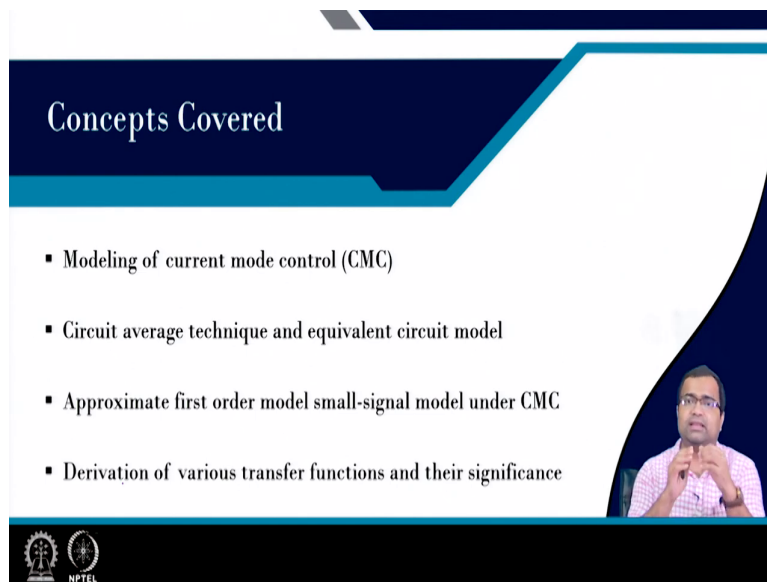


Control and Tuning Methods in Switched Mode Power Converters
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Module - 06
Small-signal Performance Analysis
Lecture - 31
Small-Signal Modeling with Closed Current Loop

Welcome back. This is lecture number 31. In this lecture, we are going to talk about Small-signal Modeling with Closed Current Loop.

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Concepts Covered

- Modeling of current mode control (CMC)
- Circuit average technique and equivalent circuit model
- Approximate first order model small-signal model under CMC
- Derivation of various transfer functions and their significance

NPTEL

Now, we want to extend the equivalent circuit concept, which we have learned for an open loop DC-DC converter. We want to extend this for current mode control where the inner current loop is closed. Because we will not consider in this lecture with the closed voltage loop that we will discuss in the subsequent lecture.

So, for modeling of current mode control, the circuit average technique is used to derive the equivalent circuit model. I will show you that it will be a first order approximate model. And then we will derive small-signal model first-order model under current mode control. We will derive various transfer functions and we want to understand their significance. Then we will see at the end whether this first-order model you know we want to open up some question.

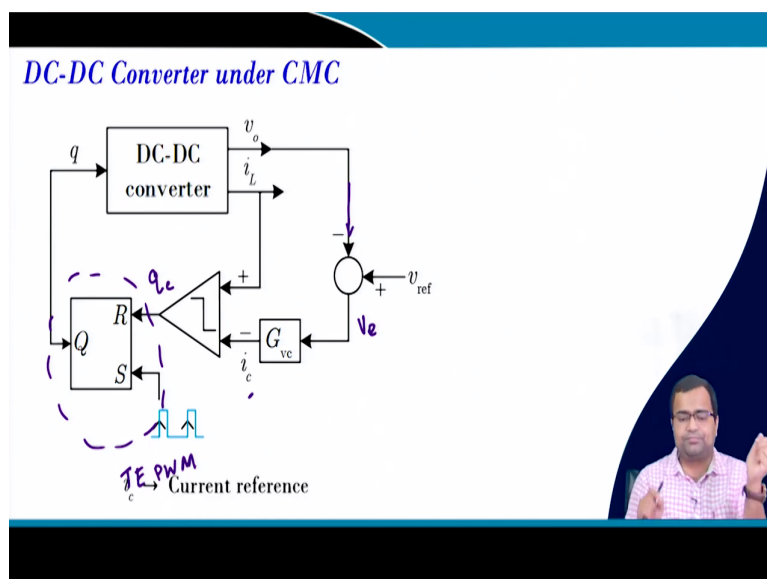
And in subsequent lecture we will see whether this first-order model is really good enough can we really use it or where is the limit. So, all these will be covered in the subsequent lecture, but today we will start with the very basic modeling approach.

Few more things I want to emphasize before we start. We talk about right half plane zero in a boost converter and we also saw that the location of right half plane zero what is the expression. Now, we also want we are interested to see and in for in a boost converter under open loop we saw there are there was for an ideal boost converter there was one RHP zero and two poles.

For a practical boost converter, there is one more zero due to ESR. Now, in current mode control we want to see whether the boost converter under current mode control do you have a right half plane zero and if we have whether the location will be same as earlier or it will be different that aspect also we will keep in mind ok.

Another aspect we will see in current mode control a load regulation aspect that also we will keep in mind. Because in earlier lecture we saw current mode control of our excellent line transient response. So, there is no problem with the line regulation, but we want to investigate what is the; what is the aspect of load regulation in current mode control?

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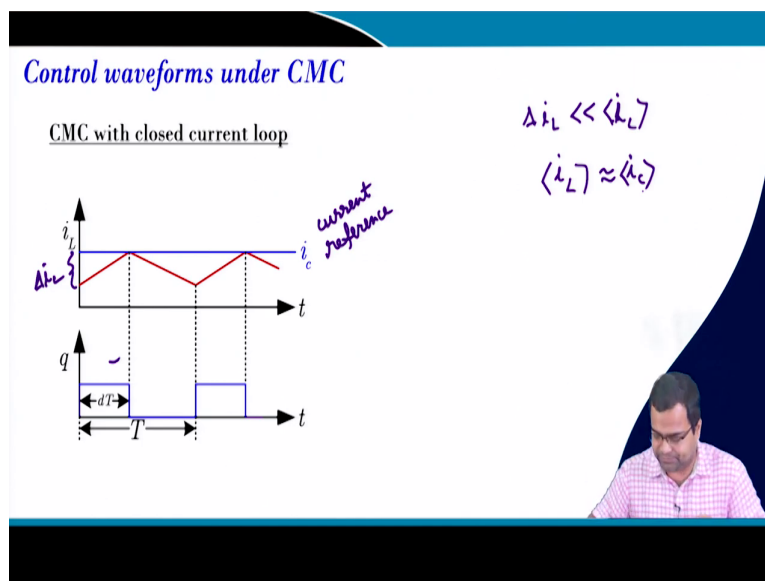
So, in current mode control block diagram we have discussed earlier where we have we will sense the output voltage here, and it is compared with the reference voltage and this is our

error voltage. It will go through a controller and the controller will generate the current reference.

