Control and Tuning Methods in Switched Mode Power Converters
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Module - 05
Modeling and Analysis Techniques in SMPC
Lecture - 28
DC Analysis Using Equivalent Circuit Model

Welcome, this is lecture number 28. In this lecture, we are going to talk about DC Analysis Using Equivalent Circuit Model.

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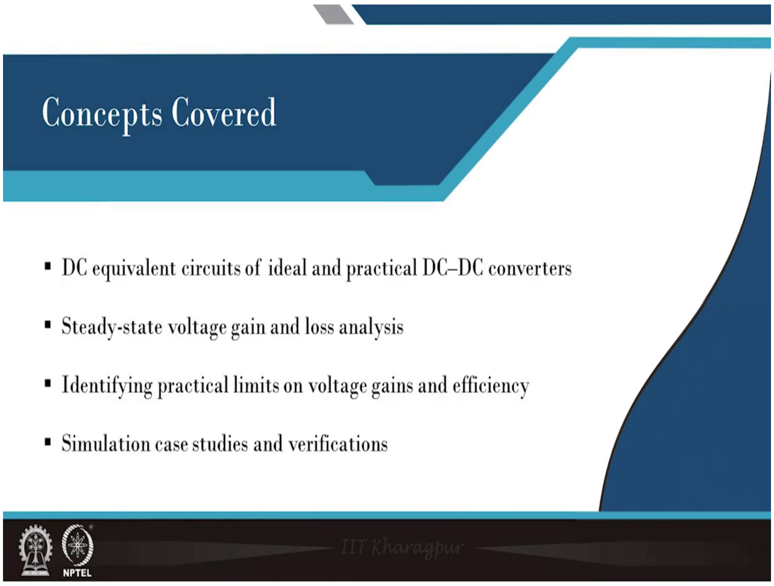

So, the concept here we are trying to derive the DC equivalent circuit of ideal as well as the practical DC-DC converters. Then we need to find the steady-state voltage gain and also we need to carry out loss analysis. We also need to find out what are the practical limits on voltage gains as well as the efficiency ok. And then few simulation case studies and verification.

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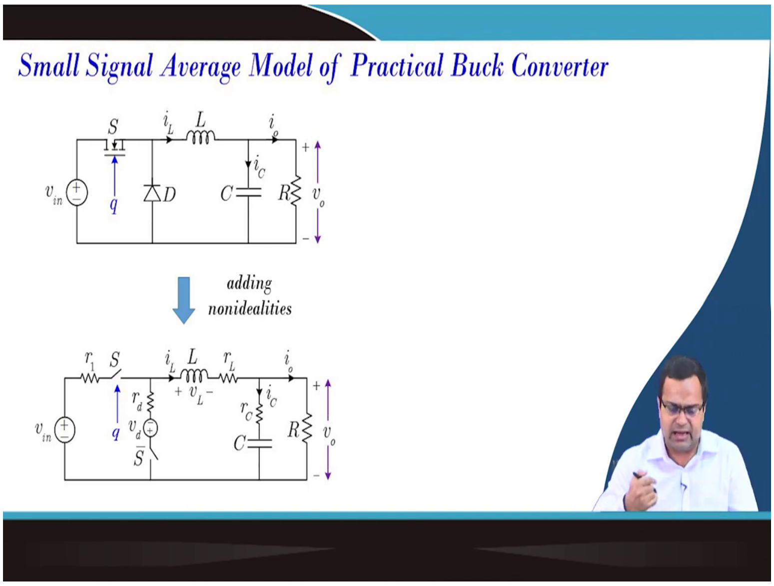

So, before we start, we need to consider start with the point. And in the previous class, we derive the small-signal model from large-single model. I will say which comprise both DC as well as AC both using state space averaging and circuit averaging techniques in the first two lectures. So, we want to continue some of the results that we have shown in the previous lecture where we have obtained some equivalent circuit using either circuit averaging technique or we also can use average switch model ok.

So, if you take a conventional buck converter, then we can write down its realistic model just considering the parasitic drop resistance of the MOSFET; and for the diode, the resistance of the diode as well as the diode drop, and also the DCR of the inductor, DCR of the capacitor.

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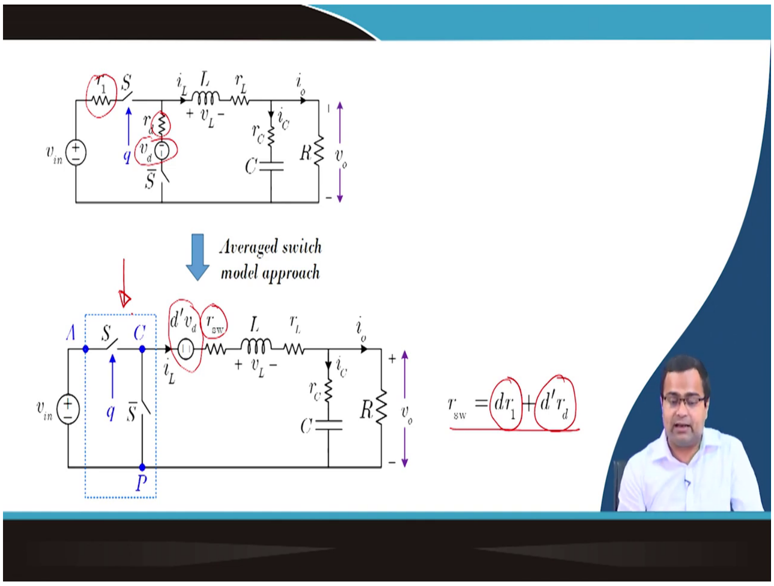

Then we also found the equivalent circuit of this conventional buck converter in the previous lecture, and this can be obtained by you know by if we want to apply the equivalent switch model, that means, sorry you know the average switch model, then we need to make this switch you know ideal switch, that means, all the parasitic has to be taken right side that we have taken.

You say this v d which is there, it actually came here with 1 minus d because this is activated during the off time. This resistance is activated during the on time, and this resistance is activated during off time. That is why you can see this effective resistance is combination of d into r 1 plus 1 minus d into r d.

So, I can separate out this term and I can make the switch ideal. Then we can easily obtain apply the equivalent switch sorry average switch model, and then we can derive the equivalent circuit model using average switch model.

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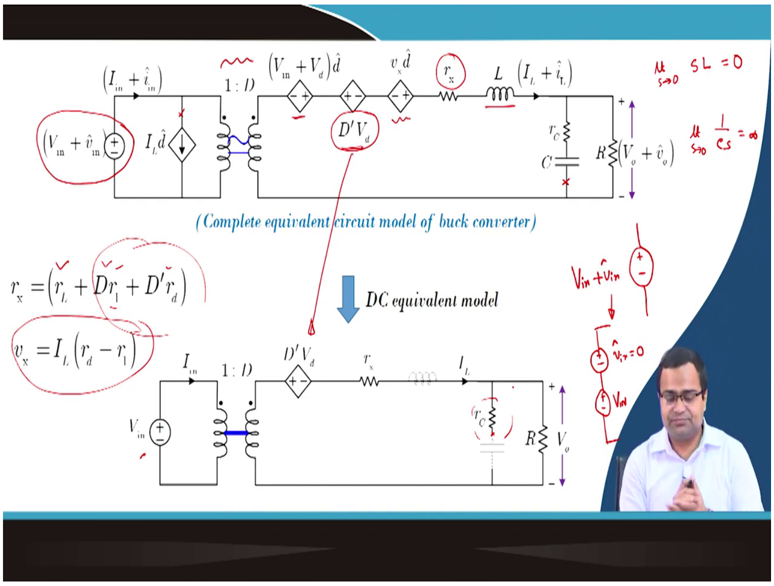

So, in this model, we already have discussed in the previous class that how to obtain the perturb term, how to deal with the DC term. And this represents the duty ratio of the buck converter using a DC transformer. Here, the r x can be shown to be the inductor of the resistance plus D into r 1 that is the on-state resistance and 1 minus D into r d.

So, now, if r 1 and r d are same, let for example, if we take a synchronous buck converter where the on-state resistance of the high side switch and the low side switch if we take nearly equal, then this r x will be simply r l plus r 1 because D plus 1 1 as d will be 1. And they are here. We can drop this diode drop in a synchronous DC-DC converter that also you can do.

