

Control and Tuning Methods in Switched Mode Power Converters
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Module - 05
Modeling and Analysis Techniques in SMPC
Lecture - 27
Circuit Averaging Technique and Equivalent Circuit

Welcome, this is lecture number 27. In this lecture, we are going to talk about Circuit Averaging Technique and Equivalent Circuit model.

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Concepts Covered

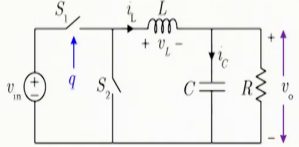
- Circuit averaging technique
- Small signal model with assumptions
- Three terminal model and average switch model
- Equivalent circuit of switched mode power converters

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So, the concept which will be covered circuit averaging technique, small signal model with assumptions, three terminal models and average switch model, then equivalent circuit of switch mode power converters.

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Objective of AC Modeling



- Reduce the effect of nonlinear characteristics for switching element S_1 & S_2
- Neglect switching ripple
- Ignore complicated switching harmonics and side bands
- Predict low frequency variation in duty cycle

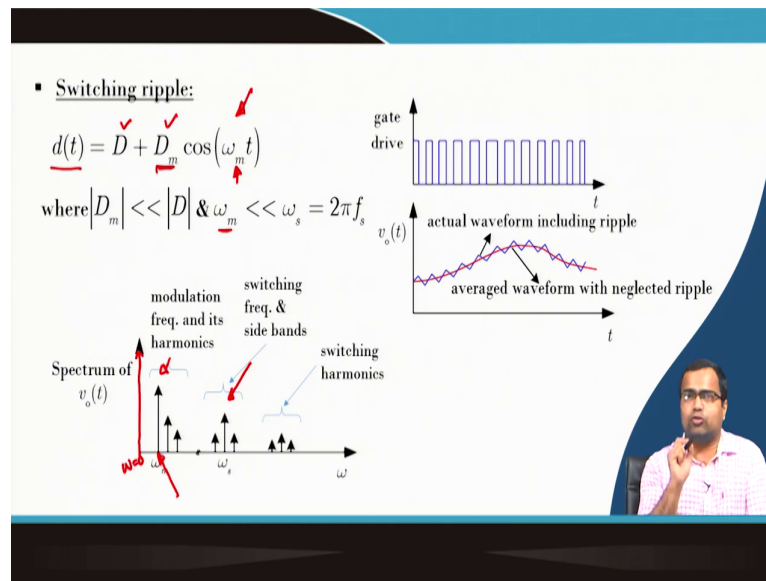


So, first we need to understand what is the objective of AC modeling. So, we are talking about a switching converter. Here I am showing an example of a synchronous buck converter. Here we have already seen that this converter is operated by a periodic pulse; that means switch signal, gate signal and the switch is turned ON and OFF in a periodic interval and there are two switches S_1 and S_2 . It has a non-linear like ON – OFF characteristics.

Now, for modeling like an AC modeling the small signal modeling, we need to neglect the switching ripple component because we need to extract the average dynamics. We need to ignore complex harmonics information as well as the sideband information we are only interested in finding the term or the frequency which is associated with the excitation frequency; that means, if we excite the converter like it is already running by a fixed frequency clock.

But, if we excite the duty ratio with a modulating frequency, we are interested in finding what is the response of the converter due to the index of the modulating frequency and we want to find out what is the component corresponding to that modulating frequency. Then we want to predict low frequency variation behaviour and we want to predict low frequency variation in duty cycle.

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Because we are talking about a switching converter, this part we have already discussed in state space averaging. If we talk about the duty ratio, we are going to operate the switching converter using a pulse width modulation technique, where the time period is fixed and the duty ratio under steady state it is fixed. But if we consider a time varying duty ratio, where the duty ratio consists of one steady state quantity plus one excited like a small AC component, which is the sinusoidal excitation.

And, the assumption the amplitude of excitation is much smaller than its steady state value and, of course, the modulating frequency or the excitation frequency is much smaller than the switching frequency.

With this assumption, if we take the behaviour of the actual switching converter, we can extract the average dynamics and we can ignore the switching dynamics or neglect sorry neglect the ripple information provided that the frequency ripple frequency or the switching frequency is much faster than the excitation frequency.

And, we know if we obtain the power spectra of the output voltage, then we will get a DC component where it is like a zero frequency where we will get the average value of the DC voltage and then since we are exciting the duty ratio with a modulating frequency we will get a side lobe ω_m and also it is harmonic.

Similarly, this side lobe will be created. So, this will be created by modulating frequency and harmonics. Similarly, switching frequency and sideband this is the switching frequency component. And, harmonic content of the corresponding to the switching frequency and around that harmonic content also will get side lobe.

Since we are considering the ω_m to ω_m to be very small, smaller than switching frequency. So, these lobes are very close to each other. And, we are particularly interested in this particular frequency, which is the excitation frequency or the modulating frequency.

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- Why Circuit Averaging is needed?
 - Remove high frequency switching harmonics by averaging all waveforms over one switching period

$$\langle x(t) \rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} x(\tau) d\tau$$
 - Replace switching part of the converter with averaged circuit components
- Modeling method:
 - State space average model → Middlebrook (Caltech)
 - Three terminal switch → Vorperian (VPEC)
 - DC transformer based

Now, why circuit average is needed? In any averaging technique, we try to eliminate the harmonic information, but if we take a generalized averaging technique where we can also incorporate the harmonic information.

But, here we are talking about standard or the traditional averaging technique where we eliminate the harmonic information and we take only the average information of you know the dynamical equation.

So, it takes only the average dynamics by averaging over a cycle switching period. That means this models as I said is valid only up to the half of the switching frequency. Then we will replace switching part of the converter with their average quantity. And then how do you model?

So, some modeling technique we discussed. So, far state space average model which is proposed by Middlebrook from Caltech, it is a very you know well known and it is a very frequently used technique.

Then three terminal switch model it was proposed from (Refer Time: 06:01) and then finally, we want to write you know even the state space model can be framed into a canonical form to obtain an equivalent circuit model; that means, using a transformer whereas the three terminal models can be used to directly obtain the equivalent circuit, ok.

And, we will see that in the previous lecture we discussed state space average model which is straightforward, but it is much more mathematical like you know to obtain all the state space model, then we need to obtain the Jacobian you know, small signal perturbation and so on.

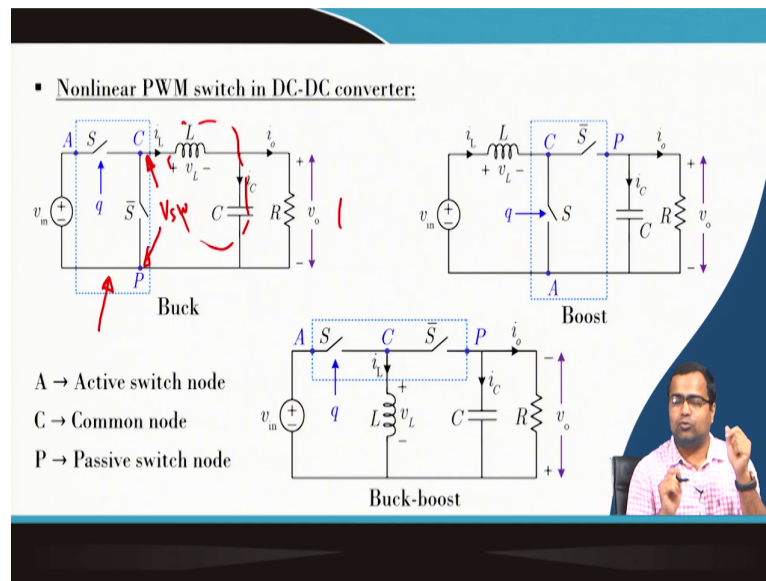
But, in equivalent switch's average switch model under certain you know consideration if we average out if we obtain the average switch model or the switching part only the rest of the circuit like a RLC part and other part we will not we are not going to touch it.