So, we will use different notation. Sometime we will use i_{ref} here I will use i_c . So that means you can use any of the reference current notation. And that output of this comparator will go as an input to this particular you know lag circuit and this whole logic is actually trailing edge PWM that we have discussed earlier ok.

Because the switch turns on at the rising end of the switching clock and it turns off when the inductor current touches the peak current ok and this logic we have also discussed it is under trailing edge modulation and it is peak current mode control, but the same analysis. What we are doing can be also applied for value and average as well.

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So, here i_c is the current reference. Now with closed current loop, if we draw the waveform, this is the current reference right. So, we call this is the current reference, and this is our inductor current the red one and this is our duty ratio.

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Simple first-order approximate model

→ Assumptions

(a) $\langle i_L(t) \rangle_T = i_c(t)$ *no slope compensation*

(b) Slope perturbations → neglected

→ Perturbed current dynamics

$\tilde{i}_L(s) \approx \tilde{i}_c$

(assuming negligible ripple effect)

Now, first thing the assumption the average inductor current is replaced by the control current. That means, if you go to the waveform, our initial assumption suppose our ripple is this ripple is negligible. That means, suppose if our Δi_L is very very smaller than i_L average, then i_L average can be approximately replaced by this current reference ok.

But in practice this assumption in now, we will see in the subsequent lecture we will actually lead to you know that model accuracy of this model is not good enough when you slowly go to a higher frequency range. This approximation makes some sense in the low frequency range. This approximation also gives problem when we talk about modulator gain of the current mode control, but here we are not and here we are also not considering any compensating rank.

So, in this model we are first no compensation ramp, ripple is negligible. So, the average current is almost equal to the current reference, but this is the starting point to remember. That means it does not mean that we want to limit our analysis. In fact, in subsequent lecture we want to investigate the small-signal model and we want to validate the time domain response of the small-signal model with the actual switch simulation.

So, assumption here the average current is same as the reference current. The slope perturbation is neglected. And perturbed current dynamics is this and here no slope compensation is considered, no slope compensation another part. That means we are not

adding any ramp to the inductor current. So, assuming negligible ripple, that we have discussed.

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Simple model – an algebraic approach

Boost Converter

Step 1:

$$L \frac{di_L}{dt} = v_{in} - (1-q)v_o$$

$$C \frac{dv_o}{dt} = (1-q)i_L - \frac{v_o}{R}$$

Handwritten notes: $q=1$ S on, $q=0$ S off

Now, if you start with this basic assumption, then we can see we will see this circuit becomes very simple. How? So, we take a boost converter; we know all the equation of the boost converter because we have already derived this equation when in lecture number 4 when we develop the MATLAB model. That means the rate of change of the inductor into L; that means, this represents our v_L that is the inductor voltage. It is nothing but input voltage 1 minus q into v_o .

That means, when q equal to 1 the switch is on, this main switch is on, then the voltage across the inductor is nothing but the input voltage. And when the switch is off then q equal to 0 q equal to 0, when S on and S off then voltage across the inductor is the input voltage minus output voltage.

One thing you will observe under when the switch is on that means the circuit will be the equivalent circuit will be like this; inductor current inductor is simply short it here and the capacitor will discharge across resistance. So, such a scenario; that means, as if we are getting two first order system; that means, this first order system consisting of an inductor that can be parasitic resistance and this circuit consisting of only capacitor.

So, originally because of two energy storing elements, inductor and capacitor, it is a second order system, but for switch configuration 1 when the switch is on two circuits get decoupled; that means, inductor and capacitor get decoupled. And it virtually looks like decoupling first order system.

And this is the problem of this boost converter because in this configuration we have an input voltage here ok. So, inductor voltage across inductor is positive ok and if we continue this for a long time, then the inductor current will keep on rising and at some point it will saturate the inductor.

And that is the danger in the boost converter. If you do not have control over current if you do simple voltage mode control and if you try to push the bandwidth height it may so, happen that during transient you can have a large on time and that large on time can really increase the inductor current and it will never come back.

And during this time, the capacitor voltage will discharge that means, it will decay. That means, on one hand the current in one hand the current will rise and on the other hand that means, this is my output voltage or another ideal condition the capacitor voltage will drop. It is continuously rising inductor current and voltage is continuously falling.

And this is the reason physical origin of non-minimum phase behaviour because here one state variable is increasing the other is decreasing even if you run it for a long time. But if you take a buck converter, you know if you run it for a long time inductor is rising and if you have a resistive load, then your output voltage will also rise. Because your average capacitor current you know because capacitor current is simply inductor current minus load current and the load current if we apply a fixed load current, ok.

If your average inductor current goes above load current, so, voltage will simply keep up rising. So, keep on rising until the output voltage becomes input voltage. Up to that point, voltage and current both will rise. That is why in a buck converter you will not get any non-minimum phase effect or any right half plane zero.

Because we saw in the small-signal model of you know buck converter there is no right half plane zero, but in boost we got it and the reason is this. That means, in mode 1, the two circuits are decoupled. But in mode 2 if you talk about mode 2 then the circuit is a parallel

RLC circuit; that means, then in that mode 2 it is on and this is off this is off; that means the RLC circuit.

In this mode, there is no problem; that means whatever energy stored during the mode 1 that should get delivered to the capacitor as it will load right. So, we need to make a mechanism so that the inductor energy can transfer to the capacitor otherwise the whole system will collapse and inductor will simply saturate and it may damage the switch because the high current can exceed the limit of the devices. So, you have to be very careful about the boost converter.

And that is why in most of the boost converter commercial product we use current mode control where we have a direct control over current and we can limit the current and turn off the switch. So, here the switch we have a discontinuous mode. Similarly, we can out the output voltage equation as I told. If q equal to 1 then $C \frac{dv}{dt}$ is equal to $\frac{v_0}{R}$, it will discharge ok. An input current in this case is same as inductor current.

So, you know in contrast to buck converter where we saw the input current is discontinuous, but in boost converter the input current is same as inductor current. So, if you take a larger inductor with very smaller ripple, then input current will be continuous. And that is why boost converter is used you know for solar application where we want to draw a continuous current from the solar panel because in buck converter we have a discontinuous current.

In that case, we need to put a capacitor because the solar panel should not be exposed to the discontinuous current. But in case of battery charging or LED driving, where the load side we need a continuous current, where we generally apply a buck derived topology where the inductor is in the output side ok.