So, this is a more generic model which can be used to derive the model for the synchronous buck converter, where v x can be obtained this v x that is r d minus r 1. So, if these two are identical, this term we will get 0 becomes 0 ok. And if these two are identical, then this term will also become r 1 which is equal to r d. So, it is r L plus r 1.

The DC equivalent circuit in order to obtain DC equivalent circuit from this model, it is very easy because we know the DC model. For the inductor, the impedance is S L; for the capacitor, it is 1 by c s we know. So, if limit s tends to 0, that means we are operating at very low frequency, this will eventually become 0. And if you put limit s tends to 0, so this will become infinity. That means this circuit should be open and this circuit should be shorted. And this is exactly what is the DC equivalent circuit. And that is the beauty of this technique that we can very easily obtain the equivalent DC circuit simply by shorting the inductor and opening the capacitor.

Next, all the perturb term should be set to 0 because if you take this particular term, for example, it consist of two terms. One is the input, which is a DC term plus V in. So, if this is our source ok, this can be divided into two parts, that means, it is basically the sum of two sources. One is our capital V in and another is our v in hat ok. So, that means, it is just the sum of two by using superposition. Now, under DC equivalent circuit, all the perturb terms is set to 0. So, for a voltage source, it will be shorted; for current source, it will be open.

So, we did it, that means, we shorted the perturb term so is set to 0, so it is simply V in. We open this part because d perturb a tilde d is 0. So, the current source will be open. We shorted this term. So, but this is a constant term. So, it is there ok. Then this term is also perturb term, this will be shorted because it is a voltage source. And this r x is already there, it is already a resistive term. Inductor is shorted, and capacitor is not there. So, this resistance has no meaning because this is not part of the circuit as a load resistance. So, we obtain the DC equivalent circuit from the equivalent circuit model.

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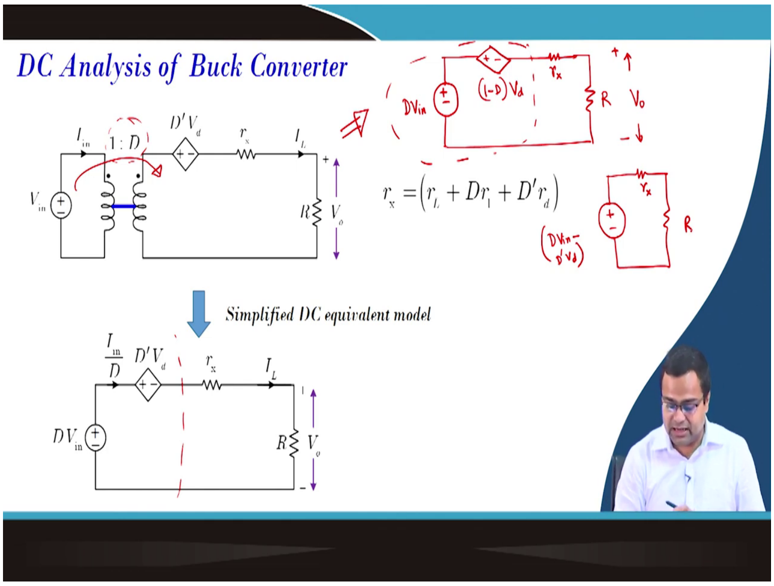

Next, once we have the DC equivalent circuit, now we want to analyze. What can we analyze? We need to find out the voltage gain. So, now, this is a simplified form of the DC equivalent circuit. In fact, if you draw this circuit, this circuit can be redrawn where r x is given as because I can take this source to the side by suitably taking into account the transformer gain – voltage gain. This will be our DC equivalent term here. It is D into V in ok, and there is another source. This is 1 minus D or D dash into V d diode drop. Then we have some r x resistance, and this is our load resistance R, and this is our V 0 plus minus.

Now, how to obtain what is the effective voltage here? This will be D V in minus 1 minus D into or D dash into V d ok. So, the simplified circuit, I have shown you that this particular term you can further simplify as this is the net voltage, which is D V in minus D dash V d. This is my voltage. And I have r x here, and R here.

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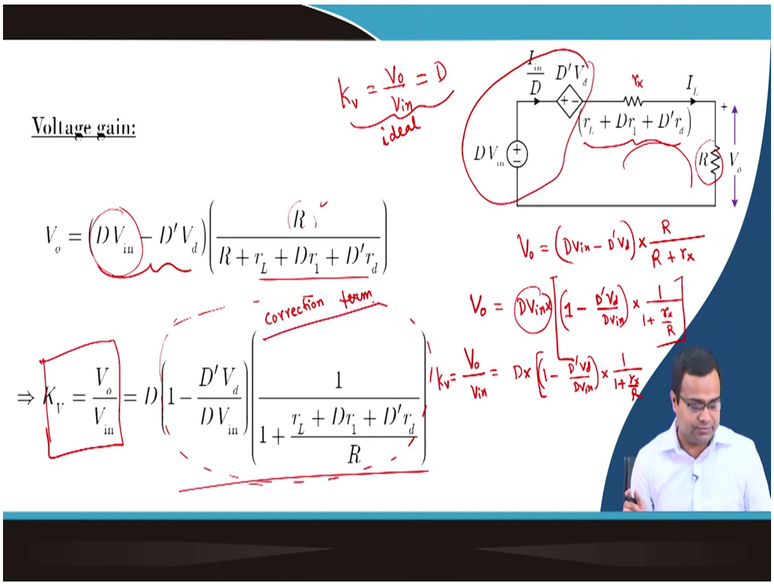

Then I can obtain this voltage, that means this voltage can be obtain by here. So, this is the term I told you the subtraction D into V in these consist of these two terms. And then this is r x which is nothing but this term. And the voltage here will be R divided by the total resistance, R plus r x. That means our V 0 will be D into V in minus D dash into V d. This is a term multiply the resistive divider R plus r x, R plus r x.

Then what is the voltage gain? We know the voltage gain of any DC-DC converter, voltage gain is the output voltage by input voltage right. So, this is exactly it is the output voltage by input voltage. And if you take V in out of I mean if you take V 0 out of this particular term, what will get? First of all you take D into V in at the beginning right. So, that means, I am taking that means, here from here to here V 0 by V 0 equal to D into V in multiplied by 1 minus D dash V d by D V in this term into now if I divide R below, so it will be 1 plus this divide by R. So, that means, 1 divided by 1 plus r x by R, simply this term.

And if we take this, this thing, so V 0 by V in which is my k v is equal to D times 1 minus D dash v d by D V in this whole thing 1 by 1 plus r x by R. So, this is exactly this. So, this is my correct turn factor because in an ideal buck converter, for ideal case it is equal to D for ideal case. That means, when there is no diode drop, there is no passive resistance, so it is simply the duty ratio. The voltage gain is simply the duty ratio. But in practical case, this is something like a correction factor or correction term. This should be multiplied to get the suitable voltage ok.

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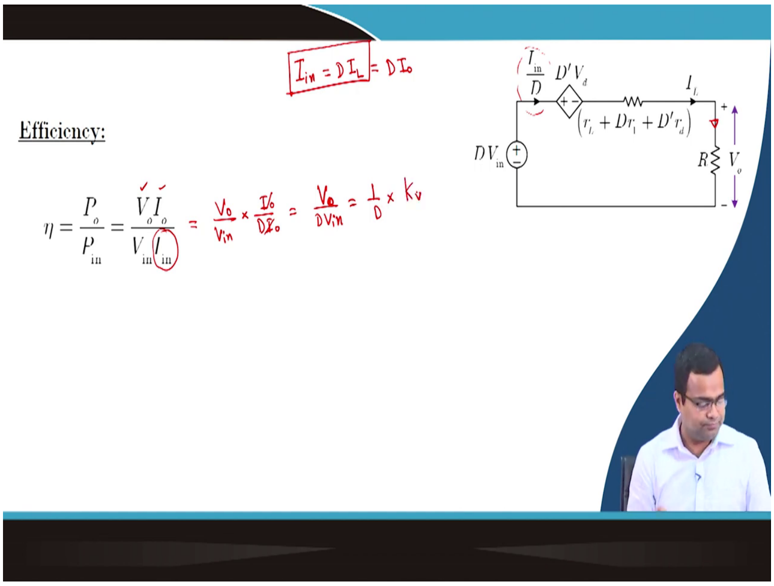

Now, what will be the efficiency? So, efficiency will be output power by input power. What is the output power? So, here the average current will be I in by D. So, what is I in? I in we can find, I in for a buck converter is D times I l average, which is nothing but the load current right. So, you know this. And this is nothing but the D time load current because it is going to the load.

