Control and Tuning Methods in Switched Mode Power Converters
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Module - 06
Small-signal Performance Analysis
Lecture - 32
Impedance Analysis and Stability

Ok. Welcome back. This is lecture number 32. In this lecture, we are going to talk about Impedance Analysis and Stability.

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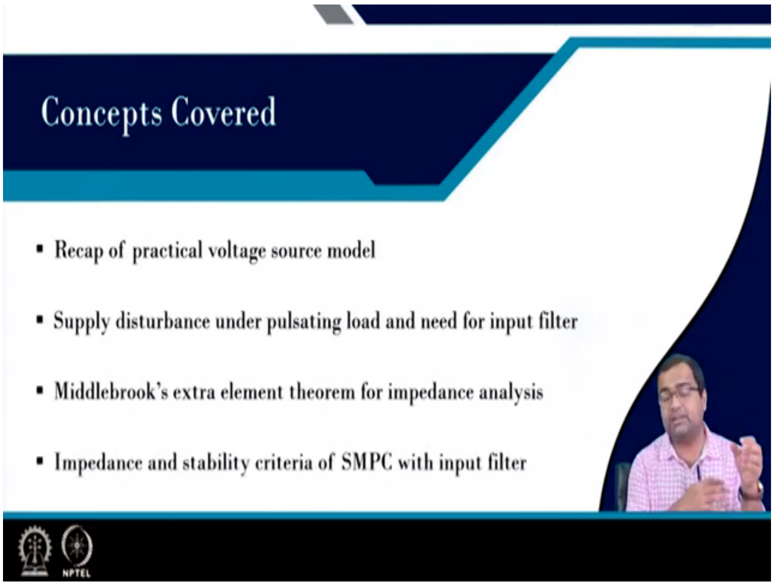

So, in this lecture we are going to talk about, we are going to recapitulate our practical voltage source model, which we discuss in lecture number 13. Then, we want to you know again demonstrate the supply disturbance under pulsating load and then we want to identify the need for input filter. Then, we want to discuss the middlebrook extra element theorem for impedance analysis with an input filter. And, then impedance analysis and stability criteria for switch mode power converter with input filter.

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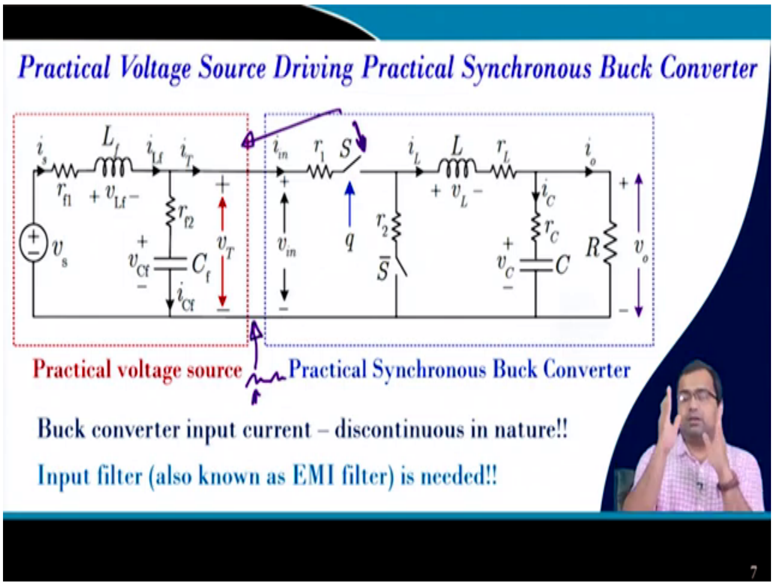

So, practical voltage source driving a practical DC DC buck converter. This thing we have discussed, in lecture number 13. And, we have identified a practical voltage source should have output impedance, which will consist of one DC term, which we generally consider in any practical voltage source, like internal resistance or a series resistance.

But, we in a practical voltage source would also have a frequency dependent term, because none of the power supply has infinite output. Sorry infinite bandwidth ok. So, this we have discussed in lecture 13 ok. And, it is driving a practical synchronous buck converter.

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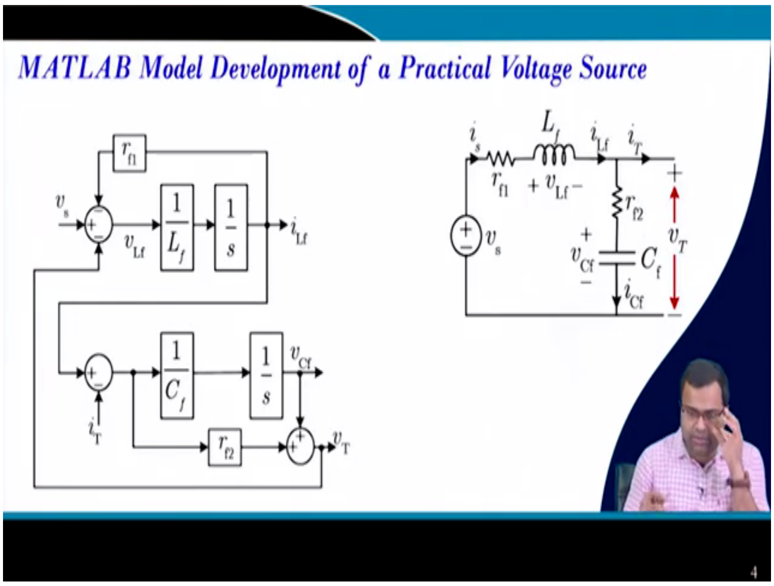

We have also developed model, and we have discussed in lecture 13, this model was developed.

