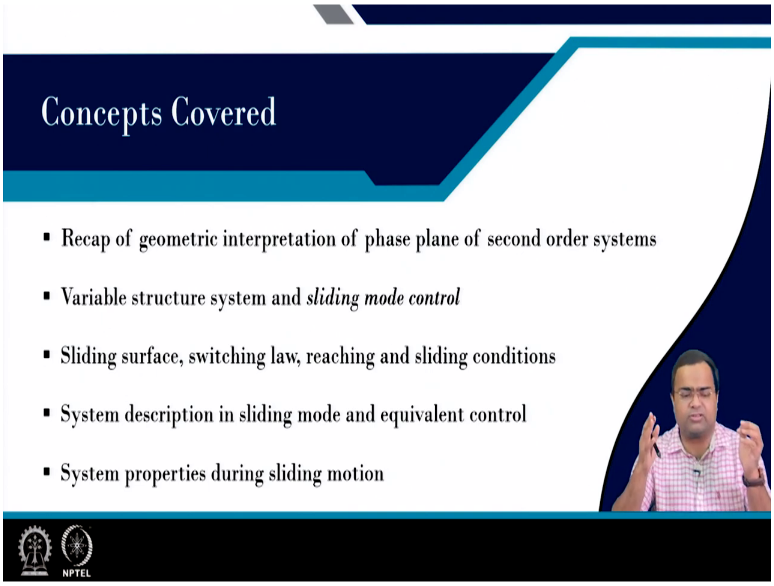
Control and Tuning Methods in Switched Mode Power Converters
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 10
Boundary Control for Fast Transient Recovery
Lecture - 45
Introduction to Sliding Mode Control in SMPCs

Welcome this is lecture number 45, in this lecture we are going to talk about Sliding Mode Control in Switch Mode Power Converter.

(Refer Slide Time: 00:35)

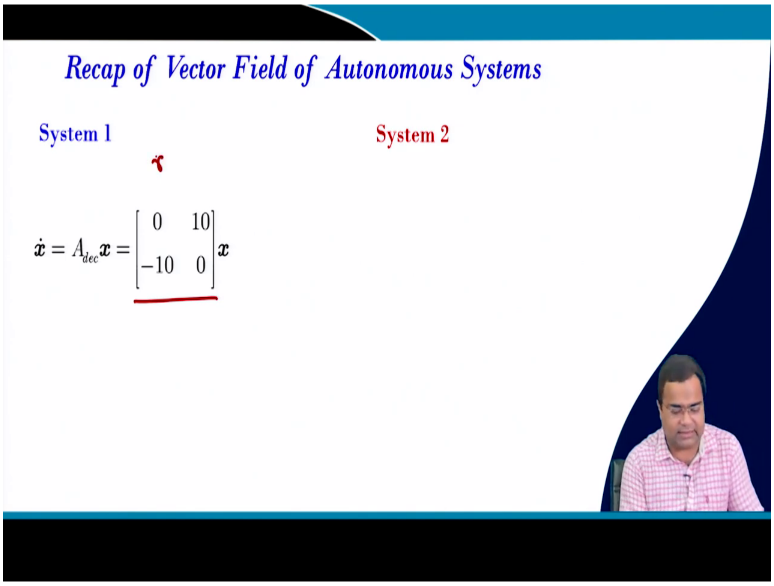


So, first you know in this lecture we are going to talk about we want to recapitulate our geometric interpretation of phase plane of the second order system, then we want to discuss variable structure system and sliding mode control.

Then we also want to discuss briefly discuss sliding surface, switching law than reaching law as well as the sliding condition and then system description in sliding motion like what will be the dynamics under sliding motion and then what is the meaning of what is the concept of equivalent control law? And finally, the property of the system during sliding motion.

So, although you know sliding mode control will require the number of lectures, but we want to just summarize the overview. So, that you know we can understand the basic concept of sliding mode control, but it requires a number of lectures hours to go in detail. So, we want to just give the basic idea and the overview of sliding mode control and the primary concept which are needed to you know apply sliding mode control in switch mode power converter.

(Refer Slide Time: 01:37)



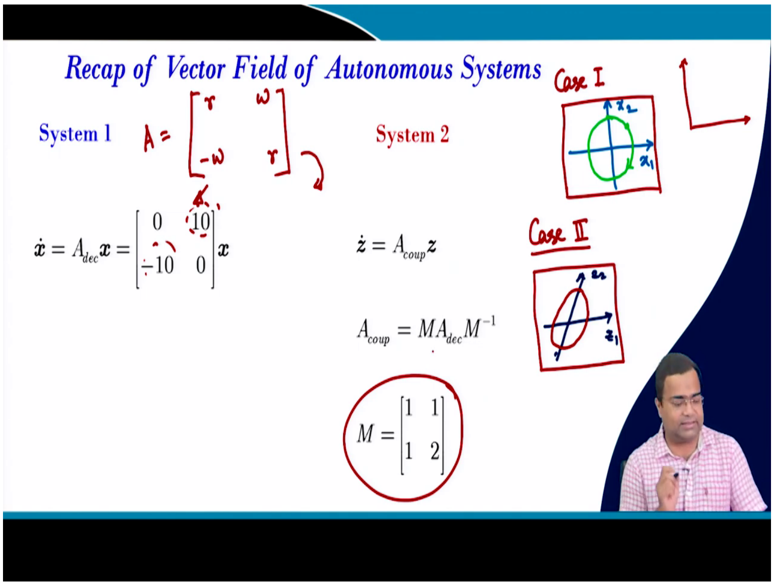
So, let us first recall our vector field concept. So, in lecture number 42 and 43 we have discussed in detail. In fact, in lecture number 43 we have talked about how to draw vector field when we consider a purely imaginary system because you know that means we have drawn the Eigenvectors we have you know we have using phase plane geometry using vector field.

We have drawn the vector field. You know real and distinct Eigenvalues for that mean the real and distinct Eigenvectors. We have also considered complex conjugate case when the Eigenvalues and Eigenvector will be complex conjugate, but in that case we have seen there is something called rotation matrix right and that will lead to a rotating motion you know.

Either it is with the same amplitude when there is real part is 0 or the amplitude will either keep on decreasing if the real part is negative. And if the real part is positive, then the amplitude will keep on increasing and that we call as a stable no stable focus or you know unstable focus and so on.

So, here again we are you know recapitulate this concept and you can see in this matrix we have considered a regular form where it is generally you know generally what we write is that.

(Refer Slide Time: 03:02)



Generally you know we take this complex conjugate case to be r omega minus omega r and this is a regular form right. We have considered a matrix in the regular form. And in this case it is the regular form where r equal to 0 and omega equal to 10 and you know if this quantity is positive and this is negative, then we know that we have seen we will get to what is called the clockwise motion that also we have discussed in lecture number 43.

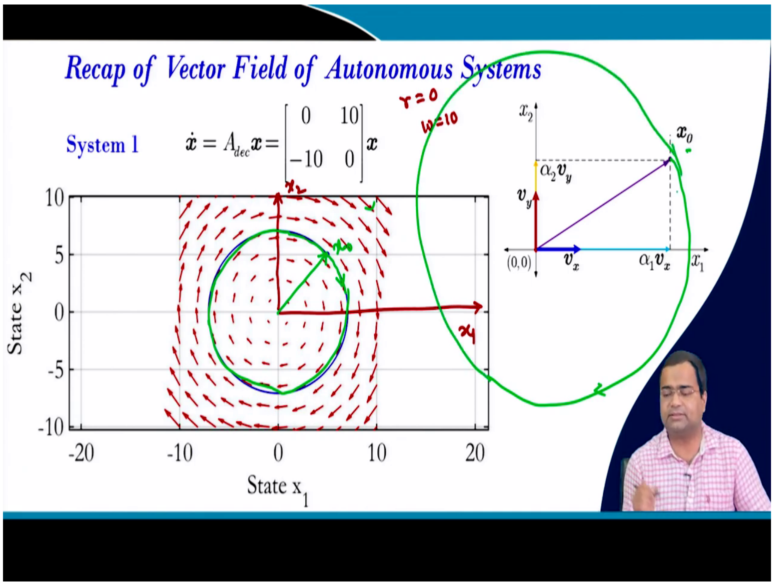
Now, we are also considering another system because in lecture 42 and 43, that one case it is in regular form and the other case we have converted into a couple form right and in the couple form we saw the vector field shape (Refer Time: 03:47) different. In fact, if you recall that you know, let us consider this case where the case 1 where the case 1 we have considered.

If we take this first case, we know the vector field will look something like this as I said that as if you take you know. So, let us first draw this axes x 1 x 2 axis this is our x 1 this is x 2. And what we have discussed that for such scenario we will get a circuit like this right this was circle and if you start with this, the motion will be a clockwise motion that is the case here.

Now, in case 2 in case 2 if we multiply with this matrix M it is a couple matrix because in this it will look like we have discussed like a basis vectors are along x 1 x 2. Now, the case 2 we have also discussed this image will look like something different you know, this image will look like you know as if one axis will be like this is my z 1 axis and the other axis will look like this like a z 2 axis.

And the shape may look like simple elliptical shape right and that we are going to see that how does the shape look like when you multiply this kind of matrix ok.

(Refer Slide Time: 05:37)

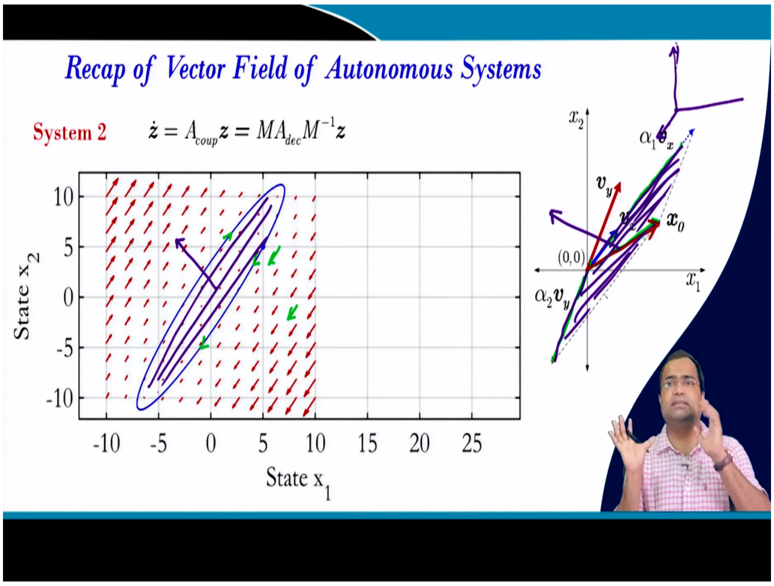


So, we have to recapitulate this vector field now this is a vector field where we have considered r equal to 0 and omega equal to 10 and this is in the regular form and where this axis is our x 1 axis and this axis is our x 2 axis right.

