**Control and Tuning Methods in Switched Mode Power Converters**
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**Module - 09**
**Large Signal Model and Nonlinear Control**
**Lecture - 43**
**Geometric Perspectives of Eigenvalues and Eigenvectors in SMPCs**

Welcome this is lecture number 43, in this lecture we are going to talk about Geometric Perspective of Eigenvalues and Eigenvector in linear time invariant system as well as we will see in switched mode power converter.

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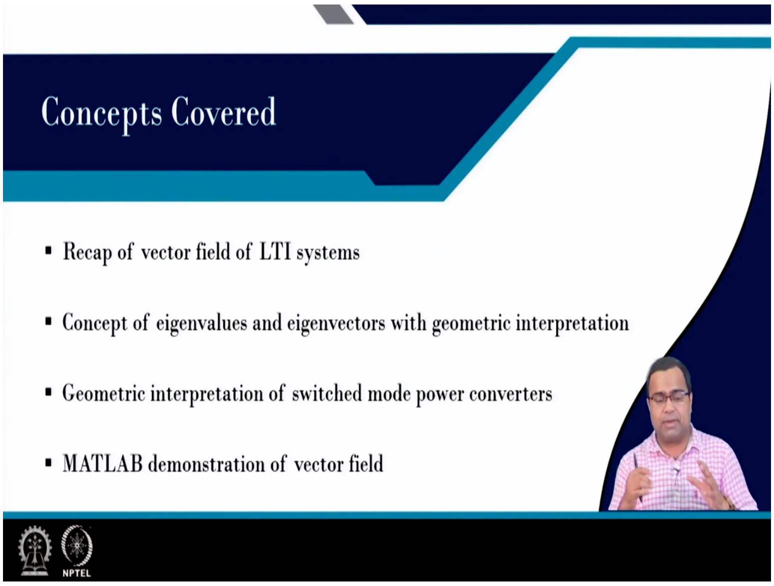


So, in this lecture we are going to first recapitulate the vector field of linear time invariant system. Then we want to introduce the concept of Eigenvalue and Eigenvector with their geometric interpretation.

Then geometric interpretation of switched mode power converter, how does a phase plane trajectory look like and this will be the stepping stone or the concept build up that will build up the concept here. Because in the subsequent lecture we will talk about geometric control or the boundary control or sliding mode control, where we need to visualize the vector field and that is why you know the previous lecture as well as this lecture. These are very important. Then we need to see some MATLAB demonstration of vector field.

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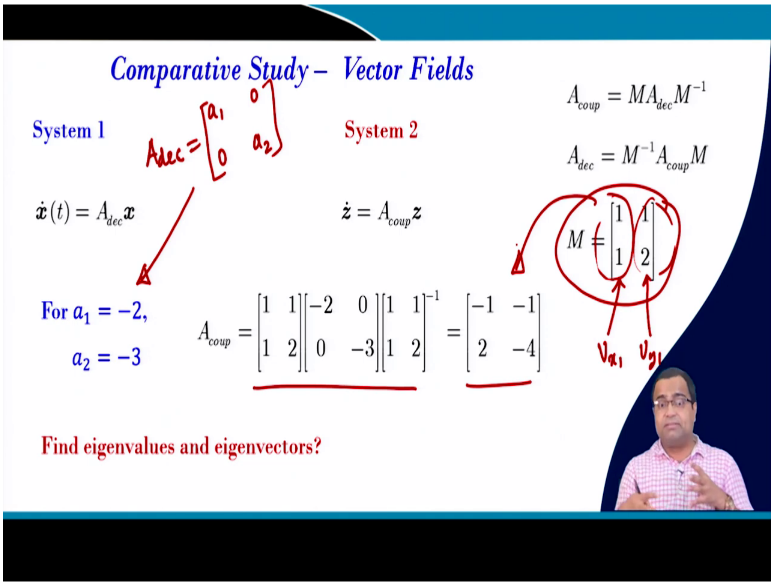


First, we will start with our last class where we left. So, where we deal with two systems system 1 and system 2, in system 1 it was a decouple system with I with a 1 a 2 matrix that we have discussed. So that means, for this system, what was A decouple matrix that we have chosen it was like a 1 0 0 a 2, where we have chosen a 1 to be minus 2, a 2 to be minus 3.

And A couple matrix we have obtained by means of a transformation and this is A couple. And what is the transformation? This transformation that matrix is basically a combination of two vectors which was V x 1 and V y 1 in my in our earlier lecture and that constitute the transformation matrix and this transformation matrix we will soon find that this will be the Eigenvectors of this particular system matrix ok. So, this a unique in that sense and it has a special property. How do you find Eigenvalues and Eigenvector and what is the physical interpretation?

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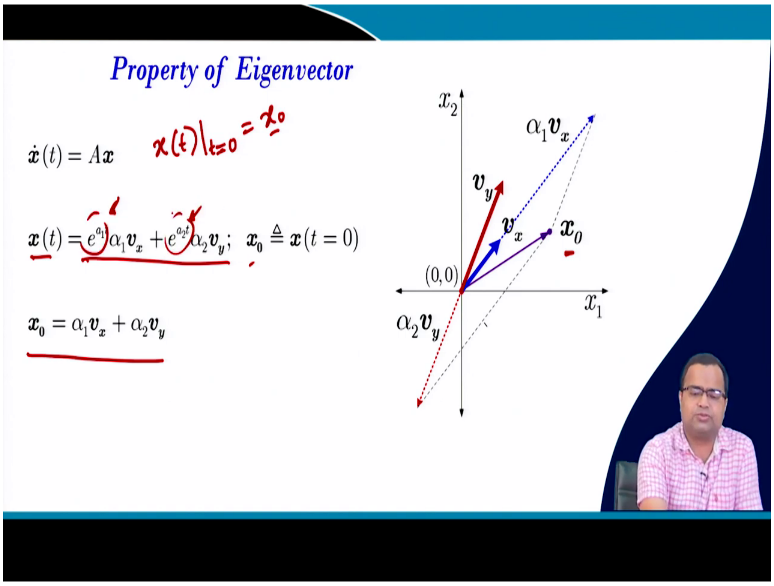


So, first the property of eigenvector; that means, we start with x dot equal to A x suppose we start with not orthogonal form any arbitrary form, but two vector v x and v y ok. Where x 0 is an initial condition and the initial condition for this state; that means, when you write a dynamical system, it I mean it has an initial value problem; that means, we need to specify x of t at t equal to 0 it is x 0, that we are taking about x 0 ok.

So, that is what if you want to solve it we know earlier if A matrix even if it is not in diagonal form we can find out a 1 and a 2 and we are going to see what are these a 1 a 2 and we can write it in this form and this is an initial condition? Now, we know how to write about initial condition, if you substitute t equal to 0 here t equal to 0 here this will be 1 and this will be 1. So, alpha 1 v x plus alpha 2 v y will be x 0.

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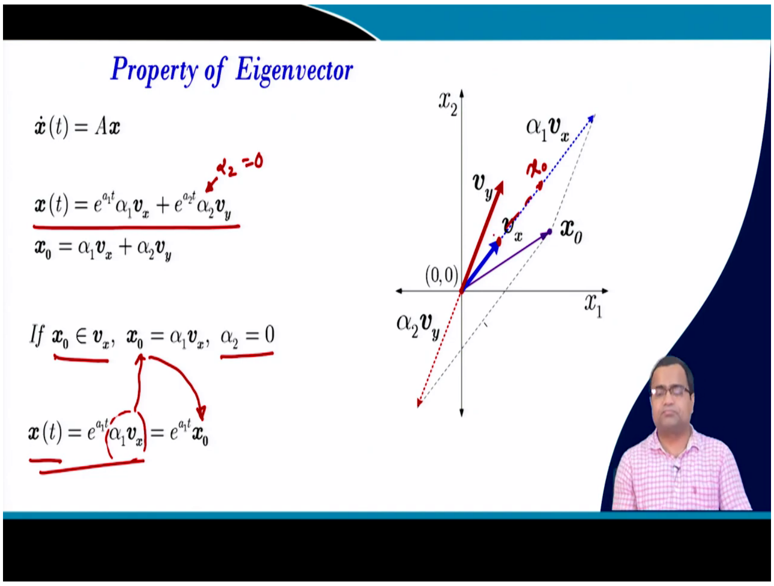


Now, again we are starting this system, what we are considering suppose we are taking the initial condition along v x, for example, you can also take initial condition along v y no problem. So that means, I am just explaining by taking one basis vector you can think of the other basis vector. So, if you take initial condition on v x naturally alpha 2 will be 0, because we are not taking any component along v y so it is along v x only.

And since v x and v y are linearly independent if you take along this axis then the other axis component will be 0. So that means, x of t x of t if we write alpha 2 is equal to 0 alpha 2 is equal to 0. So, it will be simply e to the power a 1 alpha 1 v x and we know this quantity from here. So, it will be nothing but x 0 correct, so that means we are taking the component x 0 in this case along this axis. So, suppose this is my x 0 along this axis.

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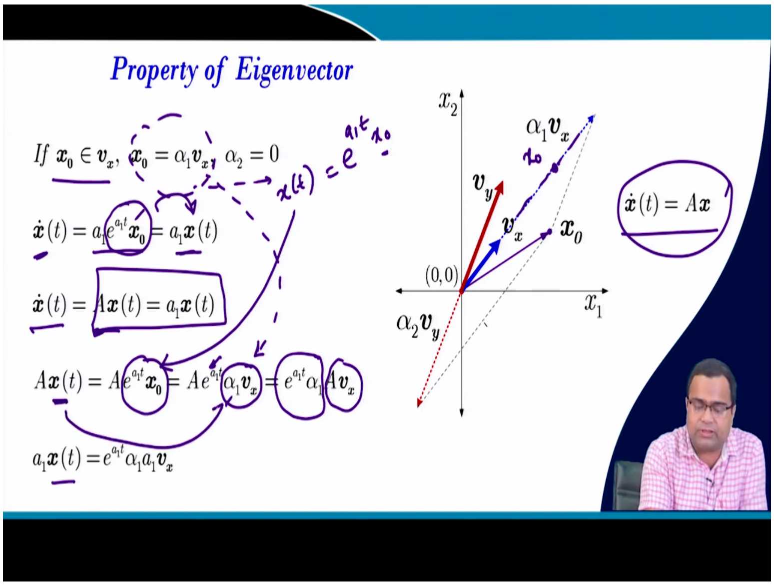


Now, we are we have considered that x 0 is along v x; that means, we have considered the x 0 is along v x. So, we are taking somewhere here is my x 0 it is along this line ok and the system dynamical equation is x dot equal to A x. Then, if you differentiate x dot you will get A into x t, because it is already the equation of motion and in this case x dot is nothing but a 1 x, because we are taking on the axis and you see if you differentiate.

What we will get? a 1 into that means, x t we know. What is our x t? What was our x of t? We have seen in the previous x of t is nothing but e to the power a t into x 0. That means, x of t equal to e to the power a 1 t into x 0, if you differentiate with this and we know that this quantity is nothing but our x of t which is coming from here. So that means, we are getting x dot equal to x dot t is equal to A x t and small a 1 into x. Now what is A x?