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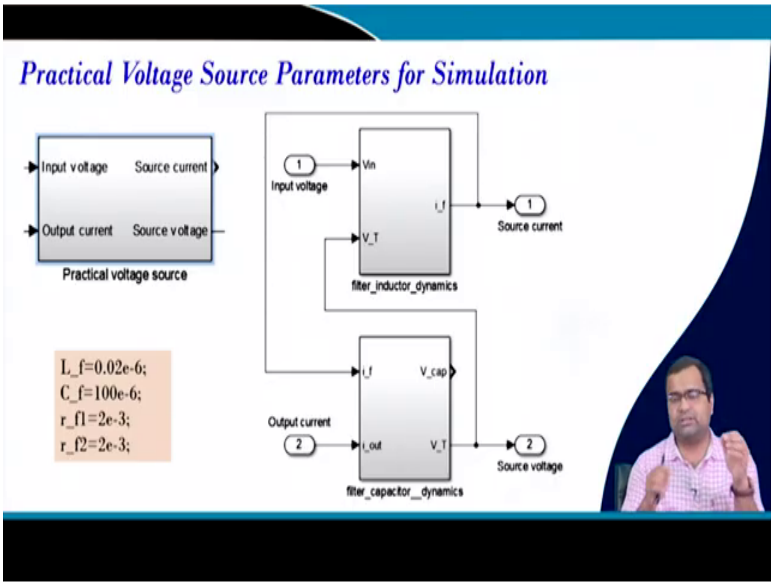

So, practical source we have also considered we build a simulink model and we have also discussed the dynamics in the block diagram to simulink and we have taken this parasitic for the input filter. That means, it has an inductor 20 Nano Henry, then it has a filter capacitor, which is 100 micro farad I took here. Then, r f1 is a D C R of the filter capacitor an r f2 is a asr of the filter capacitor, sorry r f1 is the D C R of the filler inductor and r f2 is the asr of the filter capacitor.

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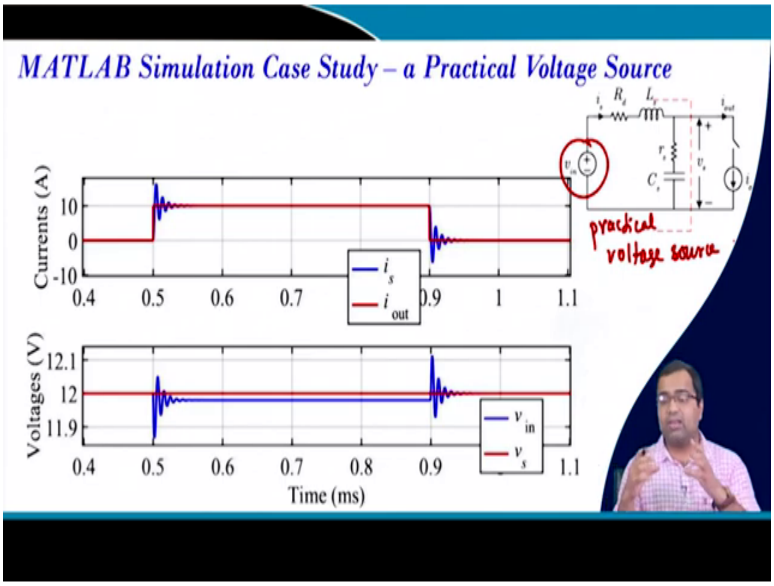

And, we have also shown this MATLAB simulation case study. What did I, what did you do? So, we consider a buck converter, where this input voltage was actually a practical voltage source that was a practical voltage source.

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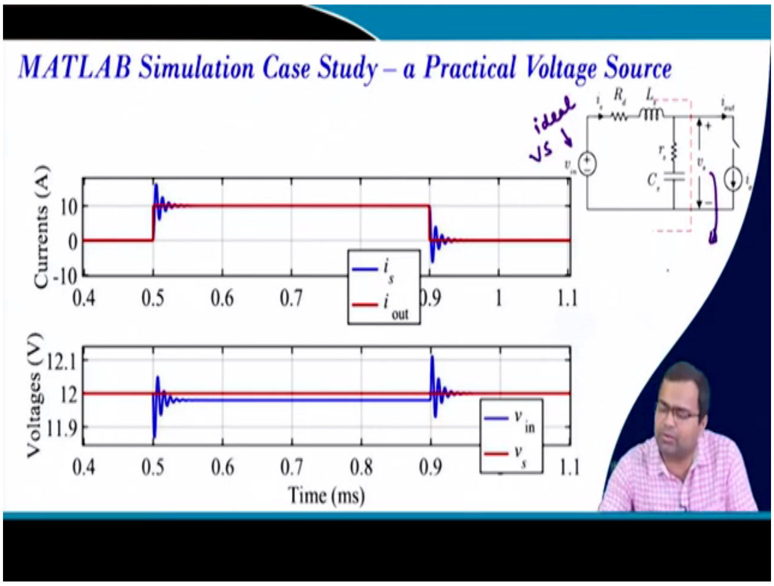

And, here we are comparing sorry this is a voltage source model. I am sorry this represents of voltage source model. So, this is an ideal voltage source. So, this is an ideal voltage source, an ideal voltage source. And, this VS, you know, this is the actual output of a practical voltage source. So, we are considering an ideal voltage source with it is output impedance.