And now if we take an initial vector, which is x 0, then we have discussed this vector we keep on rotating in the clockwise direction. And since the radius is 0 it will be a circle and which will take this path right that we have discussed that we have discussed and that is exactly is happening and you see I have drawn the vector field.

So, the vector field is actually it is showing the direction it is the clock wise notion. And this is the case when we have an x 1 x 2 where we took the basis vector x 1 and x 2. It will you know initial condition will map like this and this vector as if it will move like this is a motion right with the same radius it will rotate, it will rotate like this right, it will rotate like this.

(Refer Slide Time: 06:57)



Now, imagine what will happen in the second case. If the angle of the image right. So, initially our vision angle is same, but initially the image as if we are looking at vertically down the image appears to be a circle.

Now, we kept our eye position same, but we have inclined the plane right. So, incline the plane in such a way we want to see how does it looks like. This is like a plane is inclined right. The same thing again, it will be a periodic motion. That means, you know it will again rotate because this is a direction of vector field you can see the vector field.

So, it will we may it will go like this, but it is not perfectly a circle because as if the image angle of the plane of the image that got change and as a result it appears like an incline plane. And this is exactly what we have discussed that if you multiply with m the earlier basis vector which were x 1 and x 2.

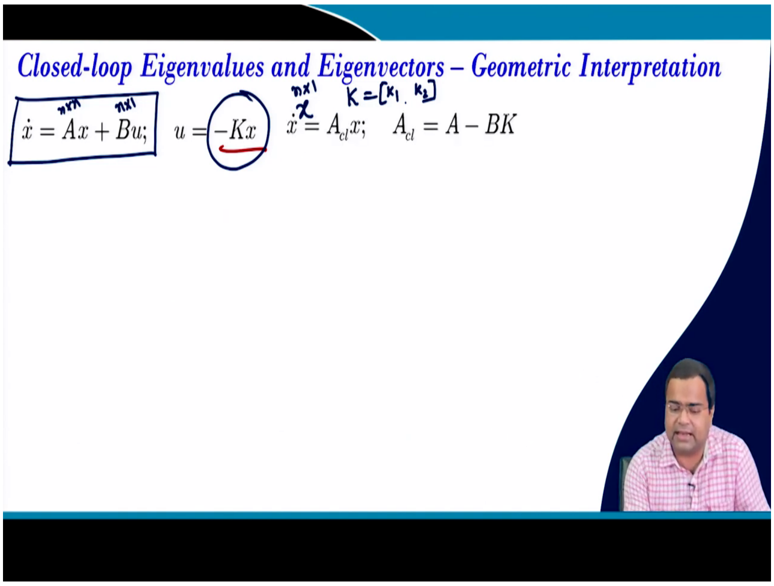
Now, it got changed to another basis vector v x and v y and that is why this initial condition we have discussed that can be mapped by spanning v x along this axis and v y along this axis. So, that the resultant will look like this, so the resultant vector will look this is my resultant vector right.

Now, you can imagine since this is now our the new one vector, is there another vector is there as if. So, this looks like a plane look like a plane and this planes look like an incline. And this is exactly showing that this looks like a plane where the plane angle actually the normal to this plane actually got changed, earlier the normal to the plane was actually towards our eye angle.

So, they were actually matching, but now there is an inclination. So, this is something similar that you can resemble. So, how to draw vector field? We can actually see how the vector field looks like. Why I am saying? Because you know in case of dc-dc converter, we will see in most of the cases the vector field is complex conjugate, the matrix will show complex conjugate.

But this may not be the matrix will not look like the regular form because that can be a kind of incline form and that we want to visualize that what you learn. So, that means, this angle plane is inclined as if this is the plane normal, earlier our normal was you know this was our normal was like this like if this is x 1 x 2 axis our normal was like vertically up, but now it has there is an inclination that got change ok.

(Refer Slide Time: 09:49)



So, that got changed now we want to go. That means we talked about Eigenvalue and Eigenvector of an open loop system. Now we are closing the loop earlier. We talked about autonomous. system you know in 40, 42 and 43 the whole lecture we talked about autonomous system.

That means we have only considered this part where we have. We derive this model from a small-signal model of a boost converter where we have ignored the perturbation the duty ratio or the control input, but suppose if the control input perturbation is considered and that control input is a function of states as if we are trying to design closed loop control.

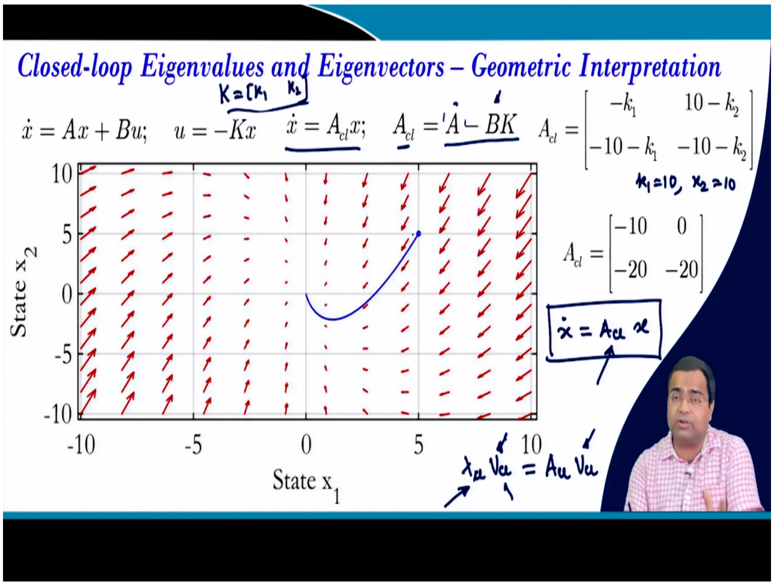
So, in that case, we have to incorporate this whole x dot equal to a xb into this function. So, this one and then what we will get is that this whole thing will be our closed loop system. Where u is nothing, but our minus k into x because it is a negative feedback system and the gains vector is positive.

So, in this case x in general can be n cross 1 that is n number of states, then A must be n cross n and if the u that means the duty ratio in our case is a single input. So, this is n cross 1. So, in our case it is 2 cross 1 because it is a second order system and a gain matrix K it is a basically a row vector which consist of k 1 k 2. That means, I can think of K to be k 1 k 2 it is a row vector ok.

Now, the question is: in order to design this closed loop control, the first thing that we have learned in our linear system theory, the system must be controllable. So, we are assuming that this concept is known. This is already covered that in other lecture. So, the controllable system that means we can design a state feedback control. And if you design a state feedback control, then the closed loop dynamics will look like this something like this.

That means this is our closed loop dynamics where close loop matrix will matrix will be A minus BK. That means, A is your open loop A matrix system matrix, B is an input matrix and K is a gain vector ok.

(Refer Slide Time: 12:13)



Now, once we have considered; that means, K to be a set of k 1 and k 2. Now, we need to choose the gain in such a way we need to place the closed-loop poles in our desired location right.

So, here I am just showing one example suppose I choose a k and k which are like k 1 equal to 10 and k 2 equal to 10 just a simple number I have chosen you can choose other because this gain will come from a desired closed loop location.

But my intention is to show that even if you choose the desired location and then you know subsequently you obtain this gain vector, then you can get another closed loop matrix. So, you will get a different closed loop matrix and for each closed loop matrix we can always draw the vector field because the closed loop matrix when you write then it becomes x dot equal to a closed loop into x.

So, it looks like another autonomous system in which the control input is a function of state. The whole thing is combined now. It will become a closed loop system. So, as if there is no external input. Now, the question is the next question will be how to that means, in the closed loop system design, what we do we generally consider the closed loop Eigenvalue right.

The closed loop Eigenvalue and again if we write the closed loop Eigenvector this is nothing, but our A closed loop into V closed loop and these are my Eigenvector of the closed loop system right. So, now, what we did? We essentially place this closed loop Eigenvalue or in other word we placed the closed loop in a state feedback control.

We are trying to place the closed-loop poles. You know sometime we interchangeably use the poles and Eigenvalues, but unless you consider an output, then there is no concept of pole 0, but in this case in state feedback control we generally talk in terms of eigenvalues. In the closed-loop eigenvalues, we can choose based on where we want to place the Eigenvalue because it you know it will indicate our closed loop performance.

That means what will be my transient overshoot undershoot etcetera there can be the specification can come from time domain specification and so on, but we can place the closed-loop pole and accordingly we can design this gain matrix, but what will not be in our hand is that how to place the closed loop Eigenvectors.

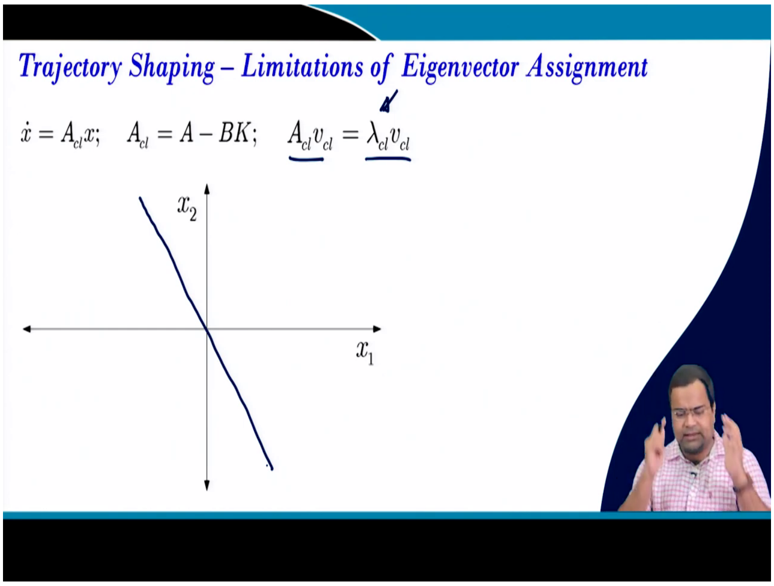
That means it is a problem of Eigenvector assignment because even though we choose a closed loop eigenvalue, our closed loop Eigenvector is not in our hand because if there are 2 cross 1 matrix second order system. So, we will have 2 Eigenvalue to be placed and in order to place that we will have 2 variable that is k 1 k 2 gain.

So, that means, for getting this Eigenvalue, I need to choose this gain. So, I have no freedom degree of freedom to place the Eigenvector, so it is the only Eigenvalue placement. So, if you do not take the Eigenvector or we cannot place the Eigenvector, the problem will be the shape of the trajectory because we have discussed that two subsystems where the Eigenvalues are same, but they have completely different Eigenvectors.

That means you know if we make a linear transformation of the system we transform this closed loop system to another plane by multiplying it is like an image because you know as if this trajectory motion if you change our view angle you will see our nature is different right, but the stability property is decided by the Eigenvalues.

