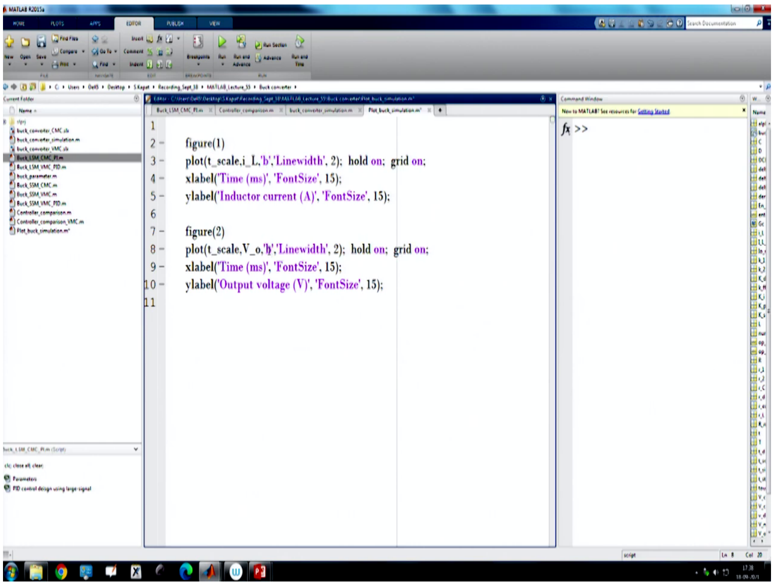

And if we change the color blue color for voltage mode.

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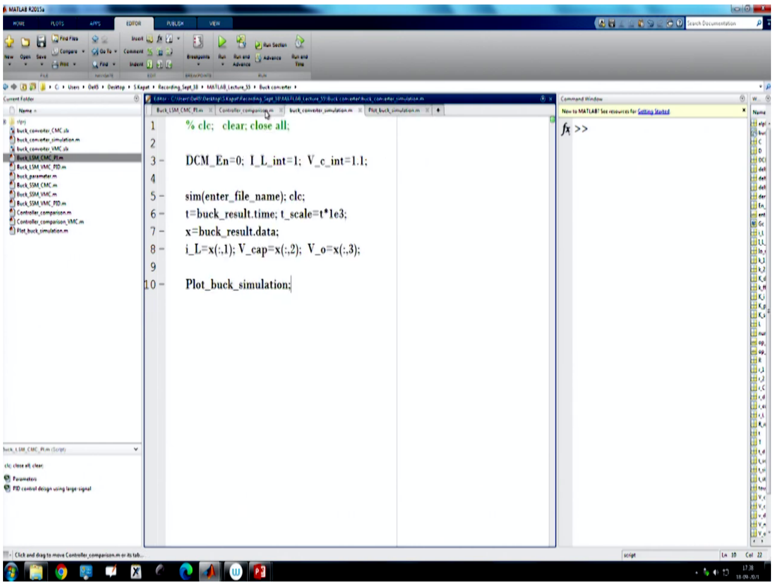

And you see it is taking the voltage mode control logic that means buck converter voltage mode control logic, which is nothing but simply this converter voltage mode control no load current nothing. It has a ramp which has a smaller magnitude, which I am using. This is an error voltage, and this is where is the compensator. So, if you run it and you are using a very small ramp and that ramp is almost you know maybe very low value of ramp you are using.

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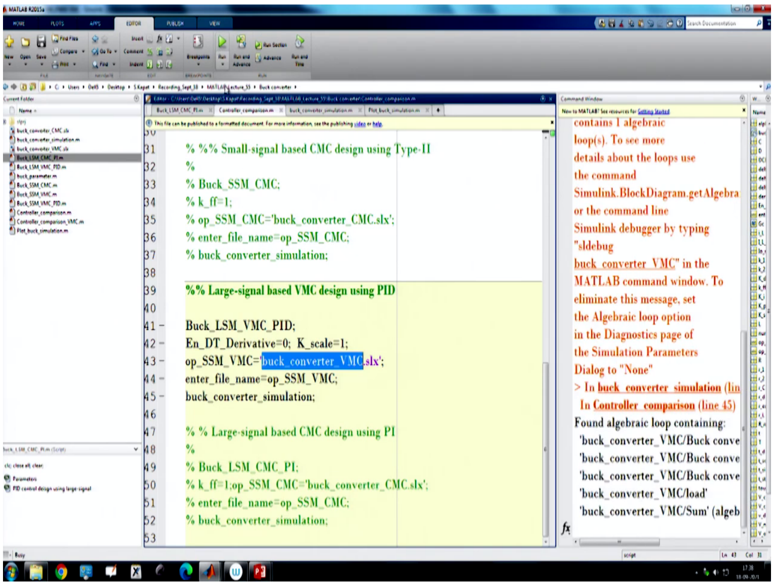

