Control and Tuning Methods in Switched Mode Power Converters
Prof. Santanu Kapat
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Module - 05
Modeling and Analysis Techniques in SMPC
Lecture - 25
Overview of Modeling Techniques with Some Motivating Case Studies

Welcome, this is lecture number 25. In this lecture, we are going to talk about the Overview of Modeling Technique and we are going to consider few Motivating Case Studies.

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So, first, we will say, we will discuss what are the objective of testing and validation in switch mode power converter. Then, need for modeling to predict transient and steady state performance, then primary modeling objective which can be categorized into three parts that means, we need to predict.

Then, we need to select the suitable control technique; then we have to design as well as we need to automate the process in order to meet certain requirement. Then, we also want to see what is the model simplicity versus accuracy and model limits on control bandwidth that also we will discuss and finally, some small signal and large signal modeling technique and then their significance ok, that also we will discuss.

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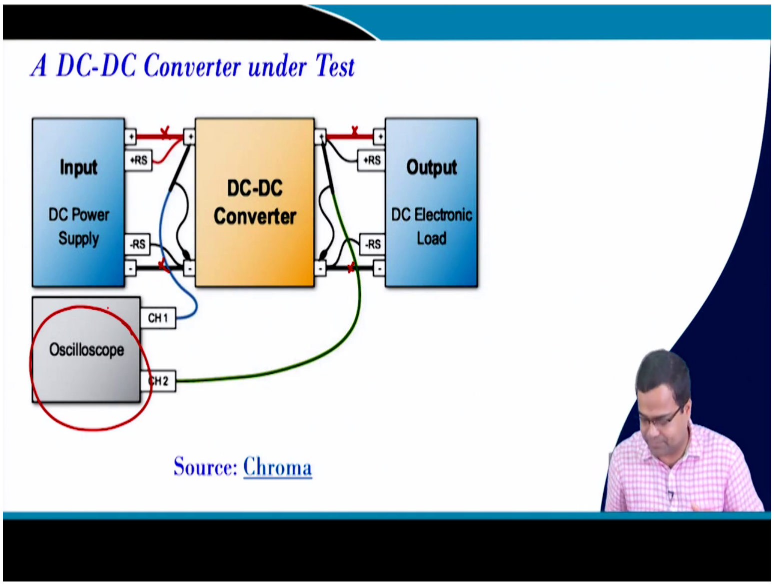

Consider a DC-DC converter under test. In this diagram, I showing a typical setup. We want to test a DC-DC converter, where the DC-DC converter can be a PMIC. We can connect the external you know inductor capacitor, but the PMIC will have its own controller and its own switches. We need to evaluate the performance of that IC, in terms of load transient performance and so on.

What are the control logic? So, that means, here the objective that we will have a DC-DC converter ok. And now, this DC-DC converter will be connected to an input terminal that means the supply. We will connect either an electronic load or a load, which can be a resistive load because in electronic load, it is very hard to do transient simulation when you go for low voltage high current applications. Slew rate may not be fast enough.

So, we need to make a special load in order to test high slew rate load transient. And also, we need an oscilloscope right. In fact, there can be other devices like a measuring instrument like we may need a digital multimeter right. So, the oscilloscope we can actually observe various waveforms of the oscilloscope whether you want to see current, voltage, then you need also current probe, voltage probe so, different probes are needed.

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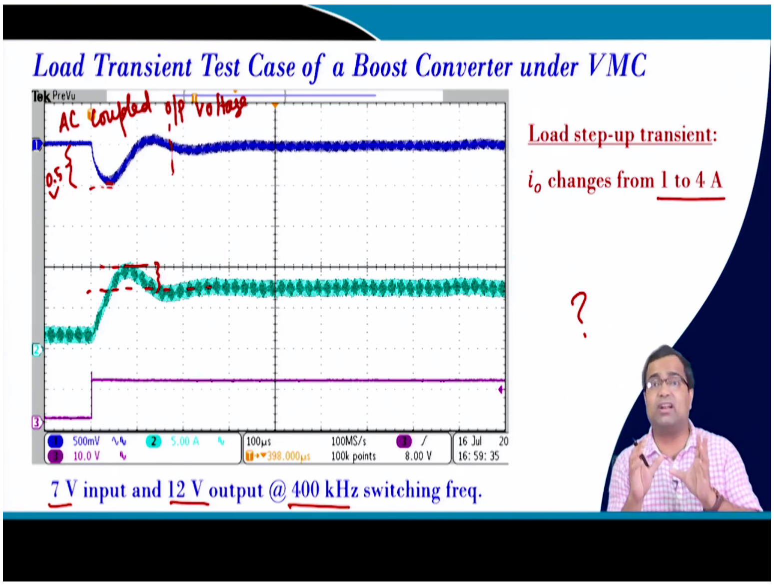

Now, here I am showing a load transient test case of a boost converter under voltage mode control. So, here, as a designer, I have designed the boost converter, but I am not revealing. Suppose I am making this boost converter and I am making it as a commercial product, for example, ok.

But now, if you want to sell it to a customer, like how that power management industries perceive. You know, the industry guys want to design a converter and its controller. It is designed based on their certain techniques, which we are not going to discuss now. As we move forward, we will see different modeling and controller design techniques. Many such techniques are followed to design such converter.

But now, once we suppose we bought an IC, this is a boost converter and we want to test in the laboratory that what is the performance so, here I am showing a load transient performance. So, here I have made I mean we have made the converter and we are also testing whether it is meeting our requirement or not. So, this is under voltage mode control, and you can see the transient performance is not good.

Because our switching frequency is 500 kilohertz, it is taking almost you know kind of 0.2 millisecond basically, 200 microsecond. This means almost 100 cycles, when there is a load change from 1 ampere to 4 ampere. When it goes from light load to high load. In a boost converter, we will see very soon that boost converter suffer from non-minimum phase behaviour. So, we cannot increase the bandwidth of a boost converter drastically, like a buck converter, because there is a physical constraint. For the time being, we are not going to discuss this issue.

So, here, we have to design a 7 volt input and 12 volt output boost converter and we are avoiding sorry 400 kHz not 500 so, 400 kHz switching frequency. Now, the question is if we apply a 1 to 4 ampere loads step, you can see this is an AC coupled output voltage so, this is my AC coupled output voltage. So, we are getting some overshoot/undershoot.

So, it is undershoot is coming almost 0.5 volt because this division is 0.5 volt and this may be too large because 12 volt though and current also have some overshoot because if you check that here to here, there is certain overshoot in the current, but how can we predict that what would be may overshoot or undershoot?

Because we have designed this converter keeping in mind certain specification, but when you are testing, we need to check, correlate that whatever we have designed. Are we going to meet or not? This is like a testing and validation this process. In this case, the processes that we know the converter, we know the procedure of design.

And we have followed that procedure and we have obtained those controller gains as well as the voltage mode control logic. We have made a closed loop control system and then, we have applied a load step transient and this is a response. So, as a validation engineer, what we will see?

We will find out whatever undershoot we are getting, is it matching with my specification that means I can design the converter either using frequency domain specification or time domain specification. Even though if you take a time domain specification, you need to correlate in frequency domain because most of the compensator are designed using frequency domain where you know the time domain you have difficulty because there will be multiple poles and 0.

In frequency domain, we have to we will consider in terms of phase margin, bandwidth, crossover frequency and so on. So, we need to check whether the obtain performance is matching with my desired requirement or not. And in often, you will find they may not match, they may match closely, and this mismatch can have you know there can be various reason that means, whether are you taking a suitable measuring step that means, are you taking the right step to measure?

If we introduce more parasitic into your converter, then measurement of actual converter behaviour is different. Whatever you are measuring that can also have some effect because we need to set you know you should not use a very long cable and that we have discussed in lecture number 13 so, we have to be very careful about the setup.

So, with followed by following all the standard you know, like a procedure, are we able to match our requirement or not? So, that means, this possess question, how do you match? Even if they mismatch, we need to find a solution. This is one way of looking at this problem.

The other way, you have been given a converter with control; you do not know what type of control it is used; you know only it is voltage mode control, but we want to check whether whatever response you are getting, can you predict what kind of transfer function or what kind of compensator is used?

So, in all these cases, we need some information about the system that means, first of all we need even the converter, we are only selecting the inductor, capacitor value right, then we know about switching frequency, if you zoom the waveform, here it is given at 400 kHz.

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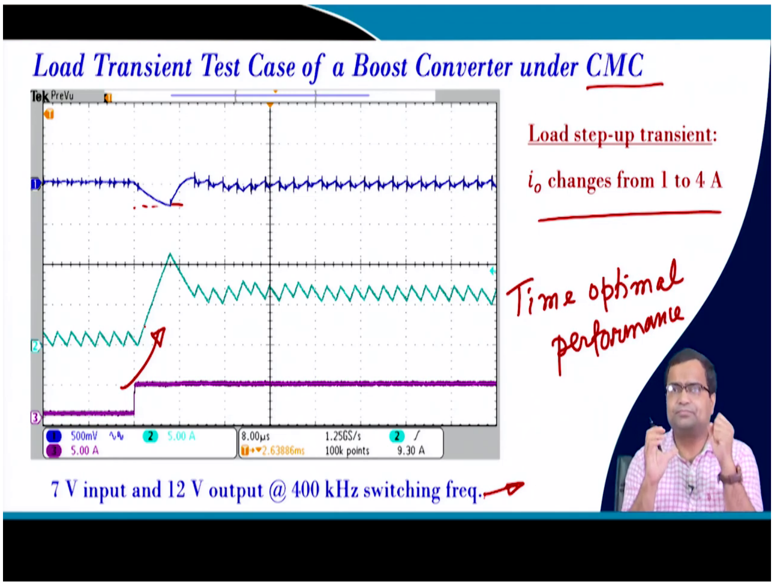

Another scenario I am showing. Here, it is again a boost converter, the same load transient, but now, it is under current mode control and here; we are using a large signal base control, not small signal. Even in current mode, your performance will be limited because of the right appearance 0, but here, we have applied some large signal based tuning controller design which we will discuss at the end of this course, and you could achieve the performance of the boost converter like a super-fast.