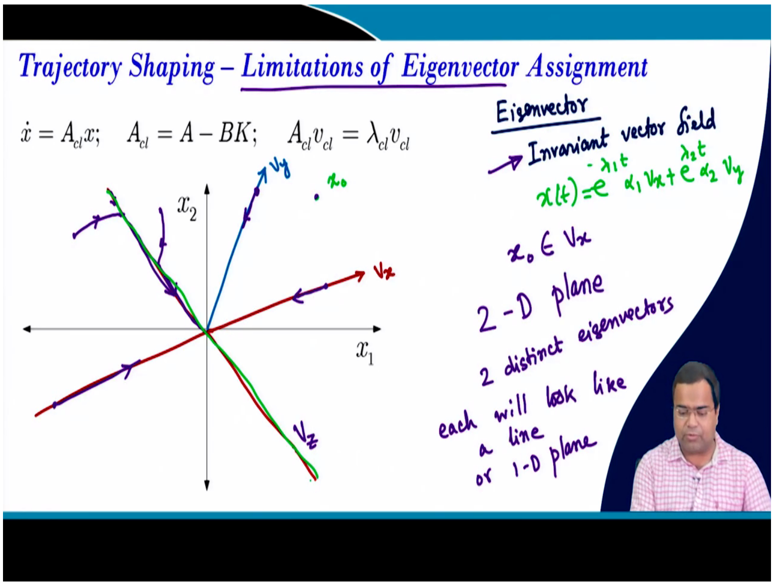
So, those are fixed that mean that is not going to get affected, but the trajectory motion, the nature of trajectory or shape of the trajectory will be different. So, this is where actually is the problem in the one of the aspect in linear control system because we have almost no choice to decide the shape of the trajectory by means of linear control because we can only place the Eigenvalues.

(Refer Slide Time: 16:25)



So, if we want to shape the trajectory that means, I talked about closed loop Eigenvalue and closed loop Eigenvector. So, when you talk you know we place these Eigenvalues then will we can find out the A matrix and from the A matrix and the Eigenvalue we can find out the Eigenvector. Suppose I want to get my trajectory something like this. Suppose we imagine that I want to shape a line straight line. Suppose this I can call something similar because you know what we discuss.

(Refer Slide Time: 17:05)



If we have discussed that, what is the property of the Eigenvector. So, we know that Eigenvector has something called invariant property. It has invariant vector field. What is that?

That means if I can define suppose for this system we got two eigenvector one Eigenvector let us say along this line let us say this is my one Eigenvector and the other Eigenvector what I imagine it could be like a v y, the other Eigenvector is v y ok. Now, if you choose an initial condition, suppose you know we choose an initial condition here x 0.

Now, how does this trajectory move towards this origin will be decided by 2 Eigenvectors right because we have learned that x of t can be written as e to the power lambda 1 t alpha 1 v x plus e to the power lambda 2 t alpha 2 v y. So, we have control over lambda 1 lambda 2 because that we are placing, but we have hardly any control over this. That is one fact.

The second thing we know the Eigenvector. If we only choose, let us say a point on this Eigenvector if it is a stable Eigenvector then the trajectory will move along this line right. It will not go out of the line because that we have learned from the something called invariant property. Similarly, if we choose an Eigen in a; initial condition here, the trajectory will move along this Eigenvector.

So, we can only ensure the movement of the trajectory along one of the Eigenvector if we choose the initial condition on it, but is it possible that if we choose a path. Suppose I want to create a path. Let us say this is my you know let us say this is my point, I want to create a path ok let me draw properly I want to create a path ok, this is my path.

And I want to make sure if I take any trajectory here I will do something. So, that it will try to come towards this and once it touches here, it should remain here it should remain here or if I take an initial condition here, it may come towards this, but once it moves, it should remain here it should go along this line. That means, it is something like because what does Eigenvector say.

That means if you take your initial state along any of this Eigenvector v x or v y the movement of the trajectory will be constrained on the Eigenvector. That means, if you have a 2-dimensional plane ok and if we have two distinct Eigenvectors and of course, they are real then each Eigenvector each will look like a line like a look like a line or in other word one-dimensional plane.

Because each Eigenvector will remember you know it will define like say if we extend this Eigenvector. That means, if you keep on extending any point you choose here any point you choose here it will move along this line. So, that means, if you choose any initial condition of this vector, it will always remain on this line.

So, the motion of this particular point, or basically the initial condition or the trajectory, will be constrained on the line. Even though you have a 2-dimensional plane, but the motion will be only along that line. So, you can think of this dimension of this line is just a 1 dimension. similarly for the Eigen other Eigenvector it will be another 1 dimension.

But we want to get for any arbitrary initial condition is it possible that by some means we want to force this and this will look like my another z which is synthesized Eigenvector looks like an Eigenvector which can be maybe a linear combination of Eigenvector or it is our own design.

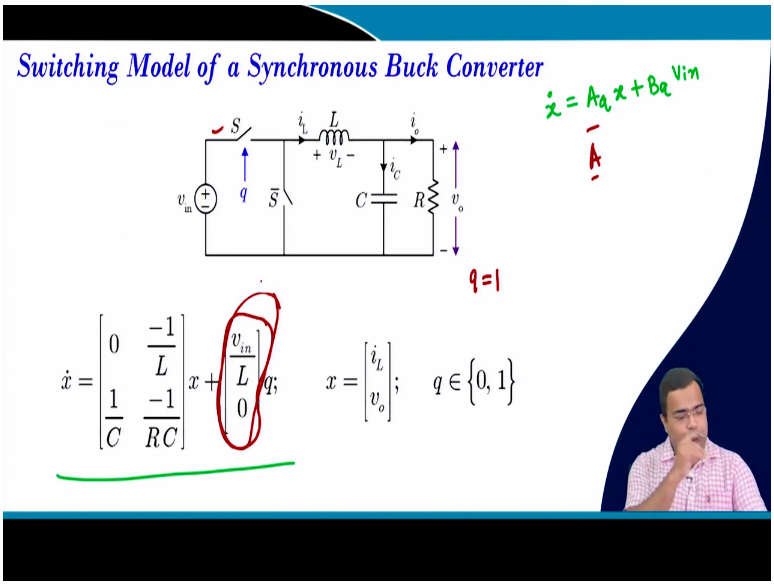
Where in the entire 2-dimensional plane the trajectory will always is force to come towards the line and it will remain on the line. And then whether it will come towards origin or go away from origin that we will discuss later, but let us say is it possible to have a scenario where we want to force and to remain on the line.

So, that is one of the objective and that probably is not possible in case of linear control system because in linear control system for a second order system we if we get two distinct Eigenvector, then each of the Eigenvector will be an invariant field, but for any arbitrary initial condition we cannot make a one dimensional motion right.

So, that is one of the problem that we are looking for because why we are interested you know we why we are interested in finding such movement? Because of this path, we want to shape. Suppose, we want to get a desired trajectory which will try to satisfy certain optimization criteria.

Suppose, I want to achieve something like minimum recovery time or we want to reduce that we want to reduce some; we want to minimize some energy something like that or we want to constrain the motion such that the fuel economy can be improved the disturbant rejection property can be improved. So, many things we want to discuss.

(Refer Slide Time: 23:30)



So, now and you will soon find that this might be possible if you incorporate the switching control. So, simply by linear control it is not possible, but interestingly a dc dc converter is a switching converter.

So, you do not need to do switching like in the linear system. If we apply a switching control; that means, it is a discontinuous control, but in dc-dc converter your control itself is discontinuous because we have a switch and that switch will turn on and off and you will get two different subsystems and each subsystem we want to drive in a way.

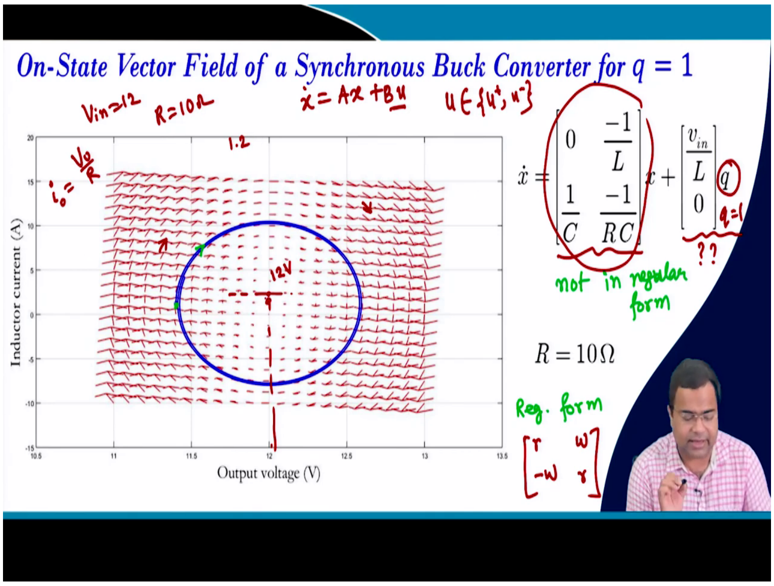
So, that our resultant motion can be along the desired trajectory that we are talking about, that means, along the green line right, that is one of the objective. Now in the switching system, this dc-dc converter we have discussed that we can obtain the state space switching model.

And we discuss that in general it is written as x dot equal to A q x plus B q v in where A q is a function of state the switching function. But in case of buck converter your matrix input matrix if you for the ideal case this A q will be A. It is independent of q as long as this is in continuous conduction mode, but this structure will be different when you go for continuous and discontinuous you know combination.

Similarly, if you go for a real buck converter with parasitic with diode drop and all this A matrix can be different and also dependent on the switching matter, but if you take a synchronous buck converter with almost identical r d S on and r l. So, in r d S on then you will find that this a matrix will be more or less same.

But the B matrix is different because in the buck converter, if the switch is on the input is connected right, but the switch is off then input is disconnected. So, the input matrix is totally discontinuous. So, you will get a discontinuous vector field.

(Refer Slide Time: 25:36)



Now, if you just take only on state trajectory; that means, you are I am talking about for this model if you take q equal to 1. Then you can get this vector will come in. Input is connected and for q equal to 1. That means, your q you can substitute q equal to 1 here. What will get you see the trajectory or the vector field will be decided by this matrix whereas, this quantity will decide your equilibrium point. That means, what is your operating point?

If this quantity is not there, then your origin is equilibrium point because we talked about x dot equal to A x plus B u. So, here you can think that u can take you know either it can take u plus or u minus two discontinuous input. So, if depending on this u term discontinuous term we will only shift this operating point right, but the vector field the nature of the vector field, will be decided by this A matrix.

So, in this case, you know it is very clear that if we take the initial condition, that means this is our initial condition and you can see it is going in the clockwise direction. So, it resembles something our concept that you know our regular form, whatever regular form, for that means it is definitely a complex conjugate case where the motion is like a circular or may be oscillatory motion.

