Control and Tuning Methods in Switched Mode Power Converters
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Module - 02
Modulation Techniques in SMPCs
Lecture - 10
Modulation in Discontinuous Conduction Mode (DCM)

Welcome. So, this is the 10th lecture and in this lecture, we are going to discuss Modulation in Discontinuous Conduction Mode.

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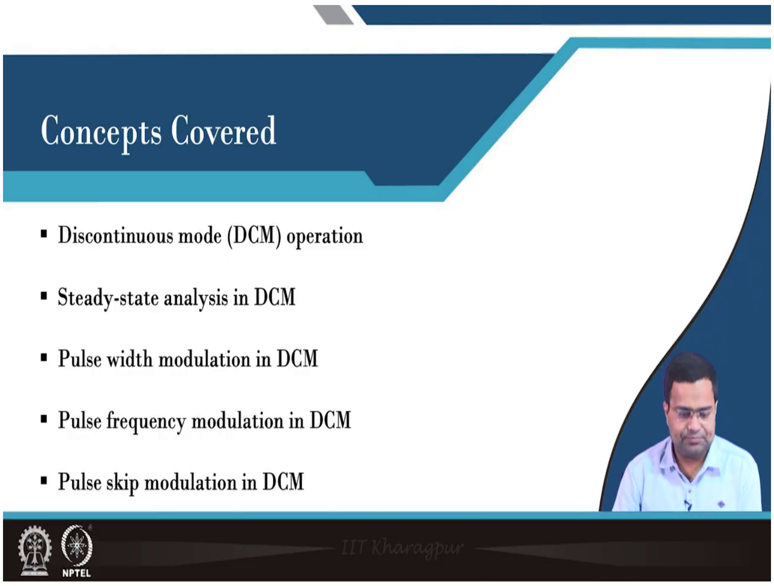

So, discontinuous conduction mode particularly, we have considered here is of specific interest because this happens when the load current decreases and we need to you know operate in a way so that we can reduce losses.

So, indeed, we first explained discontinuous conduction operation, then steady state analysis under discontinuous conduction mode, then pulse width modulation in DCM, then pulse frequency modulation in DCM and finally, brief introduction to pulse skip modulation in discontinuous conduction mode.

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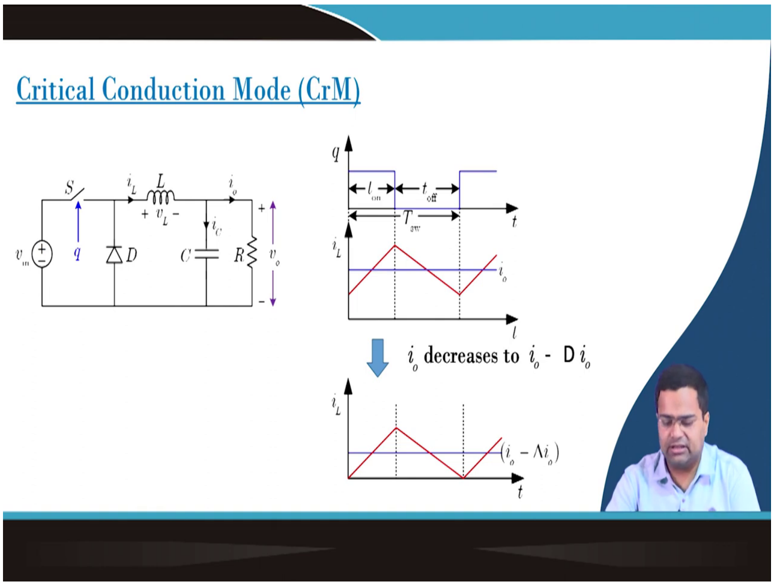

So, now, here we are considering a conventional buck converter, where we have replaced the low side MOSFET by a diode and now, the current can cannot go to the negative direction because diode will actually; diode will get reverse bias. The first scenario when the load current is relatively high then, if we choose a set of on-time and off time, then the converter will operate in continuous conduction mode.

If the load current decreases under the same operating condition, if the load current decreases from i 0 to i 0 minus delta i 0, then the average inductor current will go down, it will come down and at the critical case, the valley current just touches the 0 value, base value and this condition is called critical conduction mode.

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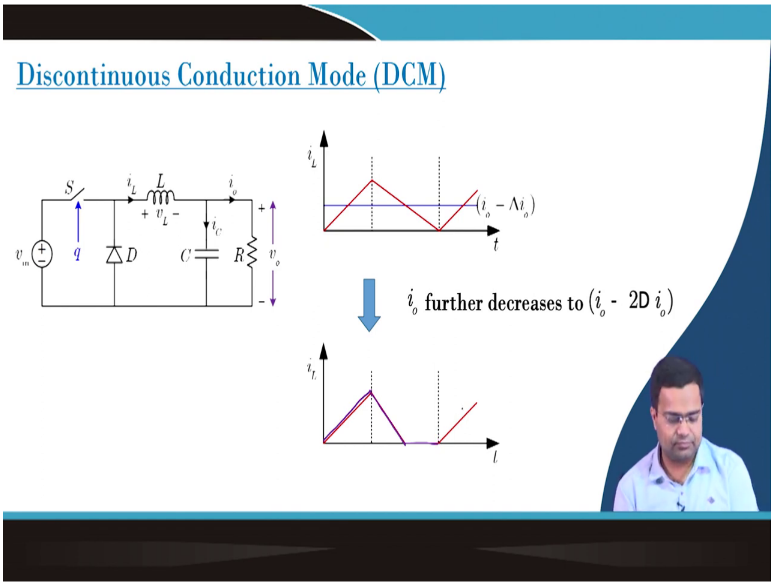

Now, if we further decrease the inductor current from i 0 minus delta i 0 to i 0 minus 2 delta i 0 so, here it should be i 0 minus delta i 0 to i 0 minus 2 delta i 0. Then what will happen? The inductor current will enter into discontinuous conduction mode during the periodic interval that means, initially it is during the on-time it will rise, it will fall and at current reaches 0, it will remain at 0 until the switch turns on.

