Signal Processing for mmWave Communication for 5G and Beyond
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Module - 02
Wireless Channel - A ray tracing model-Part-II
Lecture - 07
General channel Model

Welcome back. Welcome to the Signal Processing for Millimeter Wave Communication for 5G and Beyond. We will be continuing the channel model, but today we will be talking mainly the digital part. In the last class, we have introduced the digital part and we will be continuing that. So, this is the part II of the channel model part. So, in the part I so far we have mostly covered the RF and analog models and now, we will get into the Digital Model.

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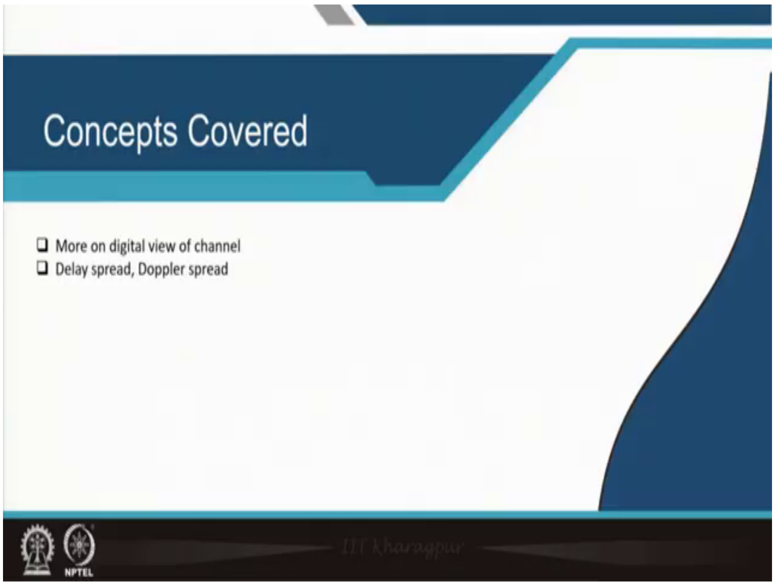

So, today what we will be covering mainly the more on the digital view of the channel and the concept of delay spread and Doppler spread ok.

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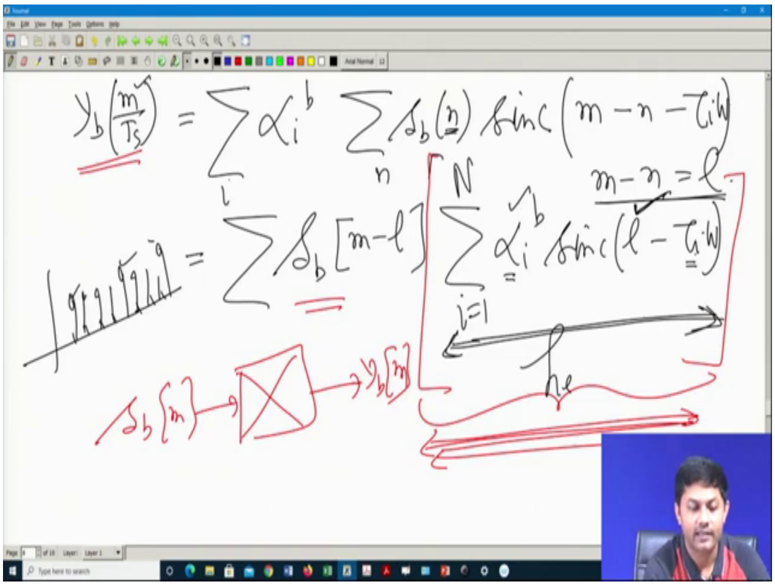

So, in the earlier class we actually came here if you remember. So, this is the complete view of the channel digital channel, right, but still we have not explicitly say where is the channel part, we have just created an input output equation, ok. So, in summary I can just write down the equation for our own benefit.