Now, this result we have discussed in lecture 13, where if we draw pulsating current like a step current from this source. Where red color is a step current, suddenly you are drawing 10 ampere current at some point. Then, this blue trace indicate, this blue trace indicate, the source voltage with practical output impedance. There will be an affect, because it does not have an infinite bandwidth, but the red one, which we consider, when we consider an ideal voltage source.

So, there is a subtle difference between ideal and practical different voltage source not only in terms of the DC, but in terms of transient behavior. And, we also discuss the difference shift in the output voltage for a practical source is due to the DC value of the output resistance or output impedance; that means, this is an output resistance, there will be drop.

And, that we consider any practical voltage source in our undergraduate course, we are taught we have been taught with this right; you are familiar. Now, the practical voltage source driving a practical buck converter, we know the input current of a buck converter is discontinuous.

So, earlier we consider a pulsating current externally, by putting a current source and turning on and off. But, now when you connect such a source to a DC-DC converter, since there is a switch and this switch turns on and off. So, the current input current of the buck converter discontinuous. So, it will appear like a load step transient for the source and that will happen twice in a switching period, because once it will turn on and then turn off.

So, it will have both undershoot overshoot effect, for each transition of the switch of this switch, of the switch and that will impact the supply voltage. If the supply is really very well regulated, very high bandwidth or may be a very large capacitor, then it will try to hold it right. Otherwise, most of the power supply now presently we are getting are basically SMPS type earlier power supply which was quite bulky ok, they are based on the linear transformer right.

That means a 50 Hertz transformer, but now most of the power supplier coming with high frequency transformer, switch mode power converter. Because, we need to reduce the size of the supply to make it lightweight. So, where I mean those supplies require very tight control, in order to regulate the output voltage and nullify or reduce a transient effect.

So, that, but still since the input current is discontinuous, so, in most of the cases we should not connect a buck converter directly to a source, because that discontinuous current can inject some you know kind of transient effect in the output of the source. And, that I will inject and that input discontinuous input current is the cause of converter EMI.

So, there will be some amount of power with this transient frequency. There is a higher frequency and that will interfere with the communication device. So, if generally we need to put an input filter in between this source and the converter so, that the source will need not see this discontinuous current ok. So, you need to filter out.

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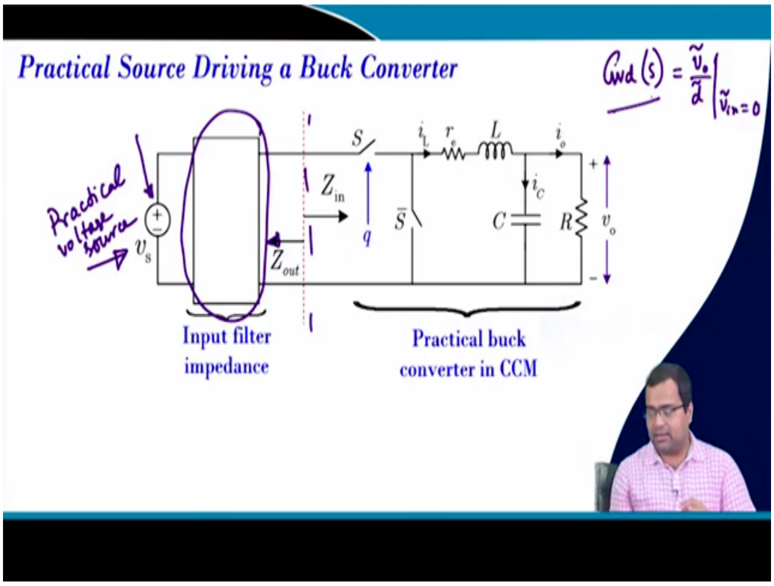

So input filter now this is a standard buck converter, where this is a practical voltage source, practical voltage source, that we have already modeled earlier source. And, in order to avoid the disturbance to the source, we are putting an input filter here. When you introduce an input?

So, please consider this Z out is the output impedance of the filter, because we have put all impedance of the source inside this block only, because otherwise it will be too bulky. That means, eventually we have as it in the modeling sense we have an ideal voltage source, source output impedance, input filter, then DC-DC converter ok.

But, here we are talking about the Z out is the output is an impedance of the input filter ok, while it is driving a buck converter. So, without input filter, we have derived the control to output transfer function, for a duty ratio control. What was that? We got v 0 tilde by d tilde, and this we obtain from where we obtain from you know our equivalent circuit model ok.

