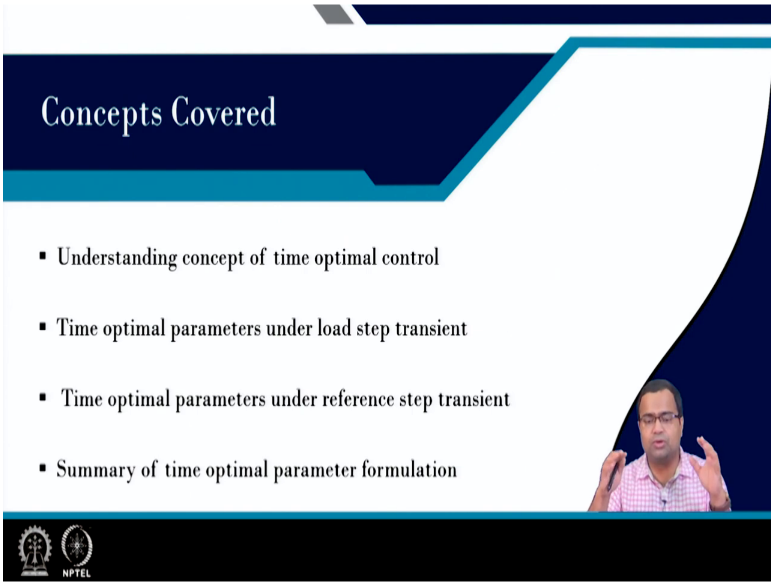
Control and Tuning Methods in Switched Mode Power Converters
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 10
Boundary Control for Fast Transient Recovery
Lecture - 48
Time Optimal Control and Identifying Physical Limits in SMPCs

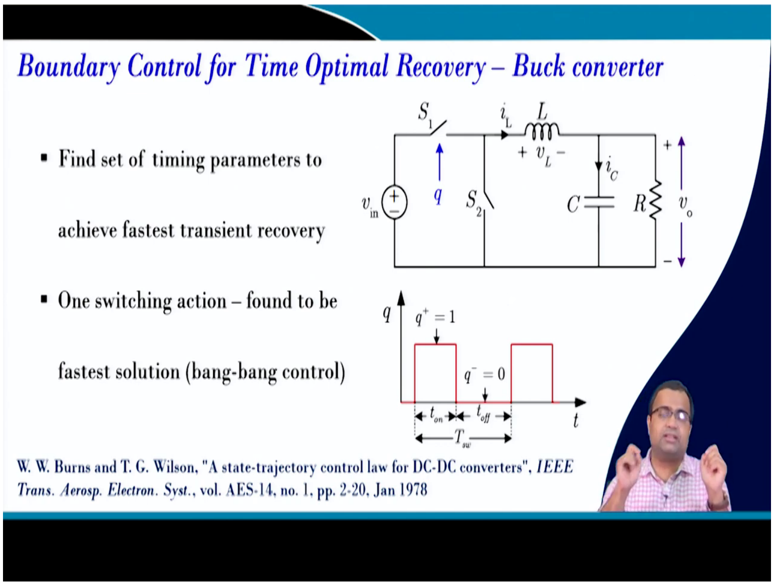
Welcome. This is lecture number 48. In this lecture, we are going to talk about Time Optimal Control and under time optimal performance, what are the physical limit and what are the parameters time optimal parameters.

(Refer Slide Time: 00:41)



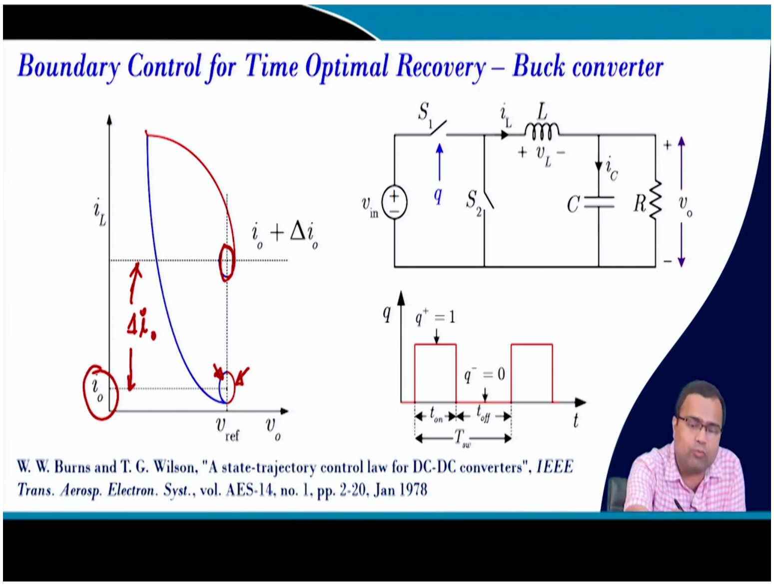
So, here in this lecture, we will talk about the concept of time optimal control, which we have already introduced in the previous lecture. Then, how to formulate the time optimal parameter and we are taking a buck converter case study under load step transient, then the time optimal parameter under reference step transient and then the summary of the parameter. And we will see you know what is the role of capacitor, inductor selection, in the recovery time, overshoot, voltage undershoot then, current overshoot and so on.

(Refer Slide Time: 01:11)



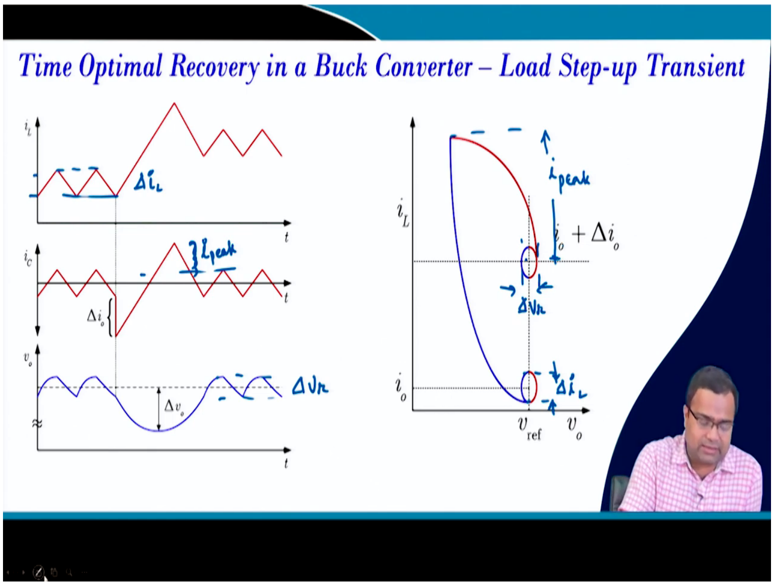
So, first we want to recapitulate the boundary control for time optimal recovery. And that we have already discussed in the previous lecture that we need to set find a set of timing parameter to achieve the fastest response. And that actually turns out to be a one switching action. So, we need to formulate all the parameters in order to ensure that our transient recovery happens in one switching action, then we can ensure the fastest transient or the optimal time optimal recovery.