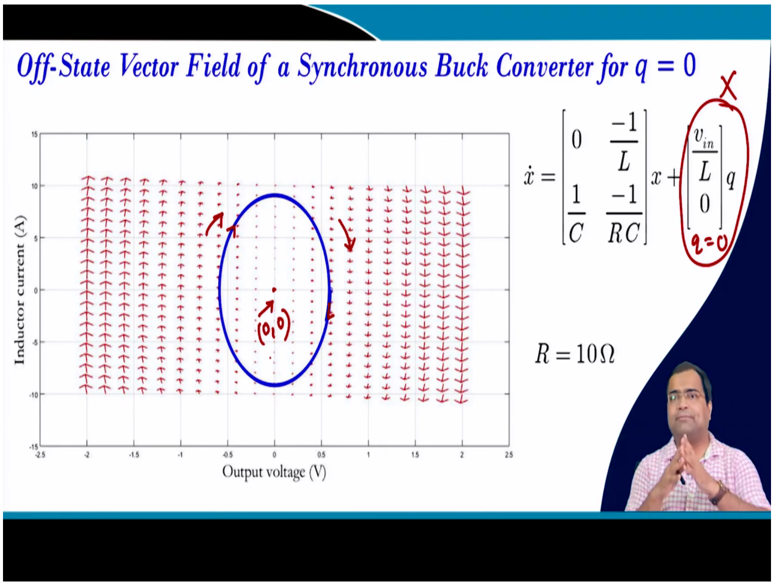
The amplitude of oscillation is decreasing when the real part is negative. But it may not be in the regular form because we saw in regular form it will look like a pure circle and in case of image version of that where it is not in regular form, but still the Eigenvalues have are complex conjugate, then you will get an inclined you know either elliptical shape or something like that.

So, here it will be elliptical shape, but since the motion is decreasing slowly decreasing. So, it is a called stable focus, since it is not in regular form it is not in regular form because it is not in terms of this R what you have learned it is not what is the regular form. So, the regular form is something like you know, r omega minus omega r it is not in this form.

But it is a complex conjugate case and, but the real part is very small. So, it is decreasing very slowly and you can see the vector field it is showing in this direction it is showing in this direction. So, it is a clockwise motion and what is my operating point because it is switched on and I took input voltage to be 12 volt, and what I choose R, R I have taken r equal to 10 ohm.

So, if you turn on fully on the switch, the final steady state voltage will be equal to the input voltage and that is exactly is a 12 volt here, here in the output voltage it is 12 volt and what will be here? Because it is current will be what in final current will be final voltage by resistance if the final voltage is 12 volt is 12 by 10. So, it is 120 milli ampere right that means, 12 by 10 that is 1.2 sorry 1.2 ampere sorry 1.2ampere ok. So, now, 1, so it will be somewhere here.

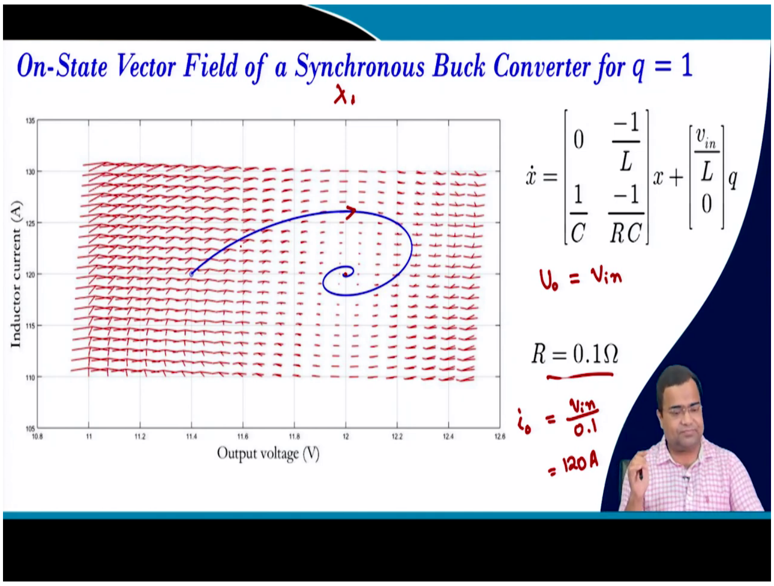
(Refer Slide Time: 29:29)



If you take the off state trajectory, the same, you know it will be again complex conjugate and you can see it is rotating in the clockwise direction. That means, it retain the fundamental nature of the clockwise motion, but slowly decreasing, but the operating point gets changed here.

Since this will be 0 q will be 0. So, this part will not be there and in that case your origin is the equilibrium point, and this is 0, 0 you can see this is 0, 0 point right, but you know the dc-dc converter we never operate either fully on or not fully off because we operate in on/off state.

(Refer Slide Time: 30:10)

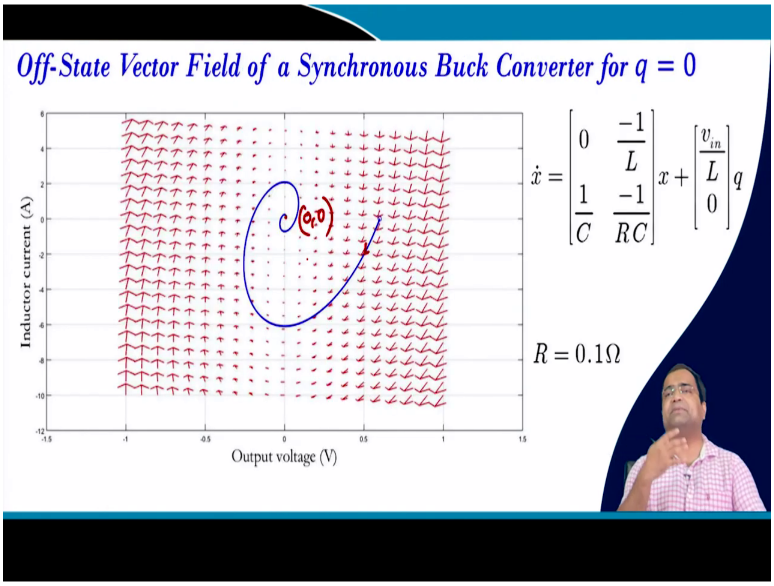


Now, if we increase the load current or decrease the load resistance, what will happen? You see the current has drastically increased because now we are using R equal to point 1 and what is my final voltage. So, if the switch is on my final voltage will be simply v in that is 12 volt and my final current will be v in by 0.1 which is nothing, but 12 by 0.1 so it is 120 ampere.

So, it is roughly 120 ampere 12 volt 120 ampere, but what has changed you see again it is a clockwise motion, but the damping ratio has increased the real part has increased. So, compared to the previous Eigenvalues and this Eigenvalue, the real part is negative only, but the magnitude has increased. So, that it is decaying much faster at the same time scale you see the oscillator is quickly decaying and coming back.

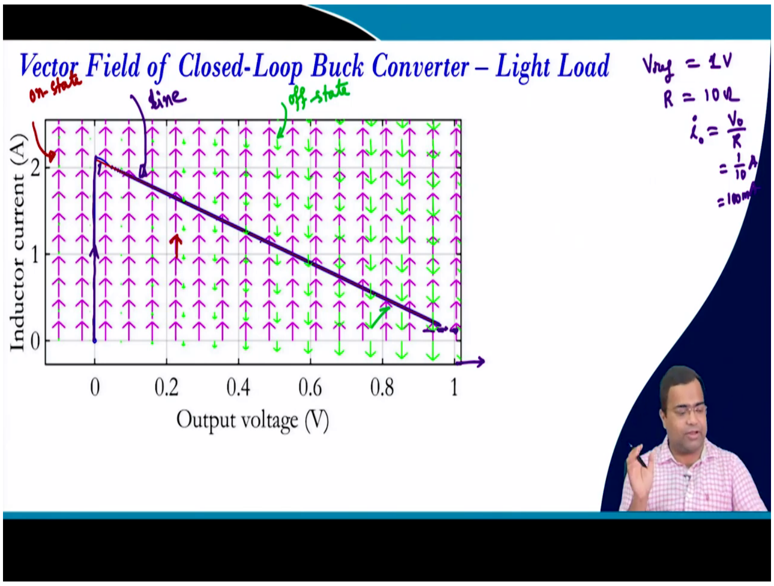
Because we will see in dc dc converter, we have seen even a subsequent lecture. Under light load condition or high load resistance, there is a damping problem poorly damped because it is a complex conjugate is a very low real part small real part and here the real part is large. So, it will quickly decay and very quickly come to a steady state. Similarly, if you take fully off the nature of the clockwise motion will remain same, it will again decay fast.

(Refer Slide Time: 31:30)



But this point gets sifted to 0, 0. So, that we have discussed.

(Refer Slide Time: 31:45)



But we never want either fully on fully off. Light load condition is really phase difficulty in terms of controlling and to get the desired response. Now, again coming back to the question of how to obtain I want to make sure my whole trajectory will look like a like an Eigenvector.

That means, if you see here this line indicate our on state trajectory; that means, this color particular color is the on state trajectory, on state trajectory and if you see the green one the green one these are the off state trajectory.

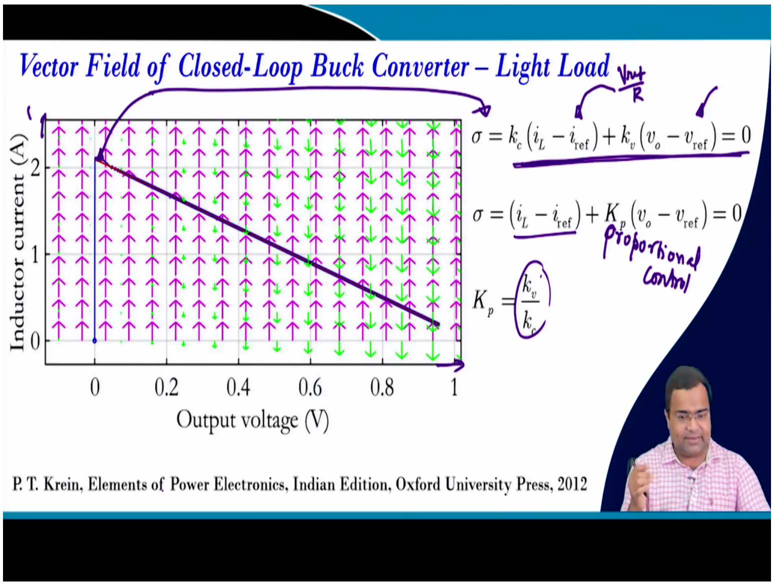
But now we have placed A something like a line. You see, this is a line straight line this I can say this is a straight line kind of or a 1 D plane or basically a straight line right. It is basically a line, this line where it will terminate? It will slowly come and it will terminate 1 because here again is my output voltage axis and I want the 1 volt to be my desired output voltage.

So, my reference voltage I want to be 1 volt and what I said load resistance in this case it is 10 ohm. So, the current will be accordingly. You know at 1 volt that means, my load current will be, you know what will be my load current? My load current will be V 0 by 10 R. So, it will be 1 by 10 ampere. So, it is something like 100 milli ampere that I want 100 milli ampere.