And in this method, the second like you know the average switch model we will see it has the benefit because if we incorporate some additional dynamic element in the switch model will remain same, only the dynamic element will be added.

So, the system the same approach only the circuit has to be expanded with an additional element, but in state space average model we need to rewrite the equation again for the additional dynamic element and then you have to follow the state for state space averaging.

So, in that way, the circuit averaging technique is more kind of generic model; that means, say you know it is more like a modular approach where the additional element can be easily incorporated into the model without re deriving the equations.

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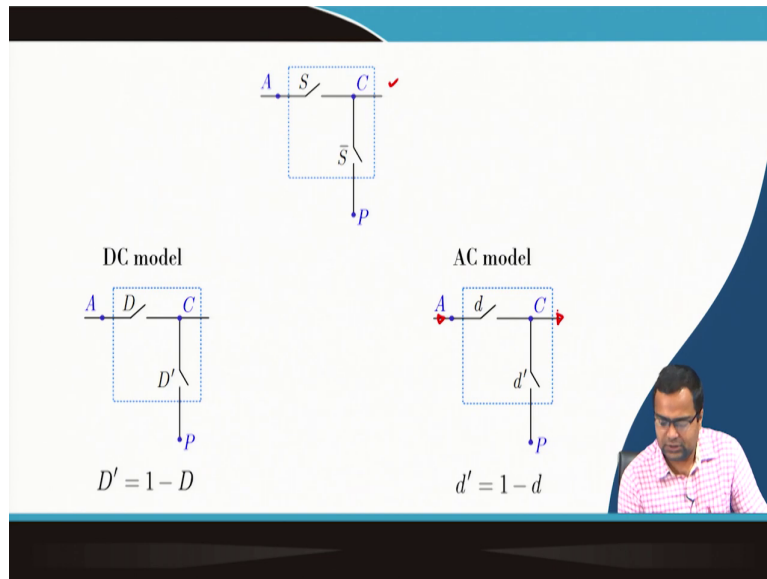


Now, if we take the PWM switching converter, the switching particular, the block which is responsible like you know this is we called this a switch node voltage. This is a switch node voltage and this voltage changes like a square wave, not a square wave is a pulse width modulated wave and this will create an excitation for the RLC circuit.

And, then you can by controlling this switch node voltage we can actually control the output voltage in average sense and it will also generate harmonic terms and if we design properly this filter element, then the component associated with the harmonic and the switching component can be substantially reduce compared to its average value. So, in this model, we take one active switch node, common node and the passive node.

Although this I know the three terminal model this is important, but we are not going to discuss details about this. You know different terminal and their you know detail explanation. We want to take extract the basic modeling information out of this three terminal or the average switch model. So, in this model you can apply to boost buck boost and buck boost by suitably considering the three active common and passive node terminals.

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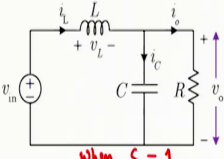
Then what next the next? If we take this three terminal model. So, this for example, this is example the switch network of the buck converter we can write a DC model where the S will be replaced by capital D, \bar{S} will be replaced by D' where D' is the 1 minus D. Also we can take AC model, where S will be replaced by small d and \bar{S} will be replaced by d' and that is shown here the average quantity.

But, this is not complete. We have to write the current and voltage equations and we have to take into account current and voltage expression associated with this switch; that means, what is the current entering into this network, which is coming out.

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Basic AC Modeling Approach

▪ Switch in position 1:




When $S = 1$

$$v_L = L \frac{di_L}{dt} = v_{in} - v_o$$

$$i_C = C \frac{dv_C}{dt} = i_L - \frac{v_o}{R}$$

Small ripple approximation:

$$v_L = L \frac{di_L}{dt} = \langle v_{in} \rangle_{T_s} - \langle v_o \rangle_{T_s}$$

$$i_C = C \frac{dv_C}{dt} = \langle i_L \rangle_{T_s} - \frac{v_o}{R}$$


So, basic AC modeling approach, if we take a buck converter first one approach which is known as circuit averaging technique in which we want to you know first write or draw the equivalent circuit for different switch configuration. For example, when the switch is on this is the equivalent circuit.

So, this circuit is the equivalent circuit when S is in one position; that means, S is 1 or basically this is in the position 1 is connected to position 1. Then from this model subsystem it is simply like RLC circuit we can write down all the equations corresponding to this switch state.

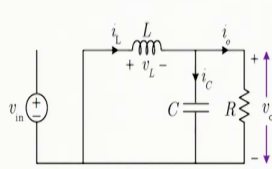
Then out of this equation there are two dynamical equations like one is the derivative of the inductor current and the derivative of the capacitor voltage. In this configuration, if we continue for longer duration so, this duration the current will keep on increasing, right and capacitor voltage depending upon the inductor current magnitude. So, it will also have some ripple information.

And, we are approximating using; that means, if we take the voltage across the inductor and we can take their average quantity. Here, the assumption is that if we take the instantaneous output voltage, it consists of both average as well as a ripple information, but here we are only considering the average quantity, we are ignoring the ripple quantity. So, ripple is approximately to be very small.

Similarly, the capacitor current also we can use in terms of their average quantity, which is a which corresponds to the actual instantaneous states; that means the instantaneous output voltage and the instantaneous inductor currents are replaced by their average quantities. So, this is a small ripple approximation.

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▪ Switch in position 2:




Small ripple approximation:

$$v_L = L \frac{di_L}{dt} = -\langle v_o \rangle_{T_s}$$

$$i_C = C \frac{dv_C}{dt} = \langle i_L \rangle_{T_s} - \frac{\langle v_o \rangle_{T_s}}{R}$$

$v_L = L \frac{di_L}{dt} = -v_o$
 $i_C = C \frac{dv_C}{dt} = i_L - \frac{v_o}{R}$



Similarly, we can take the other switch state; that means, when the switch is OFF or is it is in position 2, then again we can write down the dynamics of the inductor current as well as the capacitor voltage.

And, here also under small ripple approximation, the instantaneous current and the instantaneous voltage can be replaced with their average quantity; that means, the output voltage instantaneous and the inductor current instant are replaced by this is the average quantity like all are average quantity. That means, we have ignored their ripple information, ok.

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▪ Averaging the Inductor and Capacitor waveforms:

- Inductor voltage waveform:

$$\langle v_L \rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} v_L(\tau) d\tau \approx d \langle v_m \rangle_{T_s} - \langle v_o \rangle_{T_s}$$

- Inductor current over one switching period:

$$L \frac{d \langle i_L \rangle_{T_s}}{dt} = \langle v_L \rangle_{T_s} = d \langle v_m \rangle_{T_s} - \langle v_o \rangle_{T_s}$$

Then what we will do? Then, that means, inductor current waveform if we take, for example, this is the inductor voltage waveform ok v_L . So, inductor voltage waveform if we take the average over the inductor voltage, it can be represented by; that means, instead of if we ignore this ripple information and take just the average quantity similarly the average quantity during this cycle, it can be approximated by this.

So, this approximation makes sense, or it is somewhat it is reasonable when we assume these ripples are you know the effect of the ripple component is negligible. Similarly, the inductor current over one cycle we can average take the average value and this can be considered; that means, we are talking about this average inductor current dynamics; one switching cycle it can be represented by this.