So, this is the fastest performance that you can achieve. This is also known as time optimal control. Time optimal performance – the performance that you achieve that means your response time is the fastest. You cannot get anything better than this. But, yes, this also comes with a price, your undershoot is large, this is a non-minimum phase. If you want to reduce the undershoot, then you have to put a limit, but we are not going to discuss these.

So, what I am saying, this we have obtained using a close loop control, but how did we arrive at this close loop control, who said this? Can we obtain this performance by any like a trial-and-error method? No. So we need to follow. We need to know something about the system; we need to know something about the model of the system that means we need to develop some model and we will soon find there is a fundamental difference between this approach and this approach.

In this approach, we will view this design of the controller using transfer functions, small-signal model and, in this approach, your transfer function is not valid. We will see because we are allowing the duty ratio to saturate ratio. So, your perturb model is not valid because the perturbation is quite large and there is a duty ratio saturation.

So, that means, the small-signal model even we have familiar with even if we can find out that model is not applicable here. So, we need another level of model. Why? Because this model we will find that this model is much simpler, you can apply the linear control system theory.

And their well-known theories are there and you can play with the transfer function with achieve, desired basement there are different tool, MATLAB control system toolbox that can also be used ok, there are analytical technique. So, this actually you can represent using a linear system.

But in this case, since the actual converter is a switching converter so, here the linear model is not valid and the fundamental difference is very obvious that the response is drastically improved because here, it was taking around hundreds you know because it was taking around how much?

It was like this much time, and it is like around 150 to 200 cycles whereas, here it is taking only a few cycles maybe even 4, 5, 6 cycles or within 10 cycle. So, you can see a drastic improvement in the transient more than 10 time faster.

So, in the future power supply, we need more and more performance and here, it can be shown in fact, we will show in the subsequent lecture. If such transient happen more frequently for example, if you are driving an LED load where the LED is a dimmable LED and it is undergoing load transient more frequently because the led frequency has to be much faster so that our eye cannot perceive and under such fast transient.

If you can save energy by slewing at very fast so, you can also improve the efficiency because the efficiency will not only be decided by steady state, this transient efficiency or the energy efficiency will also become important.

So, that means, we can improve the performance as well as efficiency, but at the cost of our linear model is not valid so, we need to incorporate some non-linear model here. That means even though we try to do a modeling technique, we need to understand at what level of model we need?

Do we need a very simple model? That is ok. You know, it is reasonably easy. You can handle or you can deal with your very well or rich tool design tool of control system. But, you have a limit to the performance because it is a switching converter. If you can go a little up that means, you know if you go a somewhat up level, top level where you incorporate the nonlinearity into that model, you can actually extract the performance up to the limit of the converter because it is a switching converter.

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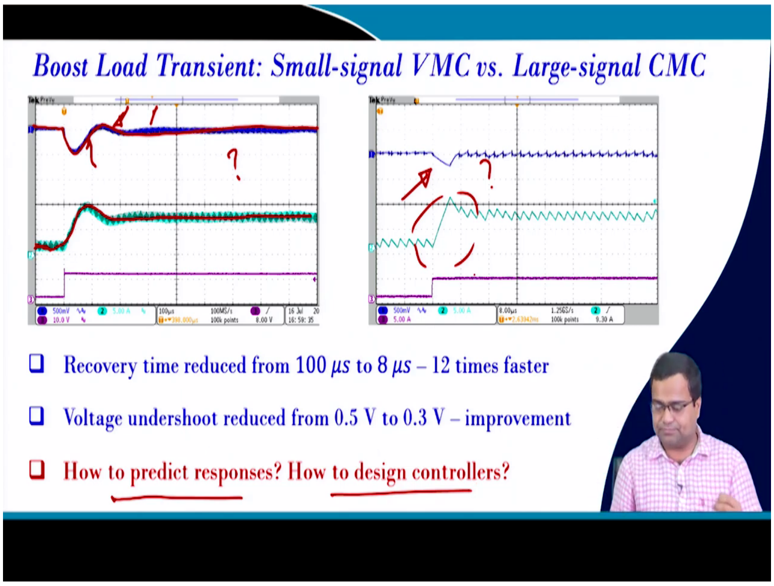

So, the question here is what I am asking. There is a small-signal based design. In fact, this you know one of the objectives of this course is to show that in future control, we will see more large signal where we want to improve the performance by many fold. But we are not going to change the modulator; the modulator remains the same ok. We will show that controller. Current mode control will also structurally remain same, but the philosophy of design is different. So, you can improve very fast.

But the question is how to predict response? How to predict this response and how to predict this response? And I said that this response if you try to draw a line in between so, this can be predicted by a linear model like there is an undershoot, overshoot like this and it can very accurately predicted by linear model.

In fact, in the subsequent lecture, I will show model matching. Whenever you design a converter, first we will design using small-signal model, we will derive a transfer function, we will design a controller using our linear model which are AC model, then we will get the AC transient response using that our linear model, then we will add the operating condition and then, we will compare with the actual switch simulation.

And you will see something similar, the red line is the result obtain from the linear model and the actual switching converter is the blue traces that we will compare in the subsequent lecture ok. But you will see this matching will start deviating when you try to increase the bandwidth of the controller so, there is a band limit, model limit in the design and if you want to go by this way, then how can we incorporate that information and how to predict this response?

What is my recovery time? Can I analytically predict for a given step size? Can I design a suitable controller to achieve this kind of response? So, these are the question that we are going to ask.

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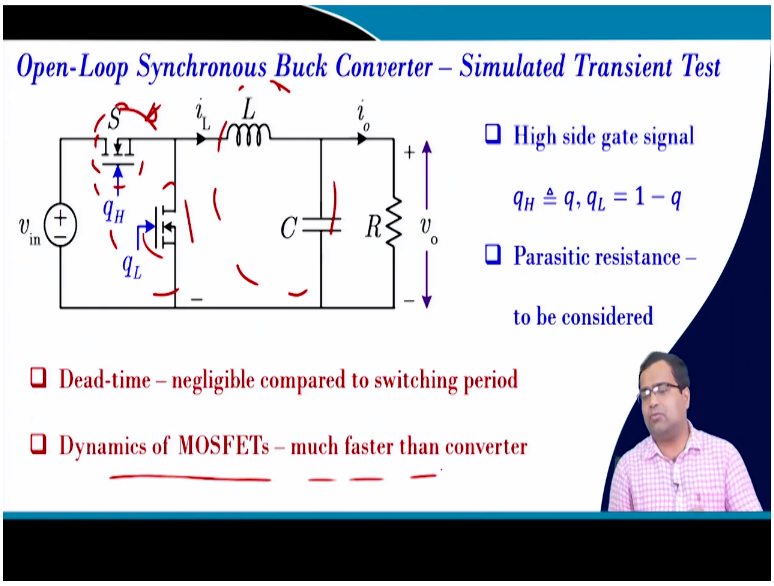

Now, coming back to an open loop buck converter. So, we talked about a close loop boost converter. In open loop buck converter, if you take a synchronous one, then we know we can take the MOSFET right, high side, low side MOSFET and we need to provide dead time for a real MOSFET, but here, the dead times are much smaller than the total time period. So, here, for the time being, we are neglecting.

Also, the dynamic of the MOSFET because MOSFET will also have turn on, turn off process, but if you look at the dynamics of the converter that mean, compared to the dynamics of the switches so, these are much faster than the dynamics of the inductor and capacitor. So, it can be framed into a singular I mean it you can, we can apply singular perturbation theory where the two systems have you know difference in their time or dynamic that means, one is very fast, and another is very slow.

So, the faster one will decay very quickly. Then, it will be primarily dominated by the slower one. That is exactly happening under such consideration. We can ignore the dynamics of the

MOSFET because once we derive small-signal model, then we will see the MOSFET dynamics are so fast so the small-signal model does not require, you need not need to incorporate that model as long as your rise time, fall times are only of only a small fraction of the time period.

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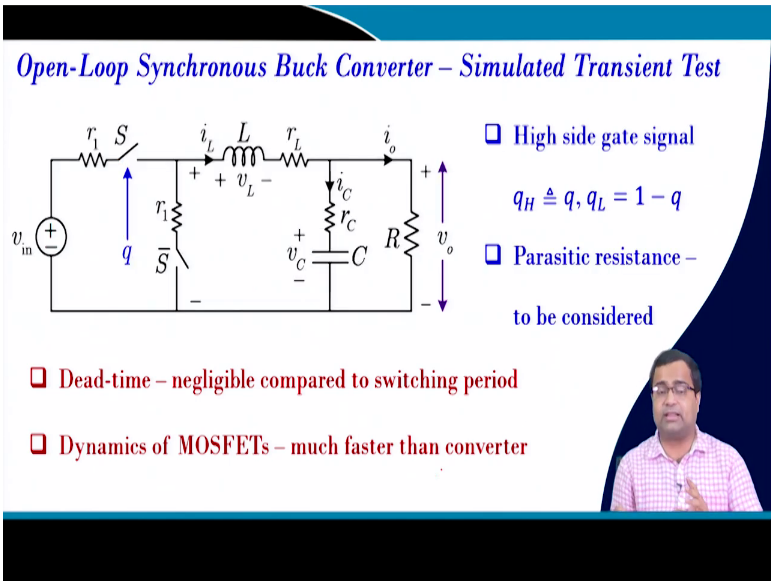

So, we are taking just a single switch and we are taking the parasitic that means, the parasitic component other than the dynamic component. This can be replaced by a parasitic model of a synchronous buck converter.