But, now all these derivations were carried out considering, there was no input, there was no the practical source I mean it was an ideal source, it was not a practical source. But, in reality, it is a practical source and we need to put a filter. When you put a filter, then this control to output transfer function will be affected by adding this input filter and this modified transfer function control to output transfer function. We do not need to re-derive.

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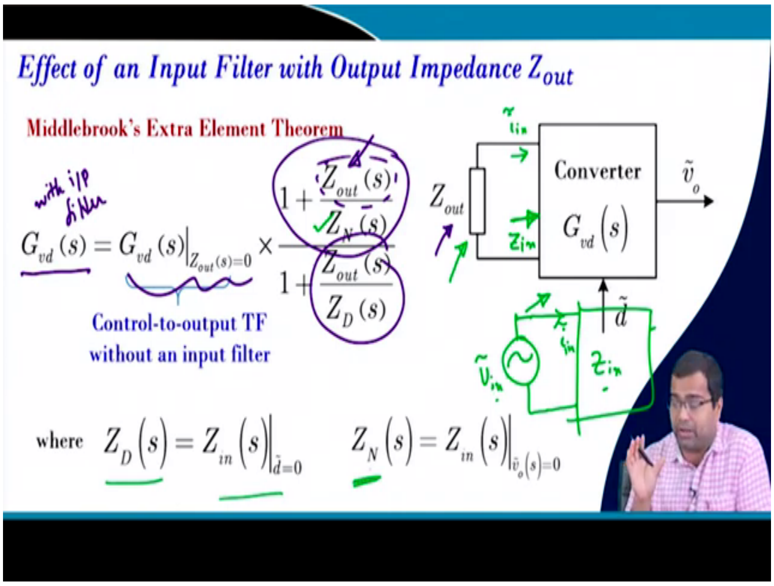

And that you know fundamental work was proposed by Middlebrook, using extra element theorem; that means, the transfer function can be updated. While considering another output impedance, by using extra element theorem, and that can be derived using this method very effectively.

So; that means, this is that control to output transfer function with filter, it is with input filter. And, this is the control to output transfer function without input filter; that means, with filter will be, without filter multiplied by this term. What are this term? Z out this one is nothing, but this Z out is nothing, but the output impedance this is already there of the input filter.

That means, this is known we no need to write ok. So, this you already know. This is the output impedance of the input filter. But what is Z_N? This is the control to output transfer function without the filter. Z D is the input impedance of the converter, while considering an open loop. That means there is no duty ratio perturbation.

That means, as if we are running the converter with fix duty ratio and we are injecting you know; that means this is the input current to the converter. And, we are measuring the impedance from looking at this side of this converter. So, that impedance would be the input impedance of the converter when we operate in an open loop. So, this is done.

Then, what was Z D sorry that is Z D, what is Z N? Z n is the input impedance of the converter when we have forced the output voltage to be perturbation to be 0; that means we need to make a control in such a way whenever we inject this current, we will not see any effect in the output voltage; that means, output voltage should not be effect affected by this.

It means that, if we inject a sinusoidal. So, if you inject an AC excitation; that means, if we apply an AC excitation I am talking about the AC model from this side, this is my AC excitation I have given and this is my input current of this converter, now it will see an impedance Z in.

Where we have a close loop control and we will control the tightly regulate the converter in such a way, that the perturbation or the frequency of excitation in the input will not be reflected in the output. That means, output will be insensitive to the input voltage supply rejection; that means input voltage perturbation.

So, this transfer function is called Z N; that means we need to have a suitable control mechanism by which we can actually nullify this effect. And how can I do that? You know we can do that even using feed forward control also that we have discussed right. So; that means, we need to place a suitable control logic to do that. We can use a simple voltage mode control with an input voltage feed forward. And, if we do that then you can reject the disturbance in the output side that is our objective. And, if you do such control technique and excite whatever impedance we will measure that will be Z in, but how to find out Z in?

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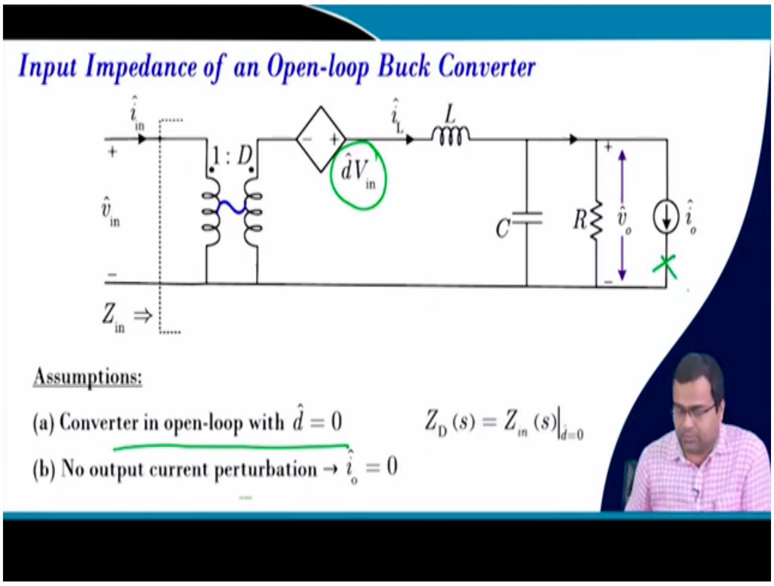

First of all, input impedance of a buck converter, we already have an equivalent circuit model and we know how to derive it right. But, in the first case, if the converter operates under an open loop, then this term will go away right, then it will be shorted no output current perturbation ok.