Again, x of t is nothing but this and this is coming from here, right? Again, what is my x 0, x 0 is nothing but alpha v x because this is coming from here, this is coming from here. Now, we can rearrange since e to the power a 1 t and alpha, they are scalar. So, we are taking out and we are forming A into v x ok. What about a 1 x t? Again we substitute x t because we have done x t and ultimately it turns out to be this, we do the same thing and what we will obtain.

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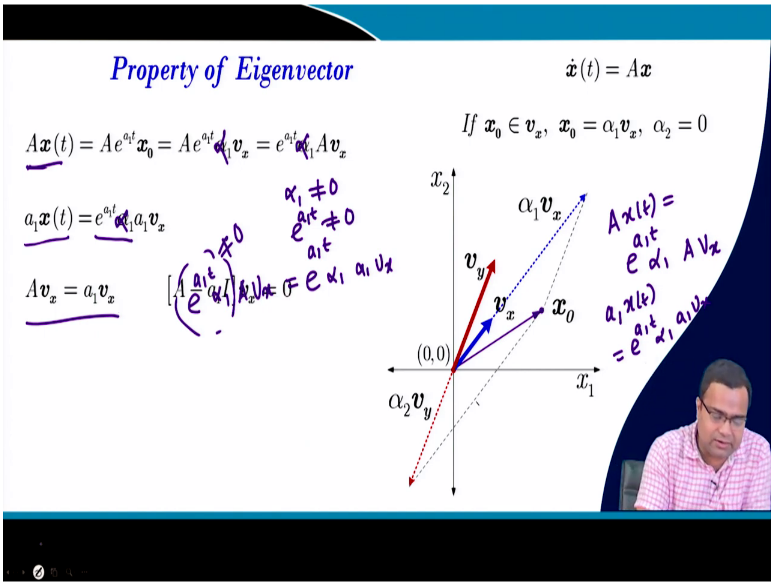


First, for x we got e to the power a 1 t alpha 1 into A into v x and a 1 x t is equal to this term right. So, this is alpha1. This is alpha 1 because this notation has to be clear. This is alpha 1 right alpha 1.

So, now since we are taking on along v x axis we are not starting at origin, so alpha 1 is not equal to 0; e to the power a 1 t since we are talking about finite time it is also not equal to 0. So that means, if we take here that means what is our A of x t what is our if I write it down what is our A of x t this is nothing but e to the power. So, what is our A of x t? A of x t it is nothing but e to the power a 1 t alpha 1 into A v x. And what is our a 1 x t?

This is again e to the power a 1 t alpha 1 a 1 v x ok. So, from these two equations from these two equations, so we are getting e to the power a 1 t alpha 1 A v x is equal to e to the power a 1 t alpha 1 a 1 v x. Since this term is not it is not equal to 0, so we will get this term ok.

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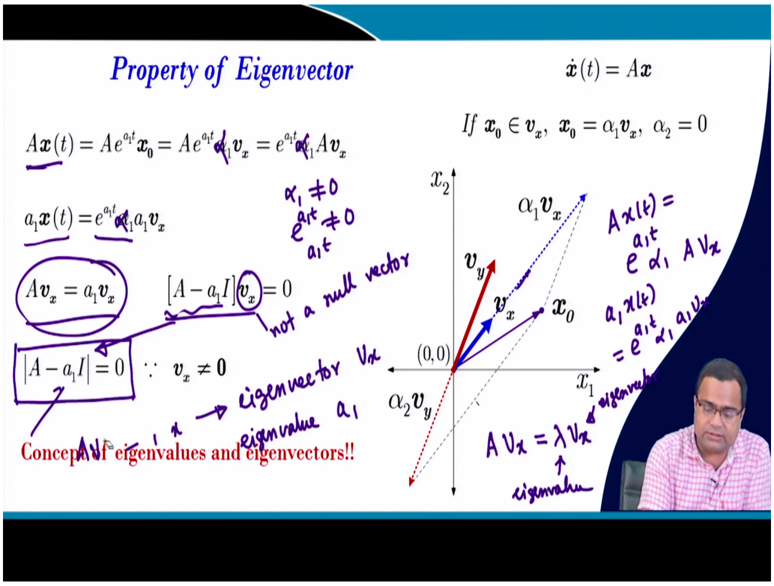


And if we that mean we can rearrange this to get this and this is exactly what we generally write to get the Eigenvalues and Eigenvector. Since v x is not a null vector, it is not a null vector.

Then this null space of this matrix the determinant must be 0 and this is exactly the equation is derived to find the Eigenvalues and A into v x equal to a 1 into v x this to find the Eigenvectors so, Eigen vectors. What is the Eigenvector? This is v x and what is the Eigenvalues Eigenvalue in this case. So, in general term we write A into v x is equal to lambda into v x, where this we call Eigenvalues and this we call Eigenvectors ok so, standard form.

So, that means these eigenvalues and eigenvector are coming from this vector field invariant vector field right. So, whatever we learned the invariant vector field, we can use that basic concept to come to the equation which we generally write. That means, the whatever we write from the standard book, we should keep in mind that these expressions are coming from the geometric perspective right. So, this is a concept of Eigenvalues and Eigenvectors the concept this is that we are familiar with ok.

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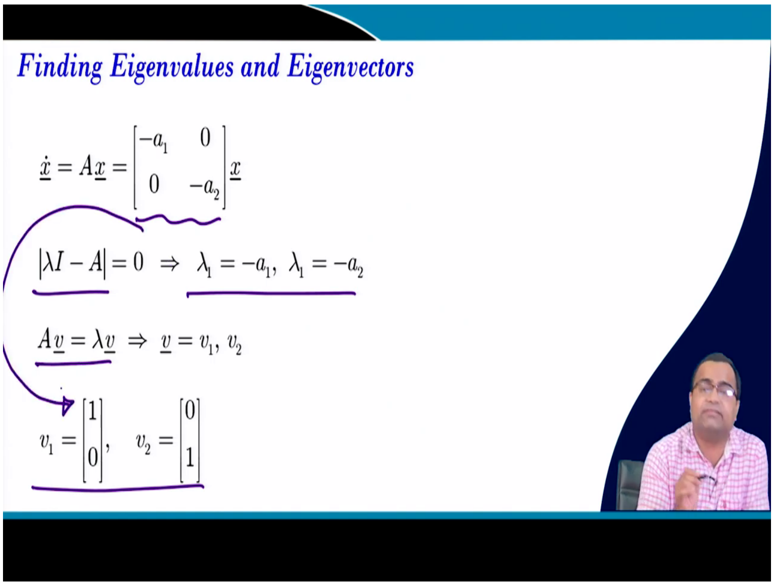


Next, how to find Eigenvalues and Eigenvector, again you can simply write lambda. I equal to you know we can write lambda I minus A if I take the determinant I will get it and this one right. So, I can find these Eigenvalues and again if you write down this equation, you can find out. So that means we started with the basis vector for this A matrix and it turns out to be those basis vectors are nothing but the Eigenvectors.

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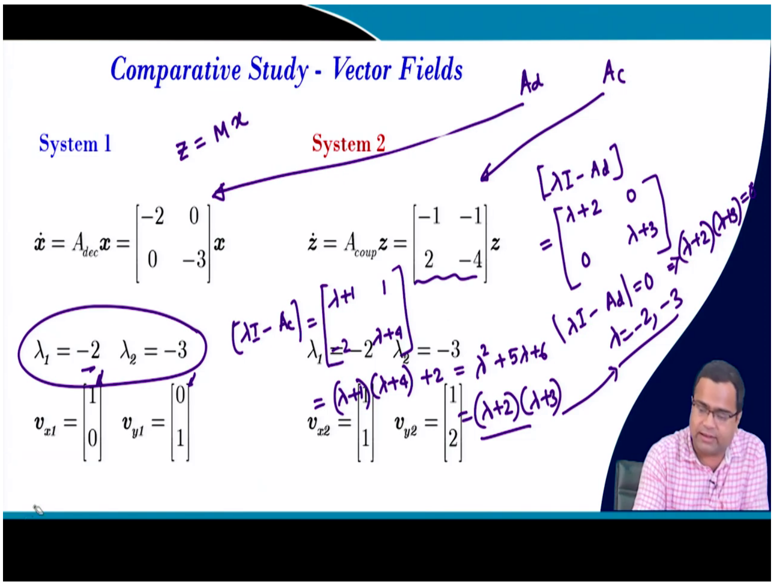


Similarly, if we take system 2 that we consider and if we find out the eigenvalue; that means, let us say first A decouple matrix. We write A decouple to represent this and A couple to represent this. So, what is the eigenvalue of A decouple A I minus A decouple? Let us try to find out first write the matrix it is nothing but lambda plus 2 0 0 lambda plus 3 and if I take lambda I minus A d to be 0 it turns out to be lambda plus 2 into lambda plus 3 is equal to 0.

So, this will give you minus 2 minus 3. So, these are the Eigenvalues two Eigenvalues. Similarly, if we consider; that means, this is a two Eigenvalues and for each Eigenvalue minus 2 you will get this one for minus 3 you will get this one that you can find out.

Now, even if we find out the Eigenvalues of this so let us say you know we can come here. So, A couple that means, lambda I minus A couple this is nothing but lambda plus 1, because A couple is this then 1 minus 2 lambda plus 4. And what we will write lambda plus 1 into lambda plus 4 plus 2. So, it will be lambda square plus you know 5 lambda plus 6, which is nothing but lambda plus 2 into lambda plus 3. So, it is the same form and it will be same.

So that means the transformation does not change because what we got z we got z equal to M time x that we got. So, this transformation does not change the Eigenvalue, but if you find out the Eigenvectors; that means, again if you find out the Eigenvectors if we rub up because I have shown the method how to find. Then you will get that these eigenvectors are nothing but 1 1 and 1 2. So, we started with that basis vector for the second case and these two basis vectors satisfied our invariant property right.

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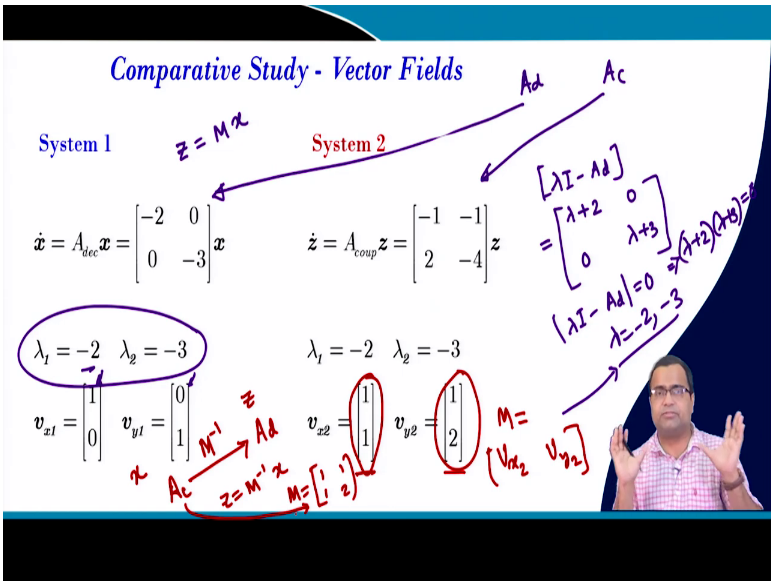

That means this v x 2 v y 2 are the Eigenvector and if we form this; this is simply our M matrix which transforms a diagonal system into a couple system or decouple system to a couple system. Similarly, if I want to convert one couple dynamics to decouple, I need to consider M inverse transformation; that means, if this is my x this is my z. So, my z equal to M inverse x and here M will be the Eigenvalues of Eigenvectors of A c matrix 1 1 1 2. So, this couple matrix Eigenvectors will constitute the M matrix and that is the transformation matrix.