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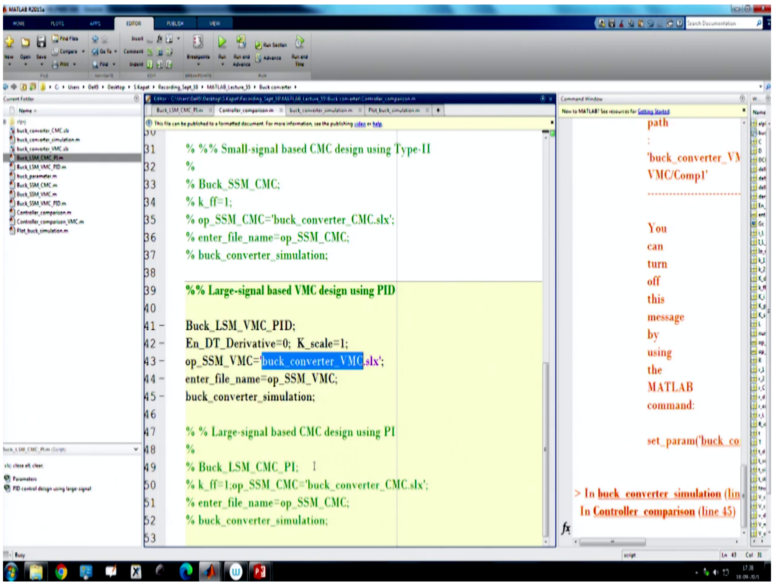

So, if you go to voltage mode control.

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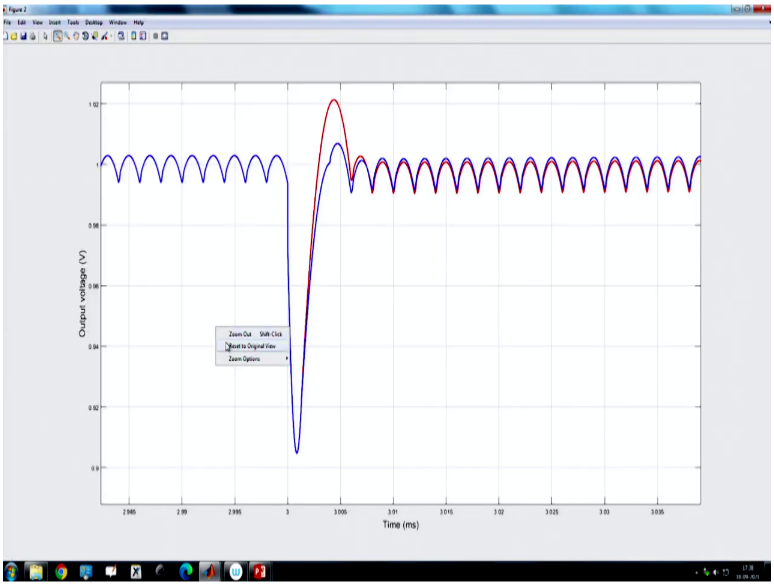

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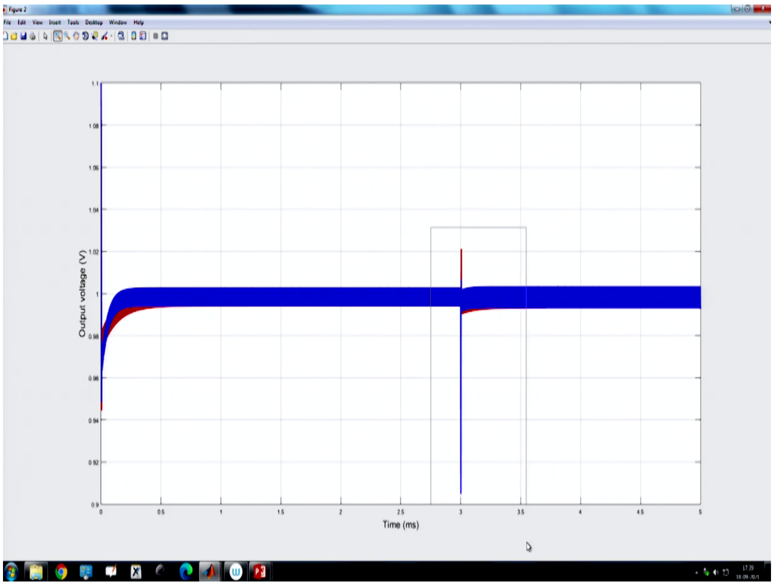