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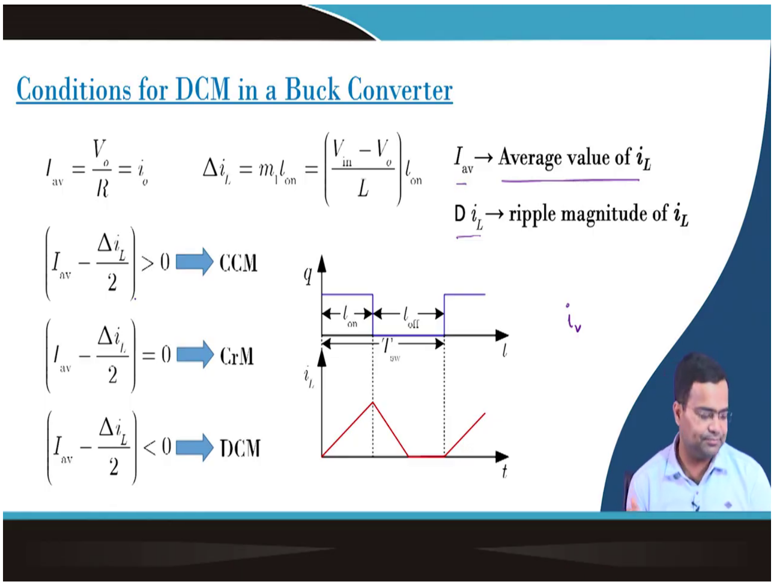

Now, during the discontinuous conduction mode, the average inductor current since it is a buck converter it is same as the average load current. We can find out the ripple inductor current. So, I average is the average inductor current, delta i L is the ripple current that we have already discussed and from this waveform in fact, we can find out what are the condition for CCM like a continuous conduction mode, CrM critical conduction mode and DCM discontinuous conduction mode.

So, I if we take the valley current, that means, if we take the valley current of the inductor, it is nothing, but I average minus delta i L by 2 in continuous conduction mode. See if this valley current is greater than 0, then the current will be in continuous CCM. If it is equal to 0 just touch the CrM and if it is less than 0, then actually it will enter into DCM.

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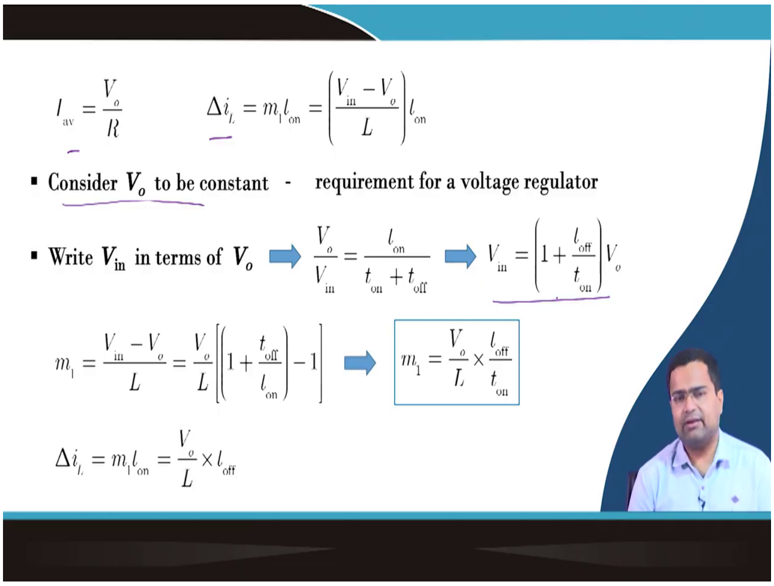

We can find out the ripple current, average current. We are considering the output voltage to be constant. This is because we are considering a voltage regulator, though you can write input voltage in terms of output voltage and we can express input voltage in terms of output voltage along with the on-time and off-time and then, we can find out the slope and we can find out the inductor current ripple.

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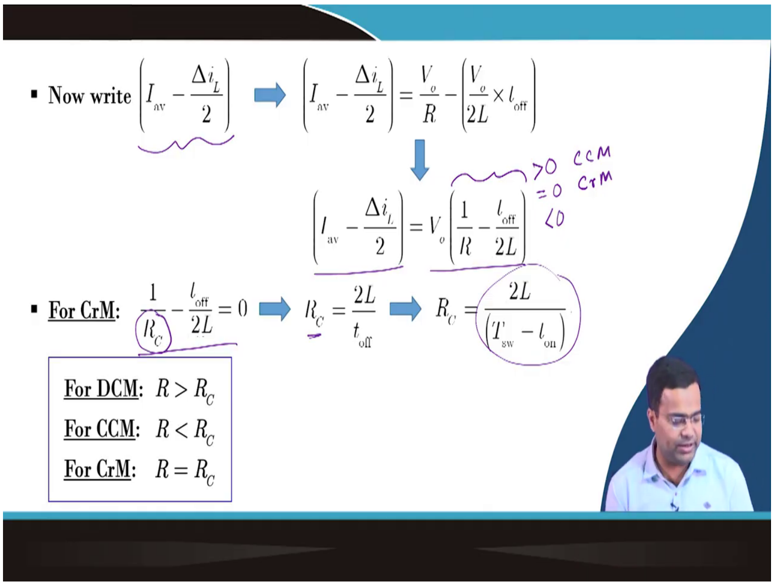

Now, what do you want to write? We want to find out this one. This one is important whether it is greater than 0, then it will be continuous conduction mode, equal to 0, it is CrM critical conduction mode. So, this can be found from our previous expression that I average minus delta i L by 2 is simply nothing, but V 0 in this term.

So, this term, if we set to 0, then this is the critical conduction mode. If this is greater than 0; if this is greater than 0, this corresponds to CCM. If it is equal to 0, it corresponds to critical conduction mode and if this expression is less than 0, then actually it has it already entered; it has already entered into discontinuous conduction mode.

So, for critical conduction mode, suppose here we are changing the load resistance so, we have to find out the critical value of the load resistance and from this equality sign, we can find out the critical load resistance is nothing, but inductance by total time period by minus on time. So, for discontinuous conduction mode, the resistance must be larger than this critical value. For continuous conduction mode, the load resistance would be smaller and for critical conduction mode, they should be equal.