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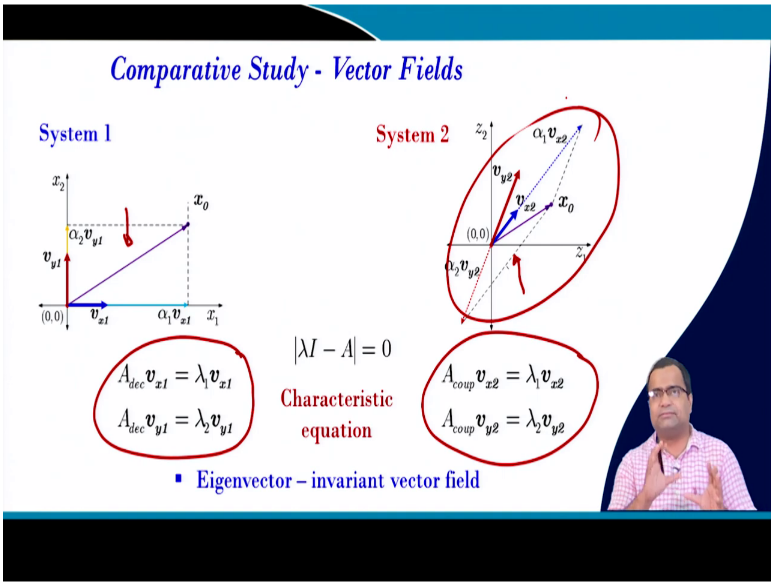


So that means, comparative study of two we can find out the Eigenvector they are the invariant property and we know about the characteristic equation and they look like this. So that means, the transformation does not change the Eigenvalue; that means, their stability property will not be affected. But, the nature of the phase portrait will get affected because we have discussed that as if this if you see vertically and if you see this from an angle.

So, it is like an image transformation right, either we rotate the page itself move the page like you know with an angle by keeping our eye in the same position. Suppose initially, the page was just in front of us right now we make an angle to the page and then it will look like the system 2. That means, this like an inclination comparative motion between I angle and the normal vector of the plane, if we change then we will come up with this kind of structure ok. So, this linear transformation is very much used in image processing ok.

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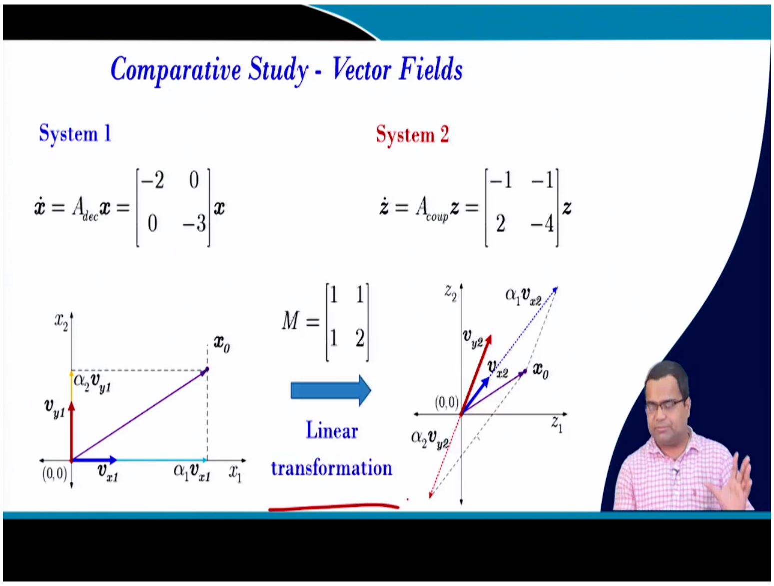


So, comparative study we have discussed these two. We got the vector field, and this is a linear transform. So, it is like image processing.

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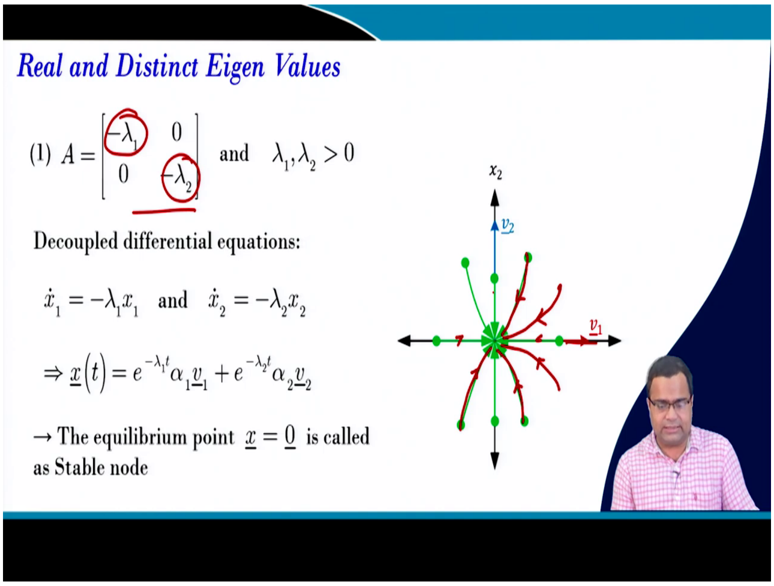


Now, we have discussed real and distinct cases, the two real cases we saw. If in this case they are negative because lambda 1 lambda 2 positive means minus lambda 1 negative.

Then we know that the vector field, if we take initial condition depending upon we know how to draw vector field, if we take here it will come like this. That means, any point you take depending upon the vector field, you can draw the trajectory for each initial condition right. So, since this basis vector for these cases is v 1 v 2 along the x 1 x 2 axis. So, we know all this because orthogonal decouple system. So, this is called a stable node.

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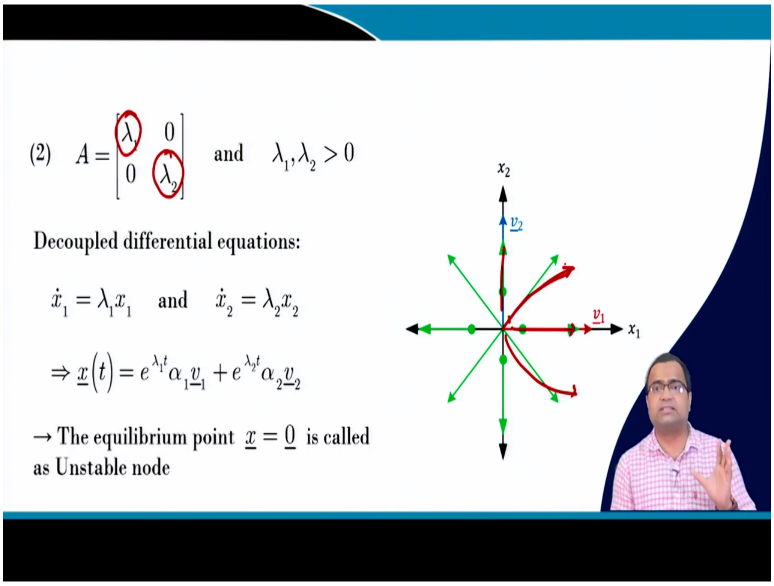


Now, if lambda 1 lambda 2 greater than 0 and this that means the Eigenvalues are positive, then you will get it is going away. It will go like away from this right. It may not be along the line, so because it is an orthogonal basis vector but it will go away from the origin.

So, the bottom line is this any initial condition if you take somewhat away from origin it will go out and the trajectory shape will depends on the vector field again it depends on lambda 1 lambda 2 nature right and we know about in the previous lecture we have discussed how to draw a vector field. So, this is your unstable node.

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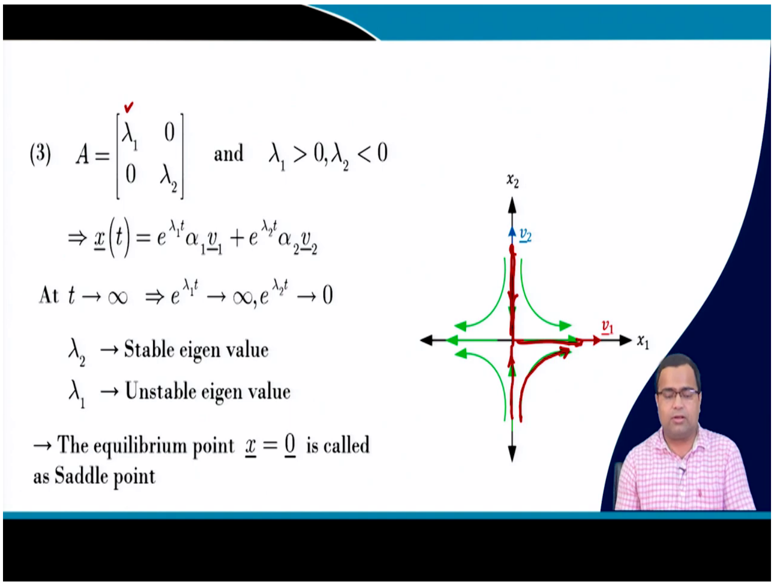


The other one; one can be stable, other can be unstable; that means, the v 1 axis is unstable here. You see that the lambda 1 greater than 0, but lambda 2 less than 0 is stable.

So, if you take any initial condition here, it will move along this line because this is the Eigenvector property, and this is a stable Eigenvector. If you take any initial condition here, but suppose if you take here it will try to move towards origin. But, then it will be influenced by the other Eigenvector. It will go away and this system is called a saddle point. This is an unstable case.

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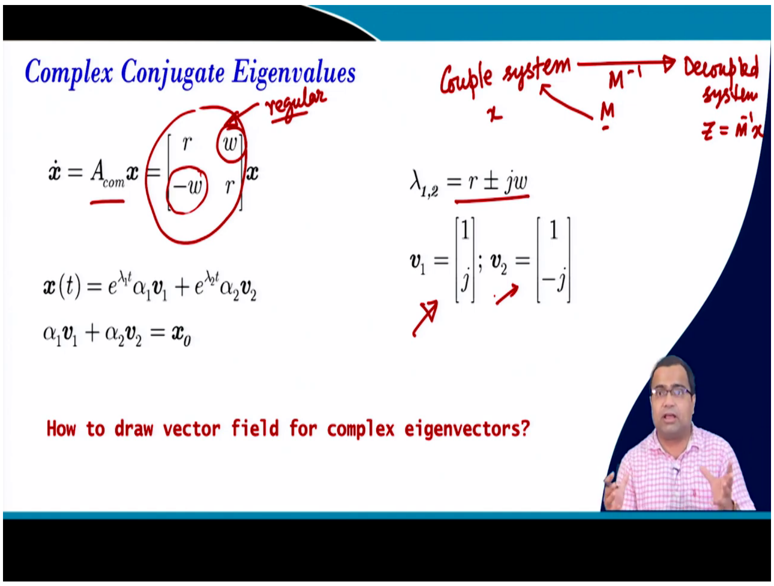

But, what we have not discussed is the complex conjugate eigenvalues. So, any matrix which has complex conjugate eigenvalues can be represented by means of a transformation.