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Step 2: Average dynamics

$$\frac{d(I_L + i_L)}{dt} = I_L$$

$$\frac{di_L}{dt} = \frac{v_{in} - (1-q)v_o}{L}$$

$q \rightarrow$ discontinuous control input

$$\left\langle \frac{di_L}{dt} \right\rangle_T = \frac{1}{L} \left[\langle v_{in} \rangle - (1-d) \langle v_o \rangle \right]$$

$$\left\langle \frac{dv_o}{dt} \right\rangle_T = \frac{1}{C} \left[(1-d) \langle i_L \rangle - \frac{\langle v_o \rangle}{R} \right]$$

$$\langle i_{in} \rangle_T = \langle i_L \rangle$$

$$\langle v_{in} \rangle = V_{in} + \tilde{v}_{in}$$

Fourier series

So, average current dynamics, you can write. We already know that $i_L dt$ over a cycle. Now, earlier, our dynamics of the inductor current were or something like just hold on. So, earlier we got the inductor current dynamics was $V_{in} - (1-q)V_o$ the whole thing by L we saw. In this case, q is discontinuous input right.

It is discontinuous input; it is a control input; discontinuous control input and, for such a system this represents a switch linear system where you have a discontinuity. So, you cannot apply any Taylor series because the Taylor series requires the nonlinearity should be differentiable right; that means, it should satisfy the Lipschitz continuity. But it is not in the current form when you have a discontinuous control input.

But, if we average over a cycle, then this q as if you are taking the average over a cycle of q then if you take the q ; that means, q you know if you draw the q waveform t if you take the q it will look it is a gate signal right. So, if you it is a periodic signal ok with a duty ratio d . So, it is d into T . Now, if we take the Fourier series of this q ; that means, if you take Fourier series because it is a periodic signal.

So, you will get a DC component of the q . In the Fourier coefficient, if you take Fourier coefficient square, if you plot it you will get a large spectrum at the DC component and this corresponds to your duty ratio. Then you will get at switching frequency, then you will get harmonics and so on. But, when we are applying averaging, we discuss we generally discard these switching frequencies.

So, we want to retain within a neighbourhood of the DC average value. So, that is why if you take the average of this q over a cycle in the conventional averaging, we are only taking the know the zero frequency component, not the switching frequency component.

So, in that case, this q over our conventional averaging will simply become duty ratio and we are replacing this average quantity. Similarly, we can take the average of the dynamics of the output voltage and average dynamics of the input current and input current and inductor currents are same.

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Step 3: Perturbed linearized model

$$\frac{d\tilde{i}_L}{dt} = \frac{\tilde{v}_m}{L} - (1-D)\frac{\tilde{v}_o}{L} + \frac{V_o}{L}\tilde{d}$$

$$\frac{d\tilde{v}_o}{dt} = \frac{(1-D)\tilde{i}_L}{C} - \frac{\tilde{v}_o}{RC} - \frac{I_L}{C}\tilde{d}$$

where $I_L = \frac{I_o}{(1-D)} = \frac{V_o}{(1-D)R}$

$$\frac{d\tilde{v}_o}{dt} = \frac{(1-D)}{C}\tilde{i}_L - \frac{\tilde{v}_o}{RC} - \frac{V_o}{RC(1-D)}\tilde{d}$$

$\tilde{i}_m = \tilde{i}_L$

Next, once we apply the average, then you can see. Now, all this quantity we can replace the average quantity we can replace. Any average quantity is capital plus V in tilde. Sometime we write tilde, in some reference book also we write hat. So, this are represent this represents perturbation.

So, whether it is hat or tilde these represent the perturbation. So, perturbation that means if we replace this average inductor current this quantity by I_L that means, sorry. If you take d/dt of I_L plus i_L tilde then the derivative of the steady state quantity will be 0 and derivative of the perturbed quantity will be considered.

Similarly, from this right-hand side we will get a DC quantity and then perturbed quantity and there will be a product of perturbed quantity which will ignore. In fact, this thing we have discussed in the averaging technique where we have considered Jacobian form or the Taylor

series. We have only considered the first-order term and with that we can write down all the perturbed dynamics here.

That means, the perturbed dynamics of the inductor current, the perturbed dynamics of the output voltage and here we have an inductor current average in a boost converter average inductor current is the average load current by 1 minus D and average load current is if a resistive load it is V_0 by V_0 by R. So, if you replace then this equation can be written here ok.

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Summary: $\tilde{i}_{in}(s) = \tilde{v}_{in}(s) + V_o \tilde{d}(s) - (1-D) \tilde{v}_o(s)$

$$L \frac{d\tilde{i}_L}{dt} = \tilde{v}_{in} + V_o \tilde{d} - (1-D) \tilde{v}_o$$

$$C \frac{d\tilde{v}_o}{dt} = (1-D) \tilde{i}_L - \frac{\tilde{v}_o}{R} - \frac{V_o}{R(1-D)} \tilde{d}$$

$$\tilde{i}_{in} = \tilde{i}_L$$

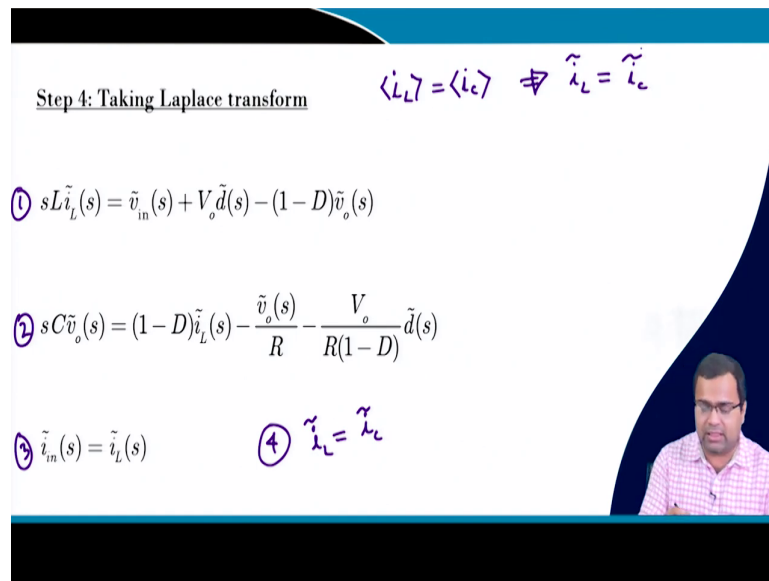
LTI

Now, we have two equations. The summary of equation the perturbed current dynamics inductor current, perturbed output voltage dynamics and the perturbed input current dynamics this three we require. Next we have perturbed which represents Linear Time Invariant system LTI because here we have only considered perturbation and this linear time invariant system.