For a continuous conduction mode buck converter, the average inductor current is same as the average load current because the capacitor average current is 0. Next, so this is fine. What about this? If you replace with this, you will get V 0 by V in into I 0 by D I 0. And if you cancel it, you will get V 0 D into V in or it is simply 1 by D into K v ok.

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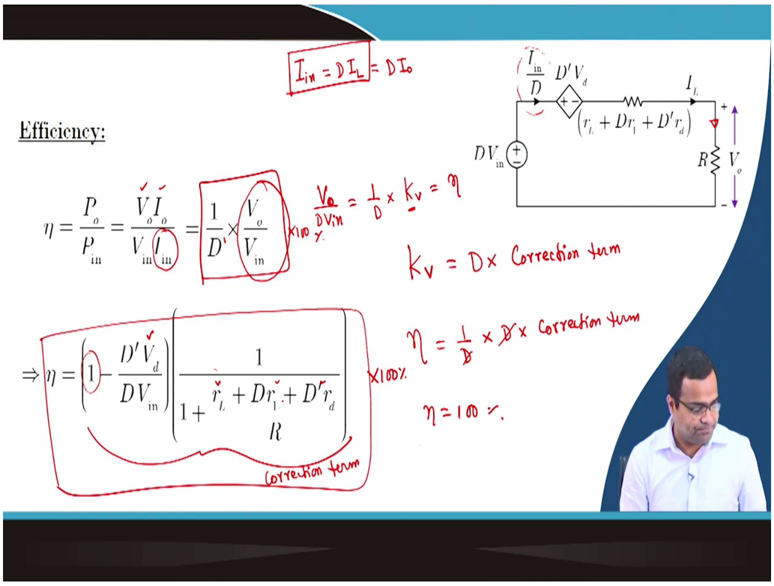

So, it is written here ok. So, I can erase this particular part. So, V 0 by V in that mean this is my efficiency. This is my efficiency. So, this term we have already obtained. Now, you substitute the voltage gain, this is a voltage gain. If you substitute the voltage gain, voltage gain already has 1 by D because we know from the voltage gain in the previous slide we saw D into the correction term right, correction term. And this is my correction term.

So, your efficiency under ideal condition you can set V D to be 0. And if you set this all this drop to be 0, so if you set V D to be 0, then this will be only 1. And if you take all the parasitic drops to be 0, this term will be 1. So, efficiency will be 100 percent. That means we should multiply with 100 in terms of percent. If we want in terms of percent, then we can multiply 100 to get percent. But in practice, it is much low due to the diode drop, due to these parasitic resistances.

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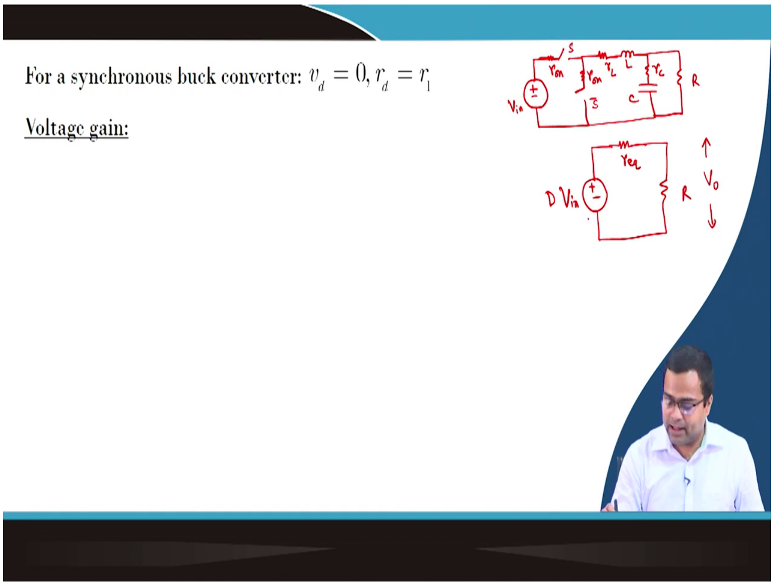

Now, if we take a synchronous buck converter, where we are not considering a diode, that means in synchronous buck converter, if we draw the actual circuit, so this is like this. And we are assuming the on resistance to be identical for both the cases identical. For simplicity, this is my switch, this is my switch bar, this is my inductor. Then this is my capacitor esr, this is my L, C, and this is my load resistance.

Now, the DC equivalent circuit will be what that we saw. So, this is my V in. So, this equivalent circuit will be my capital V in, sorry capital V in. And this resistance is my r equivalent, and this is my resistance, and this is my voltage.

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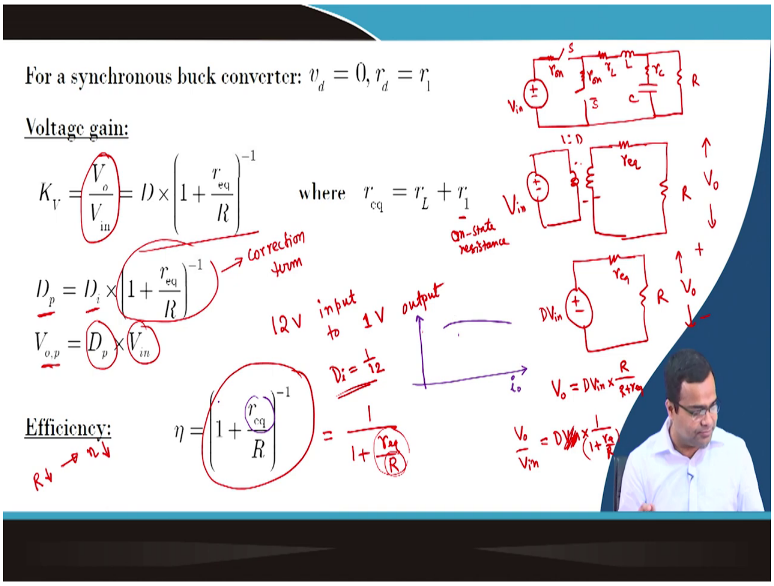

Sorry, there is a transformer term because actually, yeah. So, here there is a factor a DC factor which is 1 is to D, and this side you have V in. That means, if you transform it will be D times V in r equivalent to R, and this is my capital V 0 ok. So, that means, my voltage gain which is V 0 by V in. So, if you write V 0, what will get? You will get D into V in my into R by R plus r equivalent right of which is nothing but D times V in to 1 by 1 plus r equivalent by R ok.

And this is exactly what we are showing because we are taking V 0 by V in if you take. This term will get cancelled. So, we are taking the left side, so D times this. r equivalent is so these are on-state resistance.

So, here it is the on-state resistance of the, on-state resistance ok that means my practical duty ratio is D is the ideal duty ratio multiplied by this correction factor. This is again the correction factor, the correction term. And so what we can expect the practical output voltage will be input voltage into practical duty ratio because suppose I want to achieve 12 volt input to 1 volt output. For ideal, case I tend to set 1 by 12 is a duty ratio.

But practically for ideal converter, you will get I will show you 1 volt output, but practically you will never get when particularly the load current changes ok. And the efficiency of this converter can be simply the correction factor. So, that means, you can see the efficiency because this is nothing but 1 by 1 plus r equivalent by R. That means, if the load current increases, this term will decrease. And if the, this term decreases, this term will increase. And as a result, efficiency decrease ok.

So, that means, we can predict that if for R decreases, that means, higher load current implies the efficiency also decreases, that means, for higher load current efficiency decreases. And that is why we have discuss earlier that if we consider a load current plot, that means, if we have resistance if you plot load current under continuous conduction mode, it will decrease because this is due to the conduction loss and which can be model by this equivalent resistance.

But this is a DC term. We have not incorporated the ripple current, only the DC term ok. So, for accurate loss, we need to go for you know RMS current calculation and all, but this is just the DC equivalent, ok.