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o Similarly, average Capacitor current :

$$\langle i_c \rangle_{T_s} = \left(\langle i_L \rangle_{T_s} - \frac{\langle v_o \rangle_{T_s}}{R} \right) \times (d + d') = \left(\langle i_L \rangle_{T_s} - \frac{\langle v_o \rangle_{T_s}}{R} \right)$$

o And average Input current :

$$i_{in} = \begin{cases} \langle i_L \rangle_{T_s} & \text{During switch in position 1} \\ 0 & \text{During switch in position 2} \end{cases}$$

$\langle i_{in} \rangle_{T_s} = d \langle i_L \rangle_{T_s}$

Then what we will do? The average capacitor current also you can obtain. The average capacitor current you see the capacitor current is nothing but the inductor current minus the load current, ok. Now, the average inductor current, that means capacitor current during; so, here the inductor is always connected to the capacitor. So, it will be common for both ON state and OFF state.

So, we can take the average and then some of this duty ratio, which is 1 and which is nothing but the average capacitor current expression. Similarly, average inductor current it is same as the average input current. It is same as we are talking about the average inductor current over this period it is same as the average inductor current during the interval which is sorry it is not the average I think.

This is the input current when the switch is on it is same as the inductor current when it is in position 1 and it is 0 when the switch is OFF. So, if you take average input current, then average input current should be equal to d into average inductor current that is because it is only inductor current input current is equal to inductor current when the switch is ON and it is equal to 0 when the switch is OFF, yes. So, average quantity is written by this.

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Small Signal Model – Assumptions and Steps


- Considering some superimposed small AC variations over DC quiescent values

$$\langle v_{in} \rangle = V_{in} + \hat{v}_{in}; \quad \langle i_L \rangle = I_L + \hat{i}_L; \quad d = D + \hat{d}; \quad \langle v_o \rangle = V_o + \hat{v}_o; \quad \langle i_{in} \rangle = I_{in} + \hat{i}_{in}$$

- Applicable for small-signal perturbation compared to DC operating conditions

$$|\hat{v}_{in}| \ll |V_{in}|; \quad |\hat{i}_L| \ll |I_L|; \quad |\hat{d}| \ll |D|; \quad |\hat{v}_o| \ll |V_o|; \quad |\hat{i}_{in}| \ll |I_{in}|$$

- Nonlinear circuit elements to be replaced by linear circuit elements
- AC equivalent circuit can be derived using linear circuit elements



Now, the small signal model assumptions and states considering you know if we consider the average quantity to be replaced by the DC term and the AC perturb term for all parameter and this process we did it in state space averaging also, then what can I write?

So, this is applicable; that means, when we are considering because we are talking about small signal model where we are ignoring the non-linear term; that means any product of perturbation should be ignored.

And, this is only valid when the small perturbation the perturbations are much smaller than their corresponding steady-state values. So, this is the condition for which our small signal model is valid. Then the non-linear circuit element; that means, switching element will be replaced by their average quantity, as well as their linearized quantity; that means, and we will show and then we can obtain the AC equivalent circuit model.

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Perturbations and Linearization

Need to linearize the nonlinear models by constructing small signal model

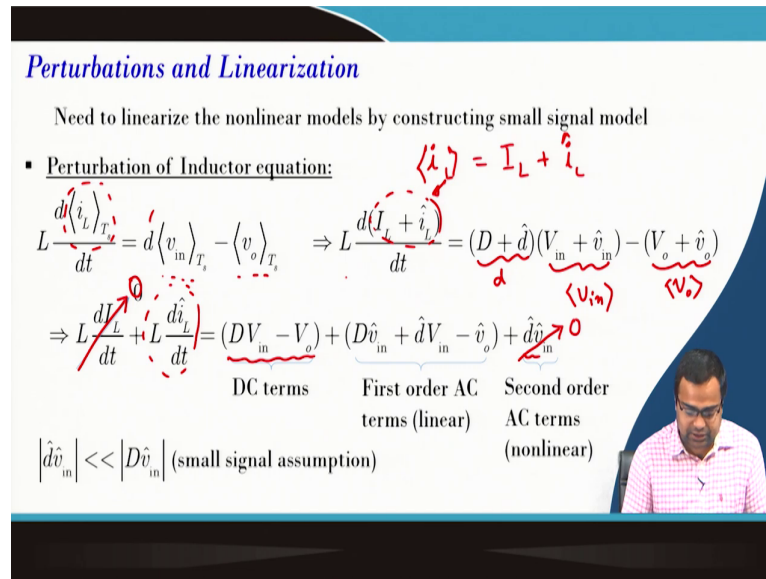
- Perturbation of Inductor equation:

$$L \frac{d\langle i_L \rangle_{T_s}}{dt} = d\langle v_{in} \rangle_{T_s} - \langle v_o \rangle_{T_s} \Rightarrow L \frac{d(I_L + \hat{i}_L)}{dt} = (D + \hat{d})(V_{in} + \hat{v}_{in}) - (V_o + \hat{v}_o)$$

$$\Rightarrow L \frac{dI_L}{dt} + L \frac{d\hat{i}_L}{dt} = (DV_{in} - V_o) + (D\hat{v}_{in} + \hat{d}V_{in} - \hat{v}_o) + \hat{d}\hat{v}_{in}$$

DC terms
First order AC terms (linear)
Second order AC terms (nonlinear)

$|\hat{d}\hat{v}_{in}| \ll |D\hat{v}_{in}|$ (small signal assumption)



Next we are going to consider perturbation and linearization. So, we need to obtain the linearized model from its non-linear characteristics and in order to do that, let us consider the inductor current average dynamics in a buck converter. Here the d/dt of the derivative of the average inductor current, which is this one the derivative of the average inductor current; that means, this is the average inductor current is nothing but d times the average input voltage and minus average output voltage.

So, now if we write this average quantity which is equal to cap you know steady state quantity plus I_L hat. So, all quantities are replaced by their steady state value plus the perturb value. So, then we will get right side equation; that means, this quantity is coming from here, then this D is nothing but this is nothing but d and this is nothing but v_{in} in the average quantity and this is nothing but the average output voltage.

Then what we will do? If we expand this, we will get. So, this is the steady-state term and if we take differentiation it will be 0. So, it only will have left side which the small signal perturb term this is our corresponding steady-state term and this is a non-linear term; that means, this should be 0 it is already shown.

These are DC term, the first order AC term only perturb term and these are non-linear term which is a product of perturbation and we want to ignore that. This also we want to ignore. Under small signal approximation assumption, this can be ignored.

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$$\Rightarrow L \frac{d\hat{i}_L}{dt} = (DV_{in} - V_o) + (D\hat{v}_{in} + dV_{in} - \hat{v}_o)$$

$$\Rightarrow L \frac{d\hat{i}_L}{dt} = D(V_{in} + \hat{v}_{in}) + dV_{in} - (V_o + \hat{v}_o)$$

$$L \frac{d\hat{i}_L}{dt} = L \frac{d}{dt} (I_L + \hat{i}_L)$$

$$\frac{dI_L}{dt} = 0$$

Inductor current equivalent circuit

Next we want to write down the left side $L \frac{di_L}{dt}$ expression and if we rearrange this term, this term and if we take this term, then we are getting this term. Then if we consider this term and this term like this term, then this will generate this result and then if we consider, this is this term ok.