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Step 4: Taking Laplace transform $\langle i_L \rangle = \langle i_c \rangle \Rightarrow \tilde{i}_L = \tilde{i}_c$

- ① $sL\tilde{i}_L(s) = \tilde{v}_{in}(s) + V_o\tilde{d}(s) - (1-D)\tilde{v}_o(s)$
- ② $sC\tilde{v}_o(s) = (1-D)\tilde{i}_L(s) - \frac{\tilde{v}_o(s)}{R} - \frac{V_o}{R(1-D)}\tilde{d}(s)$
- ③ $\tilde{i}_m(s) = \tilde{i}_L(s)$ ④ $\tilde{i}_L = \tilde{i}_c$



Now, if we apply Laplace transform, then we can get yes all these things you can you know this. If you take the Laplace transform of this it will be s into i_L s left-hand side then right-hand side will be sorry V in tilde s plus V_0 , this is a capital V_0 steady state pointing to s minus 1 minus D into V_0 tilde s ok.

So, similarly you can apply Laplace all this. So, we can apply Laplace transformation. So, you have three equation; number 1, number 2, number 3. Now, what do we want to achieve? We have 4th equation, what is that? That means, we have assumed that perturbation of the inductor current is equal to the perturbation of because we have assumed the average inductor current should be same as the average.

That means, what is our assumption? Our assumption was average inductor current was the same as the average current reference. And if we consider the perturbed term, then average inductor current is same as average reference current because their average quantities are equal.

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Step 5: Replace $\tilde{i}_L(s) = \tilde{i}_c(s)$

① $sL\tilde{i}_c(s) = \tilde{v}_{in}(s) + V_o\tilde{d}(s) - (1-D)\tilde{v}_o(s)$ *obtain \tilde{d}*

② $sC\tilde{v}_o(s) = (1-D)\tilde{i}_c(s) - \frac{\tilde{v}_o(s)}{R} - \frac{V_o}{R(1-D)}\tilde{d}(s)$ *and ②*

③ $\tilde{i}_{in}(s) = \tilde{i}_L(s) = \tilde{i}_c(s)$

Next we have to replace all this i_L by i_c because we want to we have considered this right. So, it looks like the inductor current is replaced by a control current and you will see the inductor will virtually become a control you know control current source ok. So, if you replace in this equation i_L bar.

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Step 6: Obtain \tilde{d} from current dynamics

$$\tilde{d}(s) = \frac{sL\tilde{i}_L(s) - \tilde{v}_{in}(s) + (1-D)\tilde{v}_o(s)}{V_o}$$

Step 7: Equation of input current

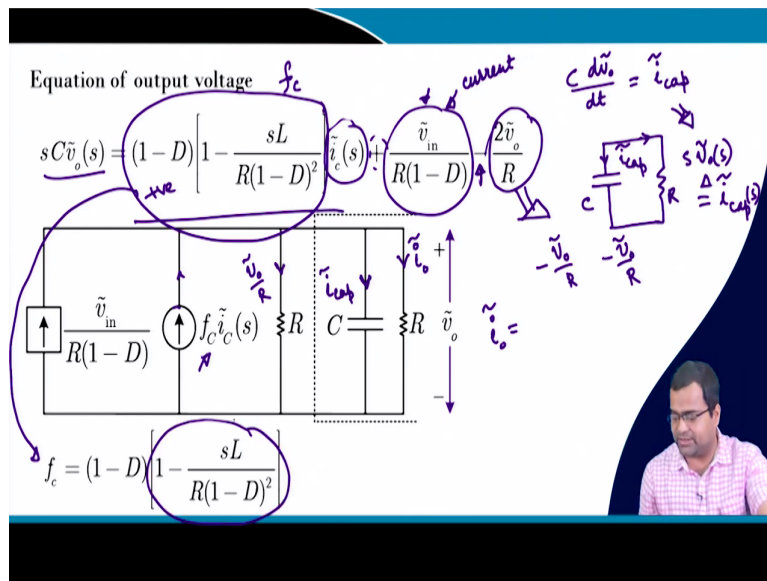
$\tilde{i}_{in}(s) = \tilde{i}_c(s)$

So, input current then we can obtain \hat{d} from the current dynamics; that means, here we have three equations; 1, 2, 3. So, if we take this equation 1 and 2; from these two equations, I

can you know all this because it consists of input, output and all. So, I can get $d=0$ or in fact, we can use one of the equation current dynamics.

So, I can use this equation, equation 1 and obtain \hat{d} from this equation. So, we can obtain here and then replace in the input current equation. Since input current equation does not have any duty ratio, so, it is simply the control current source ok. So, this is our control current source; control current source.

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Then next, we want to write the equation of the output voltage. What is that? So, output voltage we have an equation and there initially there was a duty ratio term right duty ratio term and we want to replace the duty ratio from this equation. And if you do that then we can obtain this equation that means, what does it indicate?

We know $C \frac{dv_o}{dt}$ is nothing but capacitor current. So, we will call it because the notation should be different. It is a capacitor current. And if we recall the capacitor current of a buck converter or any DC-DC converter, it has how many components? So, capacitor current; that means, I am taking the capacitor current to be positive for when it is going in because we need to give a symbol sign the direction of the current.

So, I am taking the positive direction where it is going into the capacitor because effectively it will charge the capacitor. If we take the capacitor, the positive current direction is out of the capacitor, then we need to take a negative sign here because the capacitor will get discharged.

So, here we are taking positive current direction to be going in. Next, this equation shows three things and if we take the Laplace transform, what we will get? $S v_0$, S is nothing but equal equivalent to $i_{cap} S$ capacitor current. So, this capacitor current has three components. So, this you can treat like a constant coefficient into one current right. Then you can think of this quantity virtually look like a say you can see the dimensionally this represents current because voltage by resistance and D is a dimensionless quantity.