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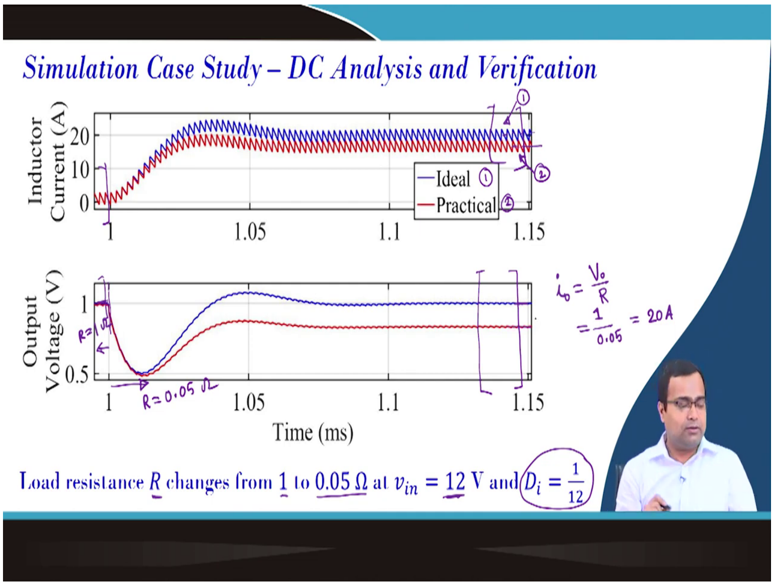

Now, I want to show a simulation case study. Here I am talking about an ideal synchronous buck converter and a practical synchronous buck converter. Where the blue trace is the one which is the ideal one and it is the 1. And the red one is the practical one, which is number 2, this is number 2. What is my operating condition? Here I made a load change here. So, I am showing you all the details. So, load resistance changes from 1 ohm to 0.05 ohm. So, this side our R was 1 ohm ok. And from this side onward, R becomes 0.05 ohm.

For the time being, do not consider this transient effect because that we are going to discuss in the subsequent lecture. But for the time being, let us consider the steady-state, let us take I am talking about this particular window, ok, this particular window. Here also I am talking about this particular window, where we can think of steady-state this particular window.

What we will find? Firstly, our objective to achieve 1 volt output for 12 volt input; and that is why I said the ideal duty ratio to be 1 by 12. But you will find for ideal condition, the output is actually coming to on average sense and our for 0.05 ampere load resistance, so what will be the load current? So, it should be i 0 should be my V 0 by R. So, if the V 0 is 1 volt and i 0 is 0.0 ohm, so it will be 20 ampere.

So, my average inductor current is 20 ampere that is shown here. But for practical converter, it is slightly it is much below, this is not 20 ampere. It is less than 20 ampere. And the output average voltage is much lower. How can we model? That means, we did already DC analysis.

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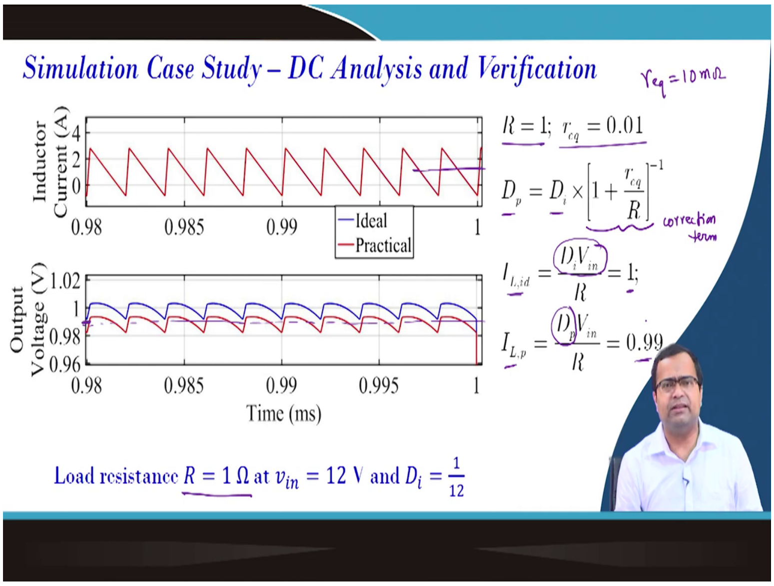

So, now, I am showing the zoom version of the condition, where the resistance was 1 ohm. Under this condition, the average inductor current seems to be more or less same, very close ok. But the average output voltage there is a slight deviation. So, can we analytically show what is the value? We know the load resistance is 1 ohm. And in our converter setting we took the r equivalent to be 10 milli, r equivalent to be 10 milli ohm, because 5 milli ohm is the DCR and 5 milli ohm is the on-state resistance. So, this is a totally 10 milli ohm.

So, what is my practical duty ratio? It is the ideal duty ratio multiplied by this correction factor right. We know about this correction factor, correction term. Then what is my practical average voltage, practical average current? For ideal average current, we know it is D times V in by R, or basically D times V in is my V 0 by R which is my average inductor current. And it is 1 ampere.

But for practical inductor current, it is D p that is a practical duty ratio input voltage and which is 0.99. And if you see the average of the rate, it is almost the 0.99, because the midpoint is a 0.9 which is more or less close to our predicted value because this is a midpoint, so which is in between 1 and 0.98. And average inductor current is almost, so sorry average inductor current, it is almost 0.98, sorry 98. So, they are more or less same like only maybe 10 milli ampere difference.

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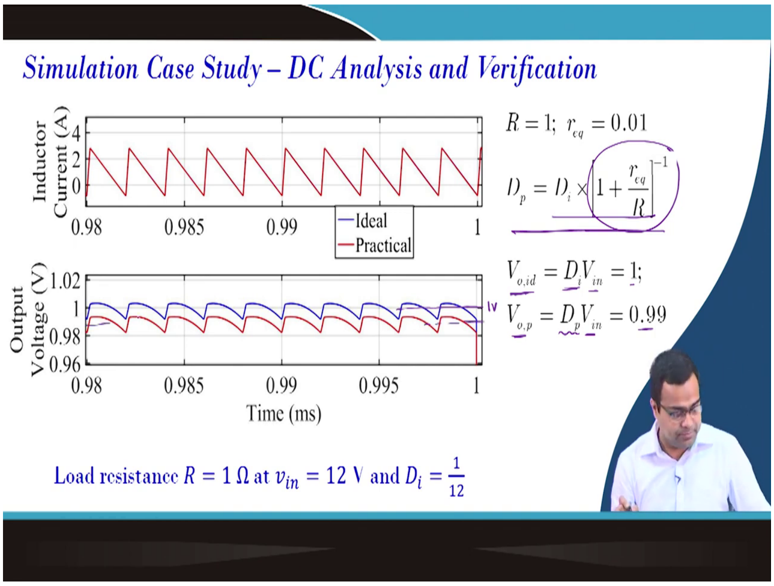

But now, under the same condition, we want to see the output voltage average. Again, the duty ratio calculation is given. Ideal output voltage is an ideal duty ratio in the input voltage that is 1 volt and that is coming exactly average value exactly coming to 1 volt. This is my 1 volt. But my practical output voltage can be calculated by V in multiplied by my practical duty ratio, which is this because this is my correction term. And it is 0.98, which is also consistent because it is also the midpoint ok, which is also consistent.

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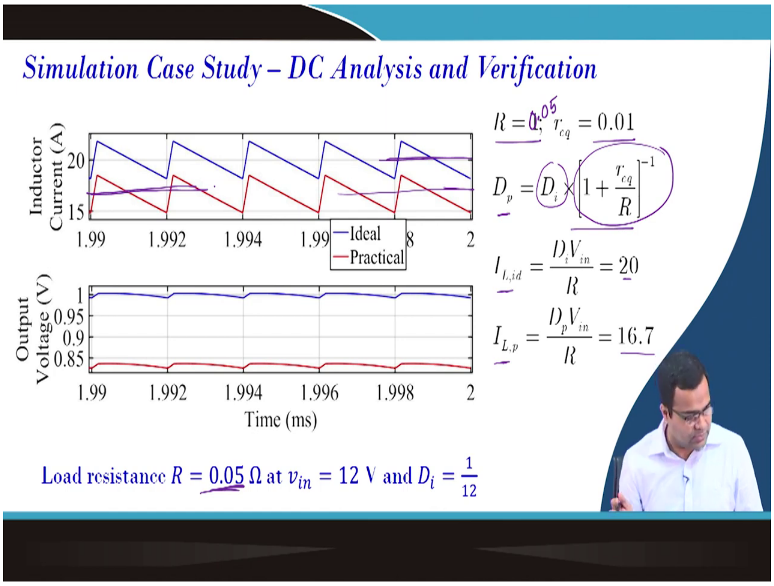

Now, can we make the same thing for the load resistance when it is 0.05 under 0.05? My R is 1, r equivalent is this; sorry here R should be 0.05 ohm. So, I can find out my D practical using this formula, this correction term multiplied by ideal duty ratio. So, my ideal inductor current is 20 ampere, which is the average value is same as this. My practical current is 16.7, which is coming to this average value average value is 16.7 which is consistent with our analysis.