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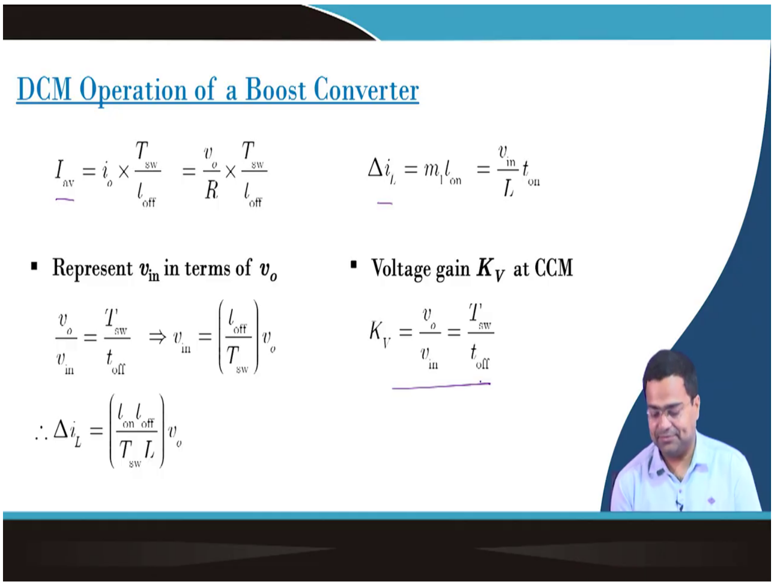

Now, for a boost converter, we can also find out the average current, we can find out the ripple current, then we can represent the input voltage in terms of output voltage and we can find out the ripple current and we can find out the voltage gain.

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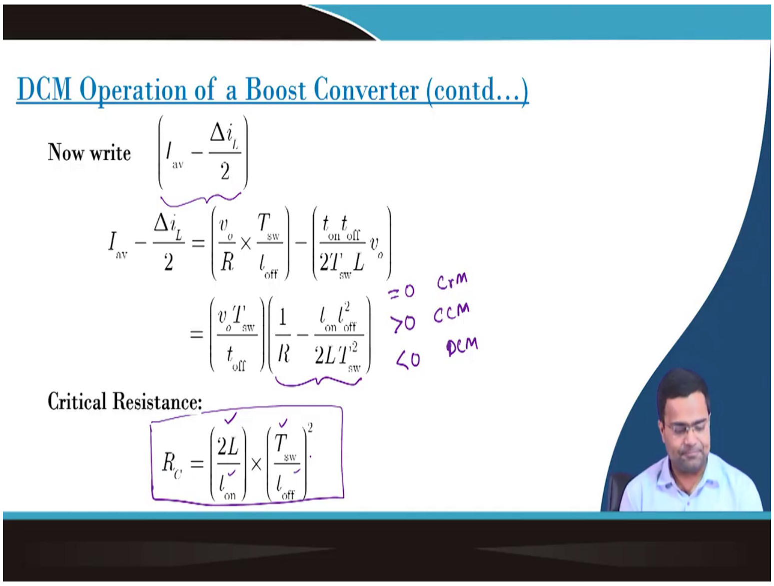

Now, what do you want to find? We want to find under discontinuous conduction mode again; we have to get this. So, for buck and boost, this expression will be different, but in general for the inductor current perspective, the current like a the notion is that the difference between the average current, inductor current minus half of the ripple that is basically the critical factor to decide whether it will go to CCM, DCM or CrM.

So, again, we can find out this expression. So, for boost converter, we need to set this to 0 if this quantity is 0, then it is CrM. If this quantity is greater than 0, then it is CCM and, if it is less than 0, it is DCM. So, we can find out the critical load resistance for a boost converter, which is a function of inductance, time period on-time, off-time.

So, now, depending upon the modulation technique, we can decide that whether on-time is constant or off-time is constant or a total time is constant because we have discussed various modulation techniques in the previous lecture.

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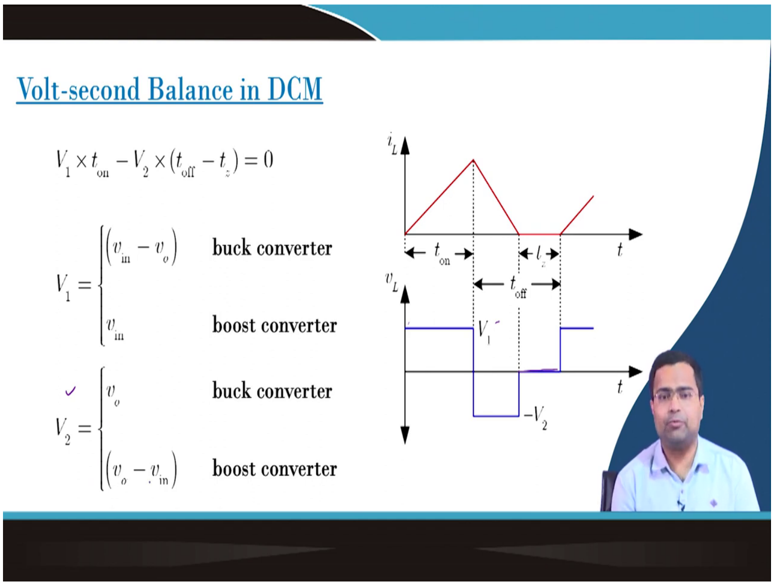

Now, volt second balance in discontinuous conduction mode that means, if we take the inductor voltage, when the switch is on, the inductor voltage let us say it is V 1, when the switch is off, it is minus V 2 and during the 0 current, the inductor voltage is 0. So, you can write for a buck converter, this V 1 is v in minus v 0 and for boost converter is v in, for what is V 2? For boost converter it is v 0 and for sorry for buck converter for boost converter it is v 0 minus v in.

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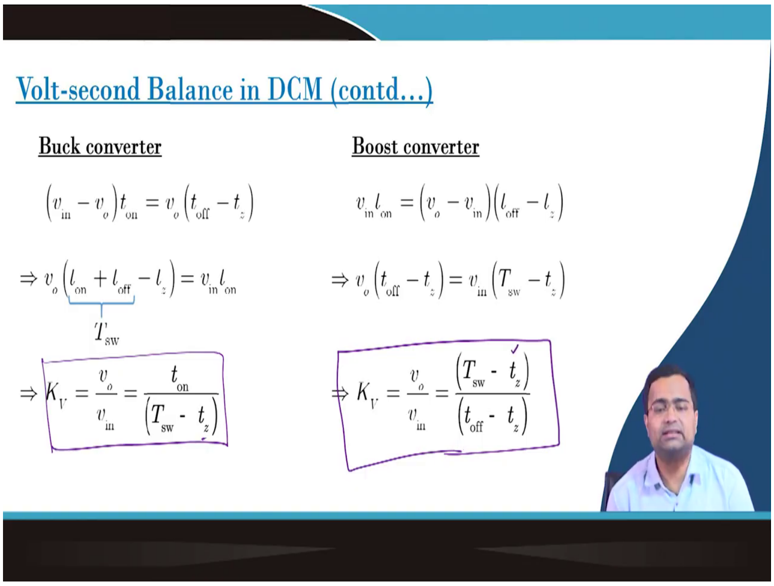

So, volt second balance in discontinuous conduction mode. If we apply this volt second balance, then you can get the voltage gain is different from the expression in continuous conduction mode. So, if there is no t z that means, if t z is 0, there is no 0 current, then is simply on-time by total time, which is consistent in CCM. For boost converter, we can again write the voltage gain in terms of this time constant. Here, the additional 0 current is coming into picture ok.