So, this is my final part now. How can we achieve this motion? That means you see the on state the trajectory is moving. This is the movement of the trajectory when it hit this line. So, there is a red line. You can see the red line and you will soon see then it remains there.

That means, whatever initial condition here, we have taken an initial condition which is 0, 0. That means the converter is starting from the 0, 0, 0 current 0 voltage. Then it turns on. When it hits, then it is moving along the line. How are we going to move? That we are going to discuss.

(Refer Slide Time: 34:19)

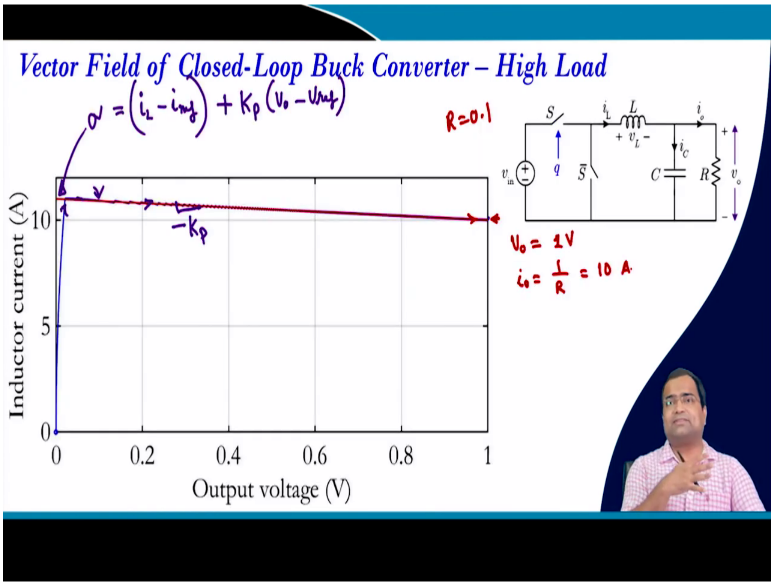


So, we are placing something called a surface. So, this red one what we are indicating here is basically you can say there is a surface a red color which is like this because a straight line here it is the voltage and this is the current axis.

So, some gain into current and a reference current is the desired current where you want. So, what is that? It is something like a v ref by R that is my reference current and what is my reference voltage that 1 volt where we have set and whenever the surface equation is sigma, which is this equal to 0; that means, if the trajectory is there. This will be simplified to 0, and it represents a 1 dimensional motion.

And we further normalize, divide by k c. It will look like a gain. It is like a current error into proportional gain. We are talking about a proportional control is like a proportional control and sigma and this you know you can get more detail about this geometric control in element of power electronics. So; that means, now we need to make some switching arrangement.

(Refer Slide Time: 35:41)

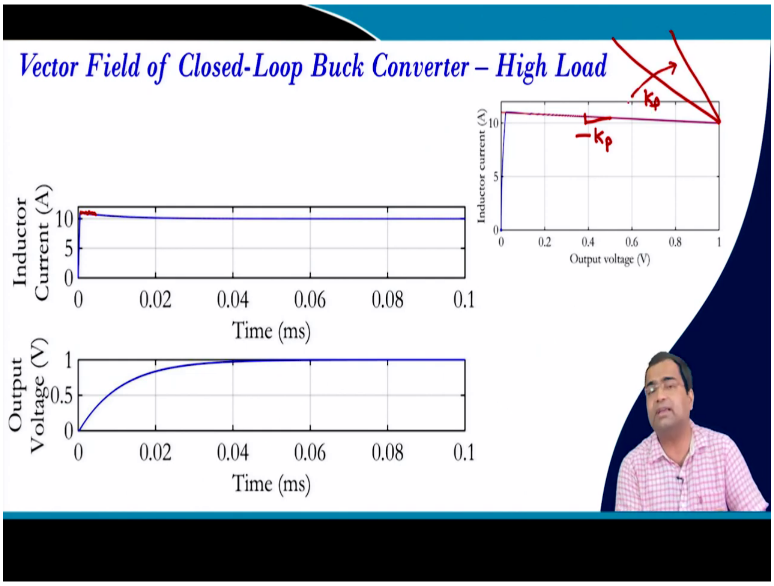


Which tells us we need a switching surface. So, this is what exactly I was telling about this is my sigma and here the sigma I have taken i L minus i ref into K p v 0 minus v ref and what is the slope of this line you can it turn out to be minus K p.

That means if you increase the proportional gain, the slope will also increase and we want to see, that means, we want to get this line in such a way it meets our desired requirement ok. So, here we have chosen some value of K p and it turns out to be like this, but remember when it hit the surface it has to turn on and off in such a way this side as if it is going towards the line and this side will turn off as if it is coming towards the line.

So, the vector field on either side should be towards this line. So, that it will converge to this line. Then you have to see whether, by virtue trajectory at all is moving towards the desired point. This is the desired point or not. So, this is the desired point where the load here is here. We have chosen R equal to 0.1. That means our output voltage should reach 1 volt and output current should reach 1 volt by R. So, which is nothing but 10 ampere and that is somewhere it is coming.

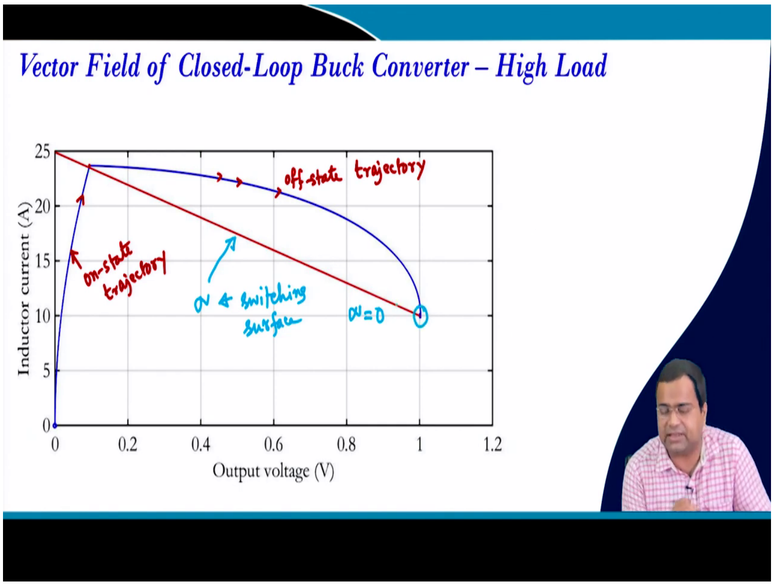
(Refer Slide Time: 37:17)



Now, what happens if we increase the slope of the line? That means you increase this gain. This is a time domain behavior. This is the trajectory or basically phase plane output voltage versus inductor current.

Now, I am showing individually how does the inductor current look like. So, initially it will turn on from here the switching operation start, but it does not look like there is switching because we have taken a very narrow band because you know you cannot keep a pure surface because your switching will be infinitely fast and that is not desirable. So, we want to give take into account all this practical constraint.

(Refer Slide Time: 38:00)

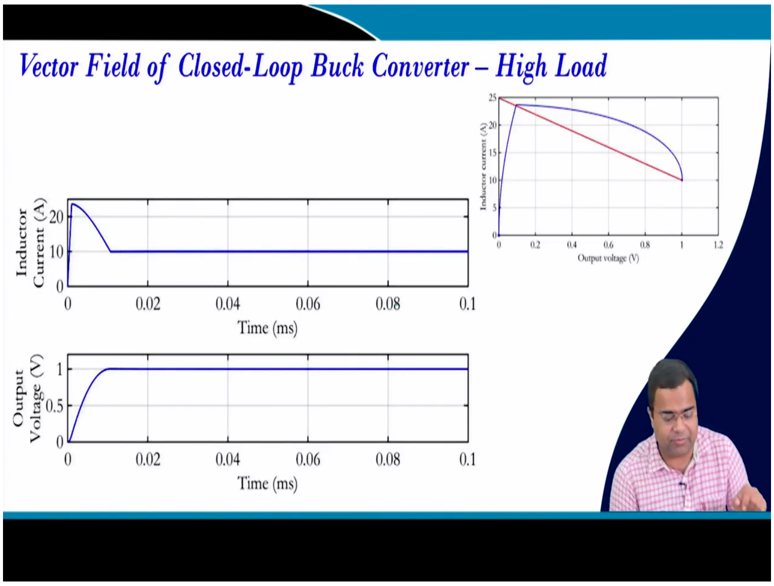


So, now this is the scenario whereas I told that this is my K p slope; that means, if I increase K p the falling slope and the slope will increase. So, gradually this line will it has to pass through this point. So, the line may look like this as you increase the K p. This line will look like this ok.

Now, with a higher line, you see first the trajectory hit here when it turns off, but it may not because this movement it is decided by the off state trajectory. So, this is the off state trajectory because we have already drawn the vector field and this is my on state trajectory and this line is our sigma or we can say this is you said switching surface.

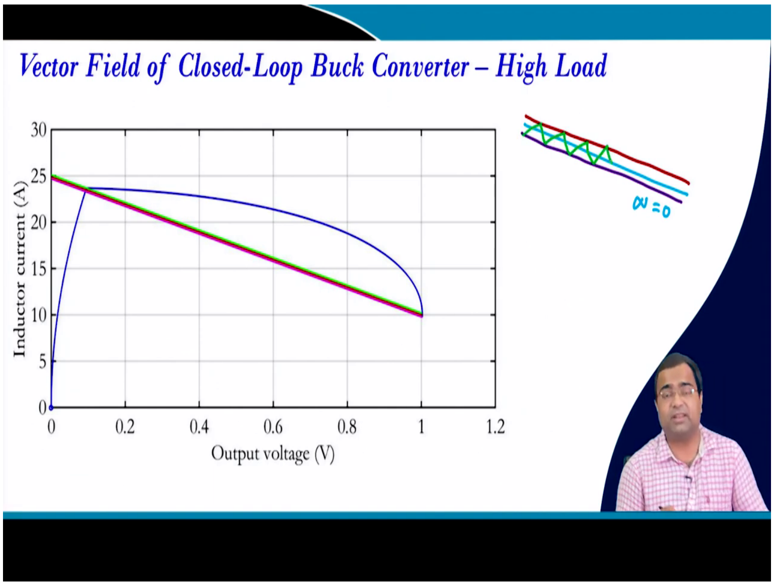
So, since it is a first 1 dimension, it is a straight line. So, in this case the surface dimension is just a 1 dimension, but here your sigma equal to 0 and here you can see the switch turns on it hit switch turns off and then it comes to steady state here.

(Refer Slide Time: 39:28)



And if you take the time domain behavior, the switch turns on here, then it completely turns off and it comes here and there will be small switching here ok very very lower switching ok.