So, you can write down the equivalent circuit this term is coming here and since here $L \frac{di_L}{dt}$ is also equal to $L \frac{d}{dt} (I_L + \hat{i}_L)$ because we know that $\frac{dI_L}{dt}$ is equal to 0. So, you can simply put them. So, that is why this is coming $L \frac{di_L}{dt}$; that means, it includes both DC and AC information. Then this term is coming here, and this term is coming here ok. The next this is the inductor current equivalent circuit.

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▪ **Perturbation of Capacitor equation:**

$$\langle i_c \rangle_{T_s} = C \frac{d \langle v_o \rangle_{T_s}}{dt} = \left(\langle i_L \rangle_{T_s} - \frac{\langle v_o \rangle_{T_s}}{R} \right) \Rightarrow C \frac{d(V_o + \hat{v}_o)}{dt} = I_L + \hat{i}_L - \frac{(V_o + \hat{v}_o)}{R}$$

$$\Rightarrow C \frac{dV_o}{dt} + C \frac{d\hat{v}_o}{dt} = \underbrace{\left(I_L - \frac{V_o}{R} \right)}_{\text{DC terms}} + \underbrace{\left(\hat{i}_L - \frac{\hat{v}_o}{R} \right)}_{\text{First order AC terms (linear)}}$$

$$\Rightarrow C \frac{d\hat{v}_o}{dt} = (I_L + \hat{i}_L) - \frac{(V_o + \hat{v}_o)}{R}$$

Capacitor voltage equivalent circuit

The next we want to write the output voltage or the capacitor voltage. So, the capacitor current average dynamics we know for a buck converter is this and if we expand this term, then you can write down this equation. This is steady-state term. It will be 0, that we have discussed, and if we rearrange this term, what will get?

So, your DC term it is the first-order term and you can rearrange I L; that means, this term and this term are added to get this term and this term and these terms are added to get this term ok. Then we can write down the equation of the capacitor voltage and we can obtain the equivalent circuit and this is a capacitor of voltage equivalent circuit.

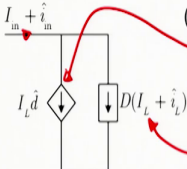
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▪ **Input current equation:**

$$\langle \hat{i}_m \rangle_{T_s} = d \langle \hat{i}_L \rangle_{T_s} \Rightarrow I_m + \hat{i}_m = (D + \hat{d})(I_L + \hat{i}_L)$$

$$\Rightarrow I_m + \hat{i}_m = \underbrace{DI_L}_{\text{DC term}} + \underbrace{I_L \hat{d} + D\hat{i}_L}_{\text{First order AC terms (linear)}} + \underbrace{\hat{d}\hat{i}_L}_{\text{Second order AC terms (nonlinear)}}$$

As $|\hat{d}\hat{i}_L| \ll |D\hat{i}_L|$ (small signal assumption)

$$\Rightarrow I_m + \hat{i}_m \approx DI_L + I_L \hat{d} + D\hat{i}_L$$


$$\Rightarrow I_m + \hat{i}_m = D(I_L + \hat{i}_L) + I_L \hat{d}$$

Now, input current also you can write. We know what is the input current the average input current is nothing but d into average inductor current. So, if we write down the average quantity can be replaced by their steady state quantity plus perturb quantity and this d can be written as this term.

And, the average inductor current here is nothing but i_L average. Now, if we expand so, we can get and you will find this is the DC term. This is the first order AC term, and this is a non-linear AC term which we have to ignore.

Then if we ignore this term because we are talking about a linearize model, then we can simply approximate this term to get this and under small signal approximation these equations can be written; that means, from here. And, then we can draw the circuit because this is a total current which is coming. The input current is divide into two part this is one current which is here and this is another current which is here ok.

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Small Signal Average Model of Three Terminal PWM Switch

$$v_2 = \begin{cases} v_{in}; & S = \text{ON} \\ 0; & S = \text{OFF} \end{cases} \Rightarrow \langle v_2 \rangle_{T_s} = d \langle v_{in} \rangle_{T_s}$$

$$i_{in} = \begin{cases} i_L; & S = \text{ON} \\ 0; & S = \text{OFF} \end{cases} \Rightarrow \langle i_{in} \rangle_{T_s} = d \langle i_L \rangle_{T_s}$$

Linearization:

$$\langle v_{in} \rangle_{T_s} = V_{in} + \hat{v}_{in}; \langle i_{in} \rangle_{T_s} = I_{in} + \hat{i}_{in}; d = D + \hat{d}; \langle i_L \rangle_{T_s} = I_L + \hat{i}_L; \langle v_2 \rangle_{T_s} = V_2 + \hat{v}_2;$$

$$V_2 + \hat{v}_2 = D(V_{in} + \hat{v}_{in}) + \hat{d}V_{in}$$

ignored $\hat{d} \hat{v}_{in}$ terms

$$I_{in} + \hat{i}_{in} = D(I_L + \hat{i}_L) + I_L \hat{d}$$

So, we can combine together the another approach we can do if we take the three terminal PWM switch, here if we take a buck converter here left side if the input voltage and it is the input current this is a switch model and this is an inductor current going through and this is a voltage across this is a switch node voltage.

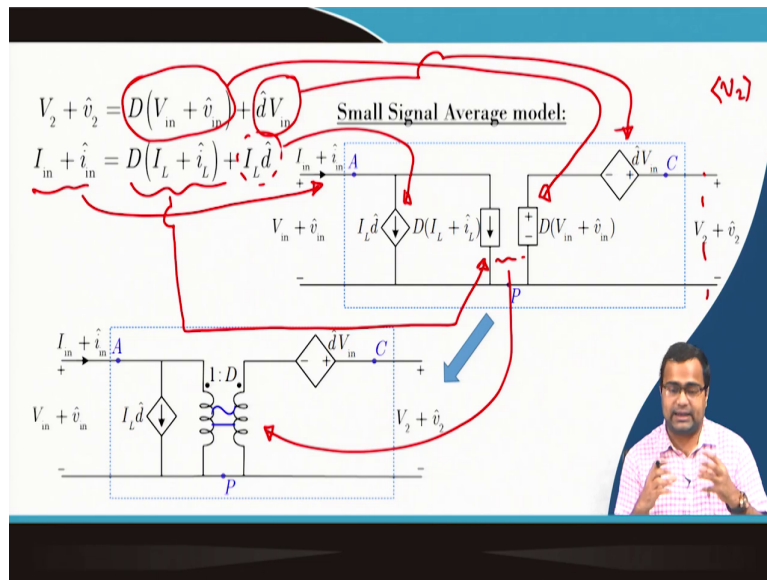
You see the switch node v_2 is equal to v_{in} when the switch is ON at a 0 when the switch is OFF. So, then we can get the average voltage v_2 in terms of d into average v_{in} and here again we are ignoring the ripple information. Similarly, if we write the inductor current average value sorry this should be input current. Input current is equal to inductor current when the switch is ON, an input current equal to 0 when the switch is OFF because this part is disconnected there.

So, you can write the input average quantity is equal to d into average inductor current, where we are ignoring the ripple of the inductor current. Then, if we linearize if we take this term so, we can again write input voltage average is sum of the DC plus perturbation input current some of the DC plus perturbation duty ratio inductor current, then V_2 voltage.

Then what can you write? If we write down this expression, what will get this where we have ignored what we have ignored here? Here, we have ignored d perturbation and v_{in} perturbation. This part was ignored the ignored term. Similarly, here we have ignored d perturbation into I_L perturbation.

So, this product of these perturbations is ignored because these are a non-linear term and since we are assuming the perturbation to be very small compared to their steady state value. So, product of perturbation will be even smaller. So, you can drop them.