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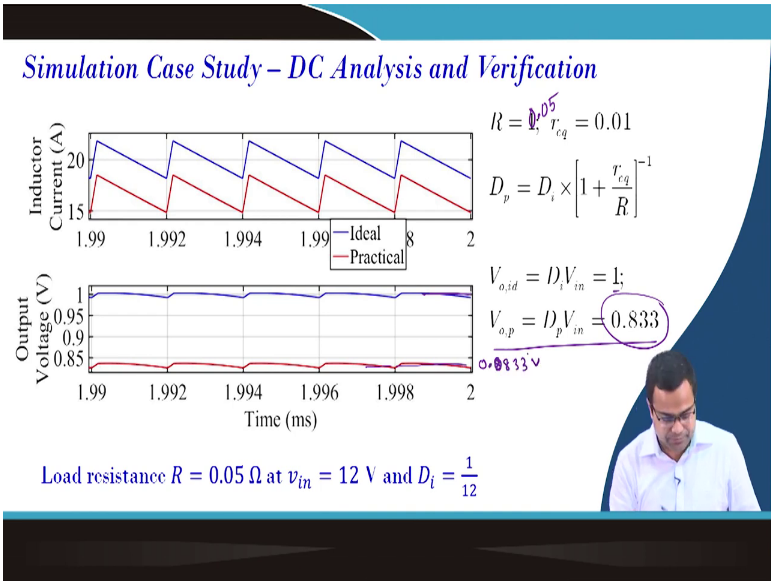

Now, coming to the practical output voltage, so, if you calculate, again it should be 0.05. Now, my ideal in output voltage is 1 volt, which is like this. But the practical it comes to be 0.833. That means, if we take this average, it is coming to be 0.0833 volt. That means we can predict eight. You know sorry so we can reasonably predict this output average voltage practical output average voltage from our DC equivalent circuit, and that we have shown and match with the simulation result.

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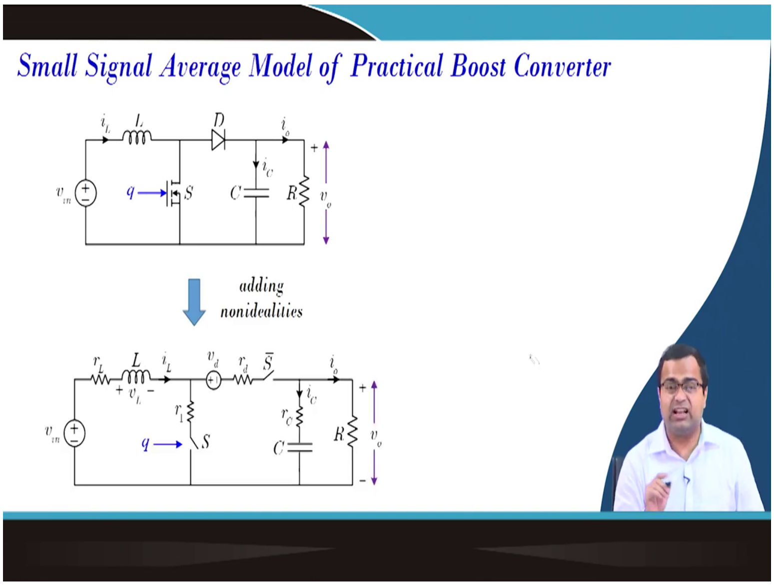

The next task we can do it for a boost converter. Again we start with the circuit. You can add all the nonideality.

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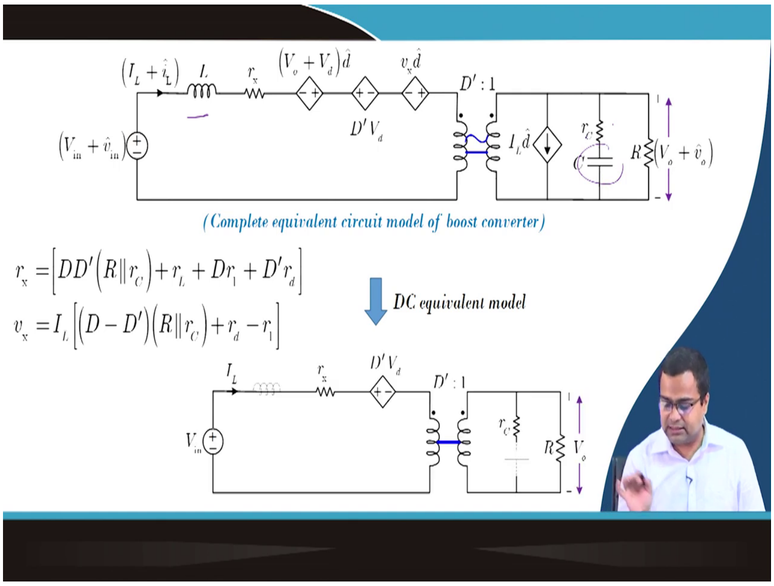

Then we can obtain the equivalent circuit that we have already derived from the previous class. So, I am not going to repeat. Then, in order to give the DC equivalent circuit, we need to short the inductor and open the capacitor, and that is the circuit is here.

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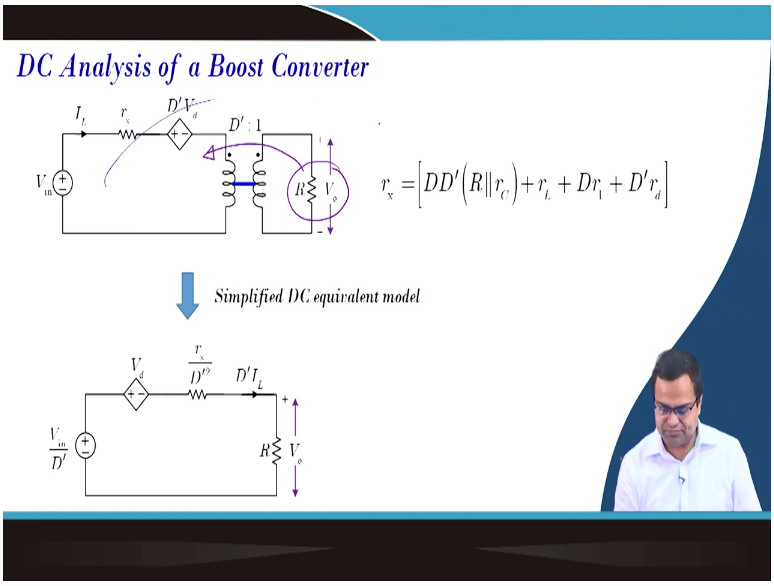

And then we need to make a suitable transformation. We can, in this case, we can transform this into this side. And then we can this is the simplified circuit, or here in this case we have transformed right to left, left to right. We can do any side.

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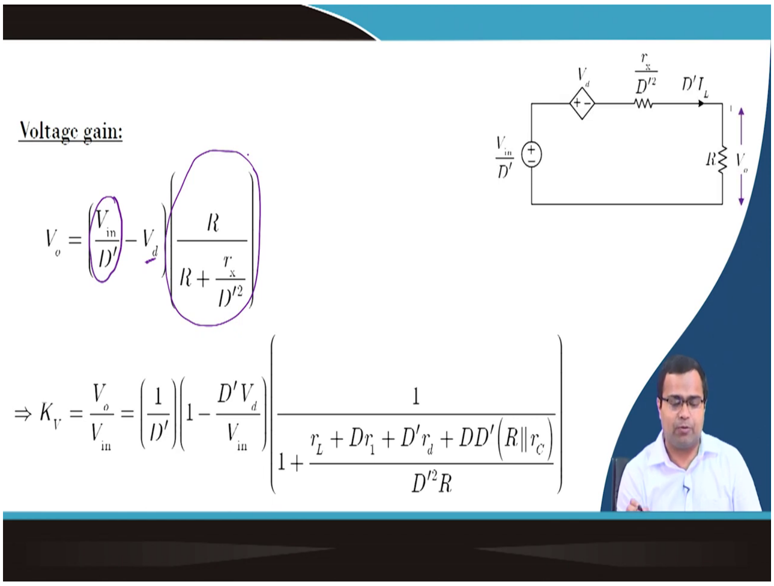

Then we can find the voltage gain in ideal boost converter, the voltage gain is V in by output voltage is V in by 1 minus D. But in practical, there is a diode drop into another term, which is due to the resistance drop of the parasitic that we also found. So, the expression can be obtain for this circuit.