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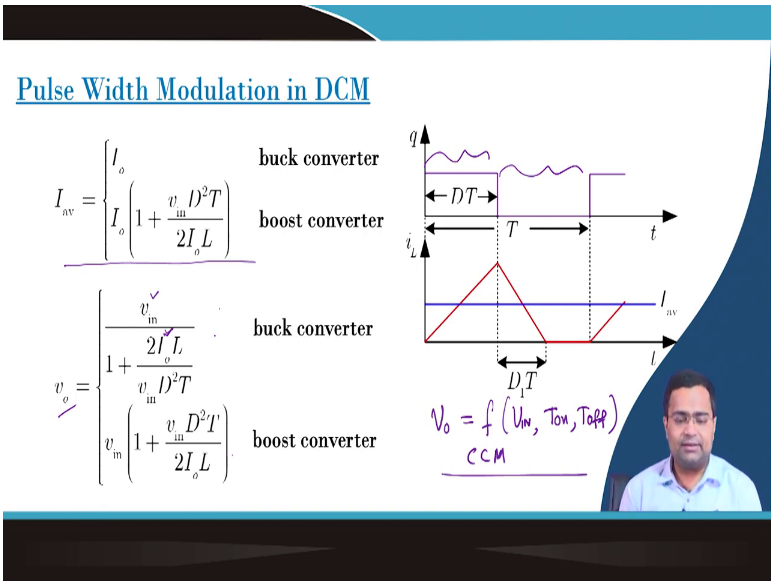

Now, we are considering a scenario under pulse width modulation. Under pulse width modulation, the total time is constant, and this is my duty ratio on-time of the main switch, which is s. So, this can be any converter, it is a generic waveform, and this is the off time of the switch, but during this off time, the current will reach 0 in between and it will remain at 0 because of diode.

So, the average current can be computed, and this expression can be obtained from a standard reference book in Switch Mode Power Converter book, in Power Electronics book and we can also obtain the output voltage in terms of input voltage, load current interestingly, even for an ideal boost converter as well as buck converter what we found? In CCM, the voltage gain that means voltage output voltage was a function of input voltage in CCM.

And of course, we can write in terms of on and off parameter and depending upon the technique you know modulation technique, we can find out what is the exact expression of f, but in CCM, we found that this was limited to output voltage input like a timing parameter sorry here it is input voltage not output voltage. So, we found that it was input voltage, function of input voltage on-time off-time timing parameter.

Now, in DCM, in addition, we are getting a load dependent term. Also the voltage gain relationship is actually nonlinear. We are getting although for boost converter you found nonlinear relationship, m for both buck and boost there is a load dependent term. This is coming because of the discontinuous conduction mode of operation and, again, this expression can be obtained from a standard textbook. So, I will not derive this expression.

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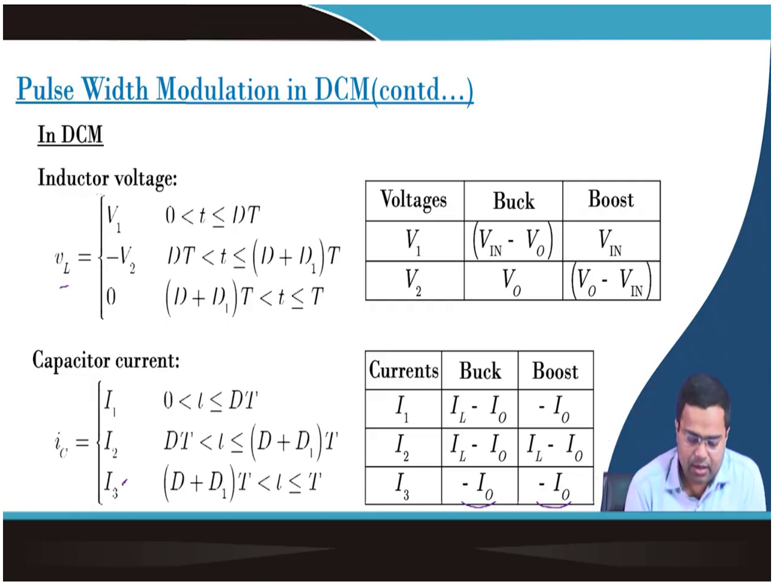

But what we are going to see? We want to see the effect of ripple voltage, switching frequency. How is it dependent on load current that we want to figure out? So, in discontinuous conduction mode, we know what is the inductor voltage and we also know what is the capacitor current that means, we can easily find out from the circuit.

When the inductor current is 0 during the 0 current, then the capacitor current I 3. This will be simply nothing, but minus of load current, this is irrespective of whether is a buck or boost converter because the capacitor is discharging to supply the load current, capacitor is supplying the load current right so, it is getting discharge because the inductor current is 0 at that time.

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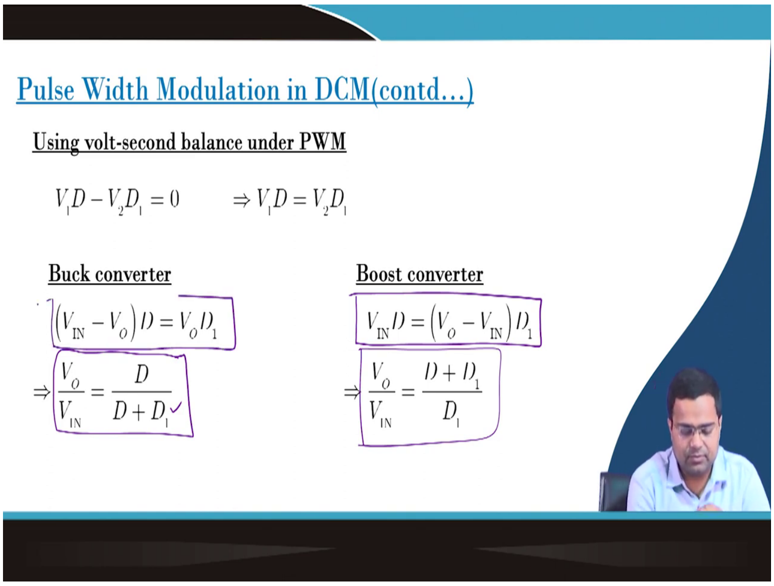

Now, under pulse width modulation, we can now apply volt second balance and we can get the expression of voltage gain for buck as well as boost converter that we got a generic expression in terms of timing parameter so, here we can get.