(Refer Slide Time: 01:45)



And this trajectory looks something like this; that means we have initial load current of i 0 here, our initial load current i 0 here and then, there is a load step transient with a step size of delta i 0 and then it will reach to another trajectory. Here again, the blue one is the on-state trajectory and the red one is the off state trajectory and it will reach to the next operating point in one switching action ok.

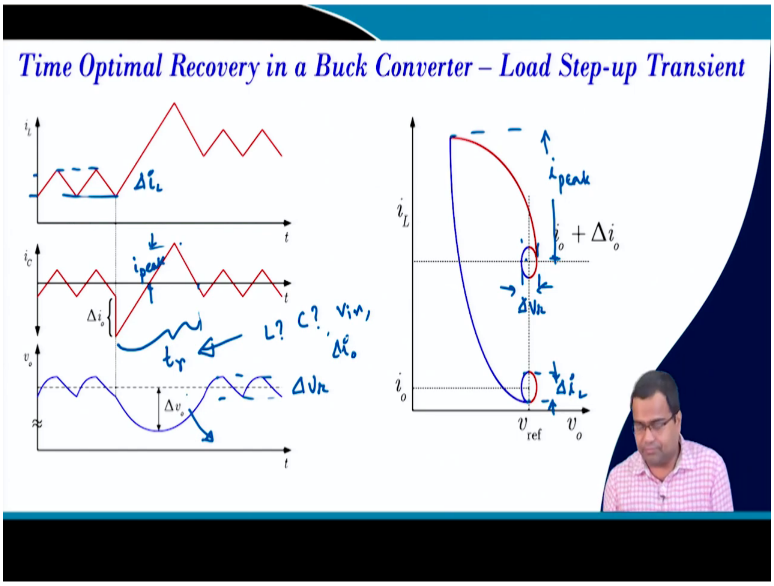
(Refer Slide Time: 02:14)



Now, the timing diagram looks something similar; that means, earlier we talked about the phase plane trajectory, where this ripples if you see if you consider the ripple here; that means, if you consider the ripple parameters here, this is our inductor current ripple and which is same as our inductor current ripple. And this is our output voltage ripple delta V r and which is nothing but our this parameter or this parameter delta V r ripple parameter.

And you can see, during time optimal transient there will be some current overshoot. So, this is my i peak current like you can think of. And this in the inductor current it is like this is my i peak current right. For capacitor current, we can think of the peak current to be almost from this current.

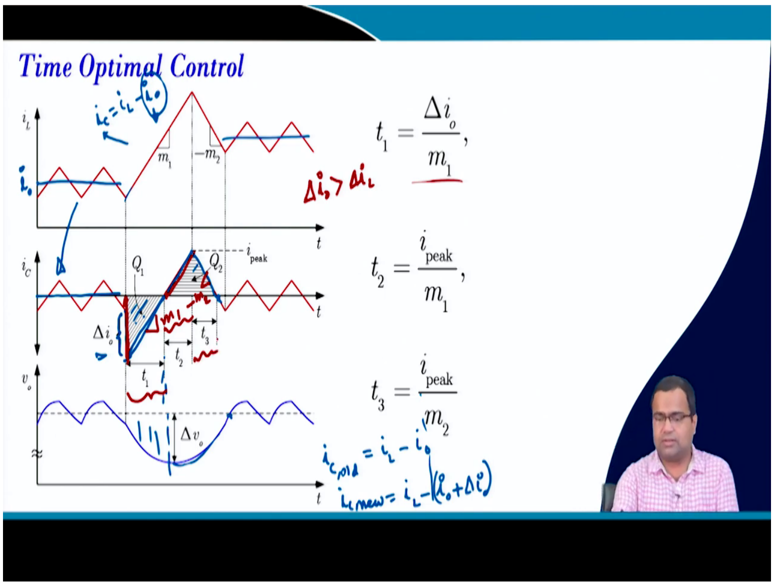
(Refer Slide Time: 03:24)



So, this is my peak current i peak that we are going to find out. So, we need to find out this peak current then, how much duration it takes that is my recovery time right and who actually I mean this recovery time depends on what parameter. Does it depend on the inductor, does it depend on the capacitor, does it depend on input voltage, does it depend on the load step size?

Of course, it should be dependent. And then how this quantity undershoot is a function of this L C V in V 0 delta i 0; all these that we are going to check and a and also the peak time peak current.

(Refer Slide Time: 04:10)



So, the time optimal recovery will start with this using capacitor charge (Refer Time: 4:14) which is a very standard method. So, you see, the capacitor current is nothing but for a buck converter in continuous conduction, because we are talking about a synchronous buck. Here, the average inductor current is the average load current right, and that is before transient and after transient, it is delta i 0 plus delta i 0.

In capacitor current, it is nothing but the inductor current minus load current. So, your average is 0 and it will take the same ripple of the inductor current. When there is a load step transient, since your capacitor current is nothing but i l minus i 0. Suddenly, if there is a load step transient, the inductor current cannot respond because we know the inductor has an inertia.

So, it cannot. It will not allow the current to change immediately. So, there cannot be any discontinuous current change in the inductor. So, as a result, any load step change which will happen in a discontinuous step load that will be reflected as a negative step load here and that is a here the magnitude remains same.

So, capacitor will directly go down and this size is same as the step size, because we know the capacitor so; that means, your new capacitor; that means, i C volt is nothing but i L minus i 0 and i C new is nothing but i L minus i 0 plus delta i 0. So, the time instant where this delta i 0 load step appears.

So, before that and after that, the there will be a jump in the capacitor current, because inductor current cannot have a jump. It will sleeve up and it will take finite time. So, because of the direct jump in the capacitor current for a load step up transient, the current become negative and once the switch turns on, the current will slowly rise.

So, until the current crosses the 0 current, the output voltage will continue to decrease. So, output voltage will decrease, because your capacitor current is negative. So, the voltage will discharge. Once the capacitor current becomes positive and keeps on increasing, then the voltage will rise ok and then, again it will turn off and it comes here.