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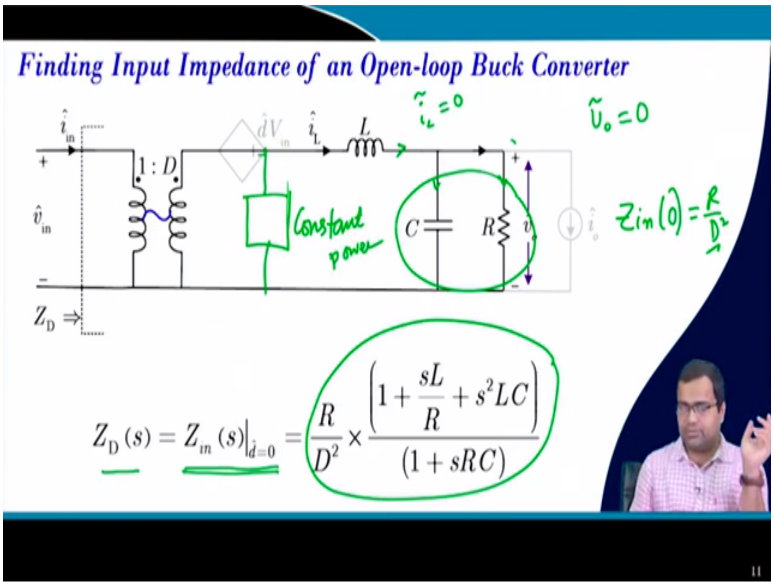

So, it will look like this, it is shorted no output current and we can derive Z D there is an input impedance of the buck impedance under open loop condition. And, this is a standard output input impedance expression of a buck converter, under open loop. This is well known.

In fact, at very low frequency you will see the whole impedance you know if you go to come to very low frequency if you substitute s equal to 0; that means your Z in 0 will eventually become R by D square ok. And, this is particularly input impedance low frequency behavior. It is important when we consider a converter. While you know it is connected to a p v load, solar load, we need some impedance matching ok.

But, solar is a very slow process and so, even though we have an input impedance frequency dependent term, but while you connect into solar, we can take low frequency behavior, which will behave like a resistive load ok. Now, so, we can find out this input impedance for open loop and this we have already derived.

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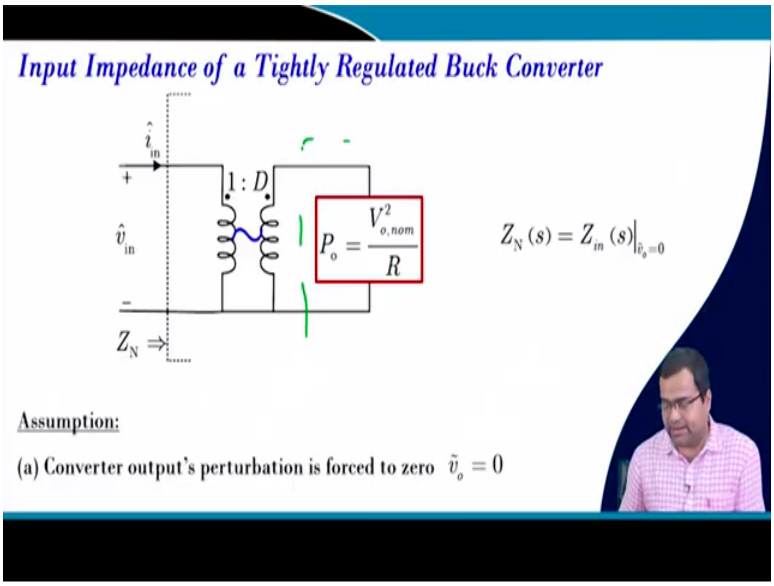

But, what will be the input impedance of a tightly regulated buck converter? Because we told you; that means, if we excite if we inject an AC excitation in the input, we must nullify that in the output, in order to get this transfer function. And, that we can do one way, by using voltage mode control with input voltage feed forward.

Even in current mode control, we want to make sure the voltage is controlled. Then, if you control it, then the right-hand side of this there is no perturbation, so that means if you go back to the circuit. If there is no perturbation in the output voltage, if the voltage perturbation is 0, which means the current perturbation to the resistance is 0 ok.

Similarly, the current perturbation to this capacitor must be 0. Otherwise, any perturbation in this you will have an impedance of this so, it will be reflected. Which means, the current perturbation of this also has to be 0; that means our inductor current perturbation will also become 0, in this case. I am not talking about this case; I am talking about the next case.