But this D 1 dash is basically it is not an independent term. It depends on D ok and it depends on the other parameter like you know what is the input voltage I mean it depends on input-output voltage. It is related to if you see from this expression, the D 1 is a function of D and also function of input-output voltage same thing also here.

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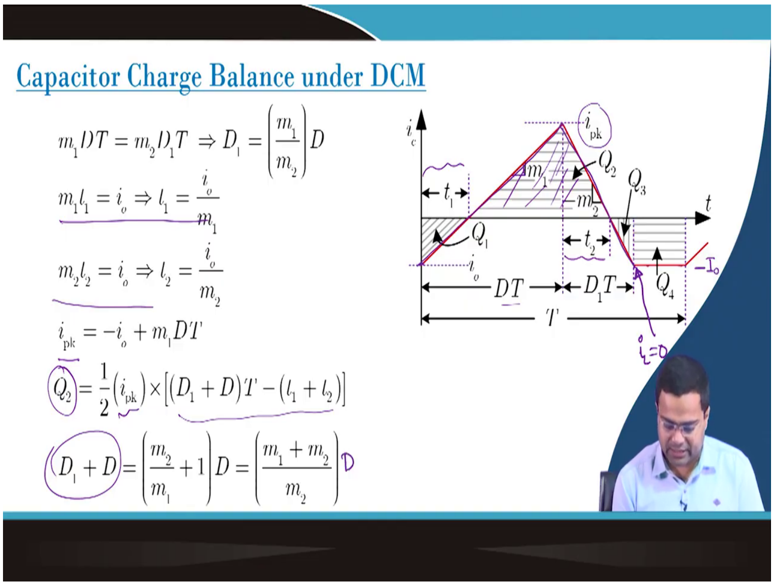

Now, we want to find capacitor charge balance. If you draw the capacitor current waveform, this is for a buck converter so, it start from negative current and the switch is on the capacitor current rises and when switch turns off, then it falls and this is the time; this is a time when your inductor current become 0. So, at that in this condition, the capacitor current become minus I 0, the load current because it is supplying to the load.

So, now, we can draw the capacitor. This is a capacitor current so, you can obtain the different charges Q 1, Q 2, Q 3 and Q 4 and these are time interval. Since we know the slope m 1 and we know the base value, we can find out this t 1 time, we can also find out this t 2 time and we need to find peak current to find the other timing like a DT and D 1 T.

So, you can find out in terms of so, these two parameters can be easily found that t 1, t 2. Then, peak current so, we can write the Q 2 that means, the area under this curve we can find out in terms of peak current and in terms of timing here it is given like DT, D 1 t and t 1, t 2 and we can what is D 1 plus D? We know that from the inductor current ripple expression, we can find out that this should be equal to D, it will be D, D is missing here.

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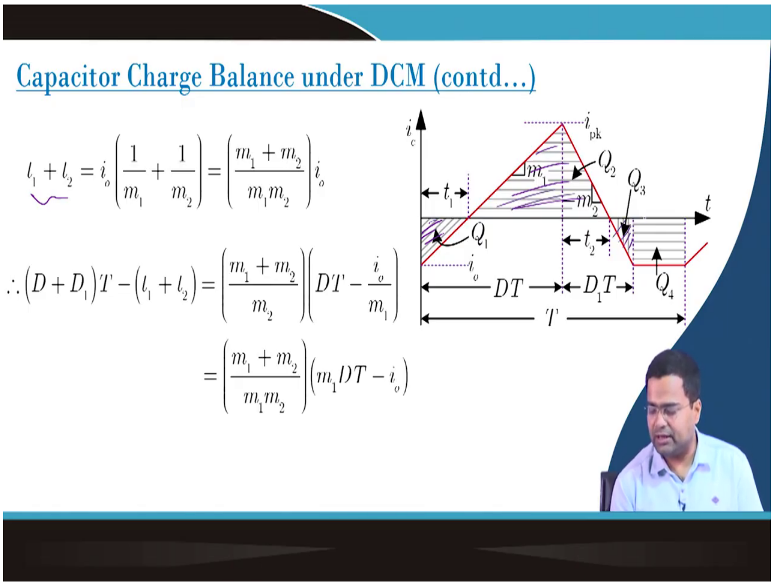

Now, capacitor charge balance under discontinuous conduction mode so, we are continuing from the previous expression t 1 plus t 2 and then, we can find out D 1 this all this time so, actually I am going step by step so, all this PowerPoint will be provided.

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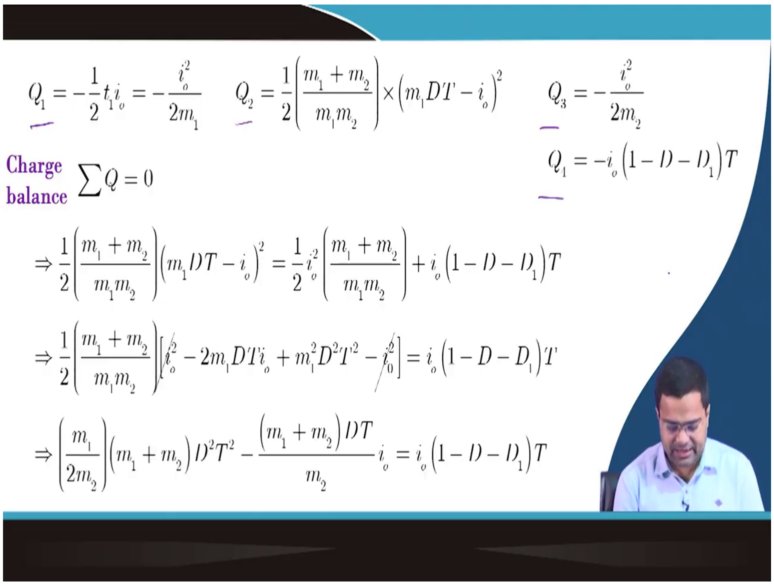

Now, if we find out the charge Q 1, Q 2, Q 3 and Q 4 from this capacitor charge balance, we can find out Q 1, Q 2, this is my Q 1, this interval is Q 2 during this is a Q 2 charge, this corresponds to Q 3 and this corresponds to Q 4. So, you can find out this Q 1, Q 2, Q 3, Q 4.

Then, sum of all these charges should be 0 that is a capacitor charge balance formulation under steady state.