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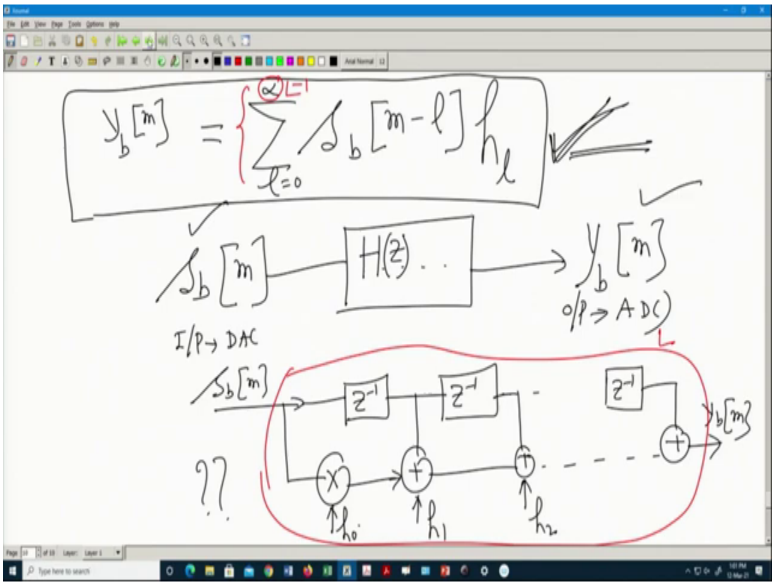

So, this is my y b m summation of s b m minus l and that whole thing this whole thing. Now, this whole thing I can see that it is actually a function of l is a function of l, right.

So, some component which is varying over l and actually it will vary over m also when we talk about the time varying nature this tau i, this alpha i they are time varying. So, when I digitize it like this here when I am sampling it this alpha i will also be impacted this tau i will also be impacted because that is a time varying nature.

But, in this case as I said we are assuming a static environment; that means, alpha and tau i they are not changing. It is completely static environment, nothing is changing ok. In that case alpha will be time invariant thing, tau [FL] meaning the delay part is also time invariant thing.

So, that is our key assumption here and we will see what happens when they become time in time variant. But, for the time being I keep it like that it is time invariant thing, ok.

So, you see this; this whole this red colored box whatever is there inside the box it varies over l it varies over l. If alpha i and tau i remains constant they are constant right in this case if there is if the whole system is a static environment they may be unknown, but it is constant. W is also constant because that is your sampling rate that is known to you.

So, apart from that the whole thing is varying over l. So, can I write it that this whole thing is some value like called h l, I just call it. So, that means, my h l this whole thing I call it h of l whole thing I call it h of l. So, what does it mean? It means that it is as if like this I am obviously, varying over l because m I cannot vary m is the index here.

So, now can you imagine this kind of input – output system? What does this input – output system says? Now, you think what is my input s b m ok goes through the black box whatever it is black box what I get y b m this is your ADC output, ok. This is the input of DAC at the transmitter side this is the output of ADC. So, this I can say output of ADC probably I will write it like that is an output of ADC and this is the input of DAC pure digital system, ok.

Now, look at this equation what should be this particular black box behavior such that given s b m when I receive y b m I receive this equation. You need to understand that. Now, can I model that particular input output system with some sort of a you know what is happening here? You see that s b m is getting delayed repeatedly getting delayed and then it is multiplied with some coefficient called h l and that gets summed up, right.

So, what can I say? It is as if like I have a s b m input ok and then goes through multiple level of you know delays. How many I do not know l number of and what is the value of l? I do not know some integer as I can say but what was the l definition? l if you remember this was the l definition m minus l m minus n equal to we have assumed that. So, that is an integer part.

So, which means that what is the upper limit of it, I do not know. So far we have not defined it, we will define it subsequently what is the value of l. So, currently I can say l equal to 0 to infinite, I can think of like that or it can be l equal to minus infinite to plus infinite whatever, but just to avoid the non-causality in my s b m I assume l equal to 0 fair enough, ok.