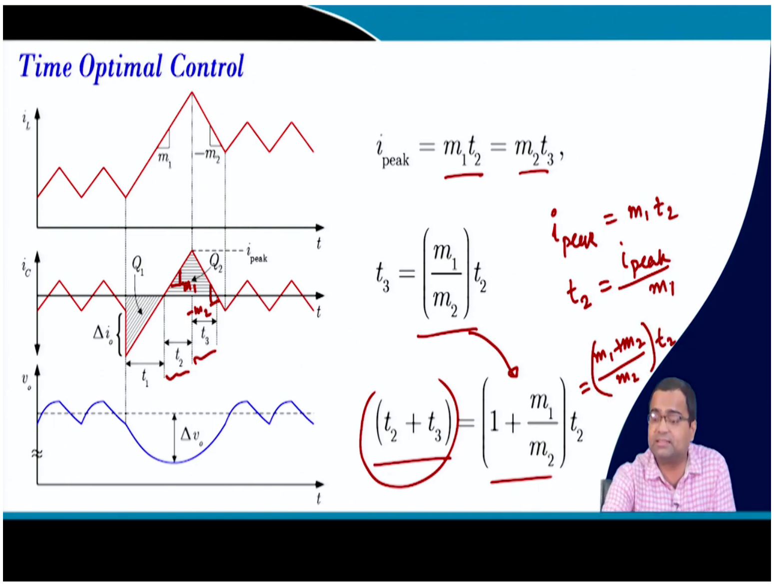
So, you can see during this recovery phase, the charge Q 1 which is the negative charge, must anticipate the positive charge so that if you can make sure then, you can expect that voltage will again come back to the original position. But it may so happen. So, you have to ensure this Q 1 and Q 2 they are equal ok so; that means, this will make this i peak unique, it cannot be non-unique value.

So, for a peak unique peak value, you have to turn on the switch and unless it reaches peak value, you should not turn off and then you turn off. Then, only the resultant Q 2 charge will balance the Q 1 charge and that will lead to our optimality condition. So, for in order to derive, we will first talk about our this time; that means, you know t 1 which is nothing, but this time; that means you are talking about this particular duration. It is delta i 0 by m 1 and we can assume more or less that this is delta i 0, because if the step size is very large.

So, your assumption is that the step size is much larger than the current ripple, then we can ignore this small part and you can assume this whole thing to be more or less delta i 0. Then you can find out delta i 0 by m 1. And t 2 this duration the current capacitor current start from 0 and it reaches i peak. So, it is i peak by m 1. So, this slope is my m 1, because it is same as the inductor current slope.

And what is my t 2 I have found and what is my t 3; t 3 here it is my minus m 2 slope. So, t 3 is this duration is nothing but i peak by m 2 that is what we got right.

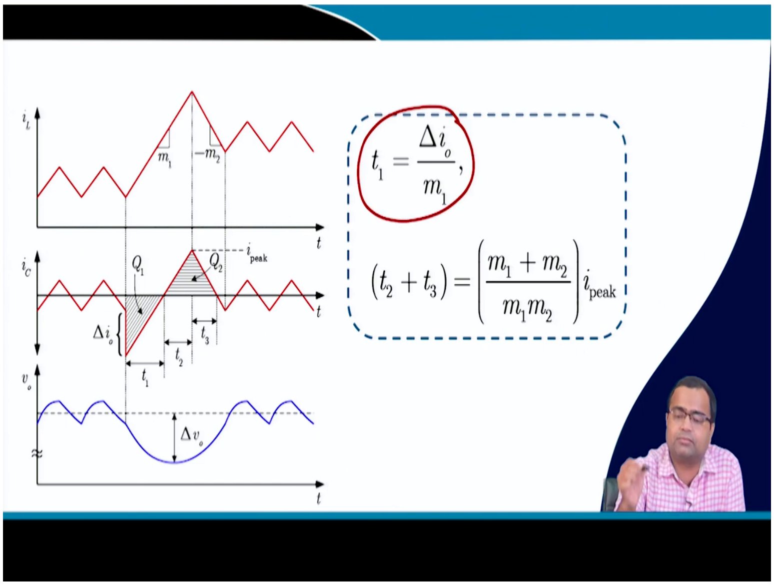
(Refer Slide Time: 08:38)



So, now we can write what is my i peak. The i peak, because this is my m 1 slope and this is my minus m 2 slop. So, I can write i peak to be m 1 into t 2 or m 2 into t 3; they are equal.

So, I will get t 3 equal to m 1 by m 2 into t 2. So, if I want to get t 1 t 2 plus t 3, this using this you can use. This expression will be needed. That is why we are competing here.

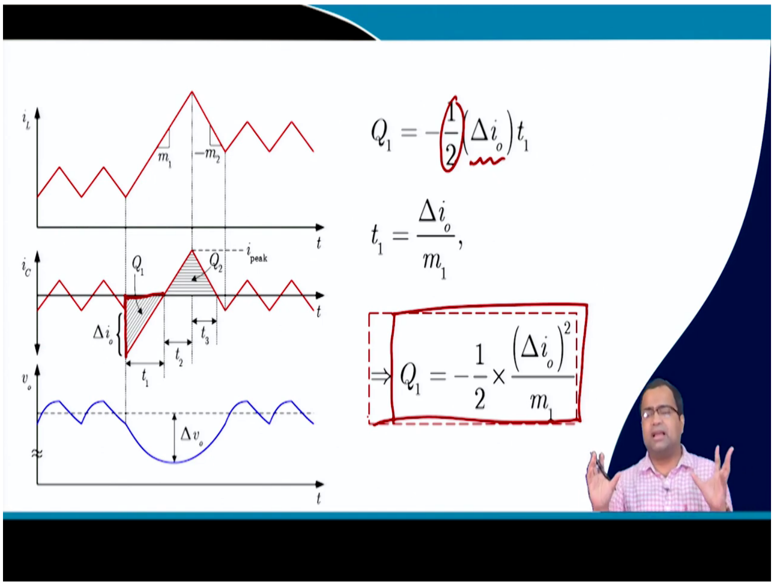
(Refer Slide Time: 09:12)



Then next what was my t 1, it is delta i 0 by m 1 and what was our t 2 plus t 3, if we go back, it is 1 plus m 1 by m 2 into t 2. And what is my t 2. From here, I can find, because I know that i peak equal to m 1 into t 2. So, I can find t 2 equal to i peak by m 1 ok. So, I have used that; that means, here it was like you know I have replace. So, this is nothing but m 1 plus m 2 by m 2 into t 2.

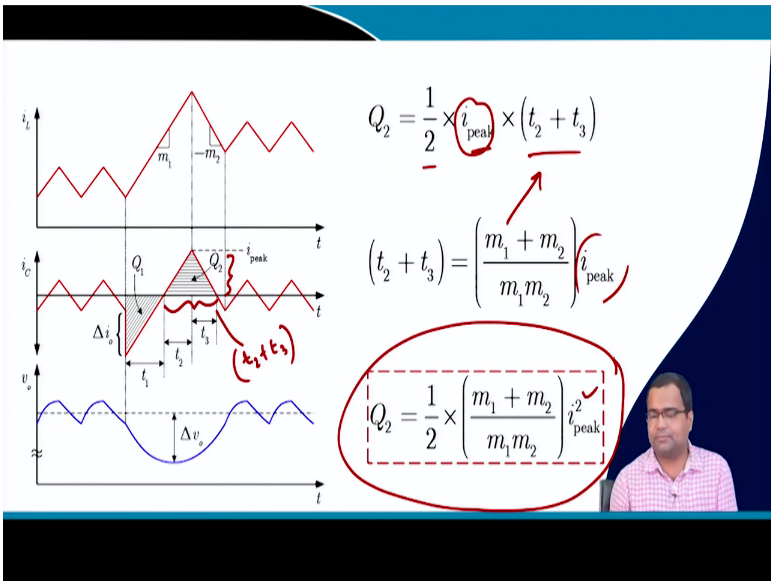
And if I replace t 2 here, then this whole thing will be coming here, m 1 plus m 2 and this t 2 is what. It is i peak by m 1. So, this m 1 m 2 will get multiplied. So, this is a multiplied term i peak right.