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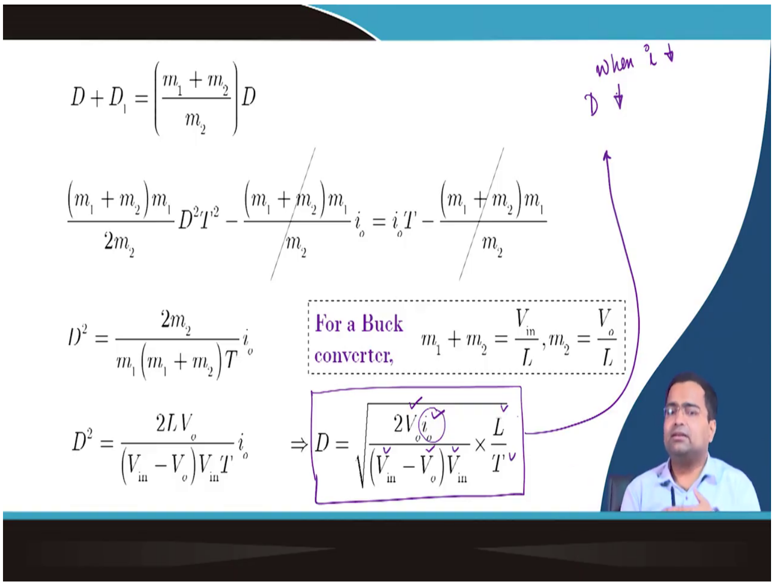

And if we can apply all this after simplification, then we can get that duty ratio can be express in a buck converter, it is a function of load current, output voltage, input voltage, inductance value and time period under pulse width modulation.

So, the duty ratio can be computed by that means. Earlier we found the duty ratio. Let us for a buck converter it is nothing, but V 0 by V in, but the same buck converter now, under discontinuous conduction mode, you will get a more complex duty ratio expression where you will get a load dependency term, it is also a function of inductor. It is also a function of time period so, all these parameters comes into the picture under discontinuous conduction mode.

So, from here, you can find out what will happen with the D when load current decreases? So, this expression shows that when load current decreases because inductor current, a load current is on the numerator is square root so, D 0 also decreases. So, if the load you know decreases and decreases, if it enters into very light load condition, the duty ratio becomes smaller and smaller.

So, such small duty ratio you know and your remaining time period, majority of a time inductor current remain 0. So, but your operating at a higher switching frequency, your turning on for a small duration and rest of the time the current is 0. So, as a result, the switching, number of switching for a given interval of time, the number of switching is pretty high because the switching frequency is constant and if the load current decreases, the number of switching event remains fixed, same that means, the driver loss remains same.

So, if the output power decreases, but the driver loss remains same, the efficiency is significantly degraded.

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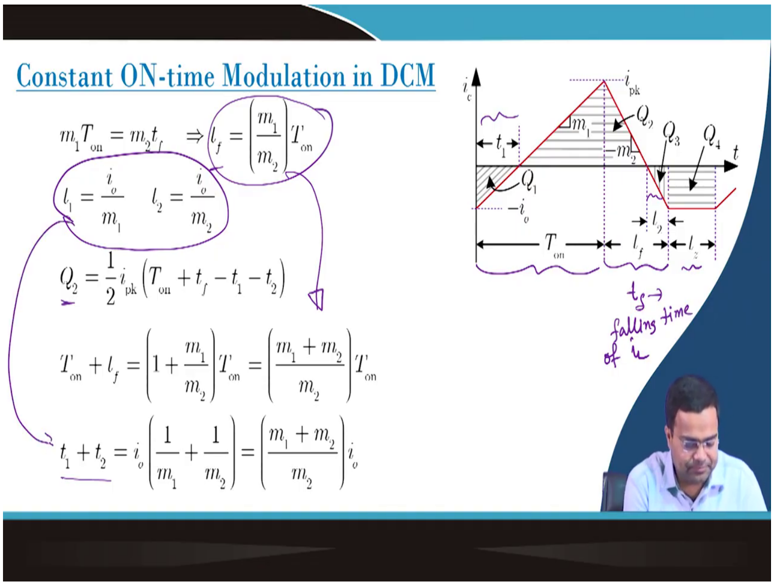

But now, if we are talking about constant on-time modulation, now we can draw the same capacitor current waveform the only thing we have to replace DT by T on because now we are keeping on-time constant and the total time period will be adjusted depending upon the load condition, input voltage, and output voltage.

So, here, we can derive the falling time sorry off time, this is my falling time of the inductor current t f so, this corresponds to the falling time of i L and t z is the 0 time when the inductor current remain 0, 0 time means t z means corresponds to time corresponds to the 0 inductor current.

So, you can find out t 1, t 2 from this curve. This is t 1, this is t 2 so, you can find out all this parameter and we can find out the Q 2 that is my positive charge in terms of the timing parameters and we can do replacement using this formula, we can also use here and then, we can write down t 1 plus t 2, from here, this expression we can substitute here.

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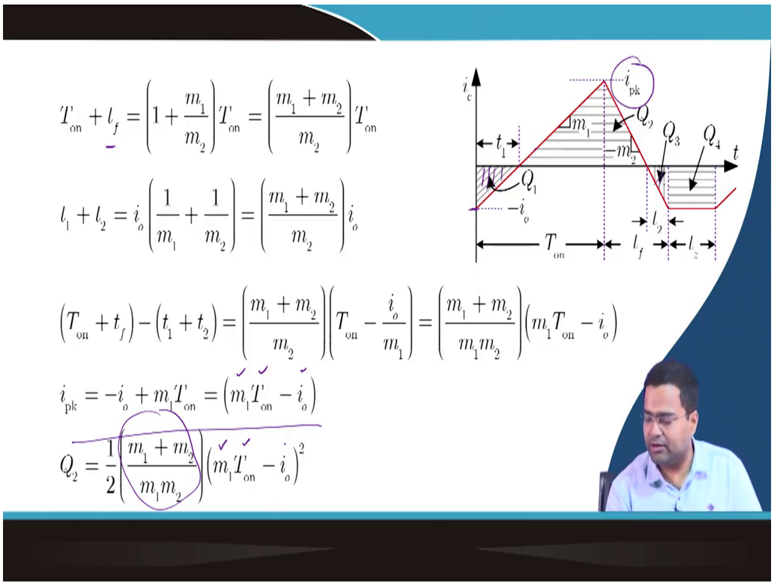

If we continue, then we can compute all these timing parameters and finally, what is our you know we need to this is just a computation, we need to find out what is my peak current? The peak current can be expressed in terms of on time, rising slope and the load current, then Q 2 can be obtained in terms of rising slope, on time, load current and the rising and falling slope of the inductor current.