Because we know the transformation a couple system, a couple system can be converted into a decouple system. We have taken this example and what is that requirement we know M inverse, where M consists of where M is the Eigenvectors of this couple system. So, you can convert it right. That means if this is my z this is my x. So, z equal to M inverse x, where M consists of the Eigenvectors of the couple system.

Similarly, by the same transformation, if we have a complex conjugate system, we can transform to represent into a regular form. So, this is something called regular form. So, it is something like a decoupled form, but since it is a complex conjugate, you will get off-diagonal element will not be 0, where it will get omega.

And if you find out, it will be r plus minus j omega right and corresponding to each Eigenvalue; that means, Eigenvalues are complex conjugates and you will find the Eigenvectors are also complex conjugate. Now, the question is how can we draw this complex conjugate Eigenvector in the phase plane, because you cannot draw because it is a real plane right.

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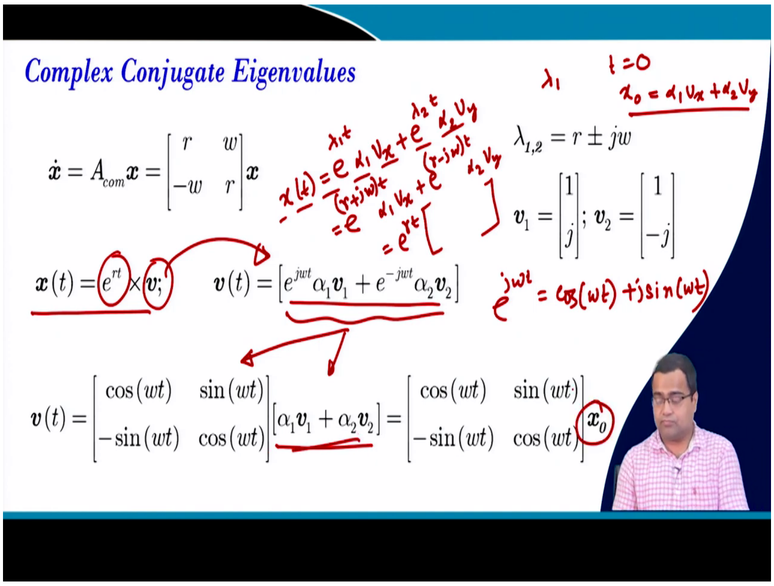


So, first let us say we found that and suppose x of t we know it is nothing but e to the power r t, because you see why it is coming like this. Suppose we know if we go back to our x of t is what x of t we know e to the power lambda 1 t alpha 1 v x plus e to the power lambda 2 t then alpha 2 t v y.

And since for lambda 1 and lambda 2 both have the same real quantity; that means, you will get r plus j omega t alpha 1 v x plus e to the power r minus j omega into t alpha 2 v y. So, I can take e to the power r t common for both cases and that is exactly what I did and the rest of the term I write as a v. Then what I did yeah it is e to the power j omega t I have written right. Now, we know e to the power j omega t we will substitute is equal to cos omega t plus j into sin omega t.

And then after simplification, this whole v M term will appear like this is a combination right and this one alpha 1 v x because even if you take this equation, the original equation. If we take t equal to 0, so this will be x of 0 will be nothing but alpha v x plus alpha 2 v y. So, this original equation remains, and it is x 0.

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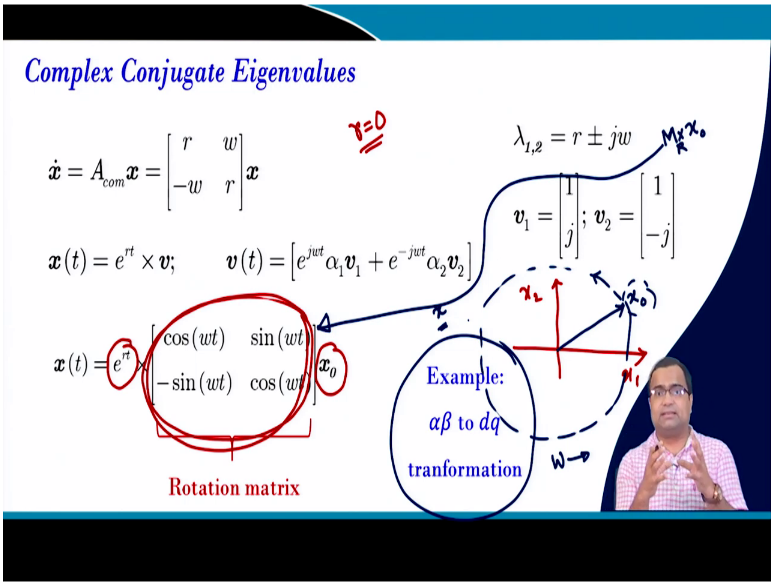


Next, that means we got this form. Now this represents something like you know if we look at this; this represents something like.

This is our initial condition, this is our real part e to the power r t. So, this is a rotating rotation matrix. That means if r equal to 0; that means it is a purely imaginary complex conjugate, then you will get e to the power 0 is 1 so complex conjugate right. So, if you multiply with this quantity you can move; that means if you take that means first let us take we have x 1 x 2 axis where in this plane we have considered one initial vector which is x 0.

Now, so this is a representation of a system x and suppose with this representation, now with this initial condition, if we multiply x 0 into this rotational matrix. So, here this rotational matrix M is like this M R; M R rotational matrix. Then what will happen? You will find this rotational matrix will keep on rotating with the same radius. Whether it is clockwise or anti clock that we will see, but it will rotate and what will be the speed of rotation the omega is the angular frequency at this frequency it will rotate.

And the good example is alpha beta to d q transformation, where we multiply with a rotational matrix to convert a stationary vector to a rotating vector ok.

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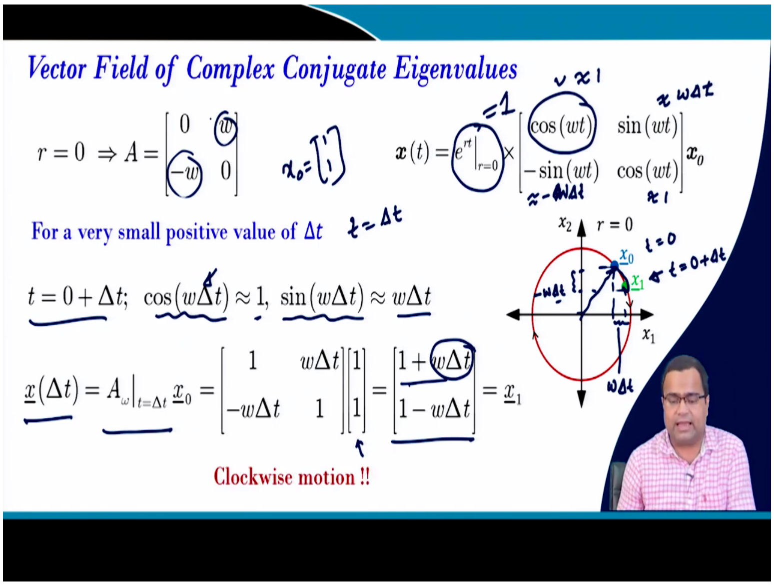


So that means, complex conjugate case if you take r equal to 0, now we want to see which direction it rotates. For example, at t equal to very small value, so at t equal to 0 we know that omega 0 all this component. But, suppose if we are taking a very small here radius is 0 t.

That means we start with time t equal to 0 and suppose we want to find out where will be the location of that means. Let us draw this is our starting point. Suppose this is a starting point where it is t equal to 0.

Now, we want to find what happens at t equal to 0 plus delta t a small time delta t. So, then we can replace this t equal to delta t right. So, let us replace t equal to delta t then it will be cos omega delta t and since delta t is very small. So, this cosine angle will be almost equal to 1 cos of this and sin of that will be very small. It can be approximately omega t. Then this matrix at x equal to delta t time since we have taken this quantity to be 1, because r equal to 0.

So, this matrix will be this quantity to be nearly equal to 1, this will be also nearly equal to 1, this will be nearly equal to omega delta t and this will be nearly equal to minus sorry omega minus delta t minus omega delta t. And if you combine because our initial state, let us say we have chosen x 0 equal to 1 1 right, then we will get the next point. That means, if we look at the x axis, along the x axis you see this quantity is positive.

This is nothing but omega delta t and if you take along the y axis this quantity is nothing but minus omega delta t negative. That means it indicate that this is rotating in the clockwise direction and if you change this symbol.

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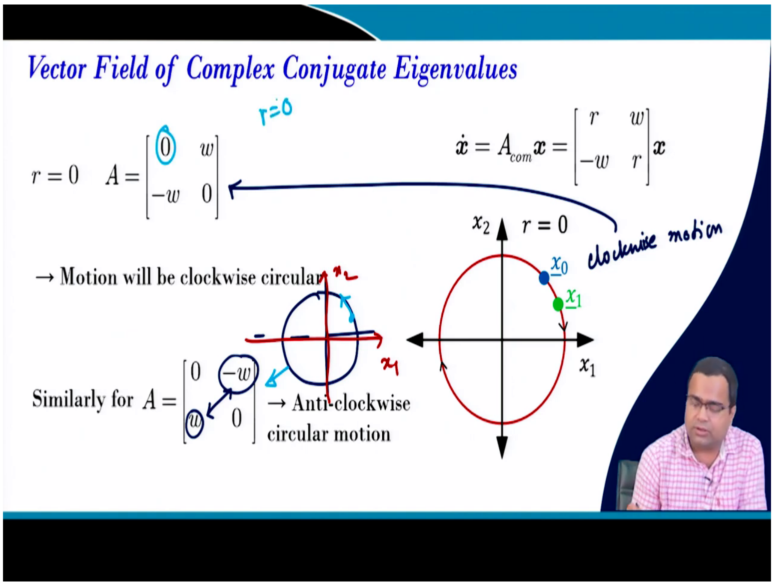


That means if you take the second case; that means, if you take if you just change the rotation angle, it is a clockwise circular motion that we have discussed.

Now, if we change it that means if you now instead of that suppose you interchange the sign you multiply with minus 1. So now, it is minus omega plus omega; omega is a positive quantity, then it will move. Sorry it should not be like this. It should be here. This is my x 1 because it will move in the anticlockwise direction. So, the clockwise, sorry it was correct because this drawing was related to the first case. So, this drawing was related to the case when clockwise motion clockwise.

I have not drawn. This is related to clockwise motion for this particular system, an anti-clockwise how does it look like. That means, if you draw it a circle like this, if this is my axis x 1 x 2 and if I start with an initial condition here, then it will go along this path, so this is for this particular case, ok.