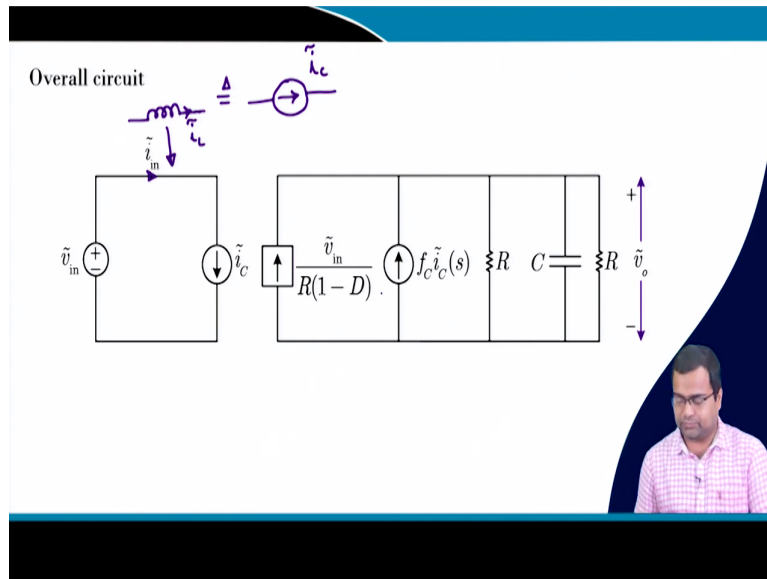
So, it represents current. Similarly, here also, but this term we can divide into two parts; $\frac{v_0}{R}$ minus $\frac{v_0}{R}$; that means, when you take the resistance which is in the output side current is going right. So, this I think ok we will write here; that means, if we consider this current, what is this current if we take this to be load current? So, load current here is nothing but now this current is positive, if we want to and capacitor current is here.

So, this is going out; that means, in the capacitor current if we apply KCL, the sum of these all current should be 0. So, this represents a positive current, which is added with the capacitor current. This is my i_{cap} , which is added. Similarly, this is also a positive quantity.

So, this and this is replaced by f of c this here and this sign is positive. So, it is also added; that means, it is going it is contributing to the capacitor current. But these two are the one which is drawing current and again the terminal voltage is v_0 . So, this quantity will be $\frac{v_0}{R}$ ok.

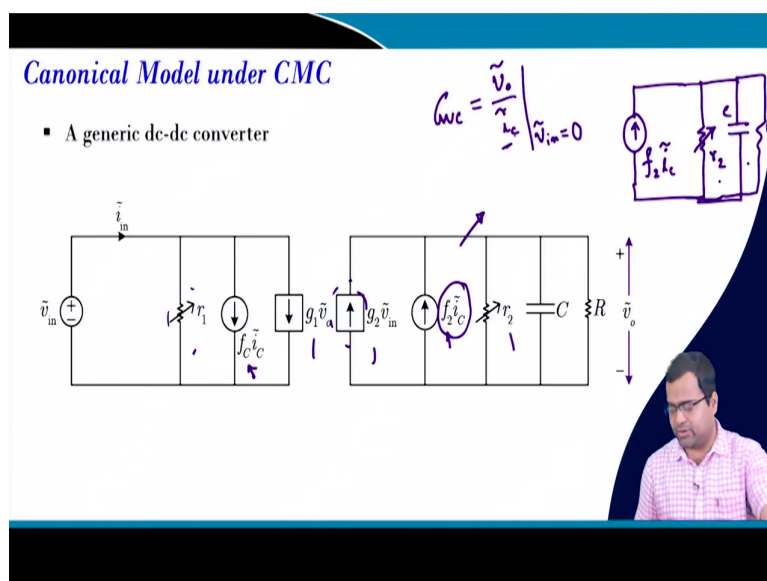
And their direction they are going out. So, there is a negative sign here. This negative sign because they are going that drawing. So, these represent the equivalent circuit, where f of c is here. So, this is my f of c , I told you ok and you will see there is a right half plane zero as well. It is a transfer function; it is just not a constant quantity.

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So, overall circuit is this. So, this is my input current, and it is coming out of the source. This is a control current. So, now you can see the input side current, the inductor which was there on the input side it was there here. It is replaced by a control current source and which is this and it was inductor current. So, inductor current is replaced by a control current source; that means there is no inductor anymore right. Output side, we saw the circuit and you know. So, we can get all this coefficient from here right.

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Now, canonical form under current mode control. So, for this method can be applied to, we did it for a boost converter. We can do it for a buck-boost converter for a boost converter. So, we can if we apply then we can get input you know current. Similarly, the equivalent, so, this is a generic representation ok.

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Canonical Model under CMC

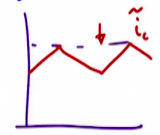
	g_1	f_1	r_1	g_2	f_2	r_2
BUCK	$\frac{D}{R}$	$D\left(1 + \frac{sL}{R}\right)$	$-\frac{R}{D^2}$	0	1	∞
BOOST	0	1	∞	$\frac{1}{R(1-D)}$	$(1-D)\left[1 - \frac{sL}{R(1-D)^2}\right]$	R
BUCK-BOOST	$-\frac{D}{R}$	$D\left(1 + \frac{sL}{(1-D)R}\right)$	$-\frac{(1-D)R}{D^2}$	$-\frac{D^2}{(1-D)R}$	$-(1-D)\left[1 - \frac{sL}{(1-D)R}\right]$	$\frac{R}{D}$

R. W. Erickson and D. Maksimovic, Fundamentals of Power Electronics, 3rd Ed., Springer, 2020.

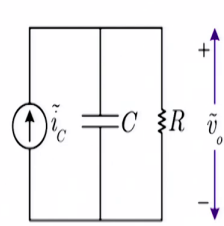
And this canonical table what are the values of this f c, f 2, this r 1, then g 1, g 2, r 2, everything is given in this canonical form and for further detail you can refer to this book ok.

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Control-to-output Transfer Function of an Ideal Buck Converter

$$G_{vc} = \left. \frac{\tilde{v}_o(s)}{\tilde{i}_c(s)} \right|_{\tilde{v}_in=0} = f_2 \times \left[r_2 \parallel R \parallel \frac{1}{sC} \right]$$


Find G_{vc} for an ideal buck converter



$$G_{vc}(s) = R \parallel \frac{1}{sC} = \frac{R}{1 + RCs}$$

$$G_{vc}(s) = \frac{R}{1 + \frac{s}{w_p}} ; w_p = \frac{1}{RC}$$

$G_{vc}(0) = G_{vc}(s)|_{s=0} = R$

So, this canonical model once we derive because this already these things are discussed in well detail. So, I just want to summarize for part of model development in current mode control. So, for control to output transfer function of a buck converter or any DC-DC converter ideal buck converter, so, if you go back to the canonical model, we have this one.