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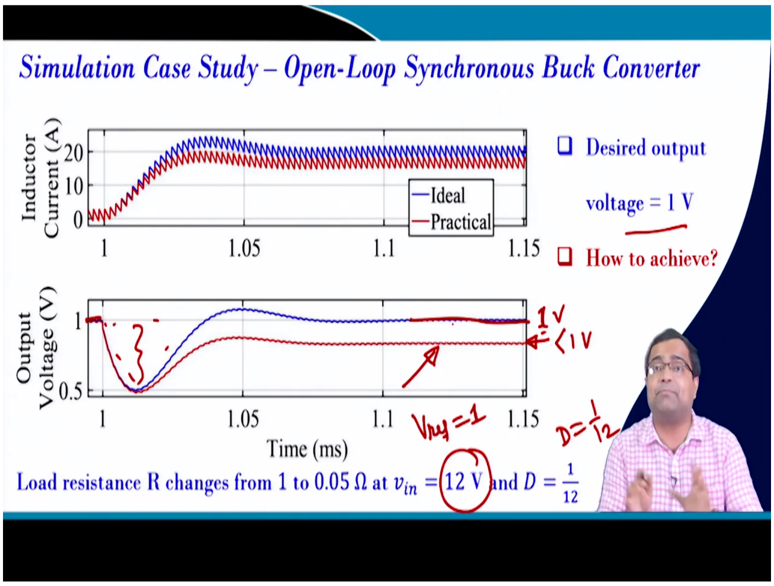

Now, I want to show in fact, this result I have shown in lecture number 13 where I have simulated a synchronous buck converter with an open loop duty ratio where my input is 12 volt and my V ref desired was 1 volt so, I set my duty ratio is 1 by 12 V ref by V in, but if it is an ideal buck converter, no parasitic drop, then before and after load transient, the average value is setting at 1 volt, it is perfectly 1 volt.

Whereas, in the practical case, it is not going to 1 volt so, it is less than 1 volt and also, there is an overshoot, undershoot in this process which also we need to identify. So, our question is that can we analytically predict what will be my practical voltage operating point for any load and input voltage condition?

Can we practically predict this undershoot and overshoot even for open loop as well as close loop converter? So, these are the question which motivate us why should we go for modeling technique ok. So, how to achieve this? If you want to achieve. Now, the next question will ask how can we achieve 1 volt output even for a practical converter ok?

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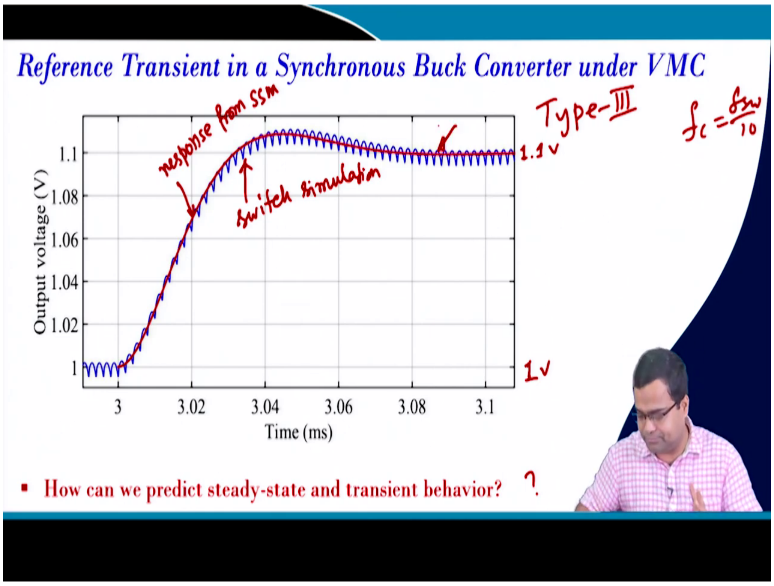

Now, I am showing another case, reference transient in a synchronous buck converter under voltage mode control. So, here, again the buck converter is running under voltage mode control, and I am not saying what kind of compensator I am using, but here, it is well-known that I am using a type 3 compensator and we will soon discuss these compensator, structure as well as this design or book buck and boost converter, but for this time being, whatever going to discuss.

So, the red color trace, the red one which is the response from small-signal model though we have not discovered small-signal model but assume that you can derive a linear transfer function out of a DC-DC converter by suitably applying averaging technique as well as the perturbation theory and the blue one is actually our switch simulation, it is coming from actual switch simulation.

And the red one which is obtained by because it is under close loop control and we want to change the output voltage from 1 volt to 1.1 volt ok and here, we have designed a compensator by setting a bandwidth crossover frequency of f sw by 10 that means, one-tenth of the switching frequency and then, we saw the response obtained from the close loop transfer function of the buck converter using transfer function and that is the AC response, then we added the DC, it is almost closely matching with that of the switch simulation in the average sense.

So, that means they are matching. This implies that we can develop a model, which can more or less capture the behaviour of the transient response of the DC-DC converter. We can predict what is the overshoot, undershoot, what will be my settling time and this thing can be predicted simply from our linear model without going into switching converter.

So, that means, if you can develop such a model, we can predict the response by without running the actual switching converter and from that process, we can set the controller in order to achieve some desired performance so that we can actually test the convertor. The next question, how can we predict the steady state and transient behaviour?

Here, we have predicted as we know that, but I am opening this question and asking how to develop this model so that we can get the red trace from our model. How can we develop?

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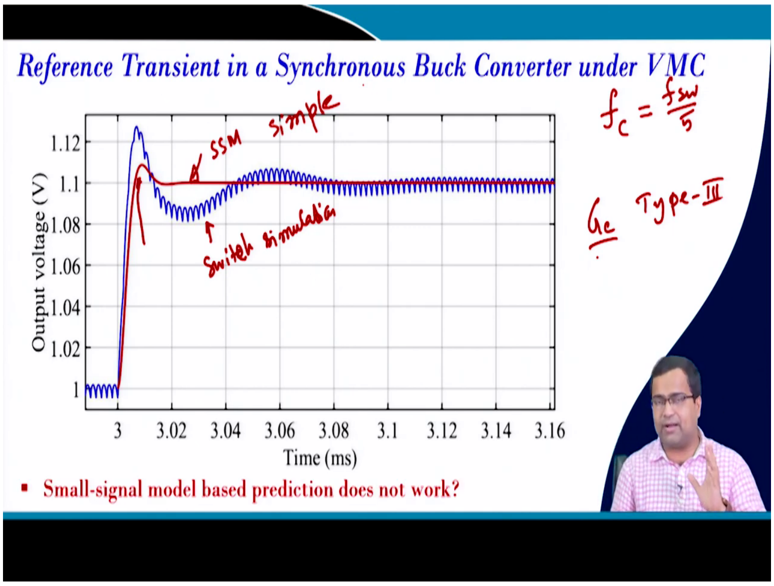

The next question, I have increased now the bandwidth; I have used the type-3 compensator. I have designed the compensator to achieve my crossover frequency f sw by 5 that means, one-fifth of the switching frequency and you see the response due to our small-signal model and this is actual switch simulation; they are drastically different, that means, even though you are able to model it, even though you are able to design it.

And from the linear design, you obtain this response in order to improve the transient performance, but when you actually plug in because this after this design, essentially we are designing a compensator and which is a type-3 compensator right. So, what are you doing?

We are designing this type-3 compensator by considering a small-signal model assuming that you have the model with you. But if you design the compressor by the model and your model shows this response, but the same compensator if you apply, if you consider in actual switching converter, this model is of no use.

Because if you cannot predict accurately with your switch simulation and your model response, then this model cannot be used or should not be used because your actual response can deviate significantly and, as a result, there is no point in validation. So, that means this will also pose another question. Even though you are able to develop a model so, there will be a certain restriction of the model. We cannot use that model for the entire range of the bandwidth that means you cannot increase the bandwidth drastically.

Though this model is simple, this model is not effective when you want to improve the performance. This motivates us if you want to do modelling, if you want to start with a very simple model that is reasonable, you can design the controller up to certain performance.

But if you want to achieve faster performance, you should not say that I cannot design because then we should look for an alternative model because it is a switching converter, it is capable to provide much faster response than what can be achieved using a linear model. So, that means, you need to find the next level of model which can be advanced version in order to speed up the response. So, the small-signal model does not work, and we need to find some alternative model.

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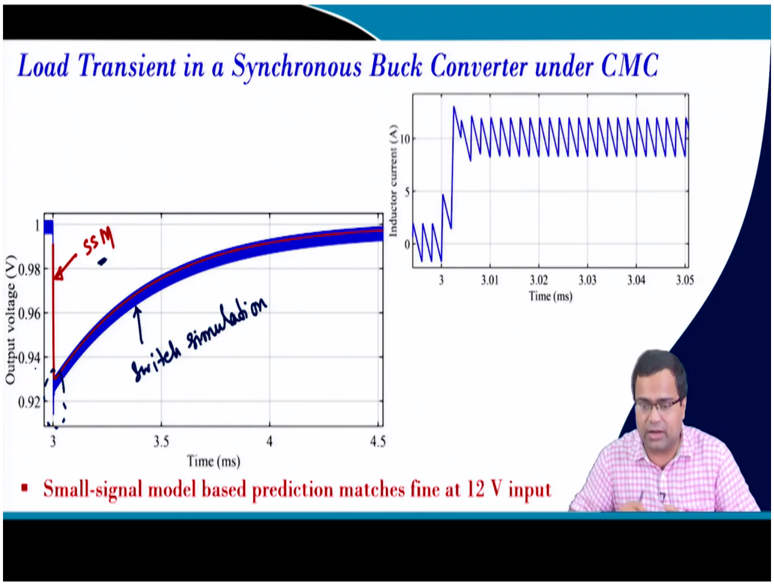

Again, we are talking about load transient response under current mode control. You can see the red trace again, which is coming from small-signal model. The blue trace is coming from our switch simulation, and I will show you this small-signal model under current mode control is very simple. It reasonably captures the behaviour. It is reasonably correct except for this portion, which it cannot capture.

It is capturing the behaviour of the converter close loop behaviour both linear model and switch simulation. So, that means, it motivates us that they are some more exist and we should try it out which can match our closed loop response of the model can match with the actual switch simulation. So, it is matches fine 12-volt input.