(Refer Slide Time: 10:26)



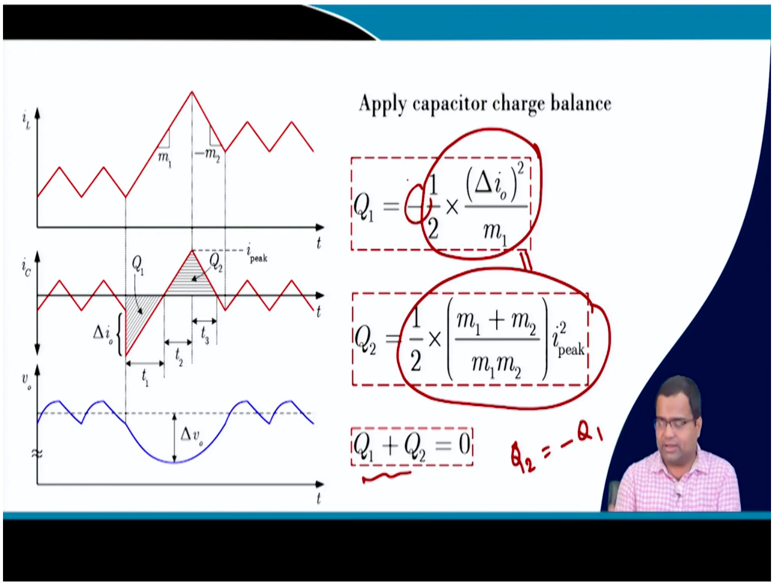
Next, what is my Q 1. It is nothing but this current height into this duration into half and since it is a negative. So, negative half of delta i 0 is my height, because I am assuming this is to be equal and t 1 is my duration. And what is t 1 i 0 by m 1? So, if you substitute, I will get my negative charge Q 1 is nothing, but half of a minus half of delta i 0 square by m 1.

(Refer Slide Time: 11:00)



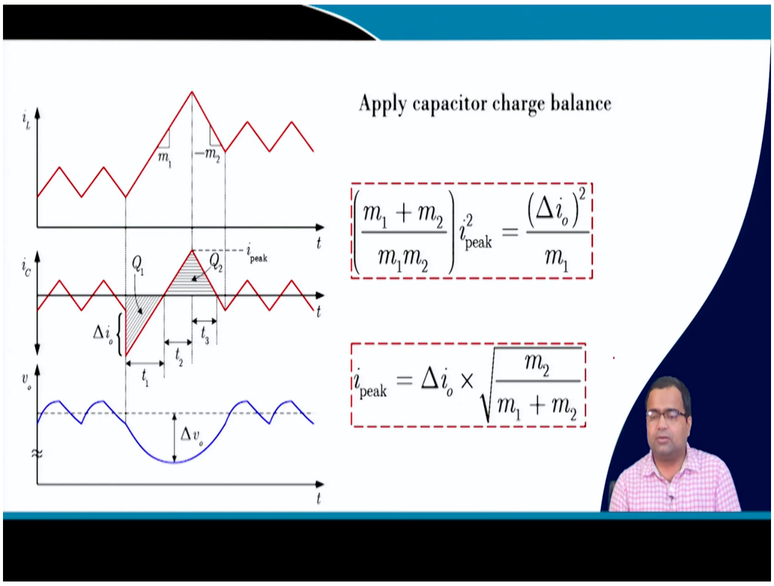
Next, what is my Q 2? Again, the height is i peak half into height i peak into this duration is my t 2 plus t 3 t 2 plus t 3 right. So, this is here and what is t 2 by t 3 that we know m 1 plus m 2 divide by m 1 into i peak. And if you substitute here, and what you will get; Q t equal to this term half of and, because there is already 1 i peak and this is the i peak. So, i peak square ok.

(Refer Slide Time: 11:38)



So, next applying charge balance we have Q 1. We have Q 2 and what is my charge balance. The sum of the two charges must be 0 capacitor charge balance and then it will balance.

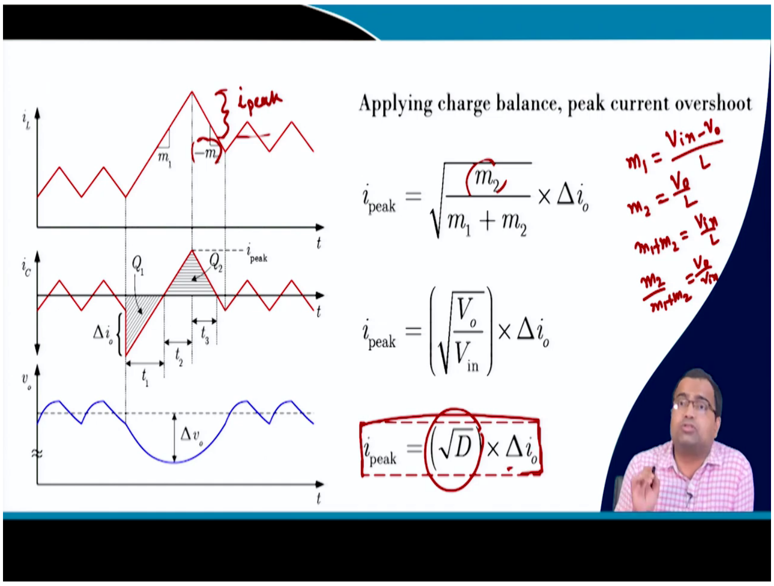
(Refer Slide Time: 11:48)



And if you do that then, what you will get; that means, you know this term if I take the absolute magnitude is nothing but is this term they are equal. So, they are equal, because if you take the sin into consideration from here; that means, from here what we are getting Q 2 equal to minus Q 1. So, it will become anyway positive here. So, these two are equal; that means, we are setting.

So, we can cancel this half half term each side. So, m 1 plus m 2 by m 1 into i peak square is nothing but delta i 0 square by m 1 and that we are writing. So, we can cancel this m 1 from either side and we can write i peak is equal to delta i 0 since it is a square root of m 2 by m 1 plus m 2 correct.