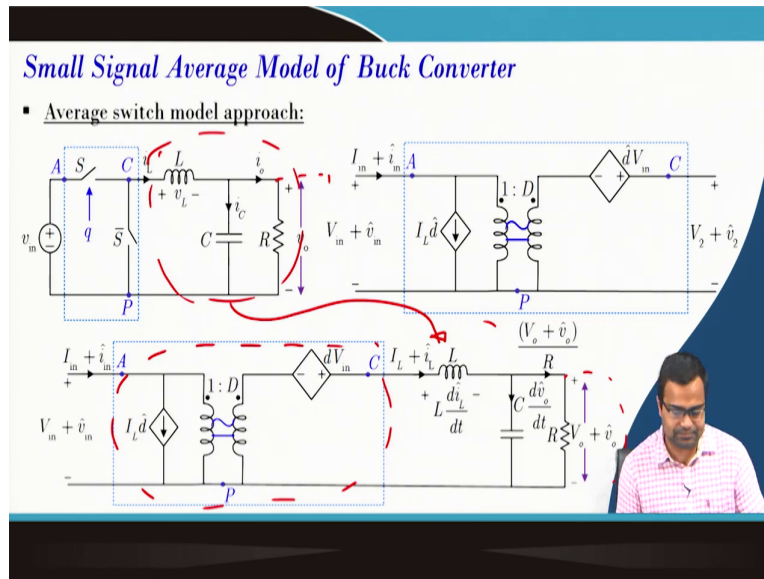
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Now, with this approximate model that we have obtain we can draw the equivalent circuit. So, v_2 was our original V_2 average, which was the terminal 2, right? So, this voltage is nothing but D times this voltage, which is nothing but this voltage and d times this voltage and they are added up.

So, we are getting this one and the input current, which is this current right. This is the sum of this current, which is nothing but this current and this current, which is nothing but this current ok. So, they are all positive signs. So, they are in parallel current path, right? Now, you see these two switches we can replace with this transformer model and then rest of the circuit will remain same; that means, this three terminal PWM switch model we can write in terms of a DC transformer and also dependent voltage and current sources.

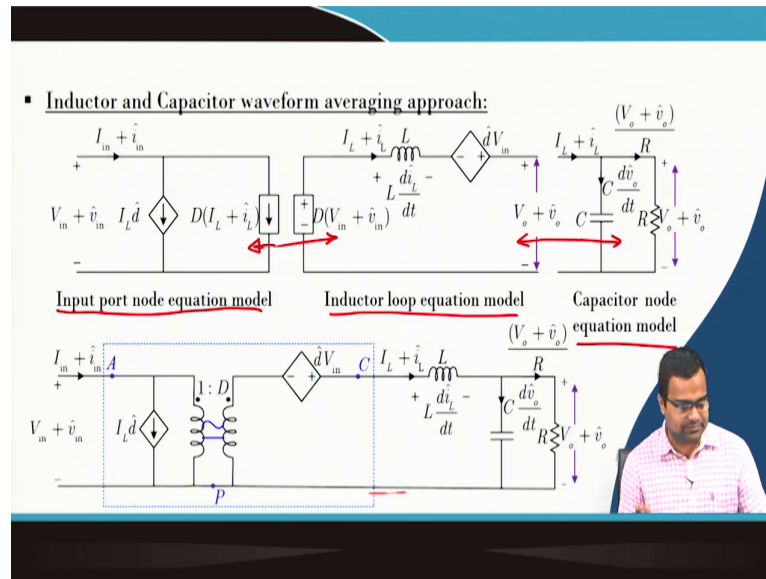
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Now, for a buck converter, this switch model we have already found, now we want to place it with this inductor. So, this part now we can join back and beautiful thing if we incorporate a current load you can also incorporate here current load if we incorporate a current load.

If we incorporate any additional impedance or you know filter element the same model only will not touch this part as long as you do not change anything then we can update this right side of the switch model without rewriting this equations and that is somewhat special in this technique compared to state space averaging where we need to rewrite all the equation every time when we add any dynamic element.

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Now, in case of the average model, you know we discuss the waveform averaging technique. If we recall that we initially consider the waveform of capacitor current, input current, inductor current, output voltage capacitor voltage and we obtain the equivalent model by averaging the waveform over a periodic cycle. And, we got three model one for the input current, the other for the inductor current dynamics and the third one for the capacitor voltage dynamics.

So, in this technique we can put them three circuits side by side and you see these two. We can replace with a DC transformer and here, since there is no ESR, we can simply connect these two terminals. So, we will get the overall circuit like this by putting a DC transformer.

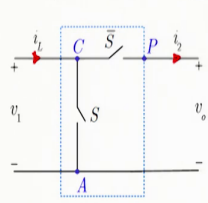
So, this model is also same which we obtain by using a three terminal switch model; that means, if we use the average switch model or if we do waveform averaging by using circuit averaging. So, we will get the same equivalent circuit model ok.

So, the equivalent circuit model can be obtain either by applying average switch model; that means, the three terminal switch model or you can obtain by individually considering individual waveform and their sub interval equation and then average out or basically sub interval waveform we can take and then average out to obtain the dynamics of the variable that we are interested in.

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Small Signal Average Model of Boost Converter

▪ Average switch model approach:



$$v_1 = \begin{cases} 0; & S = \text{ON} \\ v_o; & S = \text{OFF} \end{cases} \Rightarrow \langle v_1 \rangle_{T_s} = (1-d) \langle v_o \rangle_{T_s}$$

$$i_2 = \begin{cases} 0; & S = \text{ON} \\ i_L; & S = \text{OFF} \end{cases} \Rightarrow \langle i_2 \rangle_{T_s} = (1-d) \langle i_L \rangle_{T_s}$$

$$V_1 + \hat{v}_1 = (1-D)(V_o + \hat{v}_o) - \hat{d}V_o$$

$$I_2 + \hat{i}_2 = (1-D)(I_L + \hat{i}_L) - I_L \hat{d}$$

Handwritten notes in red: $V_1 + \hat{v}_1$, $D + \hat{d}$, $V_o + \hat{v}_o$, $\hat{d} \approx 0$, $\hat{i}_L \approx 0$

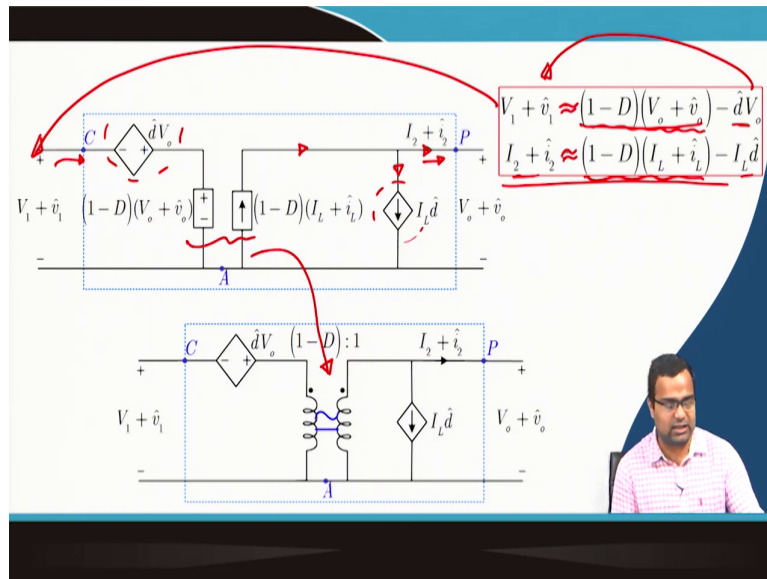
Similarly, we can do we can apply this circuit averaging technique in a boost converter. Here we can consider again the three terminal models where input side switch which is connected to the inductor is basically the input voltage is discontinuous, and the current is continuous because we have an inductor before this.