Now, it is a series of delayed value of you know delayed value of s b m Z inverse. How many such delayed value depends on what is the maximum level of l, if it is finite fine if it is infinite – infinite number of delay and then what happens? At every level what is it is happening? It is multiplied by this is the I can say h 0 I just look at the equation look at the equation l equal to 0 is multiplied by s b m and h 0. So, this is the one, then what can I say?

This is multiplied by h of 1, then what happens? This output is multiplied by h of 2 and so on. Finally, all are added up and gives me y b m, can I think of it like that? This is precisely what this equation is giving. This particular equation if you look at this equation this is what the structure.

What is the structure now? What is the structure this whole structure is? What is this whole structure? If this upper limit is finite let us say upper limit is l or it has total l number of components or l small l can take maximum value of capital L. So, it varies from l equal to 0 to l minus 1.

So, that means, a total l number of such things are there. So, I have total l number of data. So, what is the structure if it is finite number whether it is it will be finite or infinite I will describe later, but let us assume it is finite. Let us assume this limit of this summation is finite; that means, this structure will be also finite structure. So, finite number of stages of delays, adders and finally, I get the y b m.

So, what is the structure now? Can I say this is something like a filter, digital filter, a FIR filter digital FIR filter? Can I think of it like that? Yes, this is an FIR filter, provided the summation here this summation has a finite number of elements here, whether it will be finite will explain, but assume it is finite then this whole thing I can think of it has a Finite Impulse Response filter FIR filter nothing like that.

So, what does it mean? This one this black box is actually a filter. It just like a filter normal digital FIR filter. So, can I say this whole channel whatever we have you know struggled to find what exactly it is equivalent model so and so forth. It is nothing but some sort of a digital filter digital FIR filter that is what my channel is. So, it is as if like I am giving an input s b m it goes through an FIR filter of so and so coefficient and finally, I am giving I am getting y b m as simple as that.

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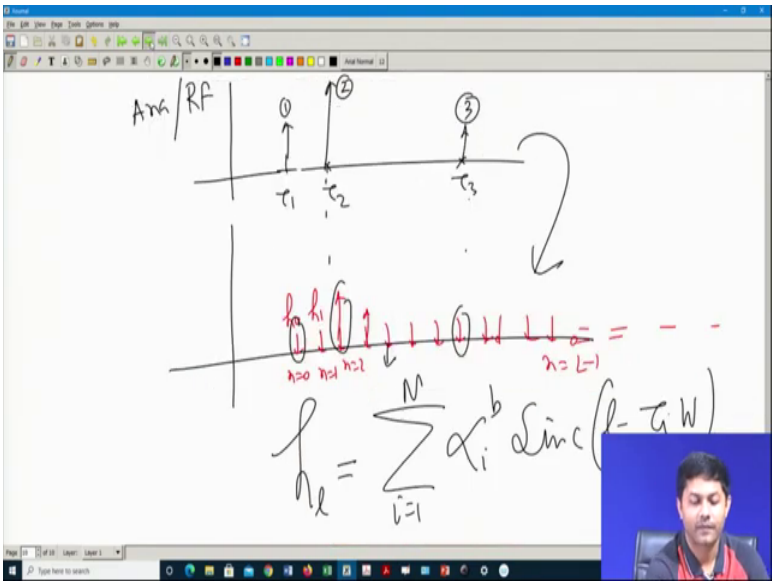

So, in a conclusion this is a very struck difference this is the struck difference between the analog view or the RF view and the digital view. So, that means, when you see digital view in analog view what happens? The datas were non-uniformly kept. So, it will be tau 1, tau 2, tau 3. This is analog or RF view of my channel.

But, when I make it digital this is not the view I am seeing it. I am seeing as if like I am sampling this whole signal at some uniform space uniformly I am sampling it, first value I am seeing it h 0, next value I am seeing it h 1 something maybe the amplitude may be non uniform, but it is as if like it is a digital signal, right.