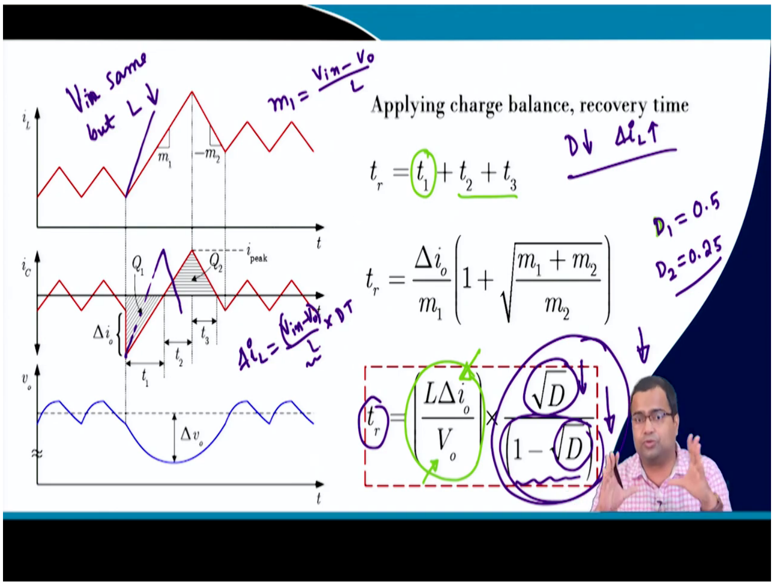
(Refer Slide Time: 12:45)



Now, what is my m 1 and m 2 for a buck converter? so; We got this relation. For a buck converter, m 1 is V in minus V 0 by L. And what is my m 2; m 2 is V 0 by L, because we are taking minus of m 2 say m 2 is V 0 by L. Then, what is my m 1 plus m 2 it will be simply V in by l and. So, if you take m 2; that means, my m 2 by m 1 plus m 2 is nothing, but V 0 by V in and this is exactly is coming square root of V 0 by V in.

And what is V 0 by V in under steady state it is the duty ratio. So, the peak current during load transient, because this peak current is computed from here, this is the peak current, but for capacitor since the average is 0. It represents the peak current. So, peak current depends on the duty ratio or I will say it depends on the input output voltage and the load step size.

(Refer Slide Time: 13:56)



So, apply charge balance. What is the recovery time. We know t 1 we have already computed t 2 plus t 3 if you substitute. And I have already told what is m 1 plus m 2. So, it will be L delta i 0 that is my load step size into V 0 and square root of D divided by 1 minus duty ratio D; that means, this recovery time I can show you.

Suppose, if your duty ratio become smaller; that means, if you take let us say suppose in D 1 in one case it is. 5 in other case, D 2 is. 25 right. So, the square root when you D decreases, this will decrease when D increases. Sorry D decreases. This will also decrease. So, as a result, this quantity will increase. And since it is in the denominator, this will also decrease. So, overall this whole quantity will decrease so; that means, if duty ratio decreases, the numerator anyway will decrease.

So, it will become smaller and smaller and denominator is also increasing. So, effectively the whole quantity is decreasing ok. So, that is why we have discussed for higher input voltage. It will respond even faster. So, it will have a very smaller equal time. But we know higher input voltage when the duty ratio decreases for a buck converter, if the output voltage is constant.

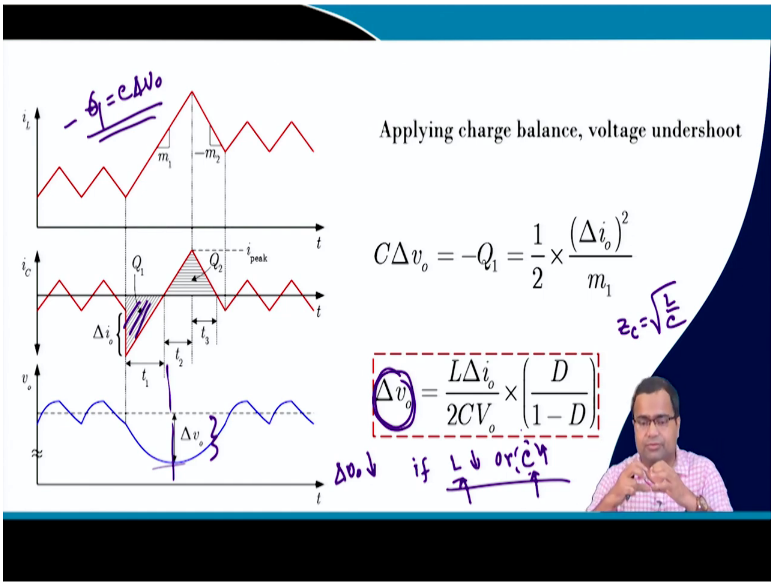
We know that as D decreases, your ripple current increases. So, your conduction loss increases and if the conduction loss increases, your efficiency also falls ok. So, that is why and also if you look at the switching loss where the switching loss is also a function of input voltage if higher input voltage; that means, the switch when the high side switch is on, then the low side switch will see the full input voltage.

Similarly, when the high side switch will off, it will see the full input voltage. So, the switch will undergo a transition from V in to 0 and if the V in is large, so; that means, the swing of voltage will be large. So, as a result, switching loss also increases. So, as a result, higher input voltage will reduce the efficiency of the converter.

So, and particularly input voltage is not in our hand, but if you go for two-stage architecture where the intermediate voltage is the input to the second stage converter and where we can optimise the voltage level, in order to you know achieve fast transient as well as the higher efficiency we cannot meet.

They are conflicting, because higher input voltage will improve the transient performance, but at the cost of lower efficiency. And if you set it lower, then the steady state efficient will increase, but the transient response will be slow. So, this trade up will be important and we will discuss this issue.

(Refer Slide Time: 17:11)



Another part. If we apply charge balance to the output, we want to compute this output voltage undershoot. So, it is nothing, but if you take this charge, because during this time the voltage will fall right. So, if you take this particular scenario. So, our Q 1 is nothing but C into delta V 0 right. If you do that Q 1. So, yeah it should be minus of Q 1, because we are considering delta V 0 magnitude; that means, it is the magnitude rather than the sign.

So, I want to compute the magnitude and we are calling it is an undershoot. So, then we can find out the delta i 0 is this and this L by C is the characteristic impedance is nothing but L by C; that means you can see the undershoot can be substantially reduced. So, this delta V 0 can be reduced if L reduces or C increases; that is why in the power supply design, we generally choose smaller inductor larger capacitor, because we need to reduce the output undershoot right.