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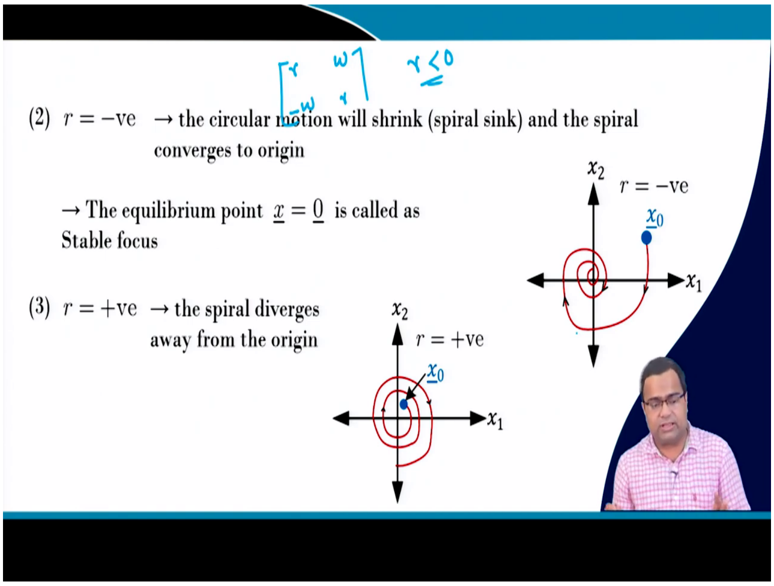


Now, consider a radius with a negative quantity. We have not considered radius. We have put radius equal to 0. Now, suppose we consider a radius here our axis is r omega minus omega r, where r we are taking less than 0. Then the radius will slowly decrease and that is exactly is happening it will slowly decrease ok and if we take positive then it will increase.

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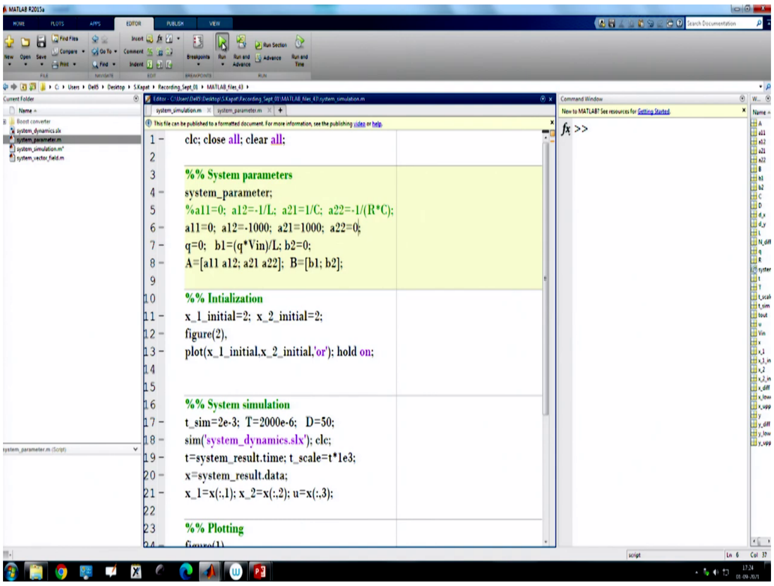


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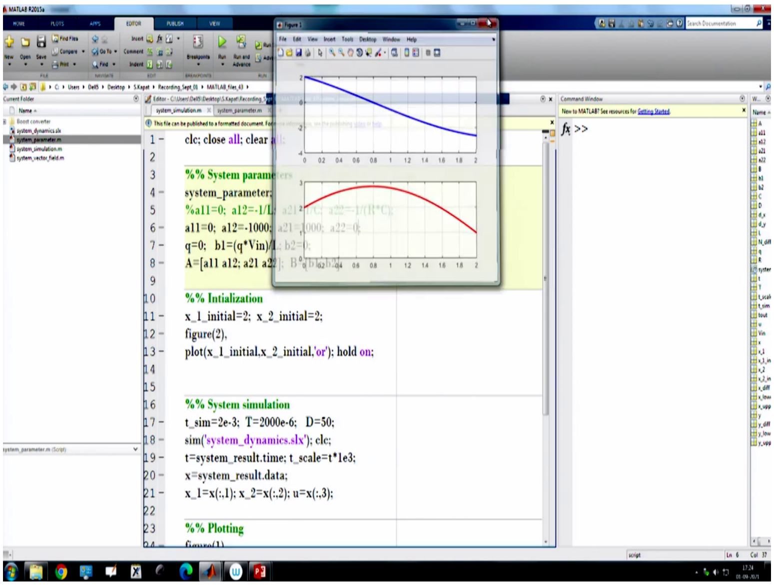


Let us say minus 1000, then plus 1000, then we are using 0 and let us see how does it looks like.

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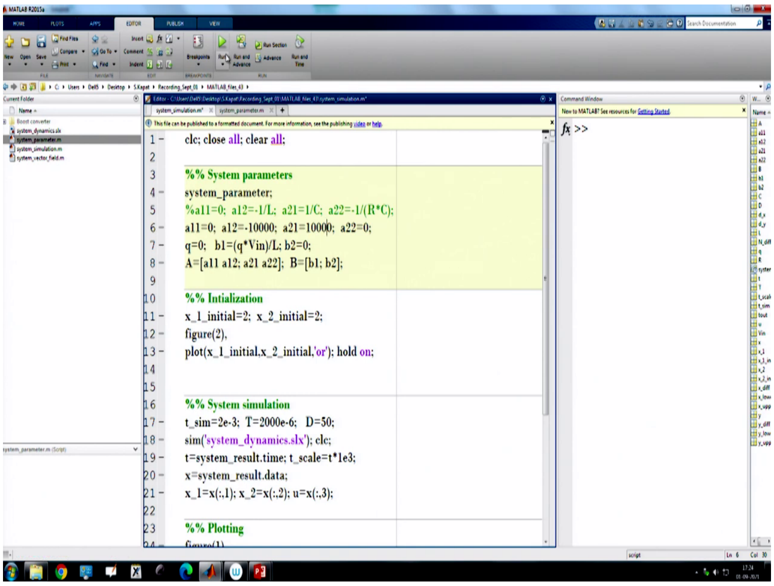


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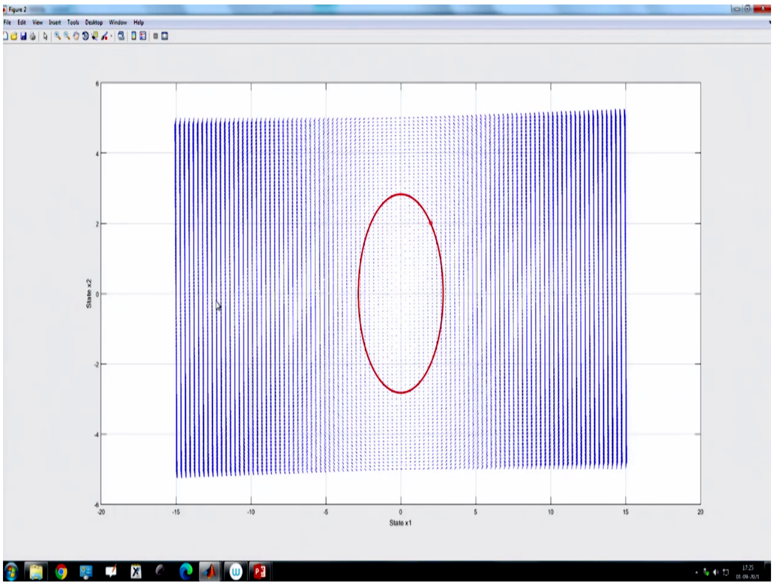

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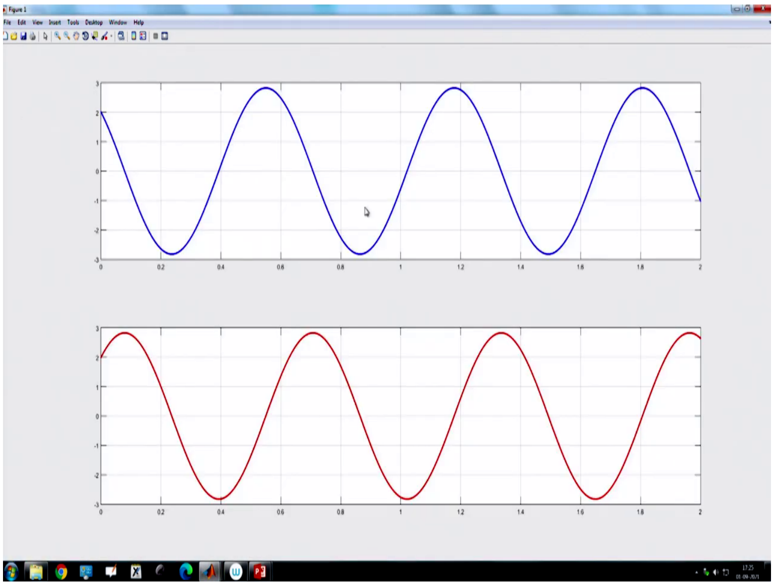

So, I think the value we have to increase the speed of oscillation is not sufficient. So, we need to increase 10 times the given time. Now, you see if you look at the waveform it is an oscillatory right sustain oscillation. Start with initial condition.

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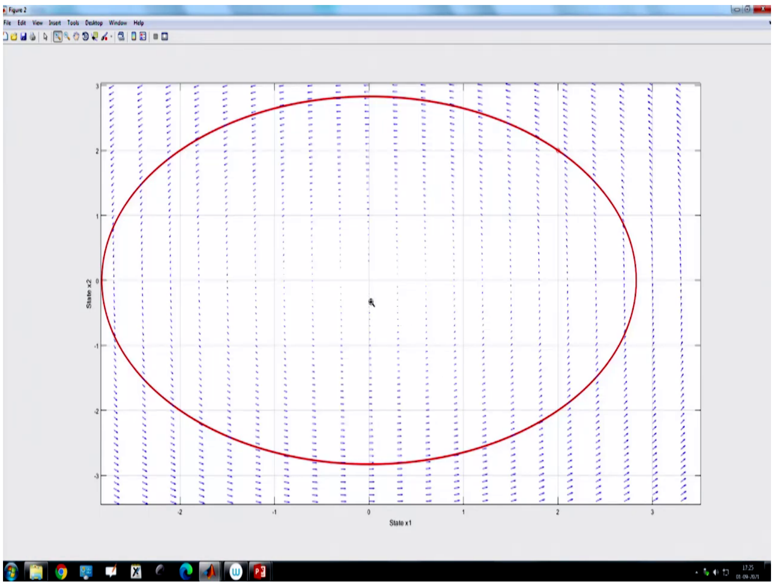

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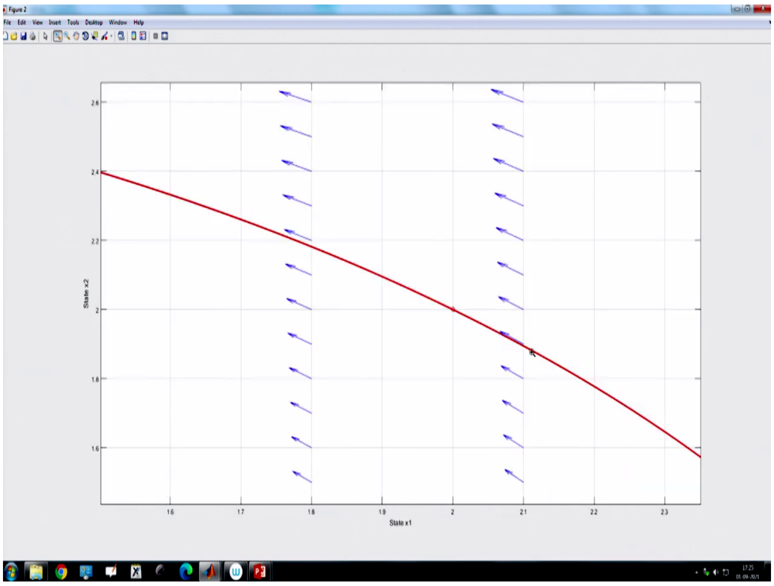


And if we look at this particular so, this will be a circle and this can be represented. And you see the vector field direction. It is the clock you know anti clockwise direction.