So, this whole model will appear, but it does not mean that it will be a short circuit, because it is still drawing current, because voltage is tightly regulated. In that case, it will appear like you know like constant power right. Because your power is constant for a given resistive load output voltage is constant and this is what we do. So, this constant power is nothing, but because the voltage is regulated at it is nominal value and what is the power? It is V 0 nominal square by R, because R is the actual load resistance. So, we can forced to 0, then Z N can be identified.

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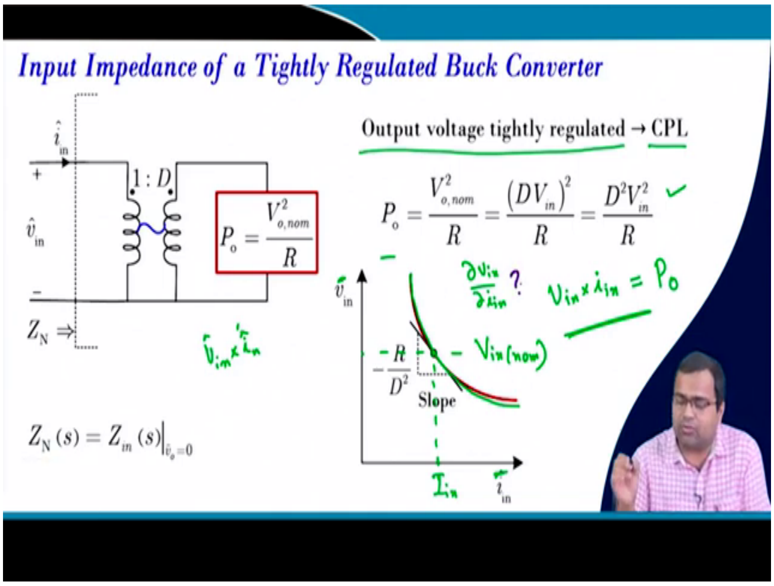

Now, if we take this constant power load; that means, it is a DC characteristic right. So, in order to find this how to start with so, it will behave like an output voltage is tightly regulated, it will behave like a constant power load. And, power magnitude can be obtained V nominal square by R and in a DC-DC converter under ideal condition, D into V in so, will get this expression.

Next in constant power load, since this product of V in into i in, you know I will say it is not the perturb term, because it is an actual term. Perturb term is 0, but I am saying that if you take actual input voltage into actual input current, they are basically constant. That means, if you take DC this is anyway constant. If you take DC plus excitation power is constant.

So, under that case, if you draw current and voltage, they will look like a rectangular hyperbolic curve right, because their product is constant. And, if you take at operating point, because this is a point where this is our nominal input current and this is our nominal input voltage, or nominal input voltage, this is my operating point.

And, if you take the slope of this curve, this will in like this will you know give us the incremental change; that means, delta I can say delta V in by delta i in. So, this is an incremental change, that we can expect this will be the slope ok. And, we need to find out what is this ok? So, you need to find out about this. So, you need to find out what is this?

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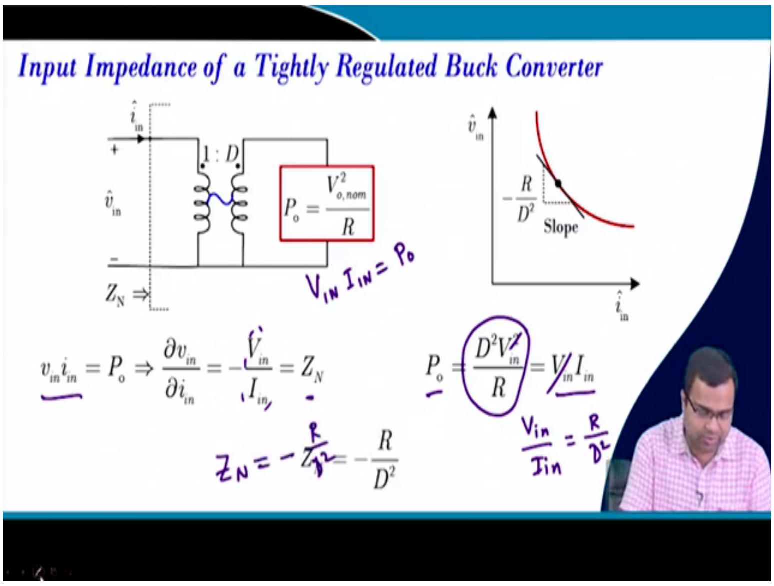

You need to find out; you know the product is constant and if you take the partial derivative it is the Z N. Now, what is v i in? We know that V IN into I IN is nothing, but our P 0 right. So, if you take substitute P 0 is nothing, but we already discuss D into V in a whole square by R and this can be written as V in into I in, because we are assuming it is loss less converter for the time being so, input power and output power are same.

From here we can find out what is my V in by I in? So, you can see I can cut this with this one so I can take this side. So, V in by I in will be simply R by D square. Now, if you substitute here. So, you will get Z N is equal to minus R by D square, this is exactly what we got right? So, Z N equal to minus R by D square ok.