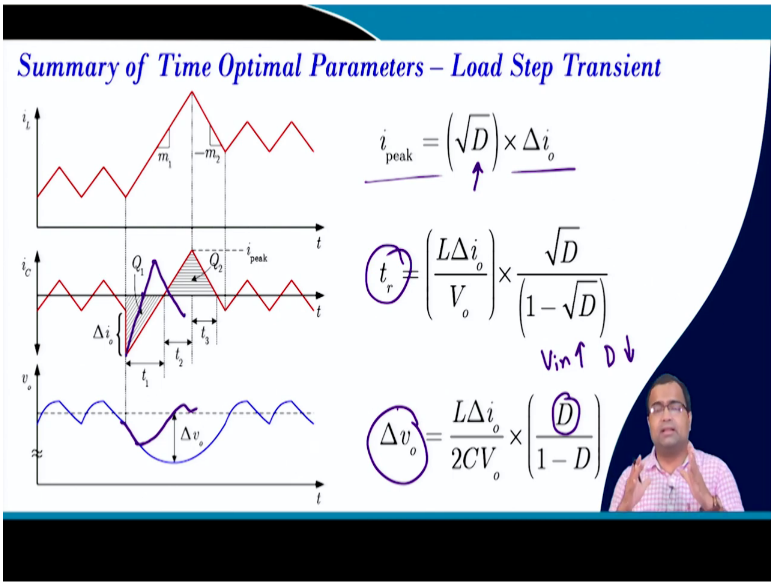
And if we see the peak overshoot, it does not depend on the inductor value, but the recovery time does; that means, if we use a smaller inductor, for example, you know suppose, if I talk about this scenario. Suppose, I have considered for the same input voltage. Now, V in same V in same, but L decreases, then what will happen. So, for a smaller L, this slop will be faster right, because this slope m 1 or this is like this 1 right. So, m 1 m 1 is nothing but V in minus V 0 by l.

So, as L reduces, the slope increases and it will respond faster, so; that means your recovery time linearly proportional to the slope. If slope reduces, recovery time reduces, but the penalty, because if you take the delta i L, it is nothing but V in minus V 0 by L into D t right so; that means, if you decrease L, the ripple actually inversely proportional. So, current ripple will increase. As a result, your conduction loss will increase. RMS current will increase.

And also, the capacitor ripple will also increase, because capacitor ripple is also a function of inductor current ripple, because it is the integration of the capacitor current right. So, that is why we have to choose this L C very carefully. Though our requirement is that we will prefer to go for smaller L and larger C. So, we have to be careful about the choice of inductor, because we need to consider the efficiency ripple and other thing.

But the large capacitor may be good, but it may not be good, because large capacitor will penalise the power density, because if you use the bigger and bigger capacitor, then what will happen with your size of the converter. It will increase and power density will decrease. A larger capacitor can have poor reliability. So, reliability point of view. So, it is more prone to failure and generally people do not use the one capacitor.

(Refer Slide Time: 20:58)

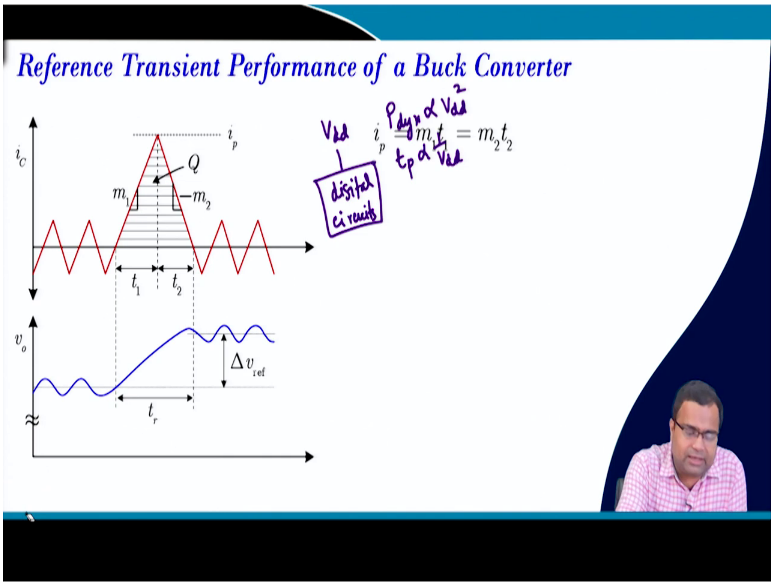


So, they we generally consider a bank of capacitor. So, summary of time optimal load transient; we know the peak current, recovery time and the load step undershoot. And this undershoot can also be reduced again if you if duty ratio decreases; that means, if input voltage decreases, then naturally this undershoot, because if you can increase this. You can see during this time the voltage will only fall and then rise, so; that means, undershoot will also reduce ok.

So, this is conceptually very clear. A smaller duty ratio can reduce also the peak current. So, it will reduce the peak current. It will reduce. You know if sorry it will reduce the peak current, it will come like this. It will reduce the peak current also, because your falling slope remains the same. It will reduce the recovery time and this. So, we have to be very careful.

But often, we do not have any choice in the input voltage unless we take a two-stage architecture and if in two- stage architecture, there are scope for optimisation.

(Refer Slide Time: 22:09)



Now, for a reference transient, because if we consider this DC-DC converter is supplying a digital processor load; that means, V d d of a digital circuit. So, this digital I can say digital circuits. So, there are millions of maybe transistor digital transistor and each of them requires a V dd. So, this overall V dd will be supplied by DC DC. Now, we know that the dynamic power the dynamic power of any cmos processor is the square of V dd.

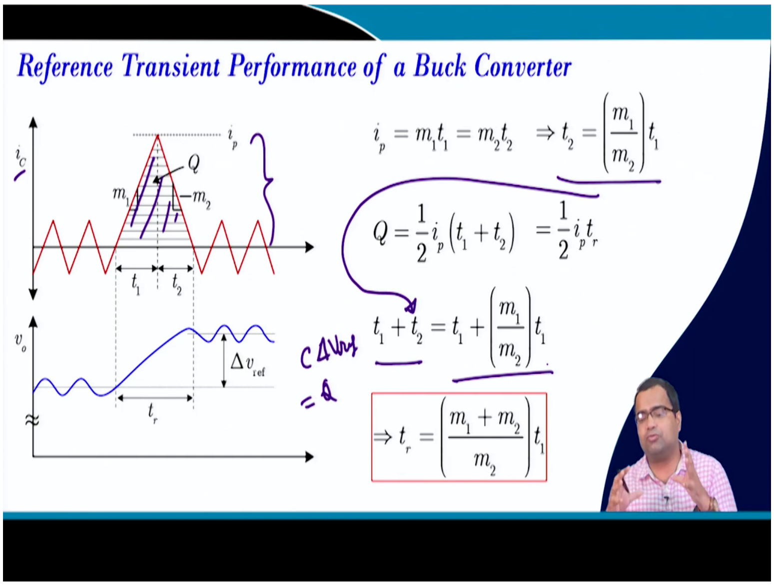
So, if you can reduce the V dd, you can save power, but this also the propagation delay; that means, if you take the propagation delay; that means, if you consider the propagation delay. This propagation delay is inversely proportional to 1 I mean it is proportional to 1 by V dd or inversely proportional to V dd.