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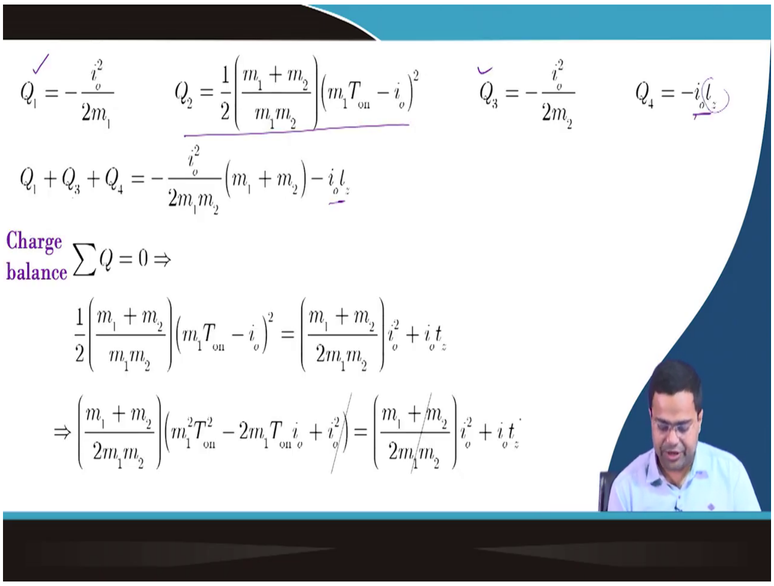

After substituting that, you can find out Q 1 so, Q 1. Also you can find because T on can be computed from you know from the because minus i 0 is known, the slope is known so, you can find out T on, you can find out Q 1 similarly, you can find out Q 2 very easily, we already found Q 2 and then, what is Q 4? It is simply i o. So, what is t z? So, if we add them together, Q 1, Q 3 and Q 4 which are the negative charges Q 3 and Q 4 so, where t z is the unknown quantity and Q t is also given.

So, after substitution, if you simplify this expression, then from here, this expression you can find out the t z term here.

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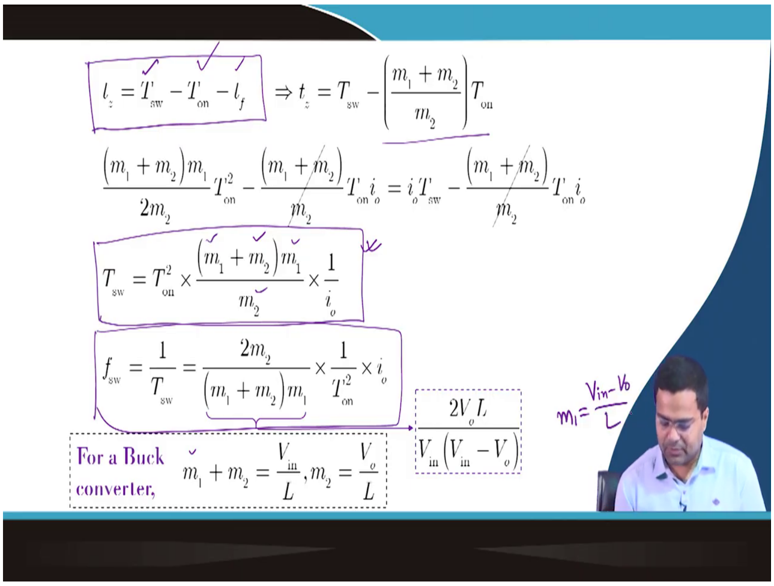

If you find out, t z can be obtained from this expression. So, this is the total time period. Again, total time period is not constant because if you vary on time, the total time period can vary. I am trying to write at steady state, we will find some time period that is T sw for a given on time, this is a falling time of the inductor current.

Now, we can we know the fall time of the inductor current can be replace express in terms of the on-time and if you substitute in the earlier equation, then the time period can be obtain by this expression for the buck converter and switching frequency will be 1 by that term.

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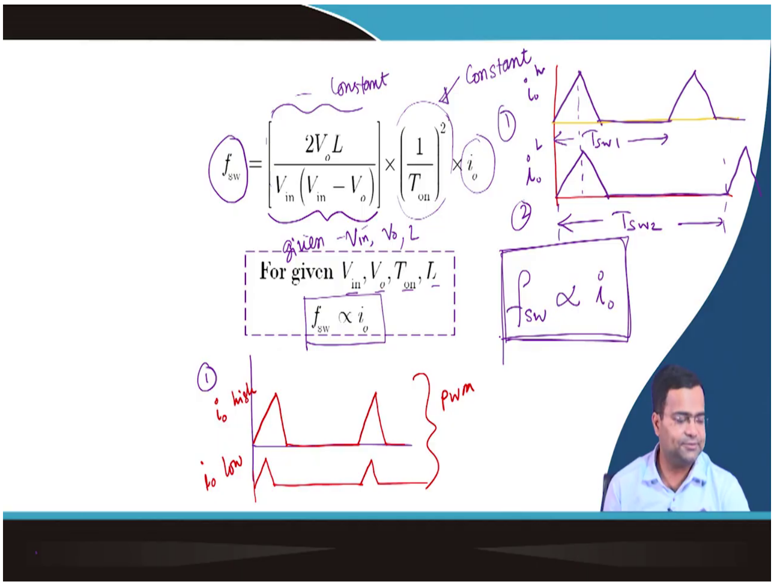

Now, if you substitute the slope of the buck converter m 1, m 2, everything is given, then we can find that the switching frequency of the buck converter. This is the time period, rising slope, falling slope, this is the rising slope, falling slope you replace with all these slope m 1 plus m 2 is V in by L, m 2 equal to V 0 by L, m 1 we know, m 1 is V in minus V 0 by L.

After all this substitution, what will get? The switching frequency under constant on-time modulation is a function of this particular voltage dependent term 1 by T on square into load current. Now, under constant on-time modulation, this term if we keep it constant, we can keep this is constant for a given V in, V 0 and L. This is also constant.

So, what we find? For a given input voltage, output voltage, on-time and L, the switching frequency is proportional to that means, f sw is proportional to load current. What does it show? That means, if the load current decreases, the switching frequency also decreases and what we saw in pulse width modulation? In pulse width modulation, the time period was constant that means, the switching frequency was constant.

When the load current was decreasing, the duty ratio was decreasing as a result, the charge balance actually you know takes place so, we can get a regulated output voltage, but. So, suppose if we draw a waveform that means, you know this is 1 scenario. If we draw this waveform, this is 1 scenario for a given load current.