So, if you take control to output transfer function; that means, the definition means v_0 to i_c which is my control current. In that case, we have to consider the input perturbation to be 0. Then what will be the equivalent circuit? What will be this circuit? So, this will be open circuit right. So, it will look like this f_2 into sorry f_2 into i_c sorry ok.

So, this will be f_2 into i_c then we have a emulated resistance r_2 then we have a capacitor and actual resistance, this is my load resistance ok, this is my capacitor. That means, if I want to get v_0 which is my terminal voltage to so, it will be a parallel combination of r_2 C R divided by f_2 that is exactly what is here. Sorry, it should be multiplied by because current is in the denominator ok, f_2 into this; all these three r_2 R C s.

Then, for an ideal buck converter, the transfer function is like this. So, it is just a simple first-order model. Remember, it is an open loop, but the inner current loop is close; that means your outer loop is open. If you go back to the buck converter, what we did here? We have kept a current reference.

It does not mean it is totally open. So, inductor current is forced to follow the reference that is it. But we are not changing this reference as per the outer loop demand because we kept a constant current reference. In that sense it is open loop ok. And we are talking about a control to output transfer function; that means, a change in i_c how is it going to affect the change in output.

And it turns out to be it will be a simple first order system with a one pole for a buck converter, where in our original model of duty ratio control direct duty ratio control buck converter we have two poles and that two are complex conjugate pole in general. Now, it becomes a first order system, and that is why we simulated buck converter without load feed forward.

You know if you go to lecture number 17, you will find the response after load transient. It becomes very sluggish, like over damped response, because it virtually you behave like a first order system because you have made the current to be a control source. So, there is the

inductor pole as if you pushed it very far to the left hand side that dynamics is not coming into picture.

But, another important point I will show you what is the gain DC gain. So, DC gain means, you have to put it that mean you take $G_{vc}(s)$ and substitute s equal to 0 and we will get simply R . So, DC gain is a load dependent term and you will see that is a major problem in current mode control. It has a pole load regulation ok. So, it is $1/R$. The pole is $1/RC$ here. This ω_p is $1/RC$, ok.

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Output Impedance under CMC

Fixed current reference

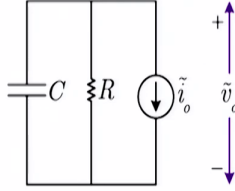
$$z_o = \left. \frac{\tilde{v}_o}{\tilde{i}_o} \right|_{\tilde{v}_{in} = \tilde{i}_c = 0}$$

$$z_o(s) = r_2 \parallel R \parallel \frac{1}{sC}$$

So, output impedance. If you consider output impedance, we know the output impedance is actually there should be a negative sign here. Z_o is minus v_o tilde by i_o tilde where we are not taking this one and we are also not considering. That means, we are keeping the current reference constant, input voltage constant, but we are exciting the converter from the external current source by seeing current. That means, sinking current ok. Since the current is sinking, that is why the negative current negative sign is coming. And this will behave like a parallel combination of r_2 , C and R .

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Output Impedance of an Ideal Buck Converter



$$z_o(s) = \left. \frac{\tilde{v}_o}{\tilde{i}_o} \right|_{\tilde{v}_{in}=\tilde{i}_c=0} = R \parallel \frac{1}{sC} = \frac{R \times \frac{1}{sC}}{R + \frac{1}{sC}}$$

$$z_o(s) = \frac{R}{1 + \frac{s}{w_p}} \quad w_p = \frac{1}{RC}$$

$G_{wc}(s) = Z_o(s)$



And it turns out to be for a buck converter. It will be simply this and this is; that means, in buck converter, the control to output transfer function is virtually same as the output impedance with close current loop with a constant current reference; that means, with close current loop only.

So, it will represent a first order system. The output impedance under current mode control also represents a first-order system, like a RC circuit, but in case of duty ratio control, we have an RLC circuit. So, the inductor was there and we are getting a getting a q peaking effect. So, it was a complex conjugate pole right it has and in that case we also got you know parasitic pole field we ignore.

There will be one 0 at origin in case of an ideal buck converter, but in case of practical buck converter, this is the output impedance. But, output impedance is heavily dependent on the load resistance that means, if there is any change in load resistance, their output impedance DC value will change shift up and down.

And that is why the major drawback in current mode control is the load transient load regulation problem. Because we know that from the voltage source concept that we have discussed in lecture number 13 that if you have an output impedance which is load dependent, then you have a real problem because the regulation will be severely affected by this. So, in that case, we can improve the load regulation by closing the outer loop and putting


an integrator, but still the response will be very slow because the loop gain will also depend on the load resistance.

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Control-to-output Transfer Function of an Ideal Boost Converter

$$f_2 = D' \left(1 - \frac{sL}{D'^2 R} \right) \quad r_2 = R, \quad g_2 = \frac{1}{D'R} \quad r_2 \parallel R = R \parallel R = \frac{R}{2}$$

$$r_2 \parallel R \parallel \frac{1}{sC} = \frac{\frac{R}{2} \times \frac{1}{sC}}{\frac{R}{2} + \frac{1}{sC}} = \frac{\frac{R}{2}}{RCs + 1} = \frac{\frac{R}{2}}{1 + \frac{s}{\omega_p}}$$

$$\omega_p = \frac{2}{RC}$$


So, control to output transfer function of an ideal boost converter if you take the canonical model if you derive.


(Refer Slide Time: 32:35)

Control-to-output Transfer Function of an Ideal Boost Converter

$$G_{vc}(s) = \frac{(1-D)R}{2} \times \left(\frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right)$$

Handwritten notes:

- $\omega_z = \frac{(1-D)^2 R}{L}$ (circled in red)
- $\omega_p = \frac{2}{RC}$ (circled in red)
- $G_{vc}(0) = \frac{R(1-D)}{2}$ (CMC - no outer loop)
- $G_{vd}(0) = \frac{V_{in}}{(1-D)}$ - duty control
- ω_z is a **RHP zero**.
- ω_z is **same as direct duty ratio control**.



And you will get the control to output transfer function. This is a constant term and one pole one zero. The pole is 2 by RC; that means the boost converter originally in duty ratio control.

There are two poles which become now a first pole. One pole because inductor is replaced by as if like a virtual current source.

But what is important here is the location of the right. There is it has a right half plane zero, so; that means RHP zero; that means, even with close current loop, we cannot avoid RHP zero because that is the physical property of the converter. By means of control you cannot get it maybe effect can be reduced.