You will find the response is close to optimal solution ok

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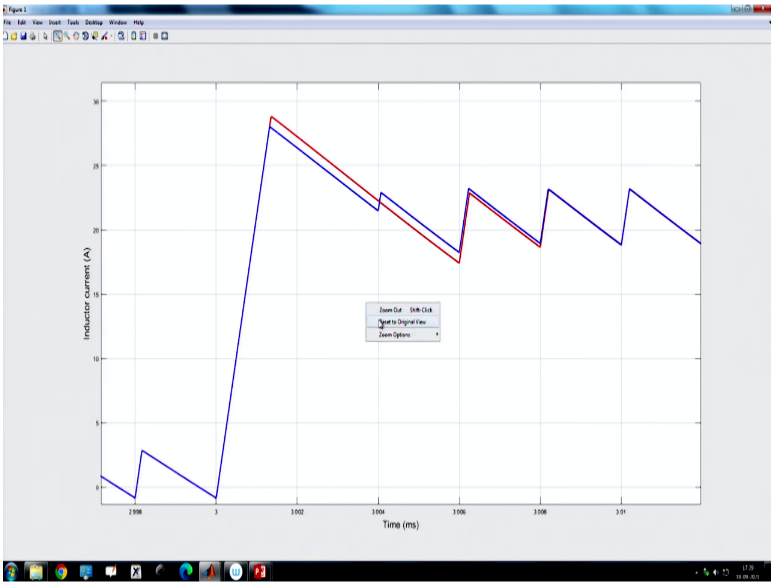

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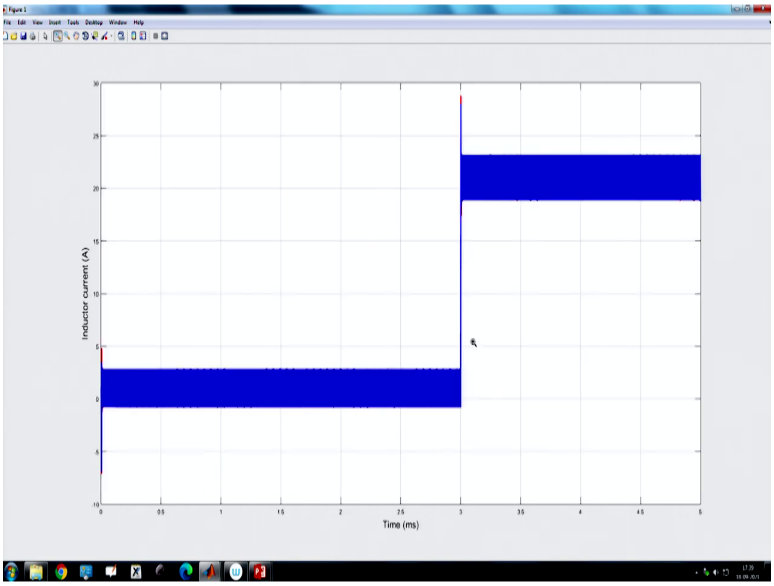

And if you go to the current response it is also close to optimal solution.

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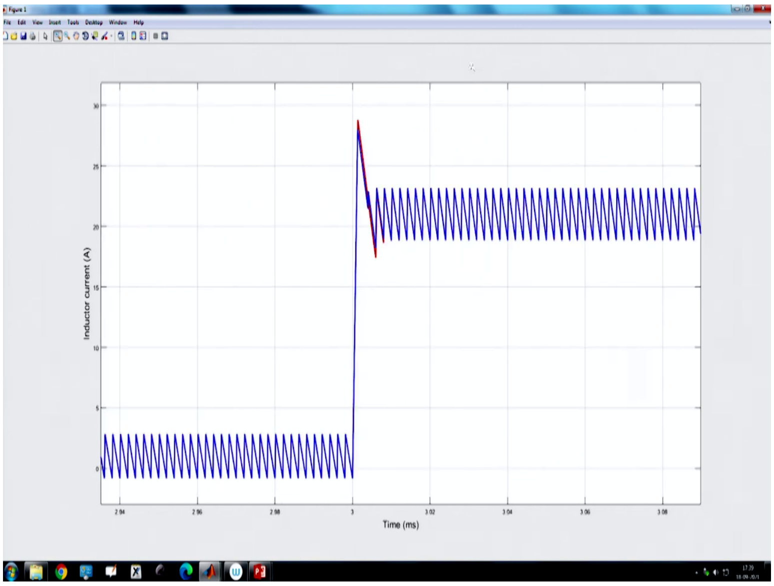

So, that means, we can get time optimal recovery for both current mode and voltage mode control.

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So, with this you know I have to summarize the recapitulation of PID controller tuning objective the large signal modeling of buck converter derivation of the tuning rule in a buck converter and that we did for current mode as well as voltage mode control and the summary of tuning parameter. So, with this I want to finish it here.

Thank you very much.