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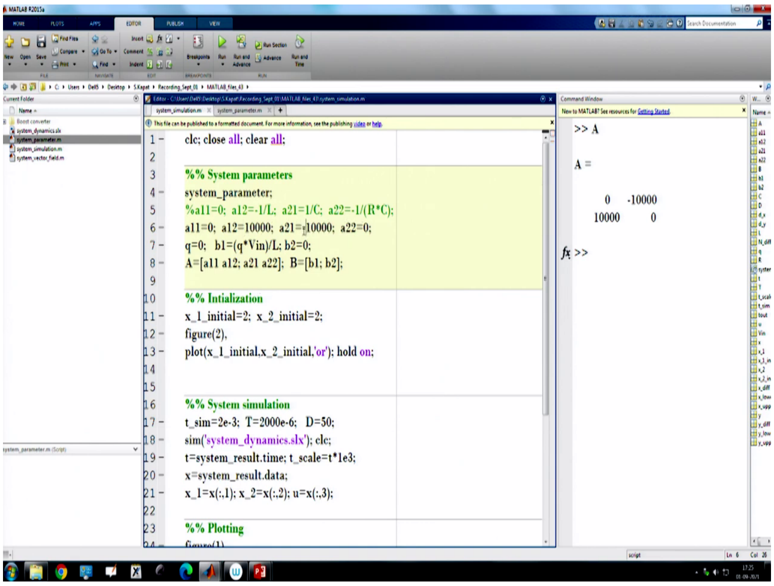


Because if we consider the A matrix, if we write the A matrix here it is minus 1 here and 0 here. So, if you interchange, that means we write 0 here and minus 1 here.

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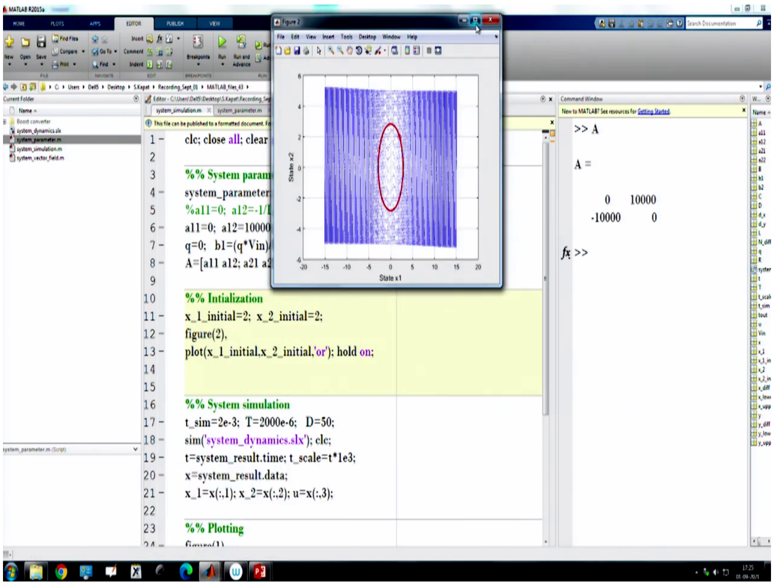


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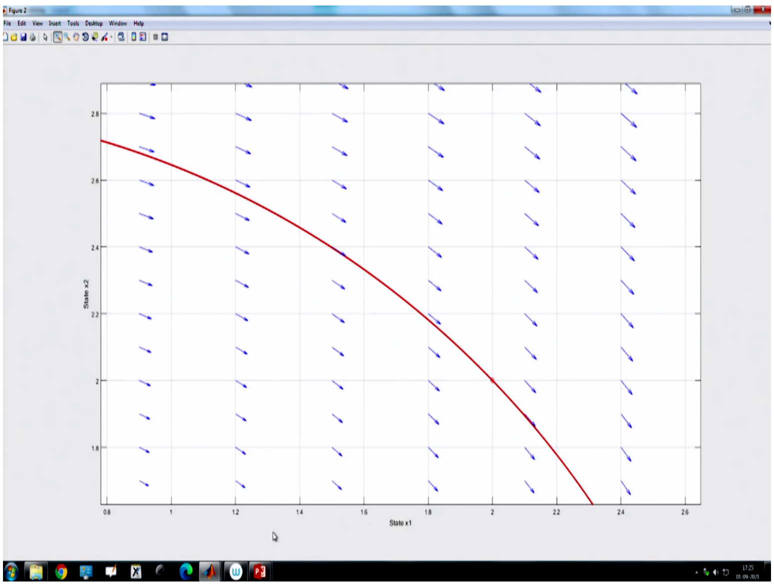


Then if we interchange, we want to draw the vector field again and now you write A it is we have interchanged and we can see the vector field if we draw the vector field. So, they are in the clockwise direction; that means it is consistent with our understanding that if this quantity negative, this is positive, then it is clockwise and if this is positive, this is negative, it is anticlockwise.

That means, in this case, we are talking about this quantity positive this negative, so it is like a clockwise motion that we have seen it is a clockwise motion. So, this is a vector field right.

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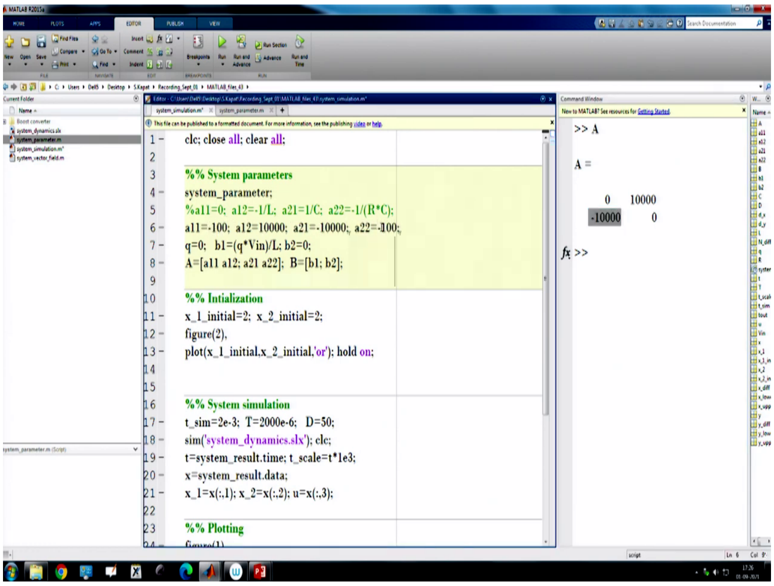


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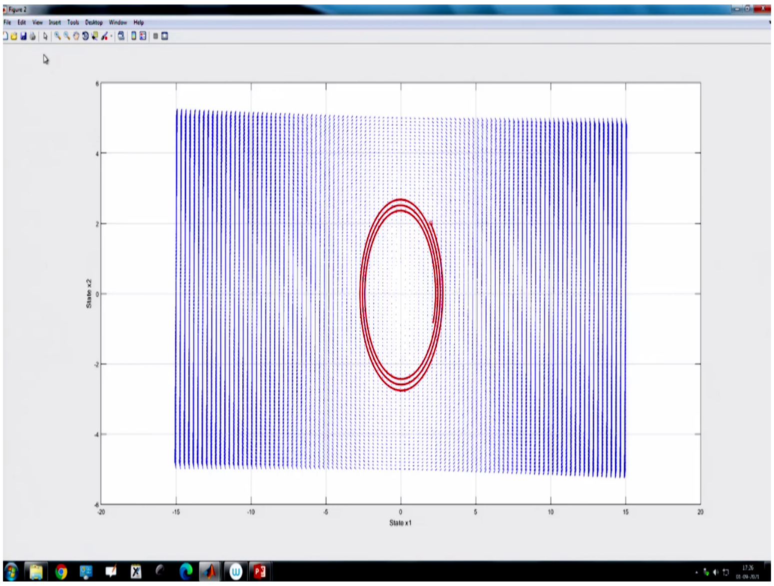

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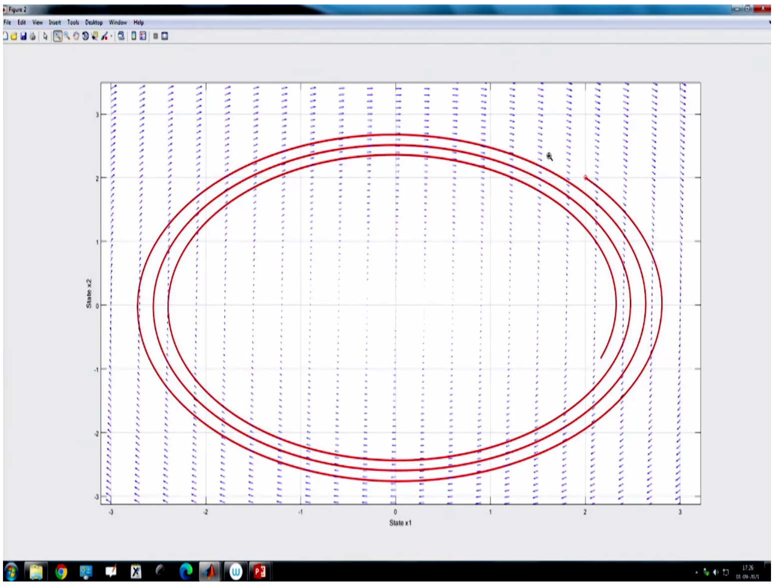

Now, so, we understood how to draw vector field. Now, we have we are introducing minus 100 term that means; we are now including r which is r equal to minus 100 and we are running. And let us see what happen you see it is slowly decreasing in the clockwise direction. It is slowly decreasing in the clockwise direction.