Now, if we take another load current, it may that means, if the load current decreases, this will be like this is like a i 0 high and i 0 low, this is under PWM. But under constant on time, what will happen? If you keep the on-time constant, this switching frequency will get adjusted automatically. That means, if you draw two-way form which you have drawn for here, in one case sorry if we take this is one case this is ok current.

Now, we are considering this current and again, this current. Now, the load this is i 0 which is high now, i 0 is low. What will happen? This on-time remain constant. Now, why? Because since the load current has decreases so, over this cycle, this is my new sw 2 and this was my earlier switching period. So, the charge balance will happen over a longer duration for the second case. So, for the same on-time like you are injecting the same charge, but it will take longer duration to discharge as a result, time period increases.

For the 1st case, we have injected the same energy from the source to the inductor, but that takes longer that since the load current is relatively high compared to the 2nd case; this is a 2nd case so, it discharged faster so, the time period is smaller, T sw is smaller and the 2nd case, T sw is larger. So, that means, the beautiful thing about constant on-time control, the time period will vary automatically, the switching frequency will automatically vary with the load current and is a proportional to load current.

So, as a result, you know if we operate a light load condition, a switching frequency automatically get reduce and your losses switching loss also reduces. So, it can achieve higher efficiency for a wide load current range.

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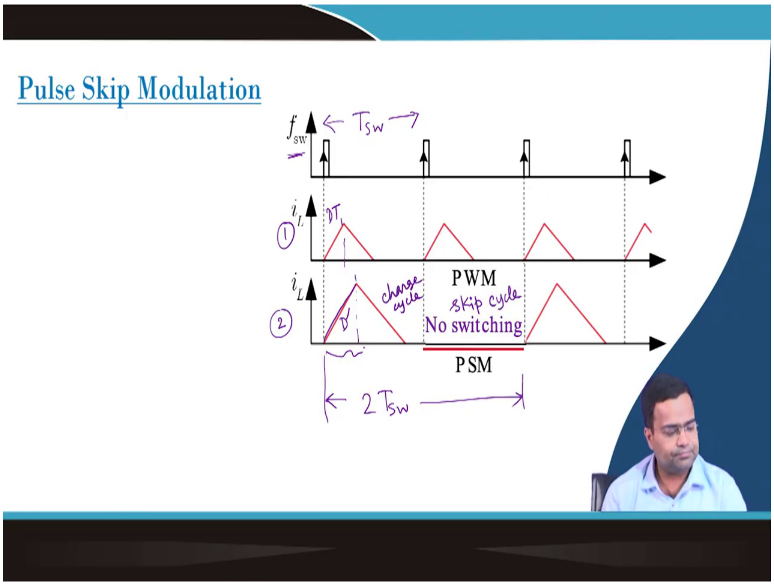

The other technique is a pulse skip modulation under discontinuous conduction mode. As I said under PWM, if the load current is low, then if say the duty ratio is very low. This is my duty ratio, but it is switching at every cycle that means, this is my clock pulse, it is switching at every cycle turning on, turning off, again turning on, turning off, but the duration of on-time is small. So, as a result, the charge balance happened over 1 cycle only.

But if you consider the 2nd case, the charge balance is happening over 2 switching cycle that means, one cycle I have completely skipped, there is no switching action. So, as a result, in order to achieve charge balance to meet desired voltage regulation, the current has to increase, or duty ratio on-time has to increase ok.

So, in that case, the duty ratio D dash is larger than D that means this is the 2nd case. This is the 1st case, the duty ratio is larger than the 1st case, but it is only getting activated for 1 cycle, the other cycle is skipped so, this is something called skip cycle, and this is something called charge cycle.

So, this technique is called pulse skipping modulation, where we can skip pulses depending upon so, how do you skip pulses that depends on different logic. So, I am not going now because we will consider this separately under light load condition. But I am just trying to give you an insight that now you if you skip one more extra pulse that means, if you activate for one pulse and then skip for two pulse, then this duration will increase, the on-time duration because we need to achieve charge balance.

So, by this way also, we can decrease the switching frequency or increase the total effective time period using pulse skipping modulation. So, this technique is also is helpful in order to improve light load efficiency.

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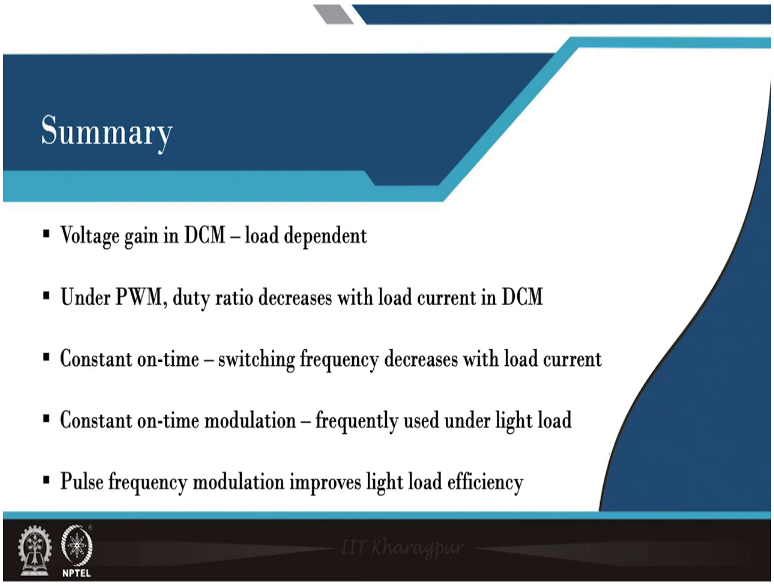

So, in summary, we have discussed voltage gain under discontinuous conduction mode, impacts some of the derivation of DCM is not included here because they are available in the you know there are very good textbooks are available so, those can be obtained from there.

Under pulse width modulation, we saw duty ratio decreases as the load current decreases, but in this case, the time period is fixed so, this may not be a very good method when the load current is low because you are driving switching losses high, number of switching is happening.

Under constant on time, we saw the switching frequency decreases the load current so, there is a proportional dependence, and this technique is very popular under light load even many commercial product uses constant on-time under light load condition in DCM.

Constant on on-time modulation frequently use in under light load condition and pulse frequency modulation improve light load efficiency and we also touch upon the pulse skip modulation, and we will discuss all these technique along with simulation case study when we will separately consider a session under like control technique for light load condition. So, that is it for today.

Thank you very much.