So, if we reduce V dd you can save power. But the penalty will come in terms of time delay, which will slow down the computation. It is important since this DC-DC converter has to meet the requirement of the processor when you know, because sometime we need to suppose we are playing a game.

So, you need a very fast processing graphics ok very fast processing then; you need to increase the V dd otherwise, it cannot compute within a time. So, you cannot play like in real life like gaming right very fast an extensive, very intensive computation. So, you need a higher V dd, because the propagation delay has to be very low. So, we do not care about the losses there, but suppose if we are just browsing internet; why should we use the higher V dd. So, we want to reduce.

So, depending upon the various kind of task you need to optimise. So, the converter has to generate the change in reference voltage accordingly. But if we want to derive the recovery time for a reference transient, we will see some interesting fact.

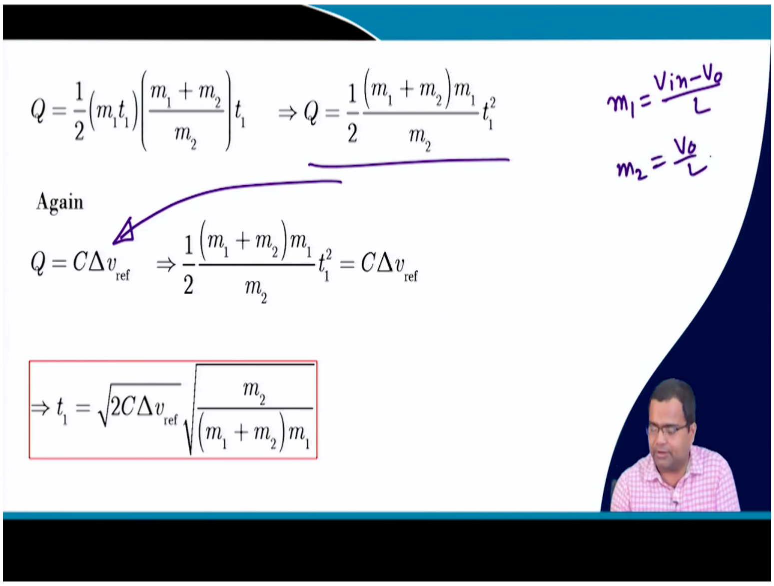
(Refer Slide Time: 24:25)



So, here also, you know we can think of this peak current. Again, the same way the peak current we are talking about the capacitor current and you see this whole positive charge will actually generate into delta V. So, we can see C delta V ref is nothing but this Q that is what we will get peak current, then we can find out Q that half of i p which is the height from here.

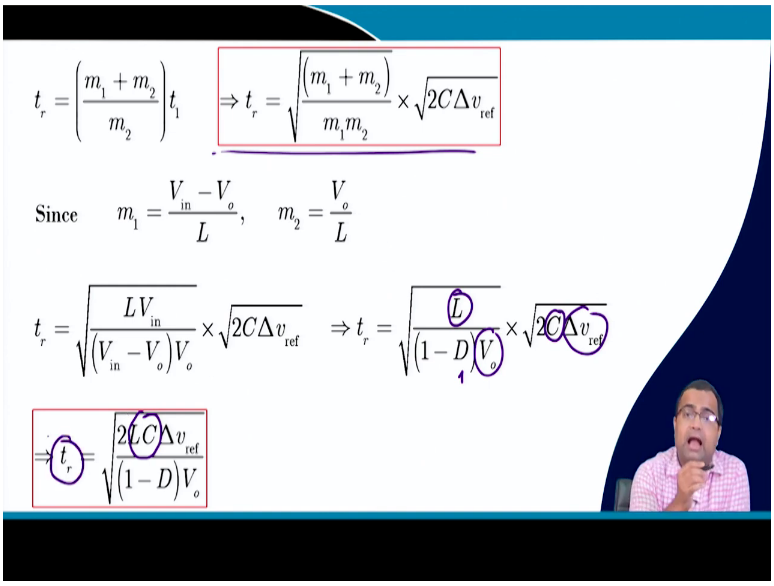
Because it is the capacitor current, this is the capacitor current into t 1 plus t 2. So, and t r is the recovery time, which is the sum of these two. What is t 1 plus t 2. So, you know t 2 is nothing but this. So, if you substitute here, then you will get the whole thing will be t r equal to m 1 plus m 2 divided by m 2 into t 1.

(Refer Slide Time: 25:13)



Now, the Q is this we can write by simplification. And we know that Q equal to C into delta V ref that we have told, because whatever charge you are injecting, it will ultimately raise the voltage level. And if we equate these two here, if you substitute here, then we will get this expression. So, your t 1 can be found from this relation and we know m 1 equal to V in minus V 0 by l and m 2 equal to V 0 by l for a buck converter.

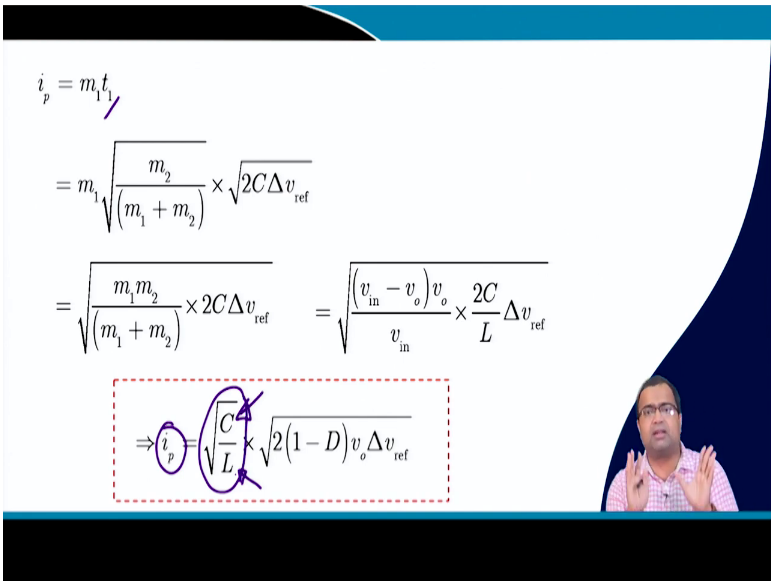
(Refer Slide Time: 25:51)



And if you substitute, the recovery time can be found in this form and if you substitute m 1 m 2, then you will get recovery time is a function of inductor function of capacitor the step size and also the output voltage and of course, D is there means it is also a function of input voltage. So, here you will see we are getting something like 2 L C; that means, if you can reduce the recovery time if you reduce the value of the inductor, but it will increase the current ripple.