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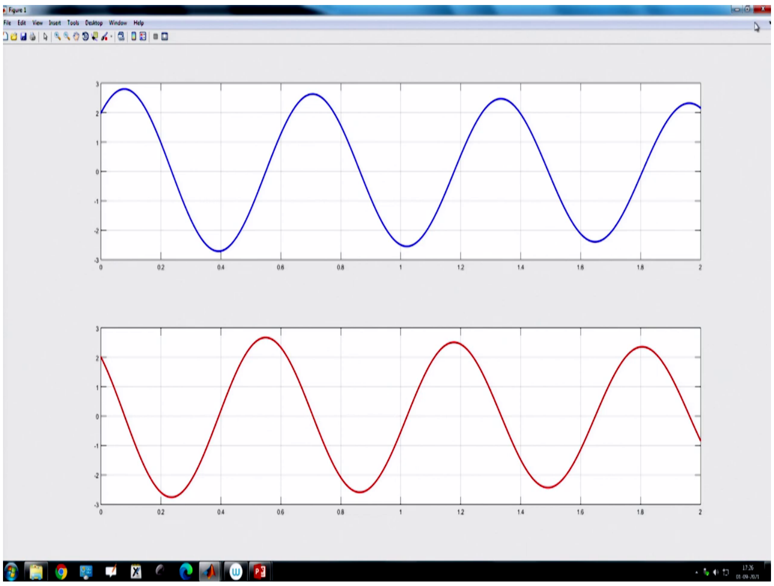

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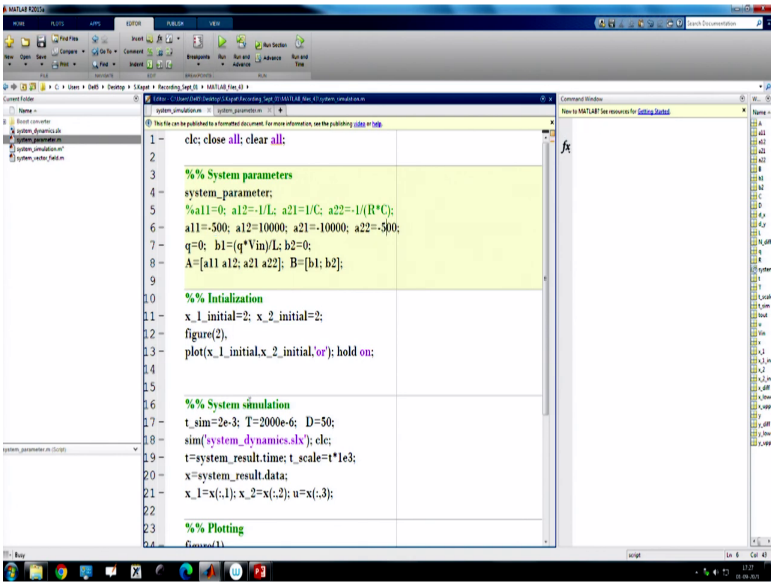


And it will be clear even the time domain waveform it is very slowly decreasing amplitude and if we run again with even higher value because the rate of decay should be faster.

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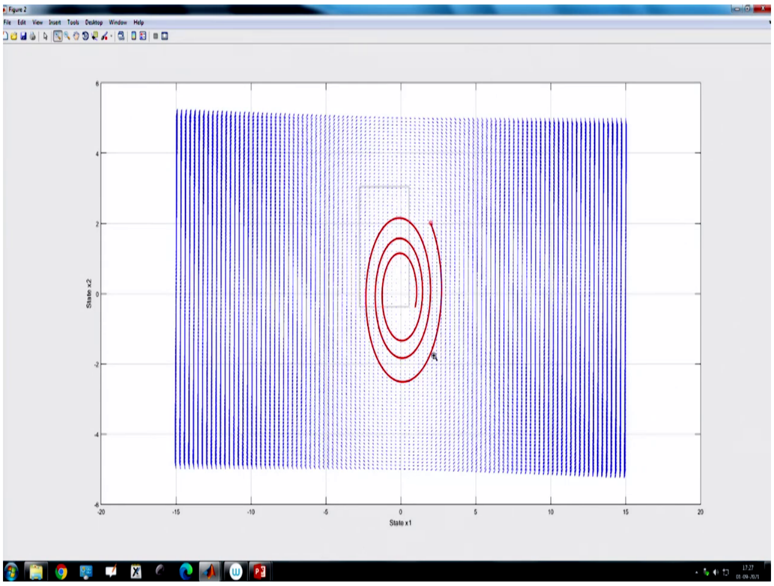

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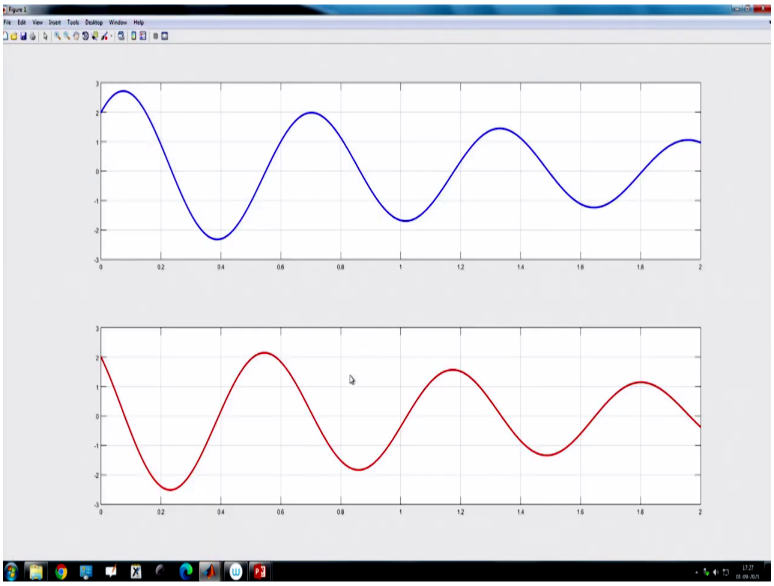

Then we will see that it is decaying faster right, so it is the speed of that amplitude is decaying faster. So, you can imagine that there is an exponential decay in the amplitude right and this will be clear. It is a stable focus, so it is decaying.

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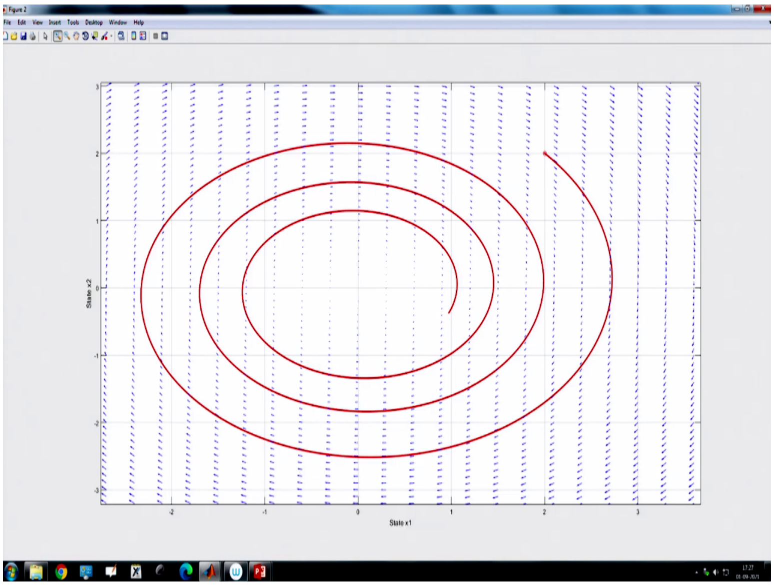

So, our concept what we learned is perfect. So, there is no doubt. Now, we can think of this spiral in spiral out and if the r is positive then it will go out right.

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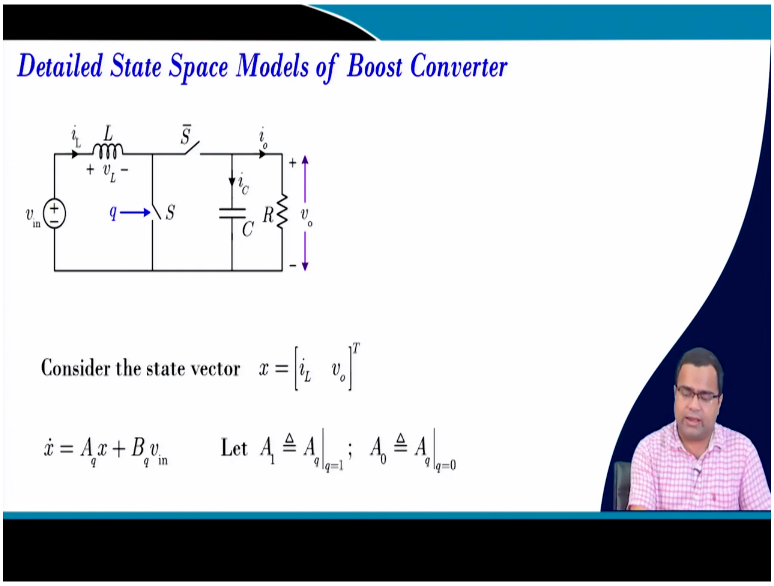


Now, we are considering a boost converter. So, the boost converter we know, the ideal circuit we are already familiar with, the boost converter we discussed multiple time. If we take the state vector to be inductor current and output voltage, then we can write the state space model for q equal to 1 when the switch is 1 and q equal to 0 when the switch is off.

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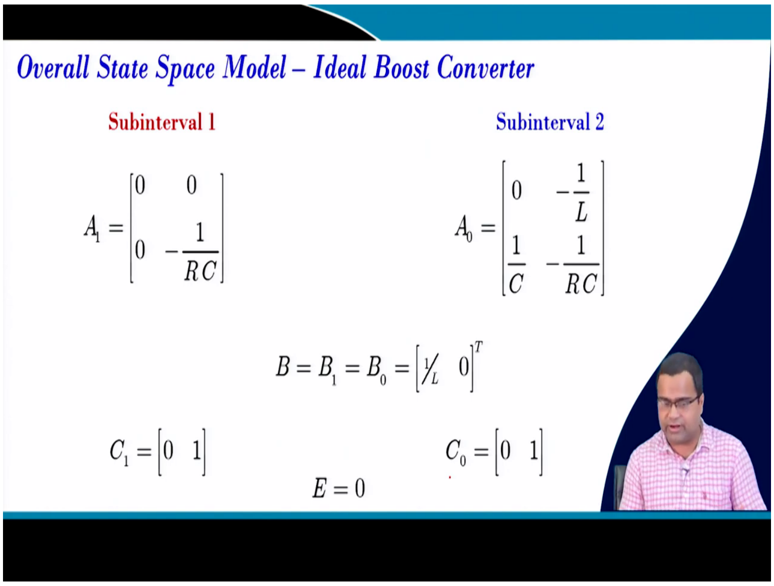


Then we can write down the sub interval 1 and sub interval 2 different A matrix, B matrix will be common, C matrix will be common because it is an ideal converter.