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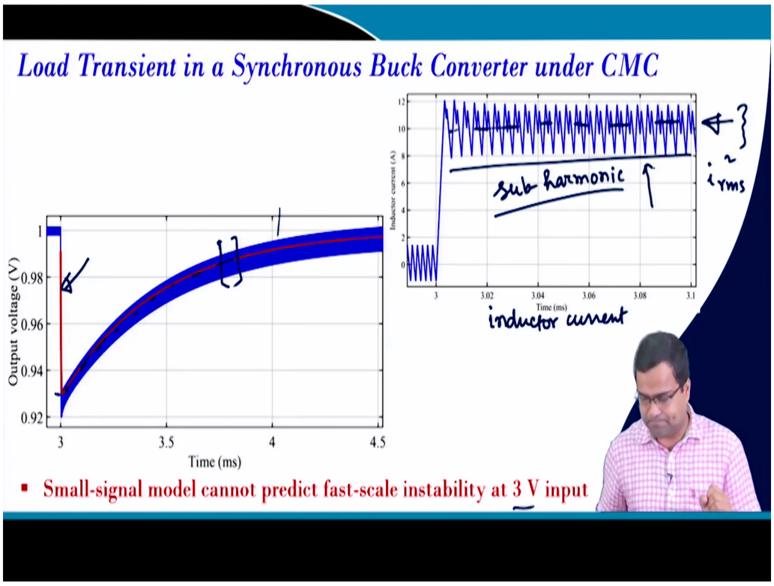

Now, if we operate at 3-volt for 1 volt output under close loop, the model seems like matching fine. It looks like in average since it is matching fine, but if you zoom this portion and corresponding inductor current I am showing, this is my inductor current, you will find. You will find sub-harmonic oscillation that means, in our common sense, I will not say it is a stable behaviour. In an average sense, yes, it is a stable behaviour, and that is why the model is predicting average behaviour.

But this model is unable to capture the sovereign instability because if you design a model, if you obtain a model, if you design a controller and you say that ok, I can achieve this much undershoot, my settling time is this much so, it is perfectly fine, but when you implement controller in average sense, it works really fine.

But you end up with this kind of scenario in actual converter behaviour which is not acceptable because your ripple has increased drastically. Your RMS current will increase drastically. As a result, conduction loss always increases and you have a sub-harmonic component where you will have a sufficient power at the sub-harmonic level and that will really create travel in your filter design ok.

That means, this model what we have developed small signal model. This model we cannot go for very high bandwidth, we saw there is a significant deviation, this model cannot be used to capture sub-harmonic instability, but there are technique I will discuss in the modeling technique of current mode control which can be used to capture this sub-harmonic instability at least to identify where it can happen ok by incorporating the sampling effect into the current loop, current mode control small signal model, but in standard traditional model cannot predict it so, then this model is also not very useful because it is unable to capture this fast scale instability.

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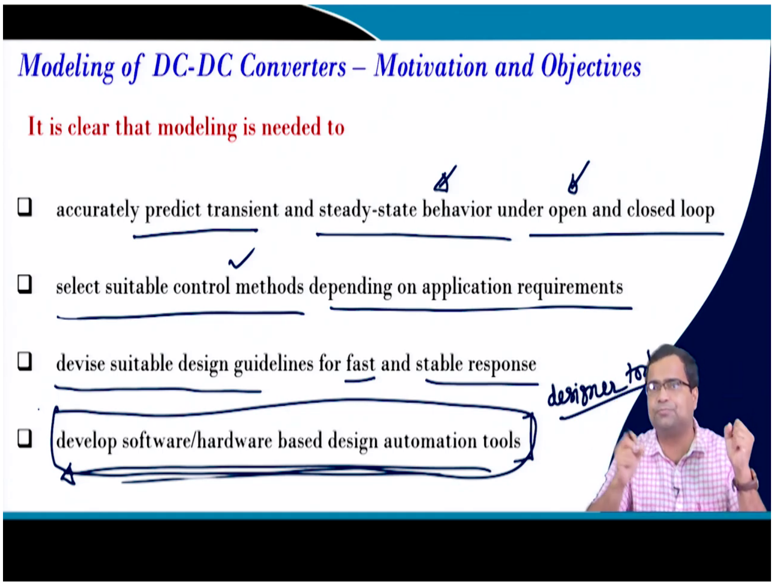

That mean this gives us the motivation. It is clear that modeling is needed to accurately predict the transient and the steady state behaviour under both open and close loop. You can first check the open loop, because ultimately, our this objective is to design a voltage source. If you recall in lecture number 13, if we want to make a very tightly regulated voltage source with a high bandwidth, that means our objective is to go close to an ideal voltage source within a range of current.

Then we should have a very high end controller and for that we need really to have a model; that means, to predict. Also, the statistic behaviour, the model should also capture whether there is a sub-harmonic or not. Then, we should, the next task once we are motivated to get a model now. Suppose, we are trying to investigate what models are available.

And why so many modeling techniques are there. Because I told you it start with the simple model, you can have a very simple model which can predict up to certain behaviour, then you should be happy, but if you want to push the performance further and that model is not useful, then have to go for next level of modeling where you have complexity of model will increase.

Once we have the model, then we need to select the suitable control method and we need to depend upon the application requirements. So, if you need a current mode control for example, in for let us say voltage regulator model multi-phase, then it is better to go for current mode and then, we need to implement; we need to rare you know consider such model and to design a very high bandwidth current controller.

Third objective is: once we have such model and once we identify the control method, the next task is to devise some suitable design guideline. How can we develop the steps to design controller in order to achieve fast transient and stable response, which you can also ensure sub-harmonic estimate will not happen. Transient performance should match with your switch simulation and actual the model.

The next task: once you have the right model and we can also optimize, we can improve performance, improve efficiency, then we can create a block that means, you know kind of library for any given inductor capacitor that means, you know if you I have discussed in lecture number I think probably 13 where you know many industry have their own call you know designer tool and the objective of this designer tool is an automated tool where you need to plug in lot of model there in that tool.

And that will ask for what is your input voltage, output voltage specification, what is your load current, then using that information it will first you know select what is my inductor value, capacitor value, what will be my switching frequency. Then, it will try to correlate in different industrial, try to correlate what is their existing product closed to that and what are the range of product that can meet certain input, output voltage specification and load specification.

Then it will give you different variants, like IC number 1, IC number 2, etc. which can meet all these specifications. But, each of them has different controller and bills of material. That means, some of them are using higher switching frequency, some of them using current mode control, some of them using voltage mode control because if you want to because you have not given any specification of the transient response.

Once you start adding more and more advanced features, then it will filter out and probably, it will try to search the right product that can meet your transient specification. It may so happen if your transient speak is very very you know stringent, you may not find any product at all. Then, you need to find your own solution. That is how the technology development happens.

So, this automated tool is very important you know to develop so that your new control technique comes in or even for existing control technique. We need not to spend a lot of time in designing from the scratch so you should develop the model and if you just plug in, the model should tell what is my compensator gain.

In fact, in this course, we will go for design and demonstration using MATLAB dot m file. If you specify some gain margin, phase margin, or even if you specify some phase margin and control bandwidth, then it will give us what are the compensator gains for a given set of inductor, capacitor and voltages. Then, you can design the compensator or you can plug in and you can also match the response of the model as well as the switch simulation.

So, that means, this step is also very important, it will try to automate, so, you do not have to spend time again and again writing code matching. So, in one code, it should be generic, which will ask for some specification, which should take the values from some parameter file.

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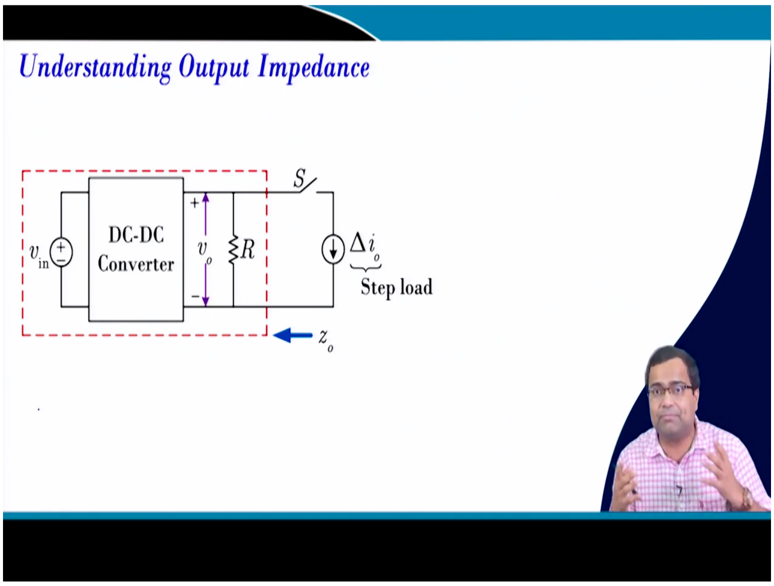

Then, understanding output impedance. So, output impedance gives us the flavour of load transient response. That means, we can predict a load transient response by looking at the output impedance ok.

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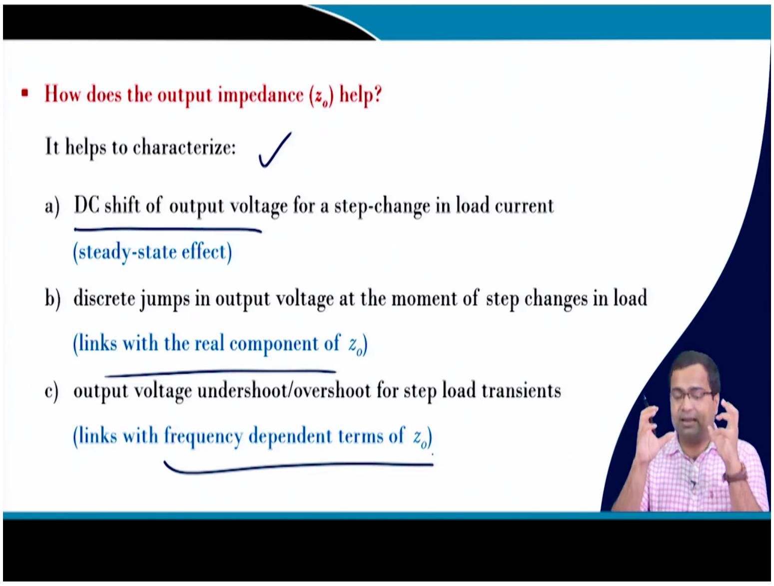

So, that means, how does it help? It characterizes if there is a load step change, is there any DC shift? In fact, this we have discussed using a case study in lecture 13, when we talk about a practical voltage source. So, a practical voltage source if there is a DC shift which indicate there is an internal resistance, DC resistance right.

So, these are steady state effect. Then, if there is a discrete jump, very high frequency, then we can also predict what is the real component of the z 0 which will cause the discrete jump, high frequency jump. And then, undershoot, overshoot can be predicted by the AC component or the frequency dependent term that means if we can get the model of the output impedance, then for an open loop converter.