So, it is say n equal to 0, n equal to 1, n equal to 2 and so and so forth. So, n is equal to up to l minus 1 after that again it will be 0 0 0. So, it is like a finite impulse response. So, what is happening here. I had this signal which gets converted to this signal. I had a samples at uniform spacing but, when I am in digital I am getting a uniform sample. See this is non uniform samples in analog and RF, but when I am in a digital domain it is a perfectly uniform sample.

So, that means, this any of this value let us call it h l the lth value of the channel; these are all like that particular coefficient that lth value; that lth value is nothing but summation of alpha i b sinc l minus tau i into W that is all. So, i is equal to 1 to N. So, this is the lth you know lth component of my channel. This is one component. The first component here, second component here, third component here. Let us say I have just three reflectors.

Now, there are so many conclusion I can make. When I am in analog and RF what was the first conclusion without getting into mathematics? If somebody gives me the view of this analog and RF channel what is the conclusion I can make? I can easily see how many reflectors and how many scatterer those things are present, right because that clearly says what about the digital? Can I conclude? Because these are all uniformly spaced.

The same three tap three you know three coefficient channel becomes so many channels we will see how it becomes because this is a sinc we will also explain it. But, just as a very simple thought will I see three component as I was seeing in the analog RF in the digital as well? The answer is no. I am not seeing three point. It is not that I will be seeing only this fellow or somewhere this fellow and somewhere this fellow. It is not that I may now see all these values also. Why is it happening? What is the methodology that I have used?

So, it is as if like I have a it is as if like I have a non-uniformly spaced data I have just done some sort of a uniform sampling on that. So, basically I have created some sort of a interpolation. So, these are the three data I have just created interpolation. This is an interpolation equation, right. This is nothing but my interpolation equation.

This is an interpolation. What is interpolation? Interpolation is you have certain number of data. I resample the signal further, then I pass it through some low pass filter. So, all these data which were 0 initially, they can be filled using these values. So, some values will be present. This is an interpolation.

What is the method? You do high sample, then pass it through a low pass filter. See this is exactly what is happening. So, these are the values of those non-uniform value, non-uniform sample pass it through a filter with a bandwidth W, it will just give you a uniformly sampled data. Say as if like it is something like this happening.

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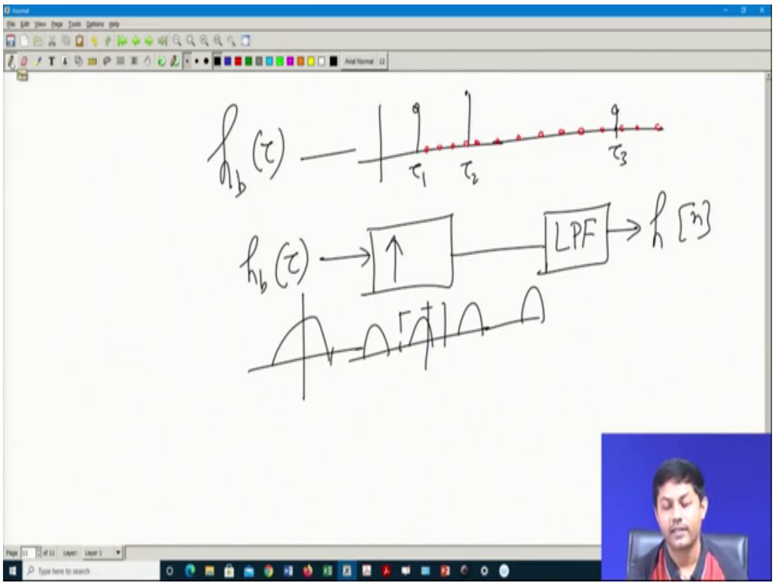

It is as if like I am giving h b tau ok which are non-uniformly sampled data that was alpha i at different tau 1, tau 2. Then what I do? I so, you have this point tau 1, say tau 2, and say at tau 3 some values are there ok.

So, what I do next? Next I just do a uniform sampling on that. So, what does it mean it is as if like I am introducing 0 in between because I am creating an uniform sampling. It is like a I am doing up sampling. So, h b t so, in a simple sense h b t or h b tau is going through an up sampler because I am introduced in between 0s. Then what I do? Up sampler meaning its spectrum will be repeated that is what the up sampler does.