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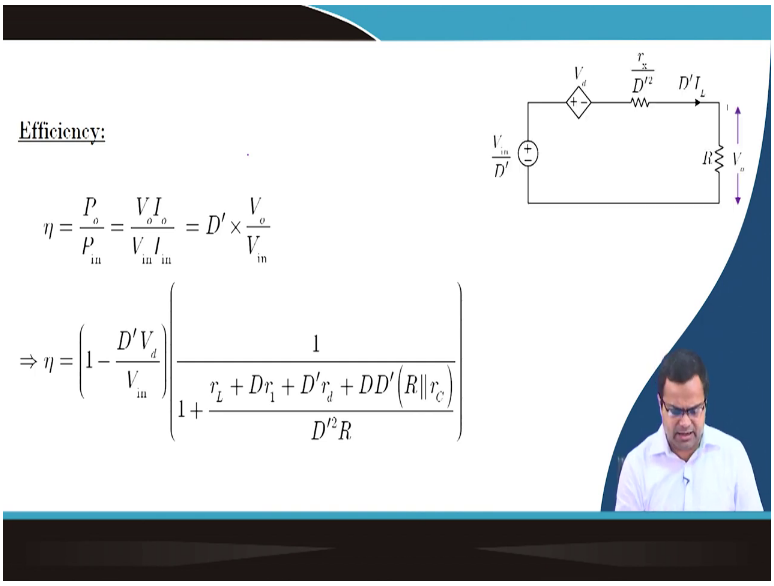

And if you do that efficiency calculation, then the efficiency will not be it will be much less than 100 percent due to this factor ok.

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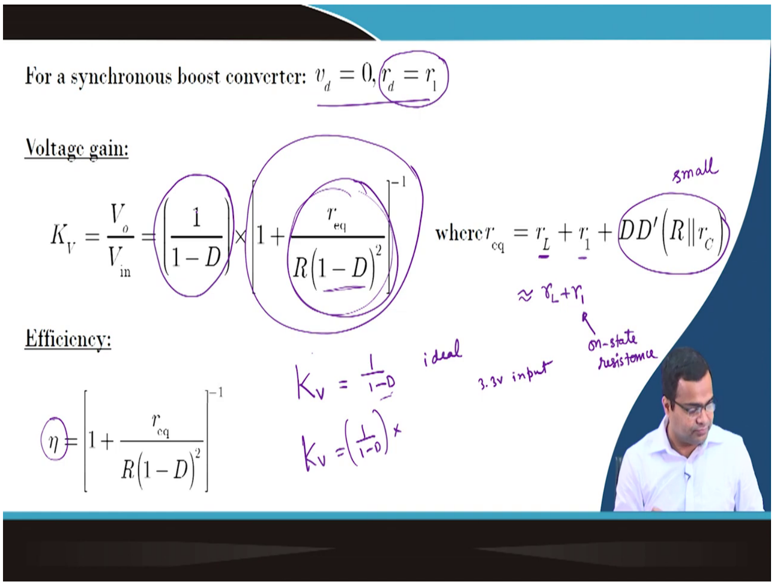

Now, another interesting point, if you take the voltage gain of the boost, so if you take a synchronous boost converter, where the diode drop is 0. And if we take the on-state resistance because we are talking about two switches high side and low side, and let us say both switches have a same on-state resistance. So, if we take that, then again we can write the efficiency here can be there is a correction factor, and this is a voltage gain. This is my ideal voltage gain of a boost ideal boost converter, but this is the correction factor.

And what is r equivalent here? r equivalent not only r L, r 1, but there is also an ESR term comes, but this is very small it is too small. So, it can be approximated as r L plus r 1, where this is my on-state resistance ok. Now, I want to show you an interesting point. Here if you see the expression the K v, for ideal converter we know K v is 1 minus D. This is for the ideal, which means if I increase duty ratio my voltage gain will increase, and I can achieve any output voltage of an ideal boost converter.

If we take, let us say, 3.3 volt is the input, then I can achieve even 300 volt in an ideal boost converter by suitably setting the duty ratio, but practically it is never possible. Why? Because in practical case it is 1 by 1 minus D it is there, but there is a strong dependency of duty ratio in this term. And when duty ratio increases, this term becomes smaller and this term becomes quite large. And your voltage gain will start dropping. And this shows a non-linear characteristic of the voltage gain.

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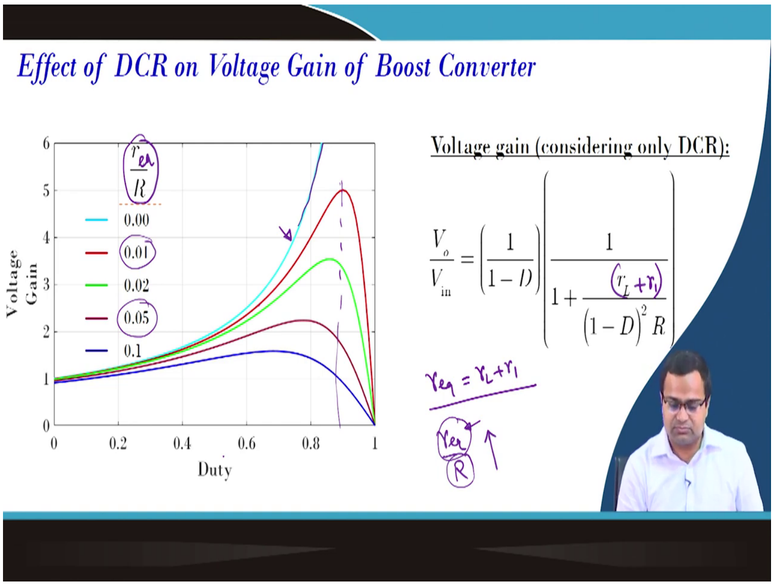

And if you plot the voltage gain for this synchronous boost converter for the ideal one which is here I am showing r L by R, or basically here you can think of r L or which is r equivalent, if you take r equivalent by R which you know we are ignoring for the time being the ESR effect. These r equivalent captures both on-state resistance as well as the inductor resistance ok. So, here this r equivalent, so I can also consider r 1 ok, so this r equivalent which is r L plus r 1.

If this is 0 that means if we do not consider any parasitic drop, you see that voltage gain can be increased like anything. But practically when this term comes into the picture, then this maximum voltage gain is achievable at this point. And these maximum points sit quite drastically, when this resistance ratio become larger and larger. And this is true when r equivalent by R, this ratio can increase if for a fixed value of r equivalent if the load resistance decreases, that means, as the load current increases the load resistance decreases the ratio increases, and then your voltage gain false.

Similarly, for the same load current, if this equivalent resistance increases, then also voltage gain fall. So, in a boost converter the practical voltage gain is limited by this parasitic drop, which is primarily due to the drop of the resistance, the drop of the on-state resistance for a synchronous boost converter for an ordinary like a conventional boost converter, it will also be a function of diode drop ok.