For example, then we can predict very accurately the transient response of the converter when we apply a load step that means, this will gives us I mean without going into test and validation, first you can say that if I apply 10 ampere load step, my undershoot is this and which is not acceptable.

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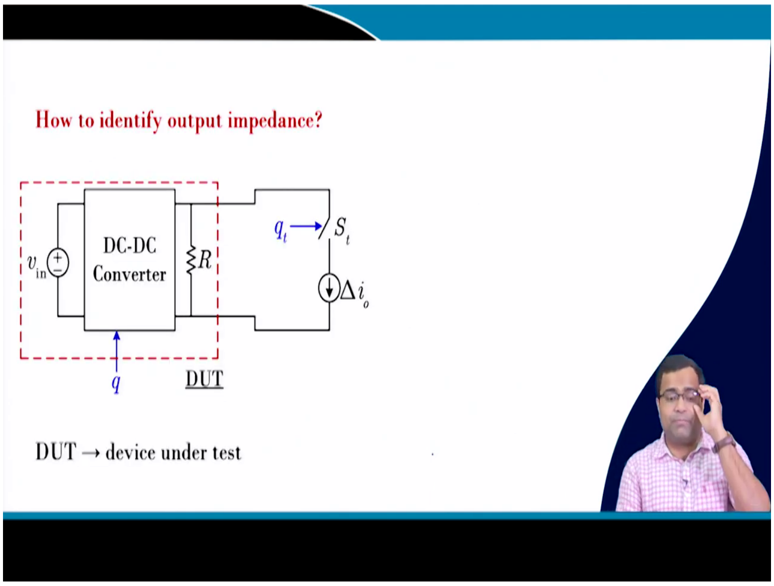

How do I identify output impedance?

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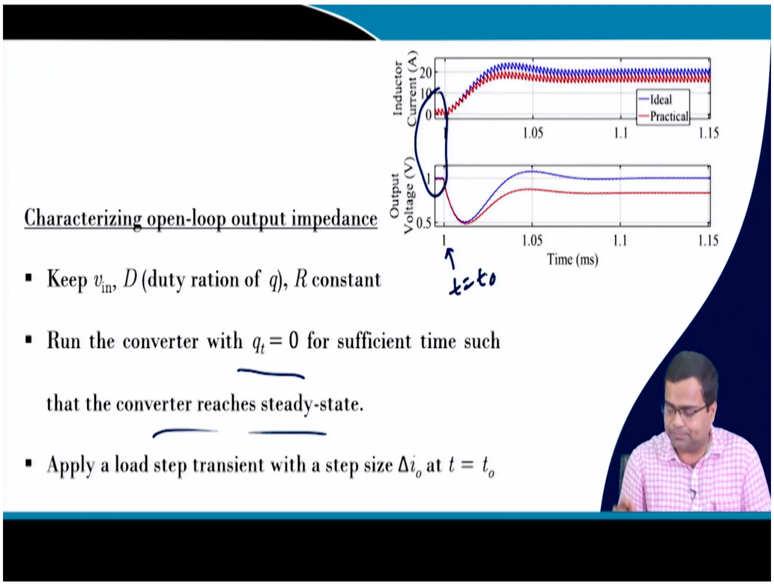

So, we have to device under test that means, this is what if you go to any experimental validation, if you have a converter, you can inject seems you have pulsating load and looking and overshoot, undershoot, you can try to match you know get the output impedance ok.

So, this step we will discuss that how to get the output impedance, but when you talk about steady state, when there is no load step transient, the converter should first reach to steady state that means we are talking about these condition, then we apply a load step transition at t equal to t 0, this is my t 0 ok.

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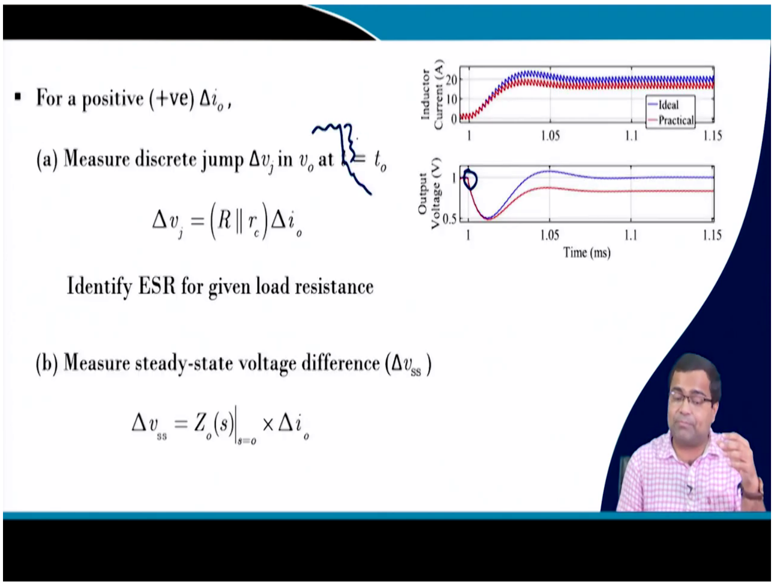

And at that time, there will be direct jump in the output voltage, which is not visible here, but there was an output voltage suddenly jump again, it will further decrease. So, this jump is basically due to the combination of R and r c. So, if there is no resistive load, it is purely by r c, high frequency jump and which cannot be avoided if you are ESR is fixed.

And you know what I know the designer is to do. Instead of using a one output capacitor, generally multiple output capacitors are used to reduce the effective ESR. So, ESR can be identified, then if you see the steady state difference, then you can get the DC output impedance.

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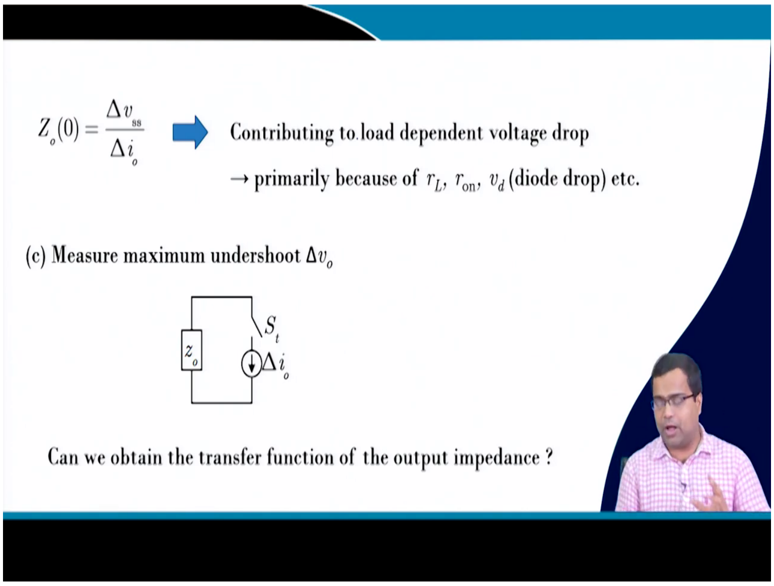

And you can also find out some AC output impedance by measuring overshoot, undershoot and that; that means, we need a model of the output impedance ok so, the model should be.

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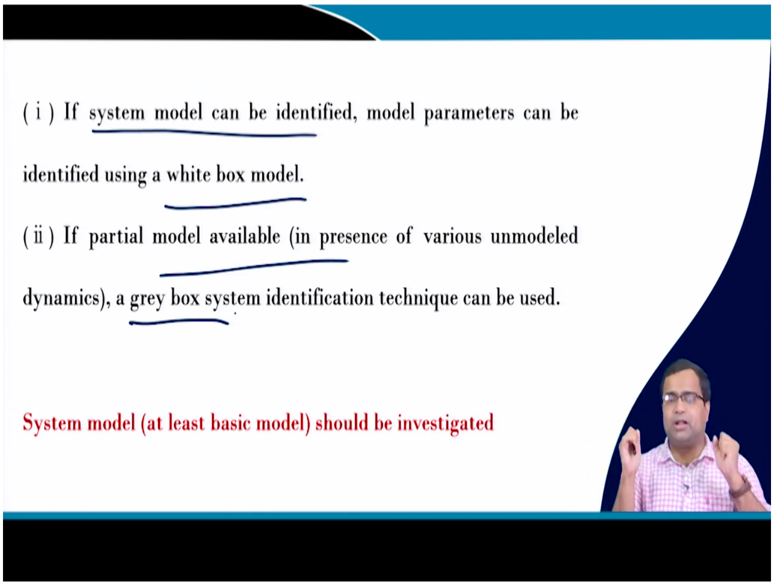

Now, it may so happen whatever model that means, you did a practical test of a converter and you try to drive because the converter is nothing is known inside except for inductor capacitor right. So, we need to obtain the close loop output impedance expression, then what we to do? We may not be able to model everything.

So, from a measured response, if you want to match so, if some information of the converter for example, in the buck converter, we know output side there is an inductor, capacitor so, we know the circuit and we can write the, we can derive the model and try to match and that is a white box model when the model is available and then, you can try to derive, it try to match the model or identify the exact parameter of the model from the measured data.

But it may so happen sometime the system is partially known because there are lot of unmodeled dynamics for example, ESL effect, parasitic effect, in the trace resistance effect, then there can be some additional delay which may not be directly measurable, but if the partial information of the model is available.

Then you can use something like a gray box model for identification and such model identifications are very important for future power converter where we are talking about digital twin where there can be automated tool. And if you just you know from the runtime of the system because there are a lot of online controller tuning method.

There are very highly cited papers are available which the tuning using digital platform can be carried out by simply identifying the model in real-time or measuring some phase margin as well as you know some parameter in real-time and you can tune the controller ok because when the load changes, you need to know what is that new load condition. So, from the identification method, we can extract that.