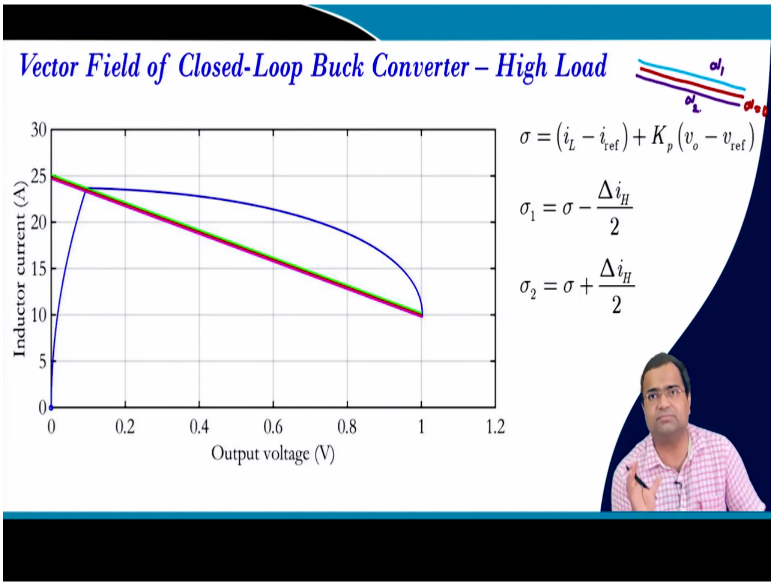
(Refer Slide Time: 39:41)



So, that means, practically there is a band, but which is sort of not clearly visible because the band is so narrow. So, on top of red that means, we have considered the actual sigma which is 0 it has there has to be some band.

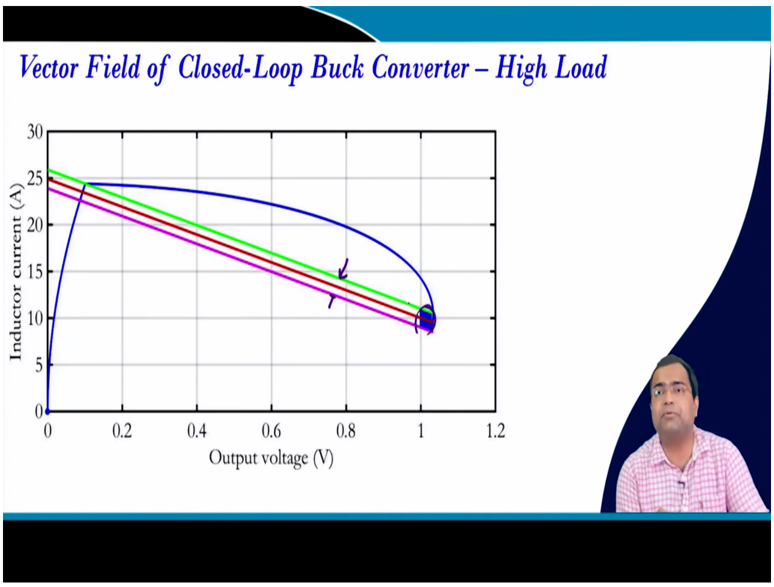
That means, it should have one band here and it should have another band here and the trajectory will actually go. It will remain on this line that means it should move like this something like this.

(Refer Slide Time: 40:18)



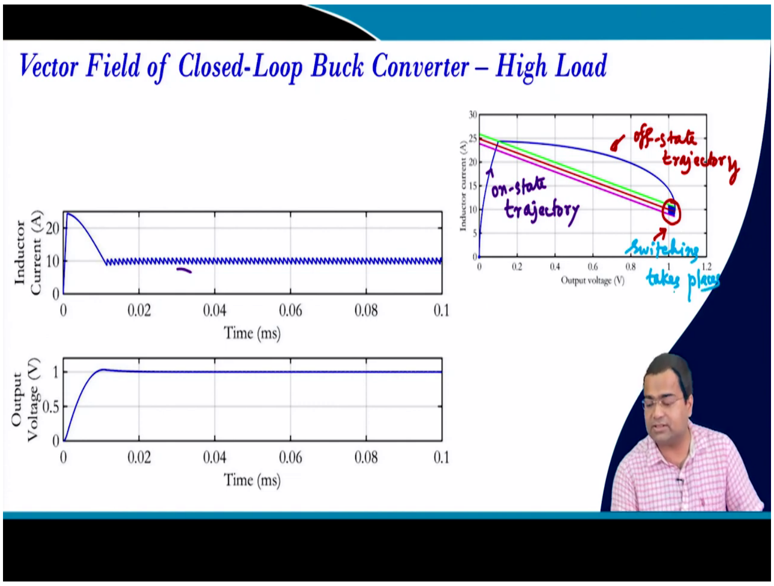
So, if we go ok now this two band one is you know as I have drawn, I can draw again this is my line this is my sigma equal to 0. The one line I have drawn is just above this and the other line I have drawn it just below this. So, this is my sigma 1 this is my sigma 2. So, sigma 1 it is just on top of this and sigma 2 just below of this ok.

(Refer Slide Time: 40:52)



And if you take a larger gap or basically band if the band if we increase, then you see this on off operation.

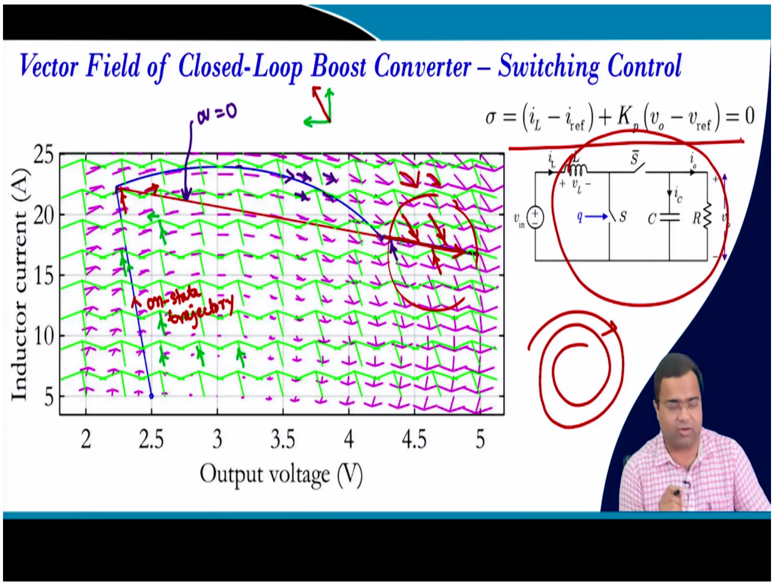
(Refer Slide Time: 41:02)



And you can reduce the switching frequency. That means you can increase the ripple and the switching frequency can be reduced. So, by adjusting this band, we can effectively control the switching frequency and that is one of the problem, but it can be addressed.

This results in a variable frequency operation. But, if you do it properly switching, it turns off. This is my on state trajectory as I said. My on state trajectory this is my on state trajectory, and this is my off state trajectory off state trajectory and this will come here. Your switching takes place. So, here your actual switching takes place switching taking takes place. Now, it is slowly comes towards this origin.

(Refer Slide Time: 42:08)



And if you see, we will discuss how to design this sigma. Similarly, if you consider a boost converter, where this green line you know if you see this green this green, these are the on state trajectory, these are the on state trajectory.

And in boost converter, when the switch is completely on you see the current is increasing; that means, it is in the upward direction the current is increasing and in this side voltage is going towards this. So, that means, it effectively shows the current is increasing, and the voltage is decreasing. So, as a result, your resultant vector will be something like this is your resultant vector, and that is exactly you are getting.

So, voltage will start decreasing and if you continue turning on the switch in a boost converter, you will find the voltage will completely collapse and the inductor will saturate because the current will increase significantly and it can damage the converter.

So, you have to be very careful about the switching mechanism of a boost converter. During off time when this L C R L C circuit will be connected, then it is similar to a buck converter where we have a complex conjugate pole. And there you will have a right that kind of spiral motion that we have discussed. That means, the trajectory will be like this kind of motion slowly decreasing motion for off state ok.

So, if you draw the complete trajectory now, the question is, how to turn on? So, this is my on state trajectory because you see this blue line is parallel to this green line right. The green line where you are actually if you look at this green line and they are almost parallel.

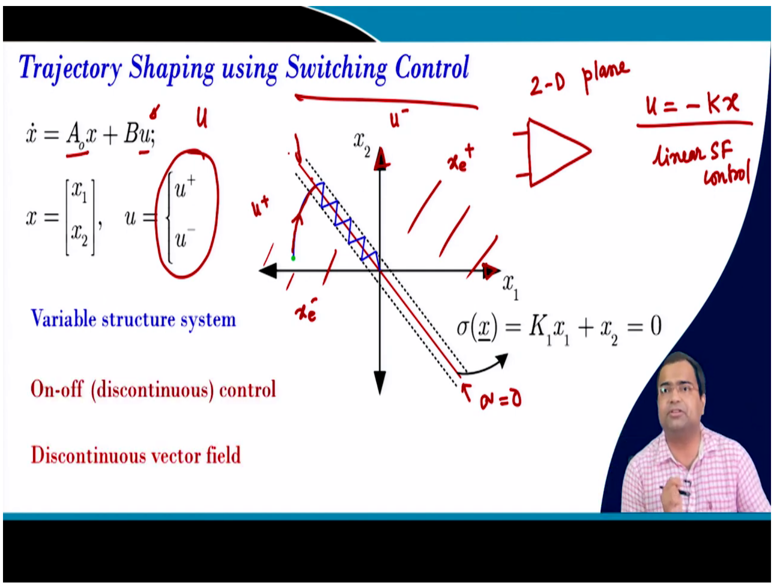
So, this indicate the on state trajectory and if you see the off state trajectory that means, if you see this off state trajectory off state this is parallel to this trajectory right this trajectory imagine the color which is off state trajectory.

So, when the converter hits this red curve or the switching surface sigma, then it turns off. If it turns off it follows the trajectory off state again, it comes and this time now in both sides because you see the green line it is towards this side and if you see this side it is towards this right. So, the magenta line towards this and the green line towards this.

So, as a result the trajectory will be will move along this path because on either side the trajectories are towards the surface, but if you see here the trajectory is going out the off state going away and on state is coming in. So, this is something called refractive trajectory where one side the trajectory is coming towards the surface and the other side is going away.

And this is the reflective or where the sliding motion exists and we will discuss this where the trajectory actually moves away. And we will discuss reflective reflect refractive as well as the rejective trajectory in the subsequent lecture yeah. So, here also we will have a surface we have discussed that is also again the same thing that we discussed for a buck converter.

(Refer Slide Time: 45:47)



So, the trajectory shaping within switching control that is a best way because that we discuss, if we want to get a motion where sigma equal to 0. So, that means, for a 2 dimensional plane for a 2 dimensional plane because it consist of x 1 and x 2. I want the initial condition will slowly come hit this red line and will remain on the line and in 2 dimensional plane we will see this switching surface sigma equal to 0 that indicate just a straight line.