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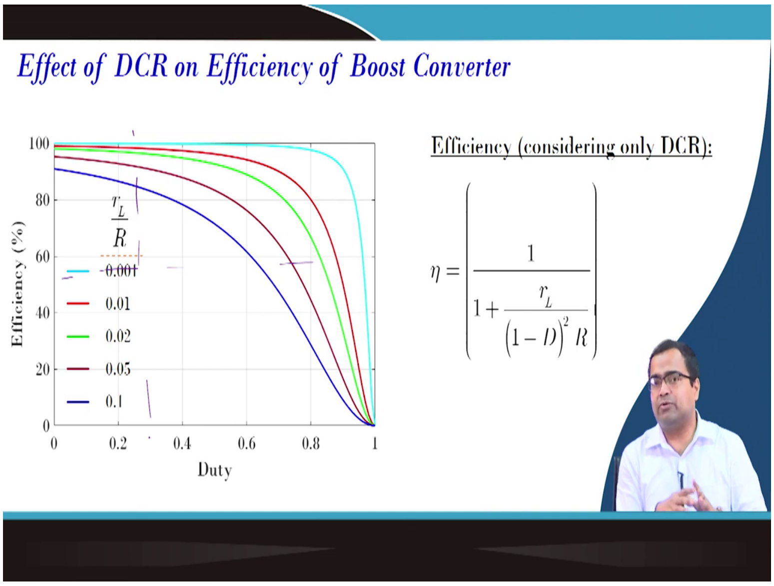

So, efficiency of the boost converter also we get significantly affected by this parasitic drop. And if you tend to achieve higher gain, you will lose the efficiency like anything, and nobody will you know because we want to achieve efficiency more than 90 percent so, which is achievable for a lower duty ratio operation. So, you have to be very careful about the selection of the voltage gain. And there are multiple research paper which is actually propose to increase the voltage gain with higher efficiency, so where the ordinary boost converter cannot achieve.

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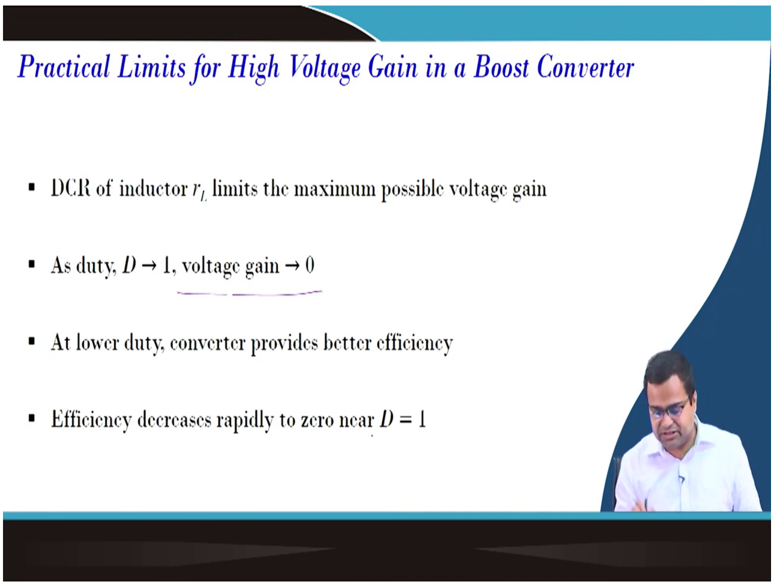

So, practical voltage gain in a boost converter DCR of the inductor limit, the maximal maximum possible voltage gain as duty ratio close to 1 voltage gain becomes 0. So, it start falling. And a lower duty ratio converter offer higher efficiency. So, for a high step of operation, there are many possibilities. One can consider of multiple cascaded state, but that also increases number of component, device, complexity.

One can go for isolated converter. And recently there is you know you know there is a research task to consider the flying capacitor boost converter multi-level where the number of flying cap if you put keep on you know adding, the voltage gain can be increase with some reasonably high efficiency. So, in a conventional boost converter, efficiency drastically decreases as the duty ratio increases that we have seen.

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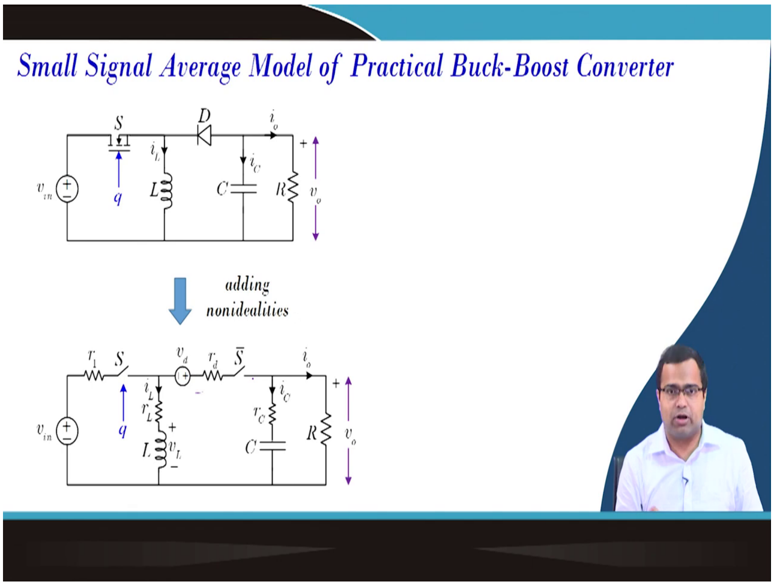

Same thing can be extended for a buck-boost converter. We have already derived the buck-boost converter. So, this is a realistic model.

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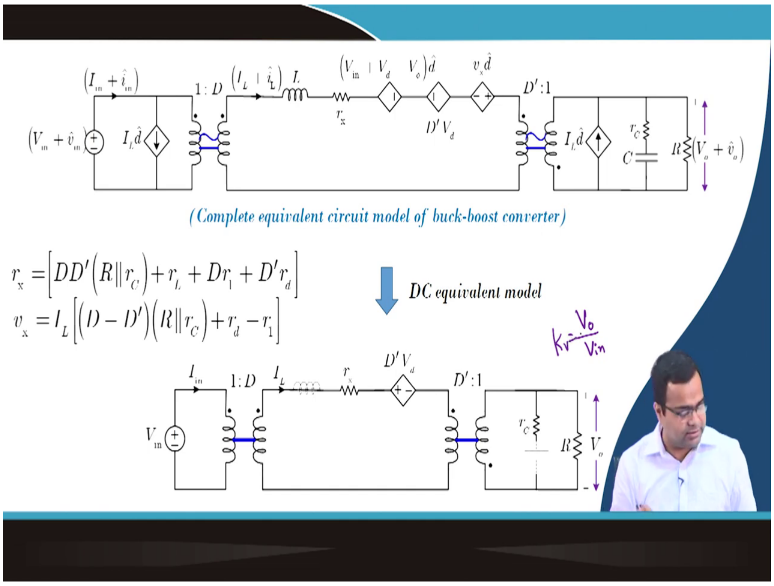

We have derived the equivalent circuit for a buck-boost converter where we need to achieve or we need to get the DC equivalent circuit because this circuit I have already shown in the previous class. Here what we need to do, we need to replace inductor by shorting it, and the capacitor has to be open. And if you do that, then the DC equivalent circuit can be obtain. And from this equivalent circuit, we need to find out what is my voltage gain, that means, my voltage gain that is my first point.

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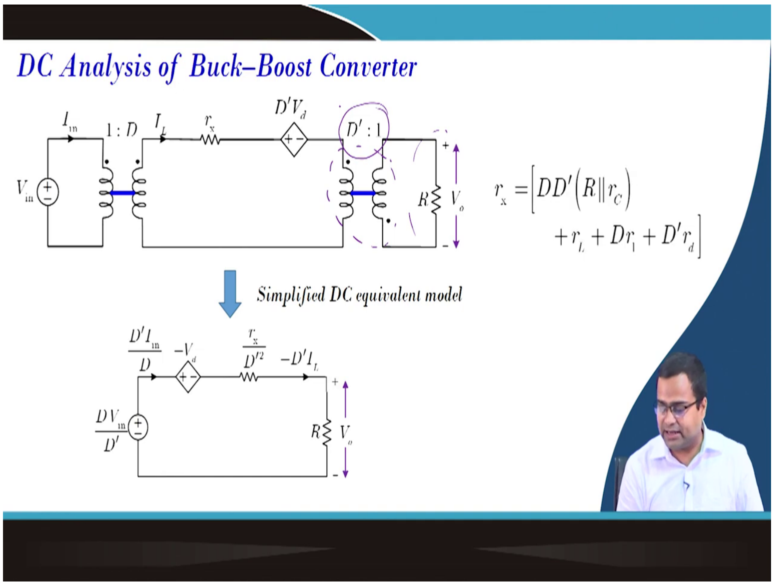

So, if we try to find out those, find the simplified circuit, so you can map back all these states to this side. And you will see there is a reverse dot convention, which takes into account the polarity. That means, if you take from right to left, the polarity has to be reversed along with the step-down ratio ok.