Similarly, the recovery time can be reduced by decreasing the capacitor. So, this will create a conflict, because for imagine if you want to change the reference voltage level and if there is an output capacitor; that means, ultimately, we are changing the voltage level of the capacitor. If the capacitor value is large. So, it will require more energy for to change from one voltage level to the other voltage level and that will come from the inductor. So, a larger capacitor will require higher recovery time it too.

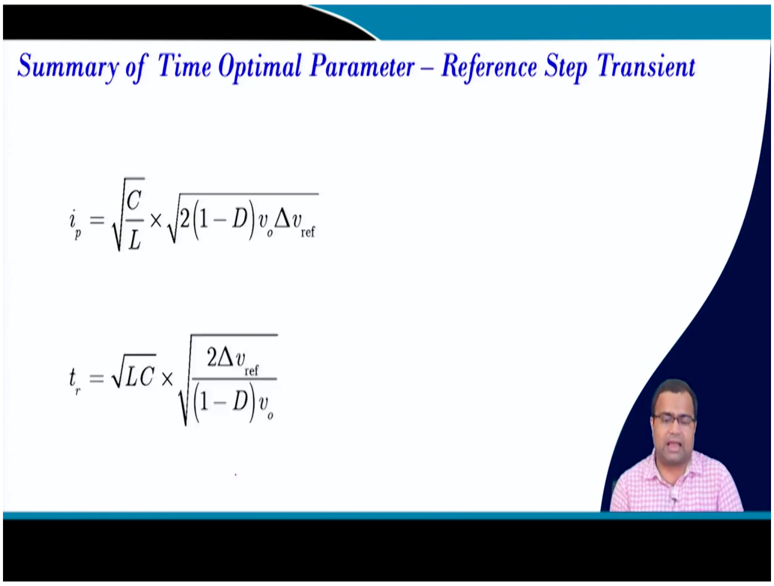
(Refer Slide Time: 26:58)



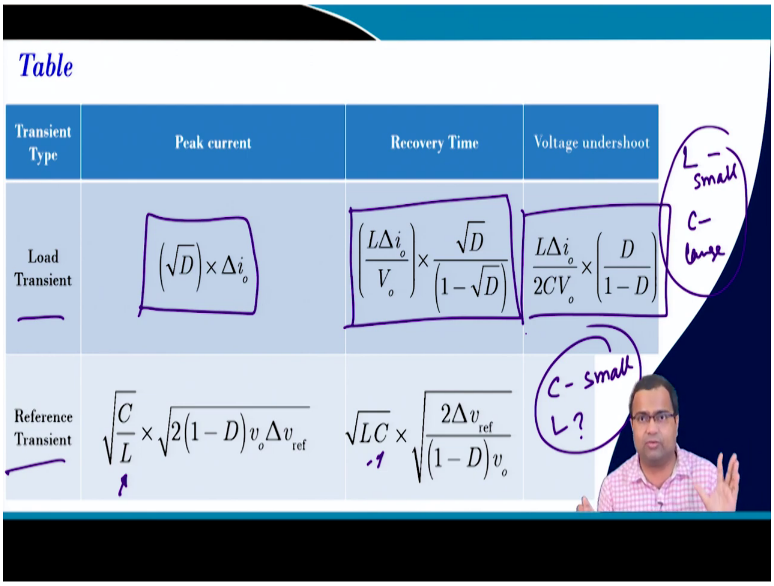
And if we compute the peak time, because we know the t 1 expression to substitute, then if you compute the m 1 m 2 if you substitute. So, we will get the peak current is nothing but 1 by L C. That means if you take a larger capacitor as I told, you need to get more energy from the source. And under optimal performance, the current overshoot has to be significantly large, because then only we can get more energy coming from the source through the inductor.

Because you need more energy to change the capacitor from 1 voltage level to the other voltage level, because your capacitor value is large. So, this higher capacitor will lead to higher peak current, but the lower inductor which you want to get earlier expression we saw, it will lead also higher peak current. So, there is a conflict right and that we will discuss in another lecture, but here we are trying to formulate the peak current.

(Refer Slide Time: 27:58)



(Refer Slide Time: 28:03)

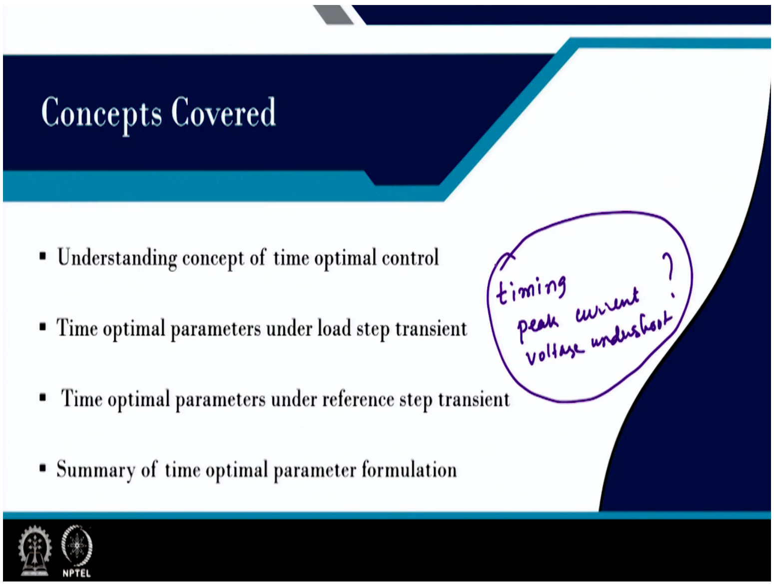


So, the summary of timing parameter, peak current and the recovery time. If you tabulate under load transient, our peak current expression is this, our recovery time expression is this and the voltage undershoot is this. So, in case of load transient, we generally want this to be small and this to be large, but that we typically design for any power supply.

But if you go for reference transient. Particularly, if you go for envelope tracking, you will just see the opposite. You use a higher lower capacitor, because. In fact, people rarely use the capacitor very small capacitor, because that will penalise the bandwidth significantly. So, your capacitor has to be small.

So, C is small, very small, and what about L. If we use the larger L, then we can reduce the peak current, but the larger L can increase the recovery time. So, you need to trade off with this L choice, but there is we can see some conflict with the L C when you are talking about this. And we will discuss this in another lecture you know and this will lead to more energy over it ok.

(Refer Slide Time: 29:17)



So, summary. We have discussed the time optimal control concept, time optimal control recovery under load step transient, time optimal control recovery under reference step transient. We have formulated different timing parameter as well as peak current. The voltage undershoot we formulated under time optimal recovery.

So, we have also summarized. And we will discuss more about this. First of all, what will be our target? How can we achieve such time optimal performance using closed loop control so that we can get fastest respond? And then, next task will be how to design power stage. So, that we do not compromise between you know this timing parameter and the peak voltage and current.

And then, another objective also in the few subsequent lectures, when you talk about load transient and reference transient how to solve this problem. So, with this, I will finish here.

Thank you very much.