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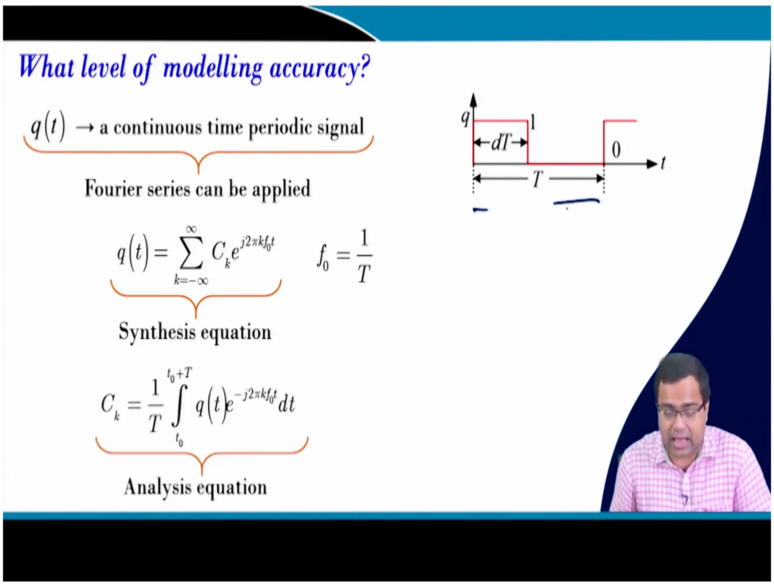

Now, the next question, we are sure that we need a model, but what level of accuracy needed? Now, if you go back to a periodic signal q of t and if we apply Fourier series, then because there is a periodic signal right under steady state, it is time period is fixed. So, we can write this q t, the gate signal and we can write the synthesis equation as well as the analysis equation and you will find there is a DC component corresponding to analysis equation.

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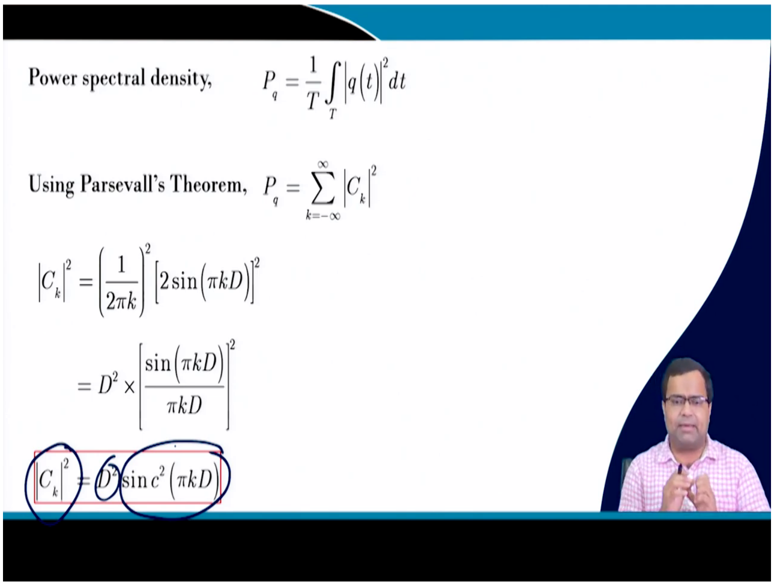

So, if you go to the power spectral density which can be obtained from the Parseval's theorem, and the power spectral density can be written by this power spectral density simply D square into the sin function square ok. So, I am not going to derive, but this can be derived, the power spectral density.

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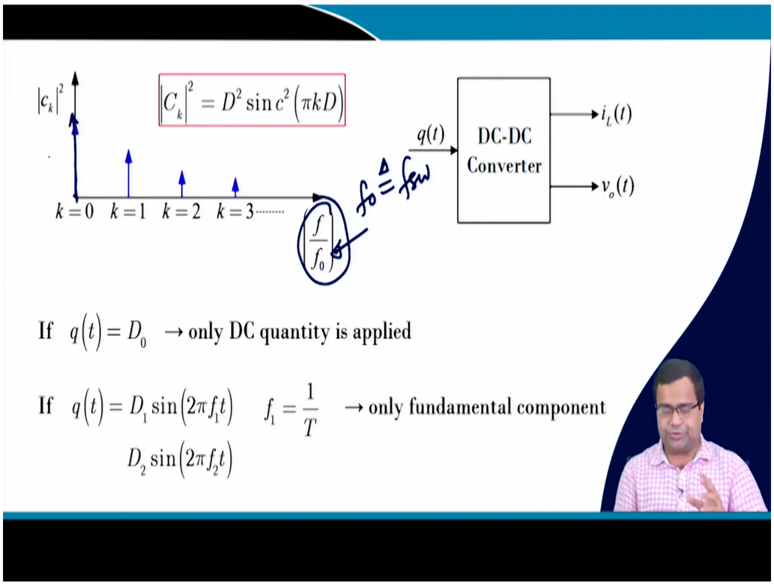

Next, this power spectral density gives us that what is the power contained at different harmonics that means, at k equal to 0 so, this is f by f 0 where f 0 is our fundamental component which is nothing, but our switching frequency in this case and 0 component is a DC component that means, this corresponds to our duty ratio D into t right, duty ratio and that corresponding power.

Now, in a DC-DC convertor, if q d equal to D 0, it only gives us the DC quantity. Then, if you find out D 1, what is the component of the duty ratio or the q for the fundamental frequency of the switching frequency? You can also find out the higher harmonics so that means, we can extract the power corresponding to different component, but you will eventually find the maximum power will retain at the DC component, which is a duty ratio.

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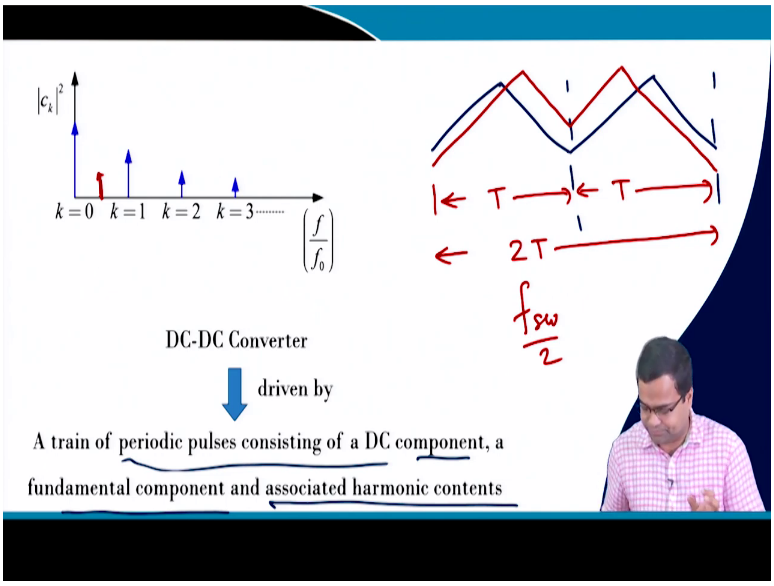

That means again DC-DC converter is driven by a train of periodic pulses consisting of a DC component and the fundamental component and a associated harmonics provided that there is no sub-harmonic oscillation. If there is sub-harmonic, suppose if you take the example of you know this is a periodic waveform I am just drawing, but for the same time period, if you draw another waveform which is something like this, it may be something like this so, which is like this yeah.

So, this is my sub-harmonic component ok. So, in this component, you will see for the red color, there will be a peak here because as if it is repeating, this time period is T, this time period is T so, as if it is repeating after 2 time period. So, it will have a component of f sw by 2 and it will have a component here. So, sub-harmonic, you will have a power component, power spectral density at sub-harmonic level.

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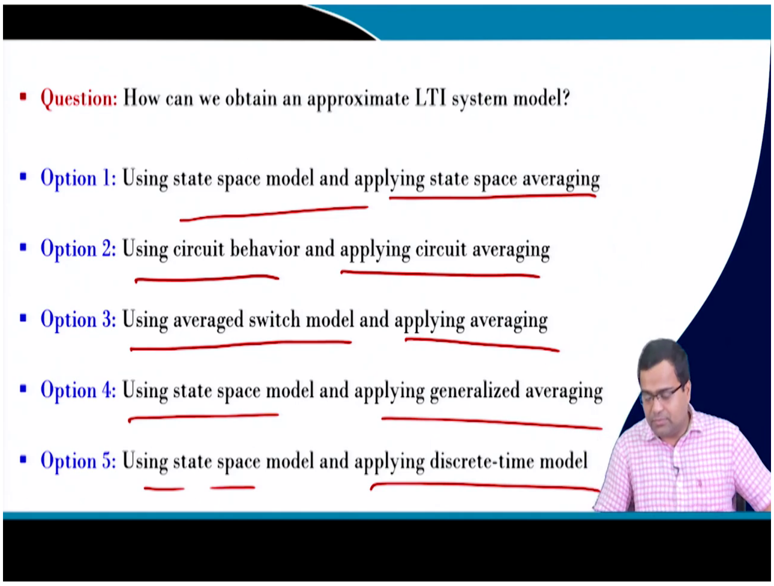

Now, the question is how can you obtain an approximate linear time model, Linear time-invariant model, LTI model because whatever we have you know model using a linear model, I think the response using a linear model I have demonstrated, how can you obtain that?

So, you can use a state space model and then you can apply a state space averaging. Then, you can use a circuit behaviour and applied circuit averaging technique. Then, you can use average switch model and applying averaging. Then, you can use a state space model and apply generalized averaging and there is state space model, applied discrete time model and there are many other options are available.

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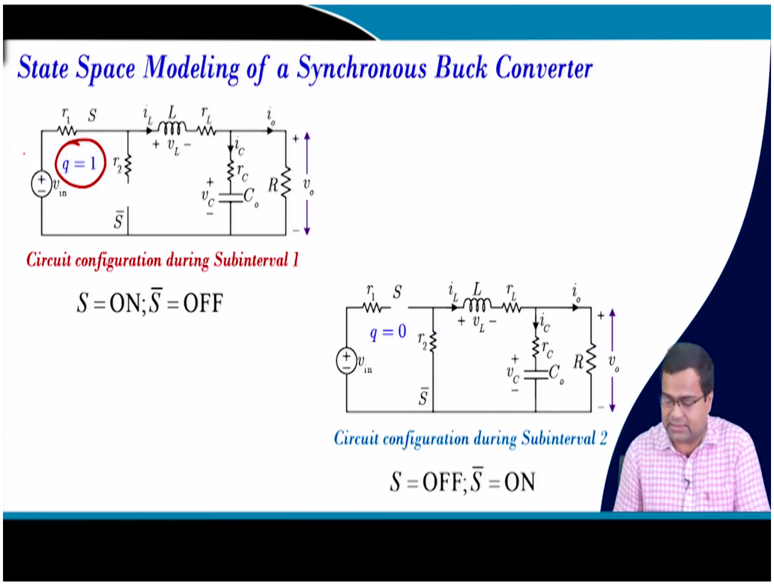

So, I will take just a glimpse of the state space model. So, if you take a synchronous buck converter, if you take the switch configuration 1, then you can write and switch configuration 2.