But in this case, the location of the RHP zero exactly the same as direct duty control direct duty ratio control; that means, when you take the duty ratio control when you talk about the control to output transfer function by varying the perturbation of the duty ratio. Here, we have closed the current loop. So, one benefit we got our current pole become a single pole, but you cannot avoid RHP zero, it is there.

But at the same time, if you see the closed loop DC gain, it is $R \frac{1-D}{2}$ and that is the problem. Because the closed loop DC gain, it depends on the resistance and if you take this is for current mode control with closed current loop. In case of that means, no outer loop ok.

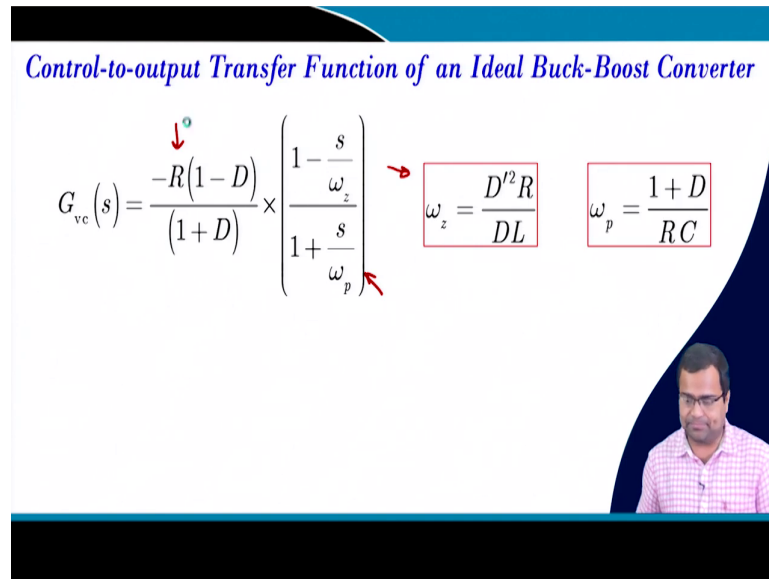
But if you take if you recall the G_{vd} of a boost converter, it can be shown that it is coming to be $\frac{1-D}{V_{in}}$ in by D^2 square. This is for duty control, where it also depends on the duty ratio, but it depends on the input voltage. Here, it is independent of input voltage, but it depends on the load current load resistance.

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Control-to-output Transfer Function of an Ideal Buck-Boost Converter

$$G_{vc}(s) = \frac{-R(1-D)}{(1+D)} \times \left(\frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \right)$$

$\omega_z = \frac{D'^2 R}{DL}$ $\omega_p = \frac{1+D}{RC}$



So, the control to output transfer function of a buck-boost converter can also be derived in the similar way and we will also get a right half plane zero and it will also have a single pole. So, the best thing about this boost converter when we will design boost converter closed loop control. Because the boost converter has in duty control has two pole which are generally complex conjugates and one right half plane zero.

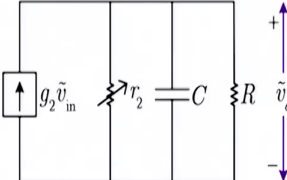
And the poles we will see because of complex conjugate pole, it has a poor phase margin, and it also has a bandwidth limit due to the right half plane zero. So, the boost converter has a real problem when you do voltage mode control and that is why it is very hard to control boost converter under voltage mode, particularly when you are considering a wide operating range.

But yes, in digital control if the life will be much easier because we have a flexibility to tune the controller. So, in ideal boost converter buck-boost converter that we can we will get the same RHP zero, but now the pole will be a single pole and you will also have a load resistance dependent gain ok.

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Audio Susceptibility under CMC

Fixed current reference $\tilde{i}_c = 0$



$$G_{vg}(s) = \left. \frac{\tilde{v}_o}{\tilde{v}_{in}} \right|_{\tilde{i}_c=0} = g_2 \times \left[r_2 \parallel R \parallel \frac{1}{sC} \right]$$

So, audio susceptibility you can derive using the canonical model.

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Audio Susceptibility under CMC

For a buck converter, $g_2 = 0$

$$G_{vg} = 0$$

For a boost converter, $g_2 = \frac{1}{D'R}$, $r_2 = R$

$$G_{vg}(s) = \frac{1}{D'R} \times \left[R \parallel R \parallel \frac{1}{sC} \right] = \frac{1}{2(1-D) \left(1 + \frac{s}{\omega_p} \right)}$$

where $\omega_p = \frac{2}{RC}$

But for a buck converter, it turns out to be audio susceptibility 0; that means, it has no response in the output when there is a transient in the supply. And in fact, we saw in lecture number 17 with current mode control when we apply a supply transient the effect was almost negligible, but it does not mean that it should be exactly 0 there will be slight effect, but it is almost negligible.

So, the current mode control inherently achieves very like excellent line regulation and audio susceptibility. So, the boost converter again we can derive you know audio susceptibility. It is not exactly 0 for a boost converter, ok. It has a pole effect. So, if you change the supply voltage for a boost converter, there will be some first order effect as well.


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Audio Susceptibility under CMC

For a buck-boost converter, $g_2 = \frac{-D^2}{D'R}$, $r_2 = \frac{R}{D}$

$$G_{v_g}(s) = \frac{-D^2}{D'R} \times \left[\frac{R}{D} \parallel R \parallel \frac{1}{sC} \right] = \frac{-D^2}{(1-D^2) \left(1 + \frac{s}{\omega_p} \right)}$$

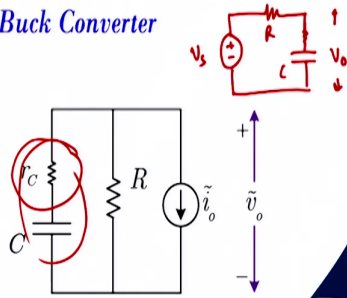
where $\omega_p = \frac{1+D}{RC}$




So, for the buck-boost converter, you can also derive.