Output side since we are not considering ESR, output side is connected to the capacitor and which is the voltage is continuous, but the current is discontinuous. So, input voltage can be written in terms of output voltage when the switch is OFF and we can take the average input voltage is nothing but 1 minus d into average output voltage.

Similarly, the output current is nothing but the inductor current when the switch is OFF and is 0 when the switch is ON. So, you can write the average output current is 1 minus d into average inductor current.

Then using the same method if we replace this quantity to be $V_1 + \hat{v}_1$ and we can replace d equal to capital D into \hat{d} and this quantity to be $V_o + \hat{v}_o$, then we will arrive this equation to this equation by ignoring the term \hat{d} into $v_o + \hat{v}_o$ that we have ignored. Similarly, from this equation we can write this equation where we have ignored \hat{d} into $i_L + \hat{i}_L$ that we have ignored, right.

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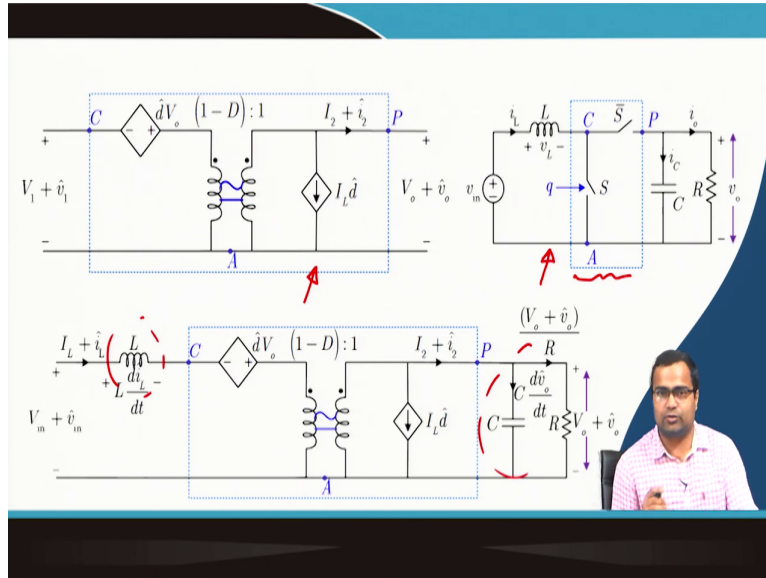


Then we have these two approximate equations, where we have ignored their non-linear perturb term and we can draw the equivalent circuit. See, this is the input voltage. If you look at this voltage, it consist of two terms; one this voltage minus this voltage. So, if you come from this to this side, it will be $V_1 + \hat{v}_1 - (1-D)(V_o + \hat{v}_o) + \hat{d}V_o$ and that is exactly what.

If you take this term to this side, then $V_1 + \hat{v}_1 - (1-D)(V_o + \hat{v}_o) + \hat{d}V_o$ that is from this circuit is equal to this term. Similarly, if we write the current, the output current consist of two part – this part minus this part and this is the minus part; that means, the current is going in this direction. So, this current minus this current will be our this current, right?

Now, from this two terminal we can represent in terms of a DC equivalent like a DC transformer where the ratio will be $(1-D) : 1$ and these are well known standard technique, but this is just for sake of you know deriving small signal model. These steps are important. That is why I am showing.

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Next, in the boost converter, we have obtain the equivalent switch model, the average switch model. Now, if we incorporate this average switch model to our original boost converter where we have obtain this model, then the full model can be obtain where we just plug in the inductor in the left side and the capacitor in the right side with the load. So, we will get the complete equivalent circuit of a boost converter. This is an ideal boost converter.

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▪ **Inductor and Capacitor waveform averaging approach:**

○ Perturbation of Inductor equation:

$$\hat{L} \frac{d\langle i_L \rangle_T}{dt} = \langle v_m \rangle_T - (1-d) \langle v_o \rangle_T$$

$$\Rightarrow L \frac{d(I_L + \hat{i}_L)}{dt} = (V_m + \hat{v}_m) - (1-D - \hat{d})(V_o + \hat{v}_o)$$

$$\Rightarrow L \frac{d\hat{i}_L}{dt} = (V_m + \hat{v}_m) + \hat{d}V_o - (1-D)(V_o + \hat{v}_o)$$

$L \frac{di_L}{dt} = \begin{cases} \frac{V_{in}}{L} & S\text{-on} \\ \frac{V_{in} - V_o}{L} & S\text{-off} \end{cases}$

A small inset shows a person speaking.

In case of if we repeat this thing using a circuit averaging technique that means what we did was the average switch model. So, in this approach, what we did? We started with a boost converter. If you look at very carefully we have first, ok.

So, we have first extracted the switch part of the boost converter; that means we have just we took the switch element and we have not touched anything like left side inductor right side capacitor that part we kept as it is and we are just extract the switch model.

And, then you obtain the equivalent circuit of the switch model and then we have connected the inductor and capacitor of the original boost converter, including the switch model we get the complete equivalent circuit. Now, can I do the same thing without using switch average model? That means, without using average switch model, how to do that? So, you take a boost converter and if we take the in average inductor current.

So, what is the inductor current of a boost converter? $L \frac{di_L}{dt}$ instantaneous inductor current it is equal to V_{in} by L when S is ON and it is equal to $V_{in} - V_0$ by L when S is OFF; that means, if I take the average because here taking L left side if you take or if I write L into this hold on if I erase this part.

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▪ Inductor and Capacitor waveform averaging approach:

○ Perturbation of Inductor equation:

$$L \frac{d\langle i_L \rangle_{T_s}}{dt} = \langle v_m \rangle_{T_s} - (1-d) \langle v_o \rangle_{T_s}$$

$$\Rightarrow L \frac{d(I_L + \hat{i}_L)}{dt} = (V_{in} + \hat{v}_{in}) - (1-D - \hat{d})(V_o + \hat{v}_o)$$

$$\Rightarrow L \frac{d\hat{i}_L}{dt} = (V_{in} + \hat{v}_{in}) + \hat{d}V_o - (1-D)(V_o + \hat{v}_o)$$

Handwritten notes: $L \frac{di_L}{dt} = V_{in} - V_0$ (S-on), $V_{in} - V_0$ (S-off), $L \frac{d(I_L + \hat{i}_L)}{dt} \approx V_0 \hat{d}$, $\hat{d}V_o \approx 0$, $L \frac{di_L}{dt} = V_{in} - V_0$

So, this is nothing but V_{in} and this is nothing but $V_{in} - V_0$. So, if we take the average of this inductor current over a cycle, T_s ; that means, I am talking about average inductor current. Then the derivative of this average inductor current is nothing but this term. This will

be nothing but d times average of this and 1 minus d time average of this and since input is common. So, it is simply the average of inductor current input voltage minus 1 minus d into the average of output voltage.

Then we can again write that we know that any quantity we have already discussed can be replaced by its steady state quantity plus perturb quantity and that we did for the all the terms we have repeated. And, we know that the derivative of steady state quantity is equal to 0, and that is why we got from this step to this step. And, this is one equation which is the perturb dynamics of the inductor current, which is derived using the average behaviour of the boost converter over a periodic switching cycle t.