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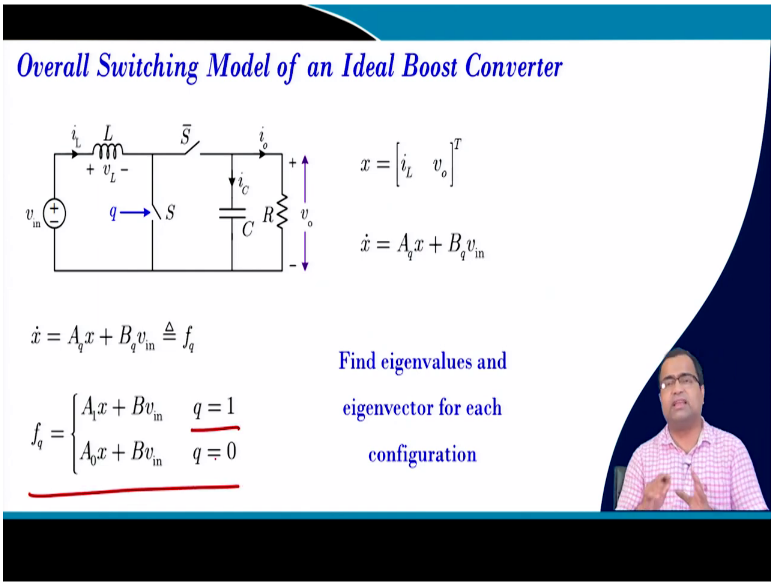


Then we can write the state space model, which will have for q equal to 1 and q equal to 0. We will get different Eigenvalues right and Eigenvectors.

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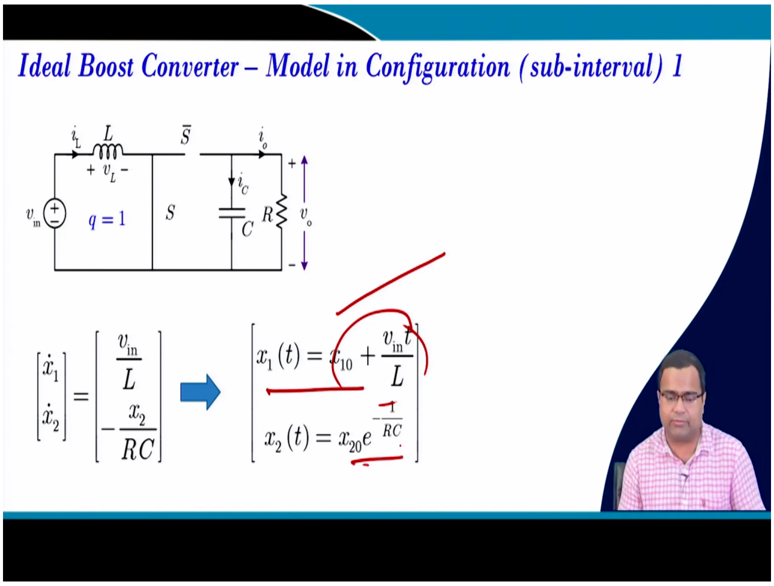


So, then we can model you know for different configuration we can get the solution x 1 dot from here v in by 2 for the ideal boost converter during mode 1. The inductor current will continuously rise because this equation says it will saturate the inductor and the output voltage will completely fall. So, we have to be very careful about the sub interval 1.

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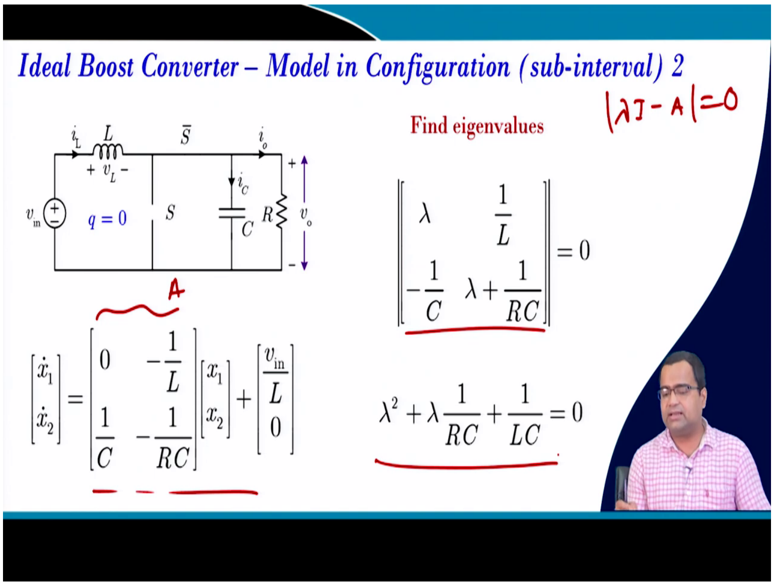


Similarly, sub interval 2 we will have this equation. Now, if we want to find the Eigenvalues in any mode, particularly we are talking about the sub interval 2. Then we will find that lambda we know how to find out like you know this is if this is my A matrix, then we know the lambda I minus A. If we take the determinant then we can find out the Eigenvalues and if you do that then we will find this equation lambda square into lambda into 1 by RC plus 1 by LC equal to 0.

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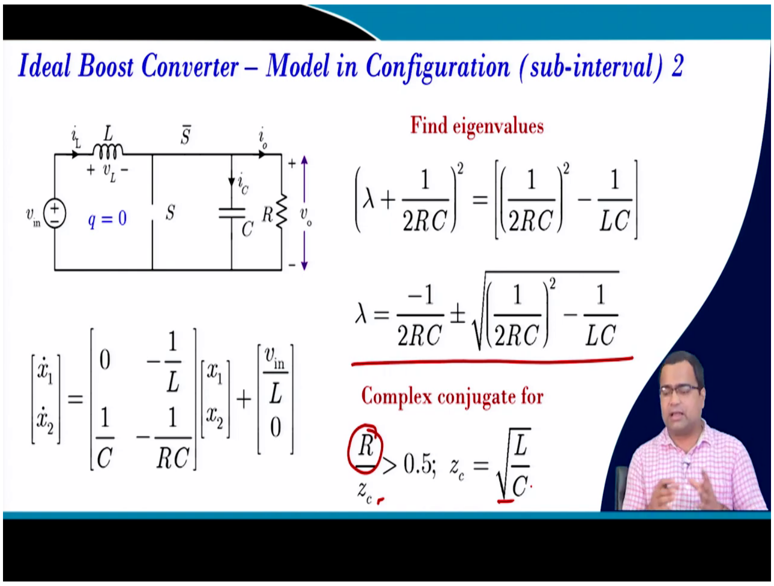

Then we can formulate this equation and we will find out the Eigenvalue which will look like this and it will be a complex conjugate if the resistive impedance is greater than 0.5 times of the characteristic impedance Z c which the Z c is here. So, since we have already discussed in MATLAB demonstration that how does the complex conjugate eigenvalue look like. So, from there we can actually draw the vector field for a boost converter both on and off state and their combination.

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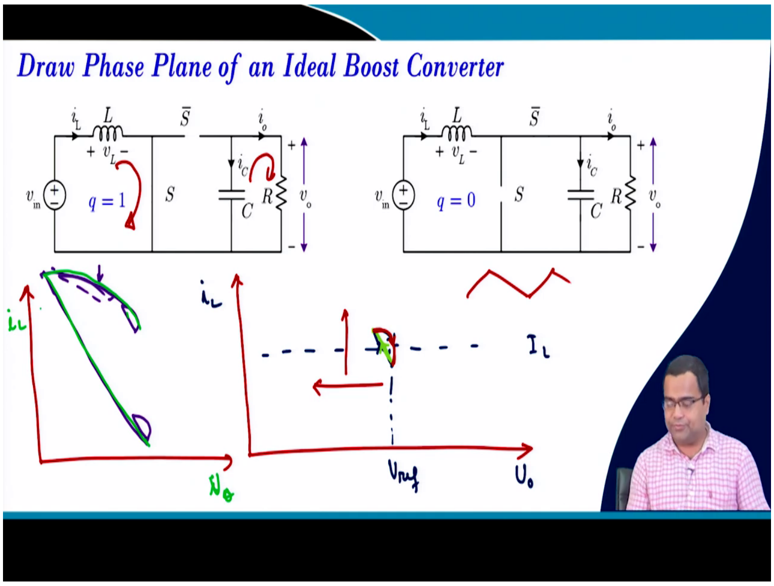


And that we are going to discuss how to draw the phase plane. That means, now if we draw because we can now draw analytically, suppose if we draw, this is my let us say output voltage and the inductor current. Suppose this is my operating voltage v ref we want to achieve this v ref and suppose this is my inductor average current where it will operate. So, it will look like this and this that means or if I draw using different colour.

So, this is the colour when the switch is on and this will be the colour when the switch is off. Because during switch is on, the voltage will fall because you see in this circuit the current will continue to increase and voltage will decrease right.

So, current will increase. That means, this direction current will increase and this direction voltage will decrease. But, during off time, the current will decrease; that means it is like this current will decrease and voltage will increase as long as your capacitor current is positive right. Because if you continue for a long time then voltage will again start falling, so this is the characteristic of the off phase. So that means, we can draw the phase plane because we need to combine the complex conjugate behaviour with real behaviour to get our desired behaviour.

And in the subsequent lecture we want to see we learn about the vector field we learn about the Eigenvectors. So, we want to reach certain desired trajectory; that means, if we take the boost converter. For example, we were here when before there is a load transient, now suppose after load transient we want to reach here. So, it will go we want to reach something like this kind of behaviour. So, how to do that? This is something like a time optimal behaviour can you extract.

So, for that we need to understand how the trajectory move during the on time and what should be our access, where we will turn off the switch and, after turning off, how does the vector field look like. So, all this thing we are going to discuss. In fact, for boost converter it will be a kind of straight line and then go. It will be something like you know the straight line, because this is the on time dynamics. So, it is like a straight line because voltage current will increase and voltage will fall ok.

So, this is my current direction. Sorry, this is my voltage, and this is my current. So, we want to reshape the transient response by using our vector concept and applying the suitable control logic from the geometric point of view.

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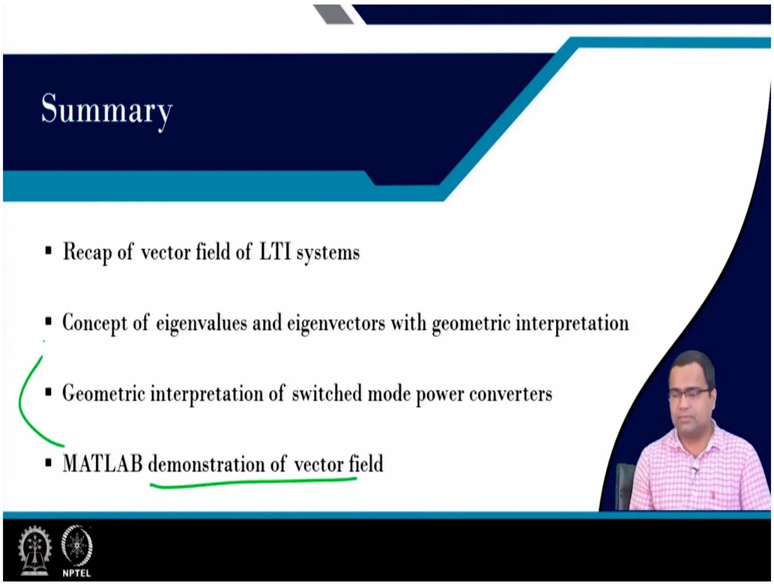

So, with this I want to summarize the recap of vector field of LTI system, we have discussed concept of Eigenvalues and Eigenvector and their geometric interpretation. Then we also discussed how does it look like for a switched mode power converter and we did some MATLAB demonstration for vector field for the linear system. So, we will take more detail vector field for this switched converter with MATLAB simulation in the subsequent lecture. So, I want to finish it here.

Thank you very much.