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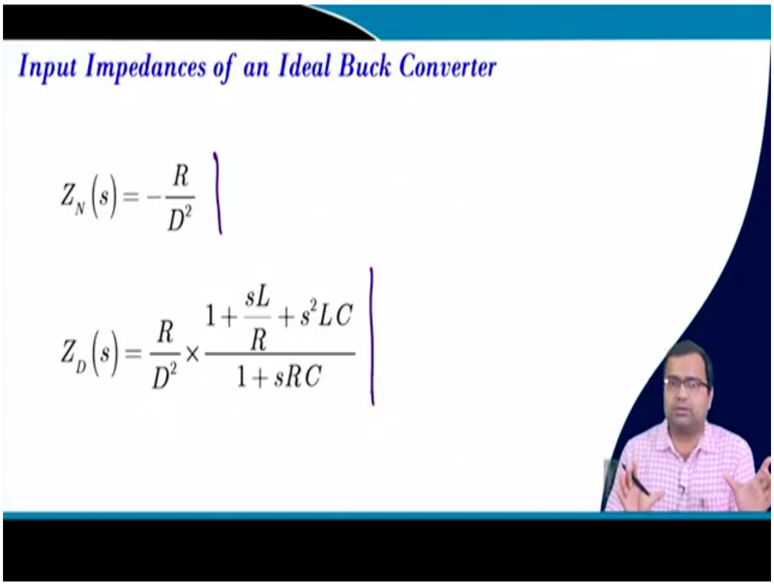

So; that means, input impedance of an ideal buck converter, there are two types of impedance; one is for the null condition output voltage, that is the tightly regulated, another is the traditional input impedance with open loop converter ok.

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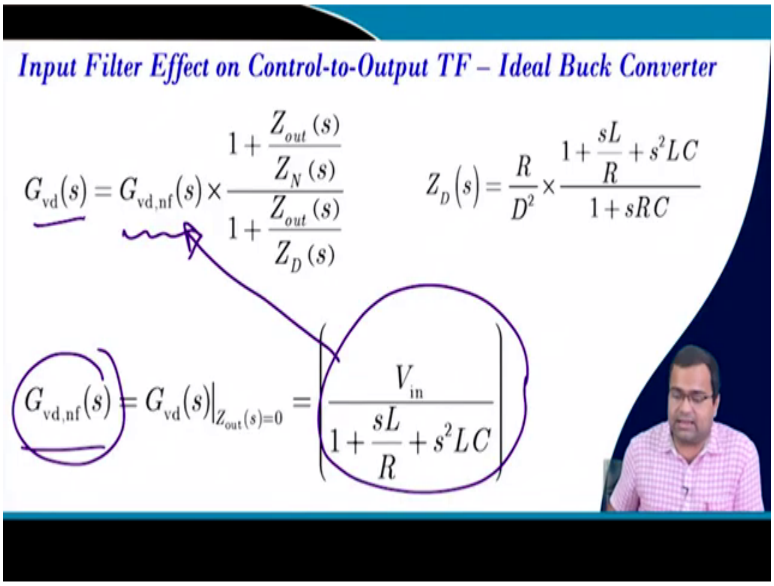

Then, effect of input control to output transformer is ideal buck converter, we have you know we know about you know; we applied the middlebrook extra element theorem and we can write it down. The converter with filter and this is without filter, we know the transfer function of the ideal buck converter. So, we can actually write down, we can, we can take it here and we can analyze. So, you can write down this. So, what are the criteria Z D all these we have to derive.

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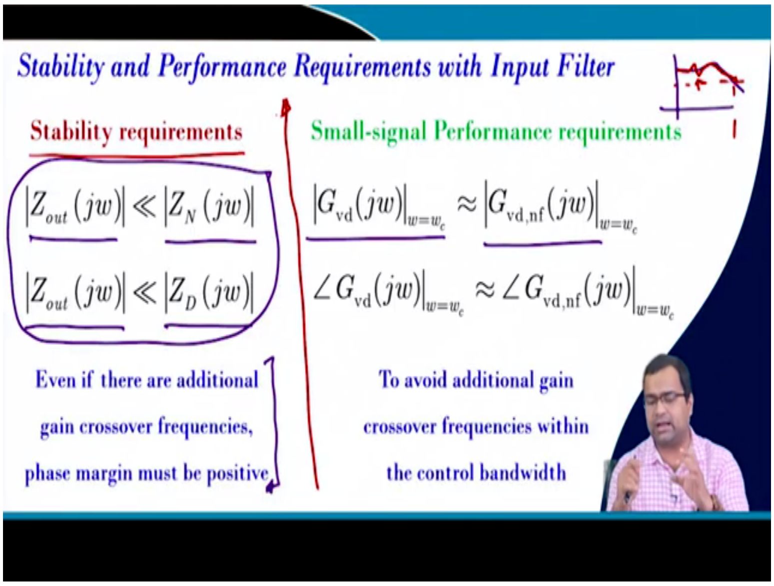

Stability criteria, the stability requirement, the magnitude of the output impedance of the input filter should be much smaller than the magnitude of the null input impedance. Similarly, the magnitude of the output impedance should be smaller than the input impedance of the DC-DC converter; that means this will make sure that their effect is insignificant.