And, here we can write down we can from this equation we can draw the equivalent circuit; that means, L di L dt. In fact, you can consider this model itself plug in. This will be the dynamics of this inductor, which is the current dynamics basically current. This is equal to V in minus this term this term that means, if I take this particular term this is here minus it is divided into two parts.

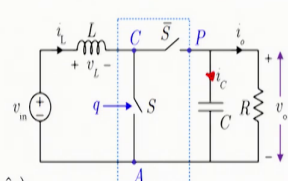
One is 1 minus capital D into this which is this term which is this term; that means, I am talking about if I take this term product with this and the other quantity is this in which we have ignored. That means, there was minus the plus d hat into V 0 plus V 0 hat that is approximated to be V 0 into d hat because we have ignored d hat and V 0 hat, this we take as a 0. So, we obtain this circuit.

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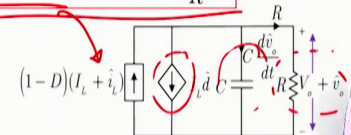
o Perturbation of Capacitor equation:

$$\langle \dot{i}_C \rangle_{T_s} = C \frac{d \langle v_o \rangle_{T_s}}{dt} = (1-d) \langle i_L \rangle_{T_s} - \frac{\langle v_o \rangle_{T_s}}{R}$$

$$\Rightarrow C \frac{d(V_o + \hat{v}_o)}{dt} = (1-D - \hat{d})(I_L + \hat{i}_L) - \frac{(V_o + \hat{v}_o)}{R}$$

$$\Rightarrow C \frac{d(V_o + \hat{v}_o)}{dt} = (1-D)(I_L + \hat{i}_L) - \hat{d}I_L - \frac{(V_o + \hat{v}_o)}{R}$$


$i_L = \begin{cases} -\frac{V_o}{R} & \text{S-on} \\ -\frac{V_o}{R} & \text{S-off} \end{cases}$



Next, let us if you derive the capacitor current equation, capacitor current is nothing but average is $C \frac{dV_0}{dt}$ here since the capacitor there is no ESR, the capacitor voltage is same as the output voltage. So, you can take the average voltage and its derivative and you can see for the boost converter the capacitor current instantaneous capacitor current is $\frac{V_0}{R}$ when S is ON and it is $i_L - \frac{V_0}{R}$ when S is OFF.

So, that is why if you take the average capacitor current this term and this term is common. So, it is simply average quantity, and this is $d \ln(1-D)$ into i_L and again we expand this average quantity with their steady state quantity plus perturbation.

So, you can write down this expression and if we ignore the product of perturbation, then we can write down that the derivative of this term will consist of this term which is here minus this term which is going down and then $\frac{V_0}{R}$ which is nothing but this voltage by resistance which is going out.

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o Input current equation:

$$\langle \hat{i}_m \rangle_{T_s} = \langle \hat{i}_L \rangle_{T_s} \Rightarrow I_m + \hat{i}_m = I_L + \hat{i}_L$$

So, you got now we can write input current for input current of boost converter, it is same as the inductor current. So, you can write all three equations; that means, input current and inductor currents are same and you already got the dynamics of the inductor current, we already got the dynamics of the capacitor voltage and we join them, then we can write replace this with the DC transformer and then we can obtain this.

And, you will find this equivalent circuit is same as the one which you obtain using average switch model. The same thing can be obtained by circuit averaging technique by averaging the waveform over a periodic cycle.

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Small Signal Average Model of Buck-boost Converter

$$v_L(t) = L \frac{di_L(t)}{dt} = v_m(t)$$

$$i_c(t) = C \frac{dv_o(t)}{dt} = -\frac{v_o(t)}{R}$$

$$v_L(t) = L \frac{di_L(t)}{dt} \approx \langle v_m(t) \rangle_{T_s}$$

$$i_c(t) = C \frac{dv_o(t)}{dt} \approx -\frac{\langle v_o(t) \rangle_{T_s}}{R}$$

Now, the same technique can be used for a buck-boost converter where when the switch is ON you will get one set of circuit, when the switch is and you can write down their voltage inductor voltage expression then in the capacitor current and you can write down their average dynamics ok, average voltage, average current dynamics.

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$$v_L(t) = L \frac{di_L(t)}{dt} = v_o(t)$$

$$i_c(t) = C \frac{dv_o(t)}{dt} = -i_L(t) - \frac{v_o(t)}{R}$$

$$v_L(t) = L \frac{di_L(t)}{dt} \approx \langle v_o(t) \rangle_{T_s}$$

$$i_c(t) = C \frac{dv_o(t)}{dt} \approx -\langle i_L(t) \rangle_{T_s} - \frac{\langle v_o(t) \rangle_{T_s}}{R}$$

That means what we did for the circuit averaging technique? That means, this is we are doing averaging the waveform. Then we can take the second switch configuration and we can write down the derivative of the inductor current in terms of their average quantity, where we have ignored the ripple of the output voltage and the ripple of the inductor current.

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$$L \frac{d\langle i_L(t) \rangle_{T_s}}{dt} = d\langle v_m(t) \rangle_{T_s} + d\langle v_o(t) \rangle_{T_s}$$

$$C \frac{d\langle v_o(t) \rangle_{T_s}}{dt} = -d\langle i_L(t) \rangle_{T_s} - \frac{\langle v_o(t) \rangle_{T_s}}{R}$$

$$\langle i_{in}(t) \rangle_{T_s} = d(t)\langle i_L(t) \rangle_{T_s}$$

$$L \frac{d\hat{i}_L(t)}{dt} = D\hat{v}_{in}(t) + D'\hat{v}_o(t) + (\mathbf{V}_{in} - \mathbf{V}_o)\hat{d}(t)$$

$$C \frac{d\hat{v}_o(t)}{dt} = -D\hat{i}_L(t) - \frac{\hat{v}_o(t)}{R} + I_L\hat{d}(t)$$

$$\hat{i}_{in}(t) = D\hat{i}_L(t) + I_L\hat{d}(t)$$

Then, we can write down the inductor current average dynamics, which is the dynamics over the cycle and output voltage average dynamics over the cycle and we can write down the input current average dynamics, ok. Then from this expression you know we can you know replace this average quantity by their steady state quantity plus perturb quantity and then write down the equations again.

And, we can we should ignore this would be capital V in ignoring the product of perturbation term, ok. So, with this we can also obtain the capacitor current dynamics here, which is C into dv 0 dt perturb capacitor current dynamics and also perturb input current dynamics.

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$$L \frac{d\hat{i}_L(t)}{dt} = D\hat{v}_in(t) + D'\hat{v}_o(t) + (V_{in} - V_o)\hat{d}(t)$$

The circuit diagram shows a voltage source $D\hat{v}_in(t)$ in series with an inductor L . A dependent current source $(V_{in} - V_o)\hat{d}(t)$ is connected in parallel with the inductor. The output voltage is $D'\hat{v}_o(t)$. The current through the inductor is $\hat{i}_L(t)$.

And, next, if we consider the inductor perturb dynamics, we can draw the equivalent circuit. Here it is very straightforward that $L di/dt$ is equal to $D v_{in}$ and this is capital V in $L di/dt$. This is added, right? So, that means, if we take $L di/dt$ will get this quantity minus plus this quantity. So, that means, it is added and then plus this quantity which is here ok.