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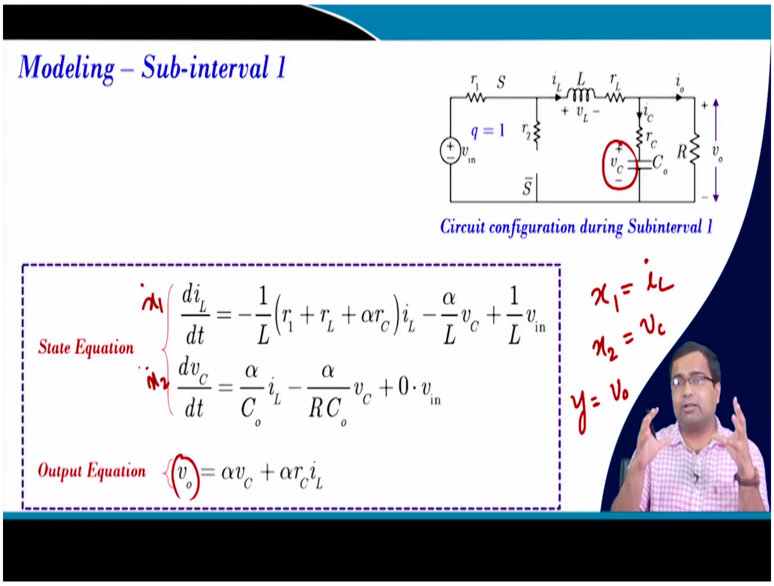

And if you take the state, one state as the inductor current that means, I am taking state 1 is my inductor current and state 2 is my capacitor voltage which is here ok and then, you can write x 1 dot, x 2 dot and their output y, y I am taking v 0 that is my output. So, you can write the state space model for two different switch configurations that mean sub interval 1 when the switch is on.

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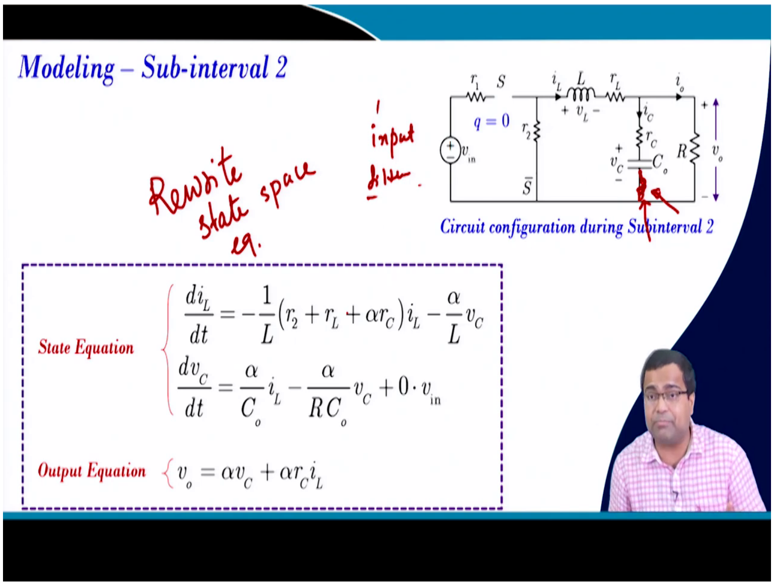

And then, we can write the same equation for sub interval 2, when the switch is off, but we are keeping the same state variable for throughout.

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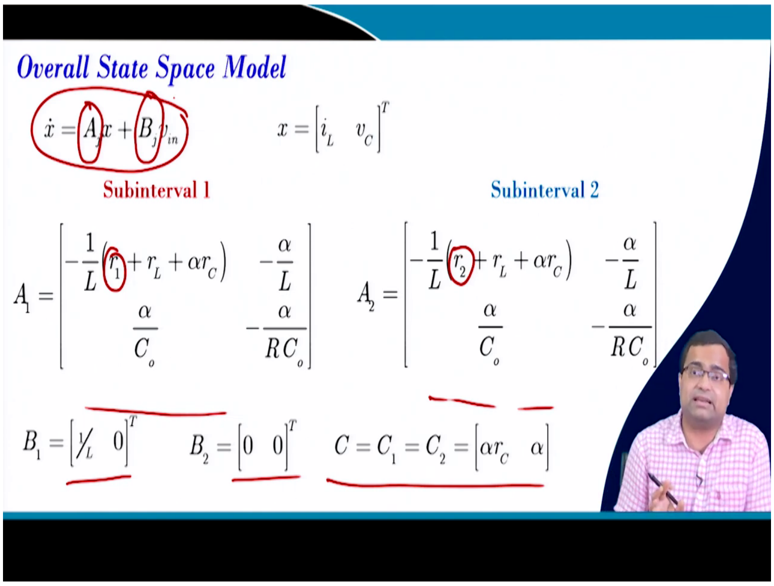

Next, overall state space model, it will have two components so, this kind of structure is called switch linear system. For within a switching interval, it is a linear system, but two linear systems are combined by means of switching. So, there A matrix is more or less common except for this difference, but their B matrix are drastically different, it is a null matrix and C matrix is common for this converter.

But if you take a boost converter, A 1, A 2 are drastically different, they are totally different, but B 1, B 2 will be identical, C 1, C 2 will be different ok. So, that means, for any generic converter, you can assume that for every sector of the sub-interval, you will get A 1 met A j, B j and, correspondingly, you will get the output voltage expression.

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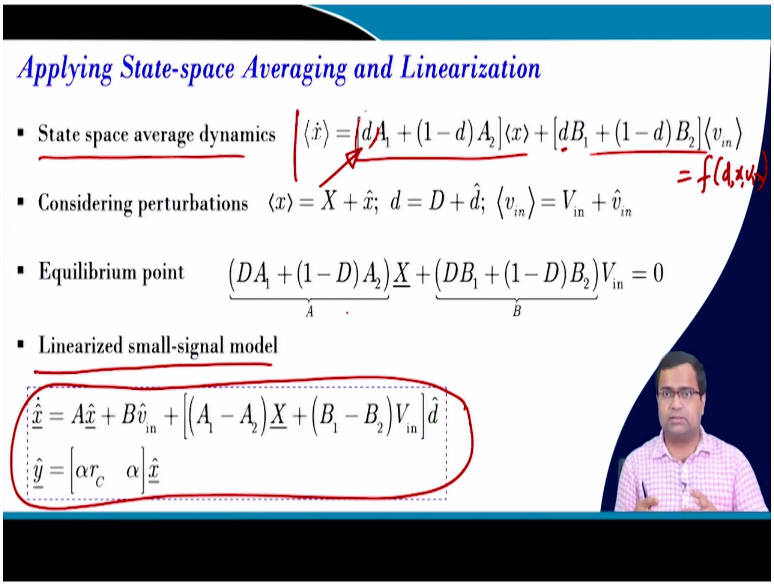

Then, once you get the switch linear model, if there are only two switching, even like switch on and switch off and switch on happened during the d into t time and switch off happened during 1 minus d into d time. So, the averaging technique you can standard everything technique we can simply average them, d times into that average state.

And then, d times a beam and after averaging because the original model, if you go, it is basically a switch linear model so, non-linear model with switching non-linearity where we cannot apply linear; linearization because it has discontinuous non-linearity. But once you apply averaging, then it becomes such smooth vector field d, it is a non-linear function; it is a non-linear function of d, x, v in, but this non-linear function is continuously differentiable, it satisfy the lifts is continuity.

Then, we can apply perturbation and we can apply, we can find out the equilibrium point, and you can apply you can get the linearized model simply by applying Taylor series linearization. In the next lecture about the state space averaging method.

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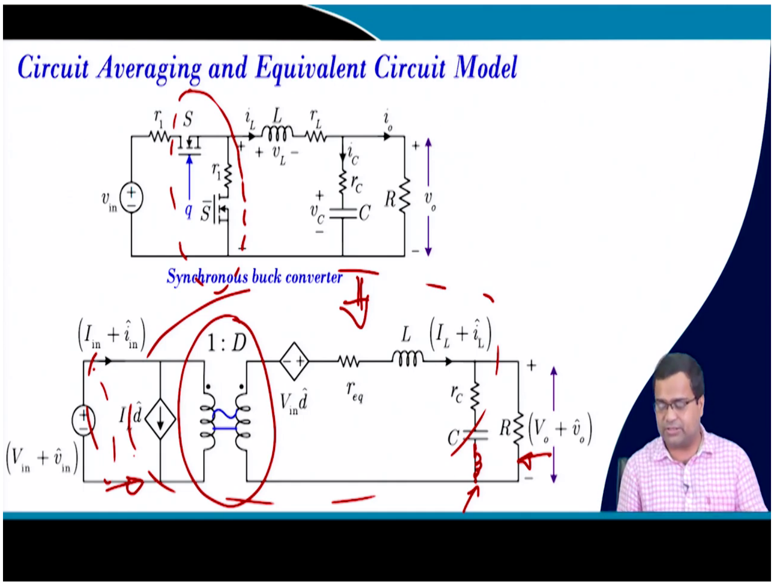

And if you go to circuit averaging method, in the earlier method, whatever you get this model, it is more like a mathematical viewpoint which is good, which is very imperative, but as a circuit designer, it has a less intuitive understanding of the circuit. The other thing in this converter you know we have taken this model ok. Suppose we add a parasitic inductance here, which is called ESN.

Then, the whole state space model dimensional increase because it is now third order because there is an energy storing element. As a result, you have to increase the order of the state by 1 so, that means, again if you consider an input filter here, suppose you consider an input filter here so, input filter has energy storing element, it will also have dynamics.

So, that means, when you write the overall equation, the overall equation in state space representation, you have to rewrite the whole equation that means. You rewrite state space equation; you have to rewrite the state space equation whenever you add any component also that means, if we even add a small resistance or you know apart from energy storing element. But even if you add a resistance, only the coefficient of the matrix will change, but if you add an energy storing even the dimension of the state space will change.