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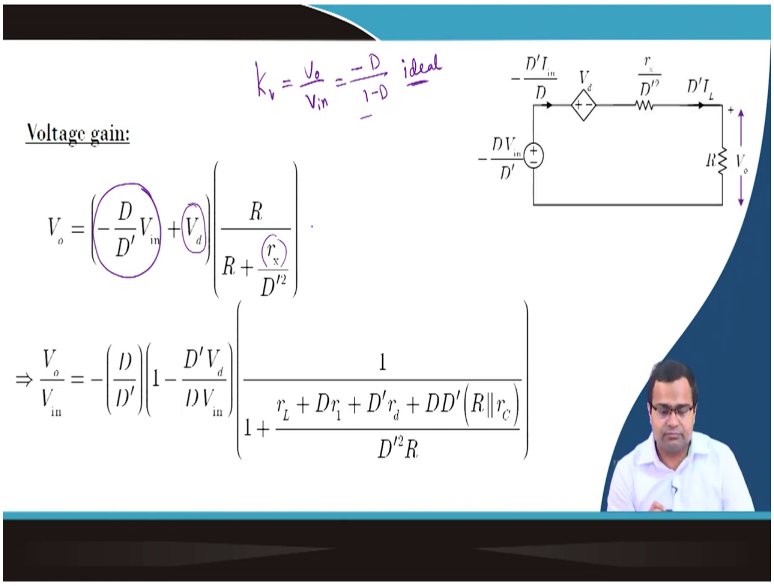

So, if we write down all this DC equivalent circuit, then we can find out the voltage gain. And here for an ideal buck boost converter, the voltage gain is simply D by 1 minus D ok for if you take K v which is V 0 by V in this is equal to minus D by 1 minus D in case of an ideal case.

And this is an inverting buck boost because there is a negative sign, and we know about this. But in a practical buck-boost converter, your voltage gain also get affected because of this diode drop, and because all of these parasitic resistance along with the duty ratio. And you can we can obtain all this expression, and we can carry out the same technique which we have done for the synchronous buck converter.

We can simulate this buck-boost converter as well as boost converter. And I will suggest that you verify the average value of the inductor current and the output voltage for different load condition using this analytical DC equivalent circuit, you can predict what will be my practical voltage when you set a fixed duty ratio, and vary the load current you can predict using this analytical formula. I can carry out the same thing by simulating we can check whether can we predict it or not.

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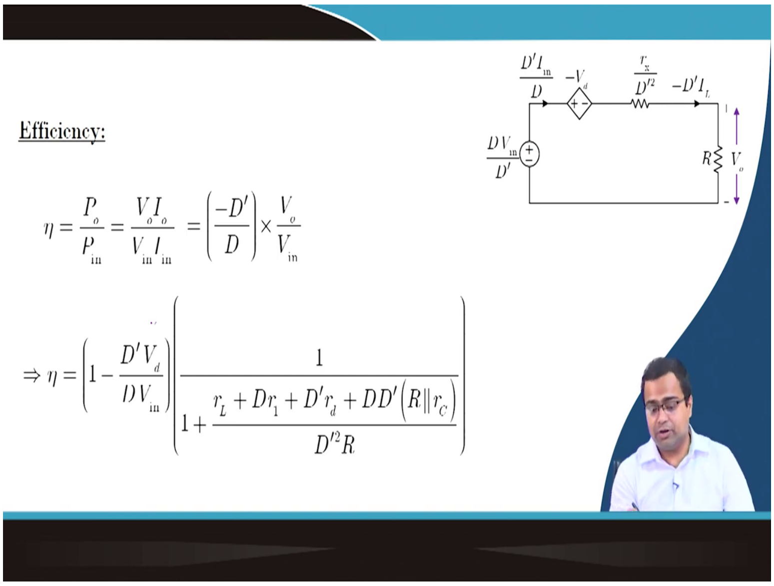

And also the efficiency of the converter, we can write the output power by input power, and we can write down all the expressions of the input current in terms of output current. And we can find out the efficiency, and it can be shown that efficiency is much lower than 100 percent when the diode drop is high as well as when the load current is high because of the parasitic drop.

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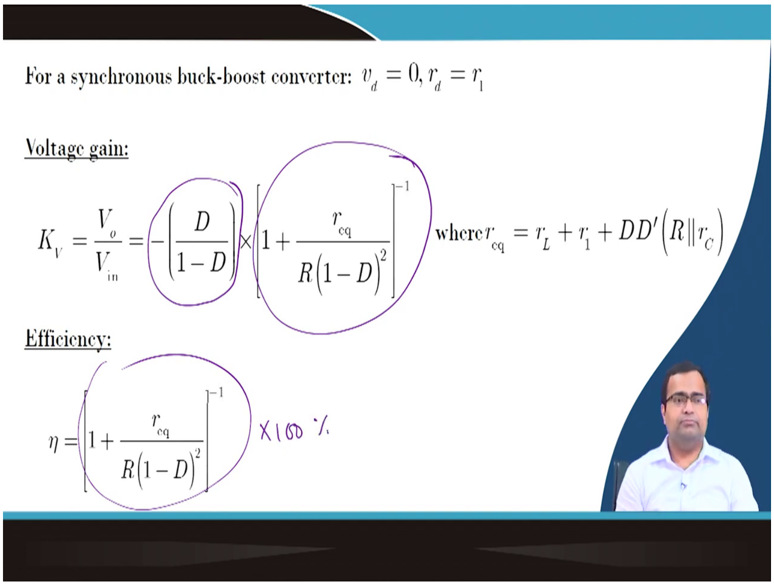

So, for a synchronous buck-boost converter, again we can write down that this is my ideal voltage gain, and this is my correction factor. And the efficiency will be simply the correction factor that we have discussed. So, this is the same thing as you know for buck boost and buck boost for the synchronous configuration. The efficiency is simply the correction factor that is the beauty. So, the if the correction factor becomes smaller, the efficiency will fall. And if you multiply with 100, then we can write in terms of percent.

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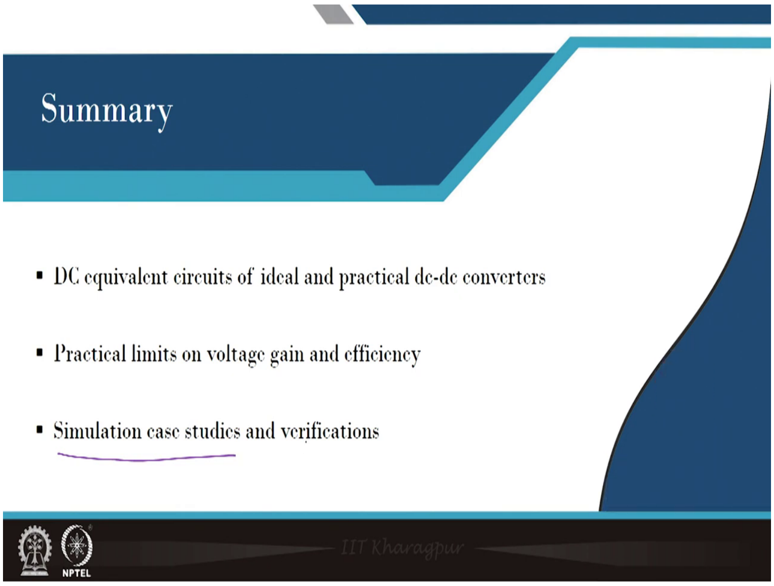

So, with this we want to summarize that we discuss DC equivalent circuit of ideal and practical DC-DC converter. We discussed the practical limit of the voltage gain though we can achieve lower voltage gain for a buck converter with this drop. But for a boost converter, we cannot increase the voltage gain beyond a certain value because of this parasitic drop. And this voltage limit will be imposed by the on-state resistance of the DCR and the resistance of the on-state resistance of the MOSFET plus the DCR of the inductor.

But even if you choose a smaller on-state resistance on the DCR by using a costly inductor, still this voltage gain will be limited when the load current increases ok, so that means, there are practical limits. And it is not recommended to use boost converter and seems like an ordinary boost converter for a high voltage gain application.

And we have shown few simulation case studies for a synchronous buck converter, and the same thing can be carried out for boost as well as non-inverting buck boost. And we can verify. So, with this I want to finish this lecture.

Thank you very much.