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$$C \frac{d\hat{v}_o(t)}{dt} = -D'\hat{i}_L(t) - \frac{\hat{v}_o(t)}{R} + I_L \hat{d}(t)$$

The circuit diagram shows a current source $D'\hat{i}_L(t)$ in parallel with a dependent current source $I_L \hat{d}(t)$. This is followed by a capacitor C and a resistor R in series. The output voltage is $\hat{v}_o(t)$.

$$\hat{i}_{in}(t) = D\hat{i}(t) + I_L \hat{d}(t)$$

The second circuit diagram shows a voltage source $\hat{v}_in(t)$ in series with a current source $I_L \hat{d}(t)$ and a dependent current source $D\hat{i}_L(t)$.

Similarly, if you take the perturb voltage dynamics; so, perturb voltage dynamics this is nothing but the capacitor current perturb dynamics the capacitor current which is positive

when it is coming in that is equal to minus of this which is this term; that means, it is going out.

And, minus of load current which is also going out plus of this current which is going in, right and we can also draw the equivalent circuit of the input current which consist of D into I L this is i L perturbation plus I into d which is this term. This is average inductor current.

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The slide contains the following equations and circuit diagrams:

$$L \frac{d\hat{i}_L(t)}{dt} = D\hat{v}_m(t) + D'\hat{v}_o(t) + (V_m - V_o)\hat{d}(t)$$

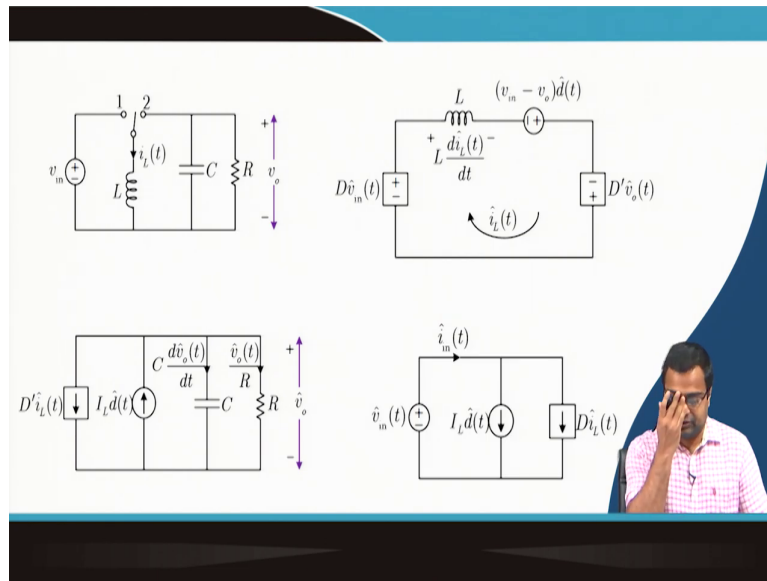
$$C \frac{d\hat{v}_o(t)}{dt} = -D'\hat{i}_L(t) - \frac{\hat{v}_o(t)}{R} + I_L\hat{d}(t)$$

$$\hat{i}_m(t) = D\hat{i}(t) + I\hat{d}(t)$$

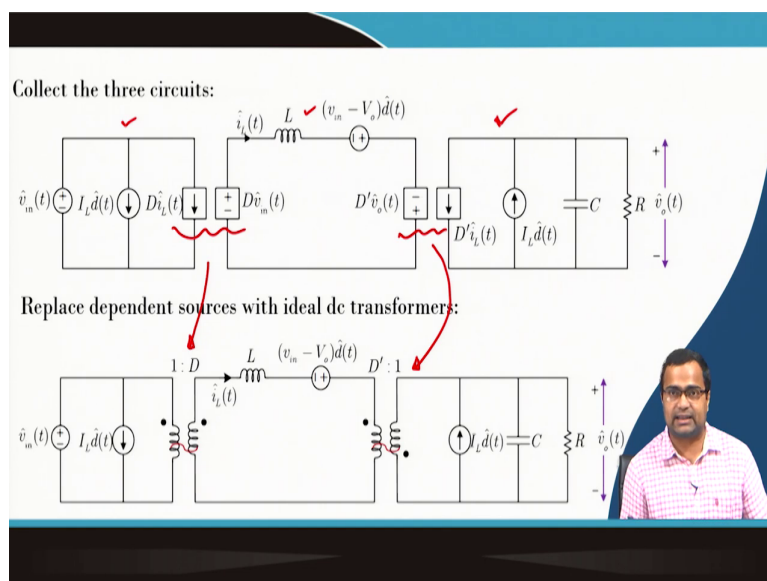
The circuit diagrams illustrate these equations. The first circuit shows an inductor L with voltage $\hat{v}_o(t)$ and current $\hat{i}_L(t)$. The second circuit shows a capacitor C with voltage $\hat{v}_o(t)$ and current $D'\hat{i}_L(t)$. The third circuit shows a current source $I_L\hat{d}(t)$ in parallel with a load resistor R , connected to an input voltage $\hat{v}_m(t)$.

Now, we can write down all these inductor dynamics, capacitor dynamics and the input current and their corresponding circuit that we have drawn already. So, this we have obtain for you know this all will be capital. We have already obtain and three individual sub circuits are the perturb dynamics corresponding to three individual like an inductor current, capacitor voltage and the input current.

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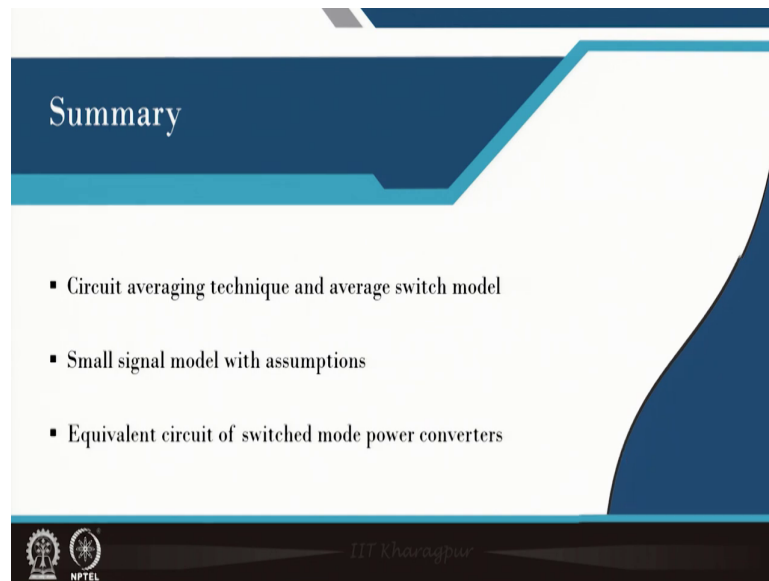
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Then we can club together all these three. This is the three equivalent circuits and then we have to collect these three terms. The first term is the input current, then the second term is the out inductor current and the third term is the output voltage.

And, you see, in between these two we can represent using a DC transformer the same thing here also and if we do that this can be replaced by a DC transformer and this can be replaced by a DC transformer. And, we can get the complete equivalent circuit of a inverting buck boost converter and that can be easily derive using circuit averaging technique.

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So, with this in this lecture we have discuss circuit averaging technique which there are two technique one we can use obtain the equivalent circuit by averaging the waveform over a periodic cycle and a second approach we can obtain the equivalent circuit by applying average switch model like using three terminals switch model. And, we saw that both results both the technique result in the same equivalent circuit.

Then, from that equivalent circuit, we can extract the AC part and the DC part. So, we obtain the small signal model with certain assumptions; that means, the perturbation is much smaller than magnitude is much smaller than is average quantity and then we derive the equivalent circuit of switch mode power converter and we derive it for buck, boost as well as buck-boost converters. So, with this I want to finish today this today's lecture.

Thank you very much.