So, the one of the major drawback of state space model is that once you add some circuit element, then your you have to rewrite the equation so, it is kind of lengthy process writing the equation, rewriting. So, you want a usable solution where we do not want to repeat this derivation process even if we add some component and that is where the circuit averaging comes into picture.

So, that means, equivalent circuit model that means, by means of circuit we will discuss if you obtain this equivalent circuit model, now if you add an ESL, this will remain same, this model is perfectly valid. So, the order of this model will increase by 1, but you do not need to re-derive this model from starting from here, you can simply add this inductor.

Even if you consider an input filter, this model is also fine because you know you can add some more parasitic left side and right side, it will work fine as long as there are certain restriction if you go to equivalent circuit model that means, average switch model, if you can satisfy that requirement, then this model is more generic and you can it can accommodate more you know more circuit element, but still the model is applicable.

And for the same parasitic inductor, capacitor, everything, the mode AC model that means, this transfer function using circuit average model and the state space model, they are identical so, that means, the state space model can be obtained from mathematical equation which is also fine, but this is this method can give more physical insight in terms of output impedance, in terms of input impedance, in terms of audio susceptibility, control to output transfer function so, it will give more physical insight.

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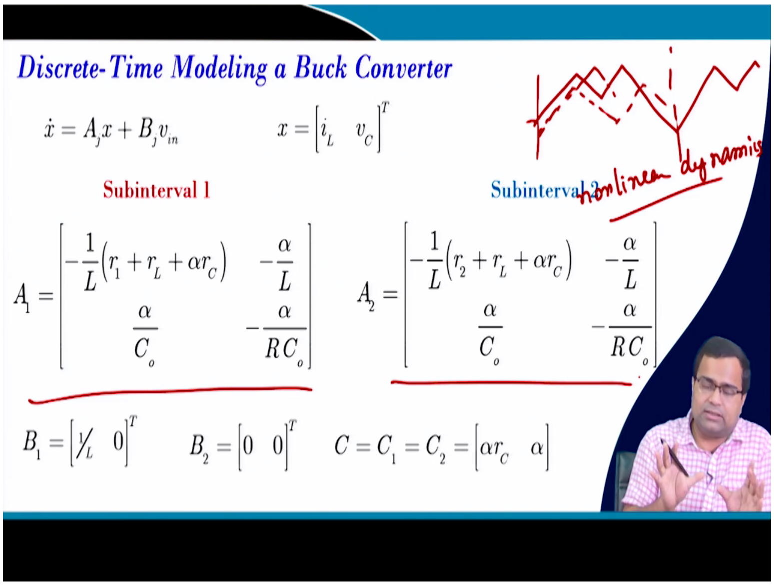

And the last one, when we talk about sub-harmonic instability because we talk about there can be this kind of current scenario which cannot be captured by a small signal model and then, we need to go for discrete time model and if we can find out under what condition this stable periodic behaviour is lost that means, stable periodic behaviour is lost and it is going to instability that can be predicted.

And if it further go into sub-harmonic instability and there is non-linear dynamics that in that context, we can go into non-linear dynamics and check what kind of behaviour we can achieve when it goes into instabilities sub-harmonic instability, what type of behaviour can be possible. So, that means, this discrete time model, it also uses state space model ok and A, B matrix that means, whatever you use in state space, the same state space model will be required here.

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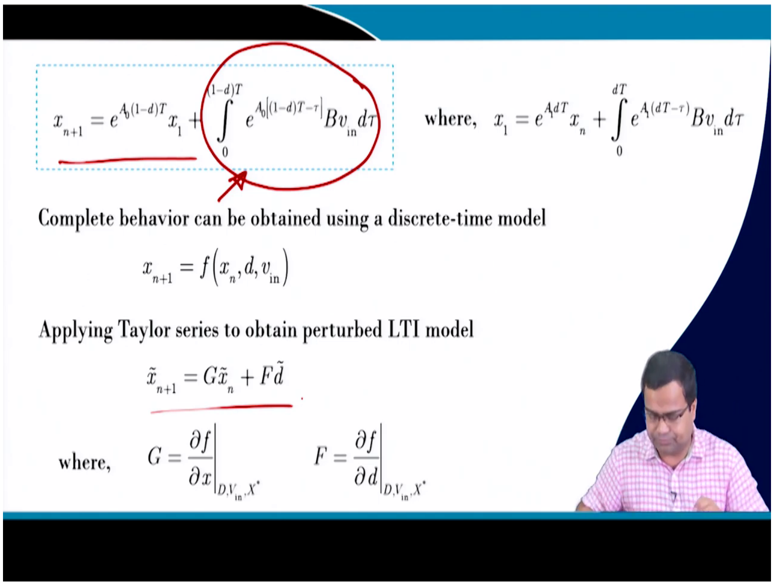

But you need to find the solution of the state space model ok and this term is particularly very important because if you want to get the solution of the state space equation, there are multiple technique to solve, to get an analytical form, but I am not going to discuss this technique because this will be somewhat advance and this may be in future you know digital control can be offered, then this can be considered or discussing detail.

But for the time being just for introduction, the discrete time model can be derived and this can accurately capture all kind of stability, but as you can see, this is mathematically complex system model so, you should use only when you want to go and investigate such kind of instability, but as long as if you ensure by a simplified way that there is no sub harmonic instability and if you want to operate at a lower bandwidth of the controller, then it is reasonable to go with a standard small signal model.

And here also, you can obtain the small signal model from the discrete time model. In fact, majority of the digital controllers are used. We use discrete time model directly to accurately design the controller, but in analog control, the small signal linear continuous time models are still popular.

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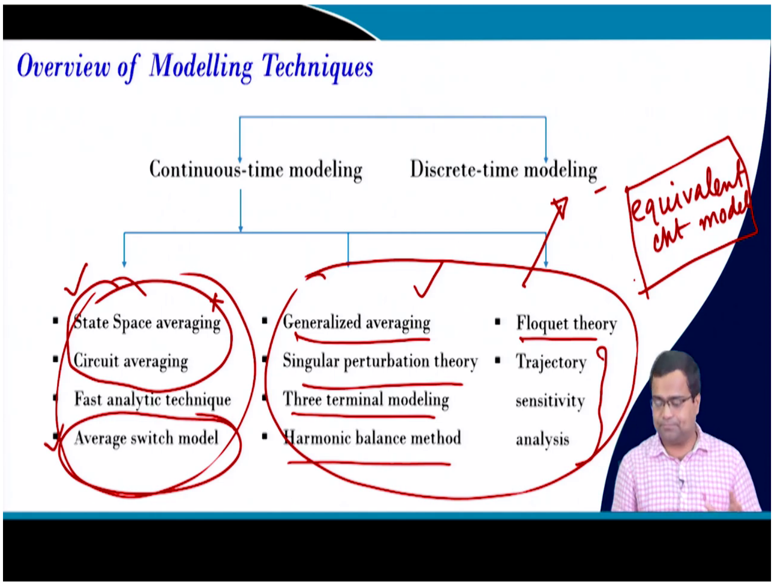

So, the overview of the modeling technique, we can divide into two-part, continuous time modeling and discrete time modelling. Then, the continuous time modelling, we can have a state space averaging, circuit averaging technique, fast analytical technique, average switch model.

We also can have generalized averaging technique, singular perturbation theory can be applied ok, three terminal model so, all these models are have been developed and they have been well reported in the literature and harmonic balance method can also be used which is a describing function method.

And also, the Floquet theory is a very powerful tool which also capture the discontinuity during the switching event using Filippov method and also, the trajectory sensitivity analysis can be used in order to get more accurate you know continuous time model.

And the discrete time model, I have already told it is a very accurate model, but a majority of these models are slightly complicated compared to the traditional model particularly, when you go for a like a Floquet theory, harmonic balance, these are slightly complicated, but these techniques are more accurate than this technique ok. So, this can give more information, but when you go to discrete time modelling, even it is more complex because of this solution of the matrix, writing matrix equation so, we need to deal with different modeling technique.

So, to start with in the subsequent lecture, we will primarily consider these two technique particularly using average switch model. So, these are the techniques that will consider extensively, and it develops small-signal model, this will give you a lot of physical insight ok and we will primarily consider equivalent circuit models that means, the equivalent circuit model.

This can be derived either by circuit averaging technique, this can be derived even using average switch model, this can even be constructed from the state space averaging model, but if we can represent in terms of an equivalent circuit, then you will get lot of inside in terms of impedance, in terms of control to output transfer function and so on which will give you a lot of physical insight, design insight.

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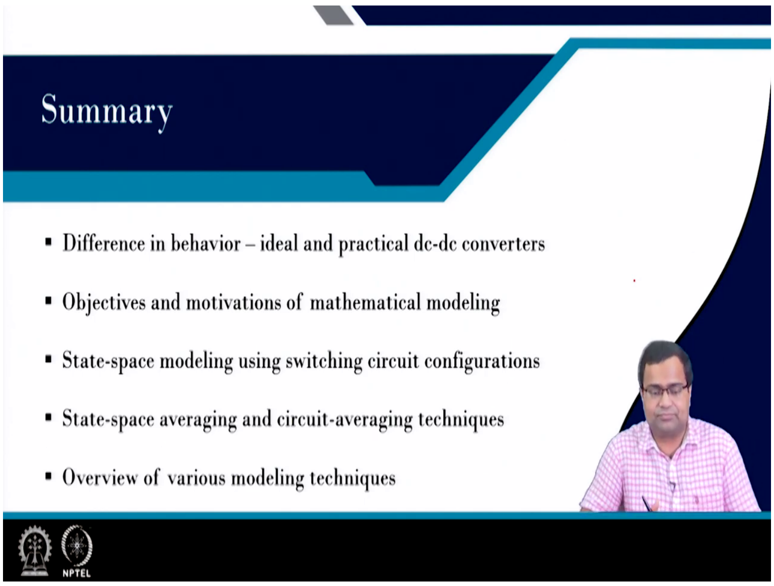

So, summary. We have discussed different you know the difference between ideal and practical converter; we have discussed in detail about the objective and motivation of mathematical modelling; we have briefly discussed those state space modeling and switch configuration, we briefly describe about circuit averaging technique and we have also discussed various modeling techniques. So, with this, I want to finish it here.

Thank you very much.