So, we want to confine the motion along the straight line and it is something similar to our Eigenvector. It becomes like an invariant vector field. That means, once the state reaches here trajectory reaches here, it will remain on the line.

So, as if it will satisfy the invariance principle and then if we make sure because the invariance principle is the Eigenvector property, but whether it is a stable or unstable that depends on you know the stability property that will come, but here we need to make sure the invariance property. So, if we can force the trajectory to move and touch this line and then it will remain on the line.

So, it is something similar to the concept of Eigenvector, but that is not possible in a linear control system. That means we cannot have a desired Eigenvector only 1 in the whole 2 dimensional plane where the trajectory will come and remain on the line.

So, that is required something like a switching control and that is exactly what and it you see when the switch is on that means you take u which takes a discontinuous control input. So, this input in case of our linear feedback system we took K equal to minus K x.

So; that means a linear function of state and the gain, but here. So, this is in case of linear control linear state feedback control right, but it is a switching control where the control input takes either some plus or minus, as if you can differentiate suppose if we are using an op amp.

So, op amp is used like a in the first case linear control op amp is using like a linear amplifier and in case of switching control op amp is acting like a comparator. So, on off, on off it just takes between plus and minus ok. So, how we can differentiate, but in u plus that means, your positive supply rail and the negative supply rail of the op amp you will get two different vector fields.

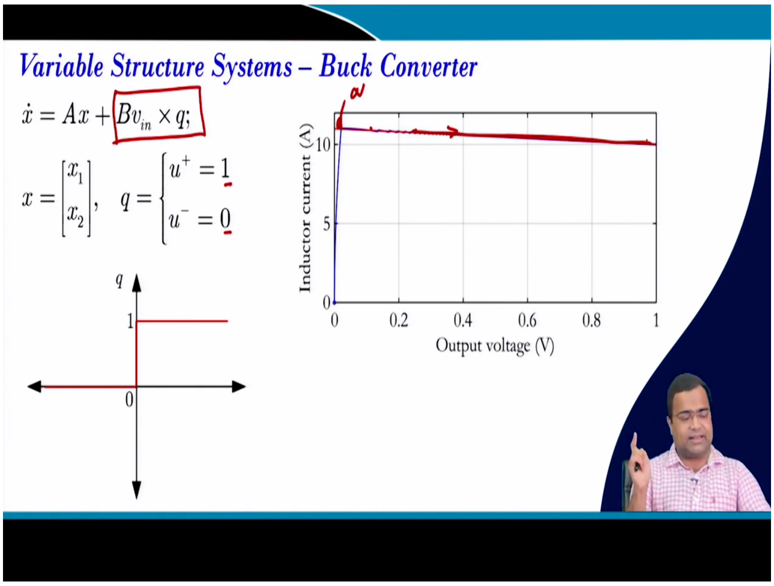
Because if the a matrix remains common we saw earlier that the trajectory in case of buck converter you see we have seen that. It was a stable focus, and it was rotating in a clockwise direction and slowly coming, but the change in u will only shift the equilibrium point. That means, we saw for on state and off state of a buck converter, the nature of trajectory was same, but only the operating points are different.

So, here also the operating point will be different. So, that is why it is a variable structure system because these two operating points create like two structures and we need to make sure that will come that if you take u plus. That means, if you take u plus your operating point where it will eventually try to move should remain in this side.

Because it should try to come towards this operating point and when it hit the surface, then it will get locked because if you take u minus here, your x e minus should be this side. So, by that we can make sure the trajectory will neither go towards x e plus 1 completely or it will not go towards x e minus 1.

So, it will finally get locked into the switching surface and we just make a switching operation on off operation. So, that constitute a something called variable structure system where we have two different vector field discontinuous vector field and the point of discontinuous will be you know just either side of the switching surface. So, on off control and this is a discontinuous vector field.

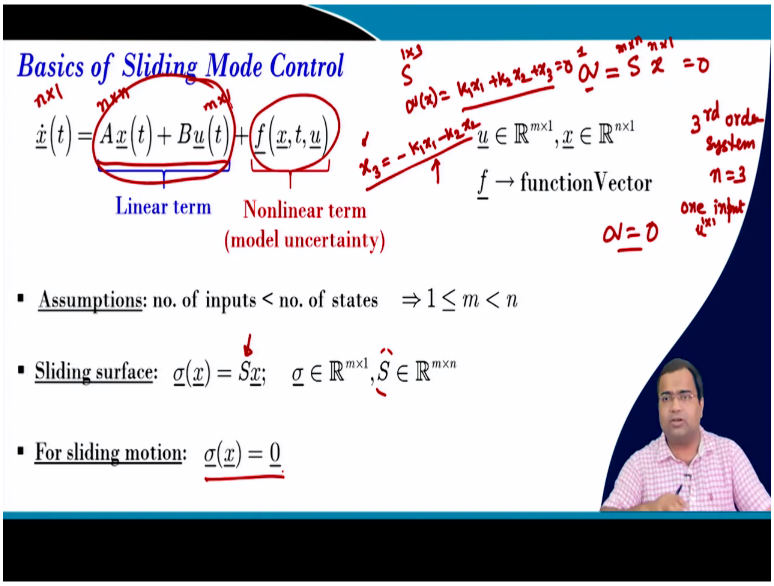
(Refer Slide Time: 50:00)



So, this start with our buck converter that we did, where this something creates our discontinuous vector field where it can take either 1 or 0 and that shift operating point, but at the same time the vector field gets shifted here right.

That means at the same you know, in the phase plane, at one particular point, the direction of the vector field will be different for two different switch states. So, it takes a discontinuous control input, and we have seen that this our is sigma right and they will move along this path. So, it looks like we are trying to shift the Eigenvector for this practical system or basically a trajectory we are trying to shape along this direction.

(Refer Slide Time: 50:44)



So, these are the basics of sliding mode control; that means you take either a linear control that we have discussed or even you take a non-linear control where along with the linear part if you separate out there will be some non-linear term.

That means, it is now we are talking about a generic term where this state space has n dimension n number of states and definitely this will be n cross n, but here we are talking about a generic where we have n number of input we talked about only single input, but you can have more inputs.

In fact, sometime we introduce more input you know like you know over actuated system under actuated system like that. So, in over actuated system we put more actuator or more control input because we want to achieve certain desired you know behavior. For example, if you go to robotics the robotic arm you know there can be you know sometime people use under actuator system.

That means there can be only one motor to control the entire movement of the hand like we have knee shoulder, but suppose you only put one motor at the shoulder, but there is no motor in the knee. So, then it will be difficult to get it, but maybe it is possible, but suppose if we put another dc motor at the knee as well as shoulder, then you can effectively control the movement of the hand ok.

But we can also place a few more say actuator in order to even precisely control in a different direction with certain objective of optimization problem or trajectory shaping problem. So, we can sometime have more inputs in order to meet certain optimization criteria, but sometime a penalty is at more actuator we have to pay more right.

So, that also we need to discuss I mean we need to take into account, but here it is a vector field and where we have already discussed this linear part for dc-dc converter for a matrix of linear. In case of boost converter A is also discontinuous matrix, there will be a functional q, but B was in buck converter is a discontinuous one right.

So, the u u that means, either u plus or u minus, but in more generic term you can also have non-linearity in the state space model ok. And this non-linearity can be inherently if that case system is non-linear or sometime we miss out some non-linearity due to the model uncertainty you know.

So, all these things can be incorporated and still this concept is applicable. So, here we are assuming the number of inputs is less than the number of states. So, that means, m is less than n, but m can be one or more. Now we are talking about sliding surface because you already discussed sigma x and where sigma x equal to 0 that we have discussed.

But we want to see what is the dimension of the sigma and that will be decided by this S and that will come because sigma is equal to S into x. So, if x is n cross 1 ok and how S is coming, S comes is a number of input. So, if it is a single input system, then this S will be. It is basically S is m cross n.

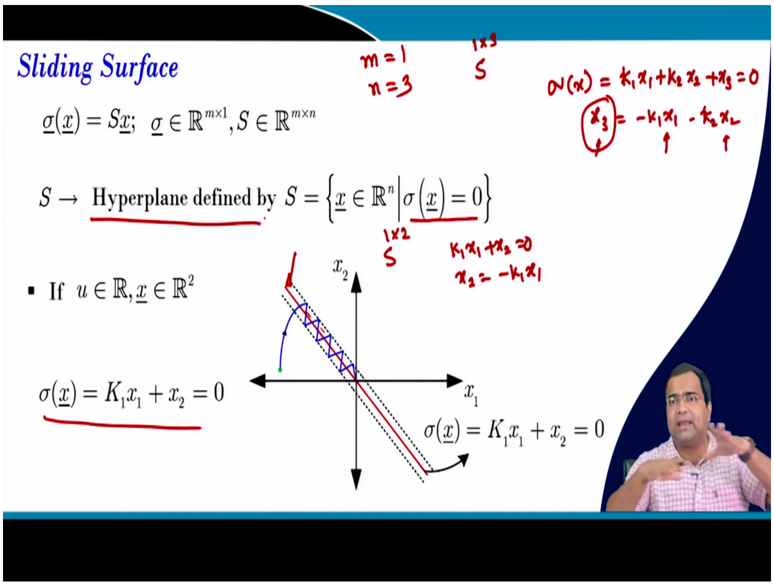
So, if the 1 input, then S will be one cross m. So, this sigma will be simply a straight line because this is a one-dimensional plane where that means, we are talking about you know sorry it is not a straight line. I will say it is a plane. If let us say we are talking about a third order system, a third order system where n equal to 3 and one input control input.

That means u dimension is 1 cross 1, then S should be what? S should be 1 cross 3 and sigma is 1. So, equal to 0 that means you will get sigma x something like you know K 1 x 1 plus K 2 x 3 plus x 3 equal to 0 and if you write this in the 3 dimensional plane.

This can be written as if x 3 equal to minus K 1 x 1 minus K 2 x 2 as if x 1 x 2 like a 2 dimension like x 1 axis x 2 axis and we can get the x 3 by combination of x 1 x 2. So, we can see that this will resemble like a 2 dimensional plane. For a 3 dimensional system this will represent like a 2 dimensional plane because it is a combination of two independent let us say x 1 x 2 is axis where the basis vector Eigenvector is along x 1 axis and along x 2 axis that we have discussed.

So, x 3 component is a weighted combination of 2 Eigenvector. So, naturally there are 2 either independent Eigenvector and the third vector is derived from the two vectors. So, it is a dependent one. So, as a result it will be 2 dimensional plane. So; that means, we will talk about this hyper plane concept and, but we need to make sure this switching surface should be equal to 0. Because we want to constrain this motion on this surface, it should be equal to 0.