But this does not say anything about the phase response; this is an amplitude response. It may so happen within the control bandwidth; that means there can be multiple crossover frequency. And, with this multiple crossover frequency, if you can ensure these criteria, the system will remain stable with an input filter and without input filter both are stable.

Because without input filter you have designed the converter it was stable, with input filter if we ensure this it will be stable, but multiple crossover frequency may lead to poor phase margin. So, you make get some oscillatory behavior. That means, we need to talk about the transient performance.

So, for transient performance, we need to match the crossover frequency with the input filter and without input filter. In a sense we want to achieve; that means, if we can get G vd like this, then with input filter you may get some peaking effect here, but it should actually otherwise match.

So, if this will not cause any extra, you know crossover frequency and if you ensure that your crossover frequency remains unaffected with and without input filter. And, phase margin at this point remains same and no additional gain crossover frequency, then you can say with and without input filter of the performance would be closely equivalent.

So; that means, one is stability criteria; that means, with input filter, we want to make sure the converter is stable, but this is not sufficient because it will impact the transient performance. In fact, we saw even the source impedance; the input of the converter was different. It was really you know we will take this in the design case study later, but we want to make sure that you know the cutoff frequency is not affected, and phase merging is not affected, then we can maintain the similar transient performance.

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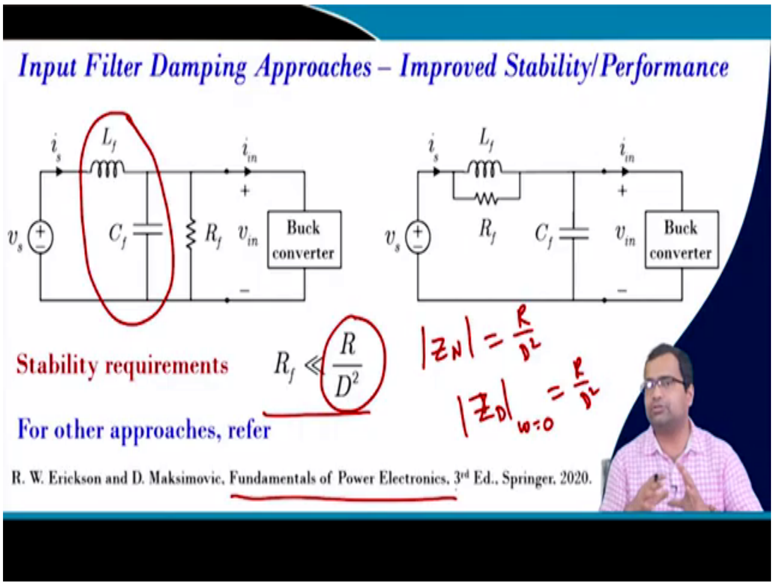

So, in order to stabilize the converter, we need to put a damping to the input filter, because if you take an ideal L C filter, this will be oscillatory and it will definitely make you know it will make the system close loop unstable. So, in order to damp one way to put a resistance in series parallel with the filter capacitor, another you can put a resistance in parallel with the inductor.

But for both the techniques, there will be loss. There are lot of research you know, that happen thereafter, but for more details, but stability requirement the filter resistance for both of these cases must be smaller than, much smaller than, R by D square which is the magnitude of the Z N, it is nothing, but R by D square. In fact, if you take Z D magnitude at omega equal to 0, it will also be same R by D square. So, this must be smaller. Now for other approach please refer to you know Fundamentals of Power Electronics book and also you can refer to more research paper.

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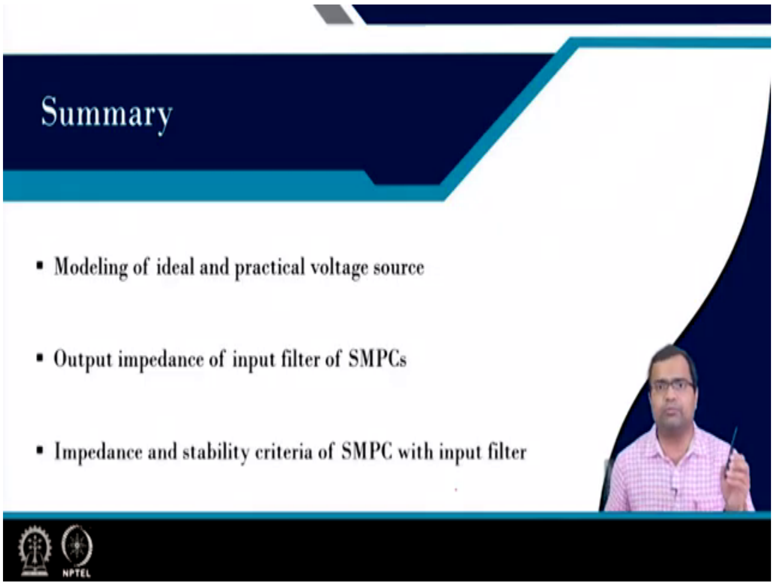

So, with this I want to summarize the modeling of ideal and practical voltage source aspect was discussed, output impedance of input filter was discussed, then impedance and stability criteria of the switch mode power converter with input filters are also discussed. So, with this I want to finish it here.

Thank you very much.