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Output Impedance of a Practical Buck Converter

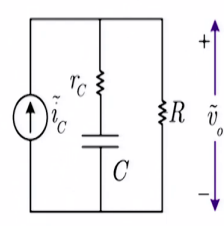
$$z_o(s) = \frac{R \times \left(1 + \frac{s}{w_{ESR}} \right)}{\left(1 + \frac{s}{w_p} \right)}$$


$$w_p = \frac{1}{RC}, \quad w_{ESR} = \frac{1}{r_c C}$$


So, output impedance of a practical buck converter with one ESR will be added. And whenever you have an ESR with a capacitor, this will definitely introduce a 0 and that is exactly coming ok. That means, in some sense, suppose if you take a simple you know source, suppose this is one circuit and if you take the output to this is supply. If you obtain the transfer function between V_0 to V_s , you will get only a single pole no 0, but suppose if we introduce a resistance in between and then you will get one 0 one pole ok and that this is exactly is happening here.

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Control-to-Output TF of a Practical Buck Converter

$$G_{vc}(s) = \frac{R \times \left(1 + \frac{s}{w_{ESR}}\right)}{\left(1 + \frac{s}{w_p}\right)} = z_o(s)$$


DC gain of the control-to-output TF $G_{vc}(0) = R$

Load dependent DC gain – poor load regulation!!

Approximate first-order model – robust compensation

So, our practical buck converter also has a ESR 0 and one pole and it has a DC gain of load resistance and same as the output impedance ok. So, the DC gain is load resistance dependent. So, it is a poor load regulation and this can be anticipated you know though can be used our integral action. But whenever there is a large load step transient, the response becomes very very sluggish even though you put an integral action.

So, in order to achieve first order transient you may have to give a very high integral gain, but that is something not desirable because that very high integral gain sometime may saturate your compensator error amplifier. But the question, the approximate first-order model because there is no you know inductance like a double pole so, it is much easy to compensate, so it is a robust.

In fact, I will show you one you know case study. In voltage mode control we can mathematically we can get a closed loop first order response, but that response is not valid

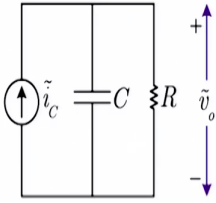
even beyond a certain bandwidth range whereas, the current mode control the first order response can be valid a wide range that also I will show.

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Control-to-output Transfer Function of an Ideal Buck Converter

$$G_{vc} = \left. \frac{\tilde{v}_o(s)}{\tilde{i}_c(s)} \right|_{\tilde{v}_in=0} = f_2 \times \left[r_2 \parallel R \parallel \frac{1}{sC} \right]$$

Find G_{vc} for an ideal buck converter



$$G_{vc}(s) = R \parallel \frac{1}{sC} = \frac{R}{1 + RCs}$$

$$G_{vc}(s) = \frac{R}{1 + \frac{s}{w_p}} ; w_p = \frac{1}{RC}$$


So, control to output transfer function of a buck converter ideal and we can derive it.

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Control-to-output Transfer Function of an Ideal Boost Converter

$$G_{vc}(s) = \frac{(1-D)R}{2} \times \left(\frac{1 - \frac{s}{\omega_{rhp}}}{1 + \frac{s}{\omega_p}} \right)$$

$\omega_{rhp} = \frac{(1-D)^2 R}{L}$


$\omega_p = \frac{2}{RC}$

Load dependent DC gain – poor load regulation!!

RHP zero location – same for both VMC and CMC!!

Trade-off between RHP zero and pole locations – better in CMC

Handwritten notes:
 $f_c = \frac{f_{rhp}}{5}$ VMC
 $= \frac{f_{rhp}}{3}$ CMC

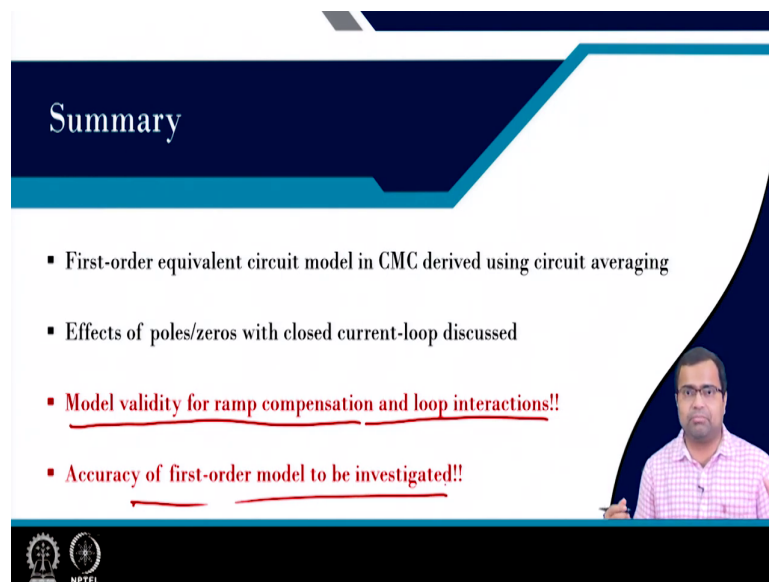


And we can derive for an ideal boost converter, the RHP zero. The load dependent DC gain results in poor regulation, RHP zero location same as both voltage mode and like a duty ratio

control and current mode control. And trade up between RHP zero and pole location better. This part will take up when we go to you know design a voltage and current mode.

And we will see for voltage mode generally the cut off frequency is kept rhp zero by 5, this is for VMC. This is a standard kind of thumb rule, but we will come to through our analytical design process why it is so, for a conservative design. Whereas, in current mode control, it is f_{rhp} by 3, maybe a reasonable choice and that also will come through the analytical design process for a boost converter. So, we can get a better trade off or bandwidth using current mode control, but it cannot get rid off the RHP zero behaviour.

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Summary

- First-order equivalent circuit model in CMC derived using circuit averaging
- Effects of poles/zeros with closed current-loop discussed
- Model validity for ramp compensation and loop interactions!!
- Accuracy of first-order model to be investigated!!

So, with this I want to summarize. The first order equivalent circuit model in current mode control was derived using circuit averaging technique. Effects of poles and zero under closed loop closed current loop discuss we have discussed. But the model validity with ramp compensation is a big concern and we will see that this model will not be valid when you add ramp compensation and when both the inner and outer loop will start interacting this model.

So, we need to you know go for improved model and also the accuracy of first-order model we need to investigate because it looks really nice first-order model, but I mean we can replace the inductor with a control current source. But we have to imagine even if you change a very first load current reference, the inductor current will not respond immediately because it has a finite slew rate.

And that practical effect as well as it will have some sampling effect we will introduce some other you know behaviour where we can improve the model accuracy by incorporating those sampling effect by incorporating the modulator gain with ramp and so on, so, that we will investigate in the future lecture. So, with this I want to finish here.

Thank you very much.