(Refer Slide Time: 56:27)



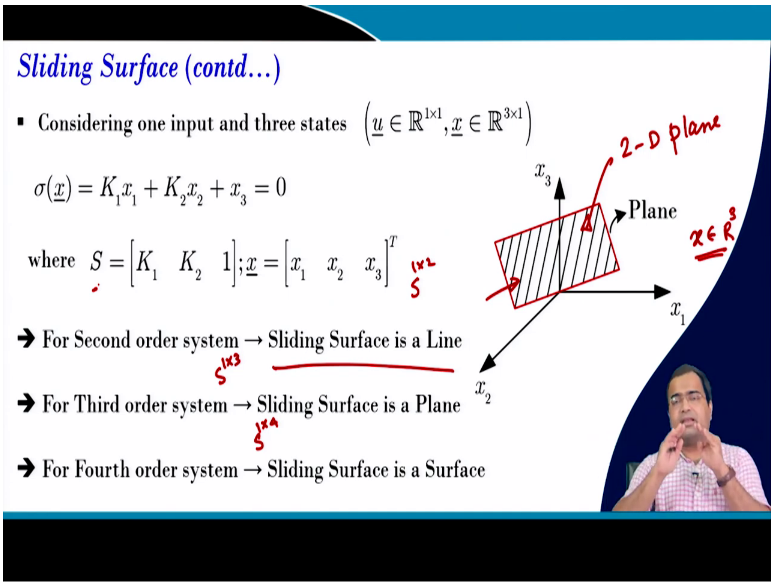
So, now if we talk about the sliding surface as I said S is a hyper plane. That means, if we take m equal to 1 and n equal to 3 then we have discussed this S dimension is 1 cross 3. Which means that if you go into the 3 dimensional place and the motion is constant along this particular sigma x equal to 0, it will look like a 2 dimensional plane. Because what is our sigma x that we have again discussed, it is K 1 x 1 plus K 2 x 2 plus x 3 equal to 0.

So, this will something similar to x equal to minus k 1 x 1 minus k 2 x 2 since x 1 is a dimension. You can think of a basis vector x 2 is another dimension. Think of another basis vector. So, the third vector is derived by a combination of two basis vectors.

So, that means it will be on the x 1 x 2 plane. It cannot have any z components because the z component x 3 is derived from these two right. So, that means, for a second order system sigma x like this. So, this will be a straight line for a second order system. S will be one cross two and sigma x will be k 1 x 1 plus x 2 equal to 0.

So, we will get x two equal to minus k 1 x 1 and this is an equation of a straight line passing through origin with the negative slope k 1 ok. So, this is called hyper plane. The hyper plane gives you the dimension of the switching surface. So, how does it look like? Whether it is just line, whether it is a plane or it is a surface.

(Refer Slide Time: 58:19)



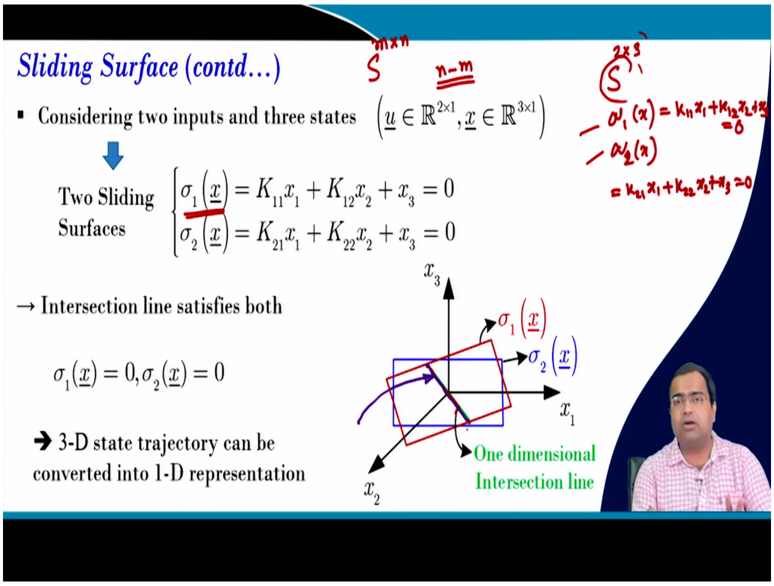
Now, if we talk about one input I discuss three states, then we will get sigma x equal to k one x 1 plus k 2 x 2 into x 3 and this will look like a plane that we have discussed. That means, this plane is a combination of x 1 x 2 or x 3, but its motion is restricted in this plane and this plane dimension of this plane is a 2 D plane. It is a 2 D plane.

Whereas the x will be x is basically belong to 3 dimensional plane. In a 3 dimensional plane, we want to restrict the motion to just a plane of 2 dimensional that is sigma x equal to 0 right. That is a switching hyper plane for a second order system. The sliding surface is just a line.

That means S equal to 1 cross 2 for the third order system for a single input we have discussed S is 1 cross 3 it is a plane this is the case and for fourth order system S will be 1 cross 4. So, it is a 3 D surface 3 dimensional surface whereas, the system order is fourth order.

So, interestingly you will find this hyper plane dimension will be 1 less than the system dimension. If the system is second order the dimension of this hyper plane will be first order first one 1 D, 1 dimension less if the motion you know the state vector is third order then this hyper plane dimension will be second 2 dimension, so 1 1 D less.

(Refer Slide Time: 60:04)



Now, if you consider two input three step two sliding surfaces. So, you know there will be S has 2 cross 3. We have sigma 1 x sigma 2 x sigma 1 x is k 1 1 x 1 plus k 1 2 x 2 plus x 3 that equal to 0 and sigma 2 x will be k 2 1 x 1 plus k 2 2 x 2 plus x 3 will be equal to 0.

So, that two different coefficient two planes are different, but each cases this sigma 1 and sigma 2 x are the 2 planes, but the hyper plane which indicate 2 cross 3 that is a combination and you can see this hyper plane is an intersection line which is a 1 dimensional. That means, each of the switching surface is a second order 2 dimensional.

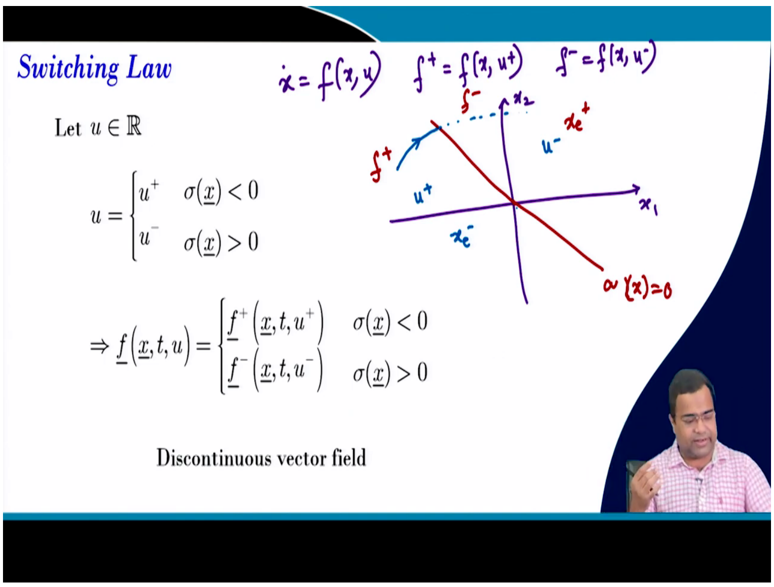
So, you can think of this hyper plane dimension when you write S m cross n you can think of the dimension of this hyper plane is n minus m that is the dimension. So, for in case of third order and two control inputs. So, n means 3 minus 2 it is a single and that is nothing, but the intersection of the two planes which is nothing, but a just a straight line.

It is nothing but a straight line passing through origin and so S hyper plane will be n minus m it will be one dimension just is a line 1 D. So, the and that means, we want to restrict the motion along this. So, that it will become 1 dimensional motion even though the system is a third order system, so 3 D trajectory.

That means we talked about in dc-dc converter the second order current and voltage. We want to confine the trajectory like a first-order hyper plane. It is just a one dimensional line, in case of there if we have another input, if the state vectors are there are three states then we can confine the motion into one line.

But whether it is possible or not that we are going to discuss, but this is a physical realization of sliding motion along what is the dimension of the motion. So, in this case, it is a one dimensional motion along the line.

(Refer Slide Time: 62:39)

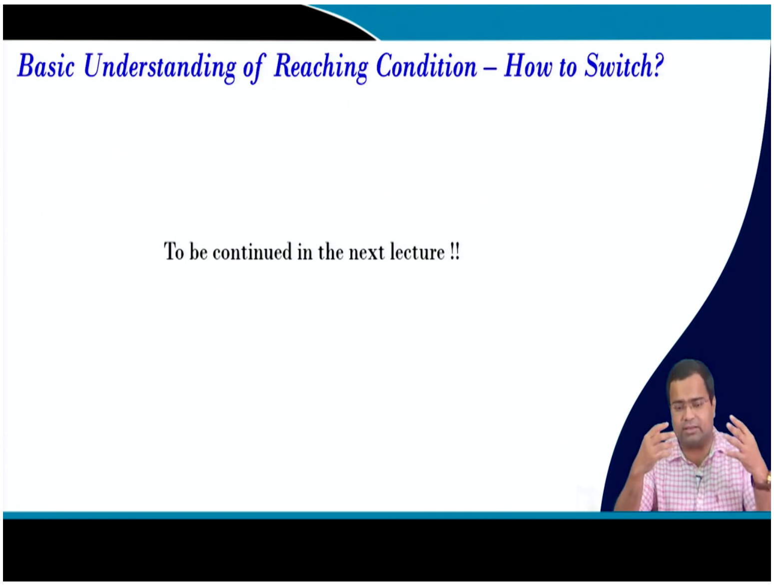


So, the switching law, if you consider switching law u plus and u minus and for each because we know that suppose, for example, x dot equal to f of x comma u. So, f plus it indicate f of x comma u plus and f minus it indicate f x of u minus ok and we are taking this in either sides of the plane. That means, if we consider our x 1 x 2 it is the second order system, for example, we consider and suppose if we draw a line like this and this is my sigma x sigma x that is equal to 0. So, this side if we take u plus, so u plus on.

So, this is my f plus, and this is my f minus and I told you for f plus the equilibrium point x e plus should be on the other side. So, that the trajectory will actually try to cross the line, it should cross the line, it should go towards this because it is trying to go towards the equilibrium point, but it cannot go because the switching line which divides the switching law.

Because here you have switching law u minus here, you have switching law u plus for this f you will have x e minus in either sides. So, these are discontinuous vector field and particularly towards the switching surface either sides you have a discontinuous vector field.

(Refer Slide Time: 64:22)



So, we have discussed the switching law and basic concept to you know switching control logic under sliding mode control. So, we will continue sliding mode control lecture in the sorry the concept of reaching condition their understanding of switching law of more detail along with a buck converter case study in the next lecture. So, we will continue this in the next lecture.