So, if I look at this spectrum it will be somewhere like this. If I pi 2 it is not pi 2 minus pi, but something like that moment I do an up sampling as if like it will just repeat up sampling, right. Then what I do? I take a low pass filter which is nothing but the interpolation filter then I will get the h b n h b the digital model of that ok.

I should not say b here because it goes through a filter may not be some digital filter will be coming into picture. So, this is precisely happening. So, you have an analog model, pass it through it is as if like it is an equivalently I can think. So, even analog model analog channel model, then it pass it through a up sampler, then get it to into a low pass filter and I get the digital value. So, this is precisely what is captured in this particular equation.

This particular equation exactly captures that it is nothing but a interpolation. So, you had multiple scattered you know different non-uniformly spaced data I am creating a uniform view of my data uniformly sampled view of my data in the digital domain. That is the only key difference, ok.

Now, a very key takeaway is that when I resample it is a unit it is basically what I am doing what I am saying is that when I resample it in the digital domain, it is a uniformly spaced sampling. Your original data was somewhere here, but when I resample it; it may be somewhere else, but it is a uniformly spaced.

But the point here is that because after LPF it is as if like I am passing it through LPF this is what I have shown it here as if like that ok. So, all these values which were not presents like if I sample it somewhere here they were 0 initially, but after the interpolation they will be all filled up. So, that is the job of the interpolation it will filled up it will interpolate the in between value given those other values.

So, which means that when I am looking at my digital channel it is a filter whose value may be L capital L. So, that mean capital L number of values I can see also there is a one more view I can think of.

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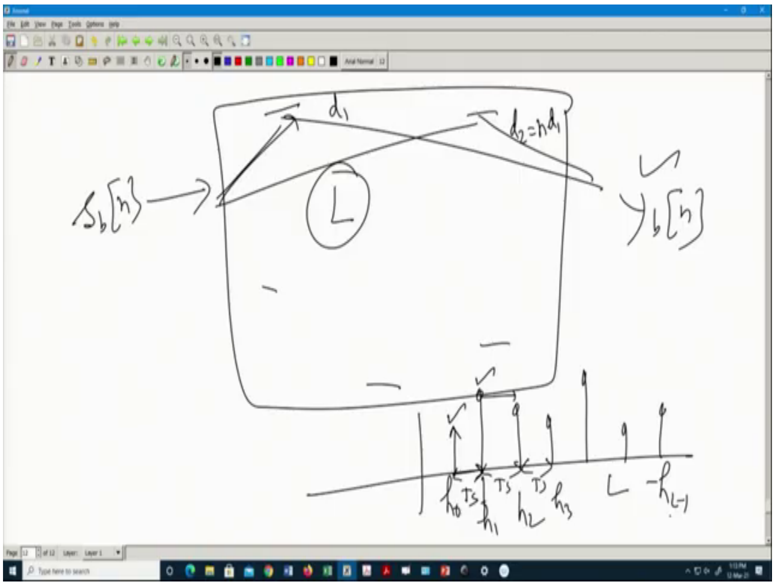

If there are capital L number of you know finite values I it is there so, what can I say? It is as if like I have capital L number of reflectors or scatterer present here. So, I have s b n transmission ok, I am receiving y b n. So, after digitization I may think that as if like from here to here this whole air or the channel has L number of reflectors or scatterer and this gets you know through L number of reflectors or scatterers I am receiving the data at the y b n and the delay of all these elements are all integer multiple as if like that, ok.

But, actually it may have a lesser number of physical reflector, but after digitization the view is as if like there are L number of reflectors and scatterer present in my system and the delay of individual paths delay of every individual paths are exactly integer multiples. If this one has tau I should not put tau because there is no longer a tau.

If it is some other parameter say d 1 delay this fellow is exactly some integer multiple of d 1. So, everything is an integer multiple of everything as if like that. So, if I plot it what will happen? So, it is like first I will get h 0, after exactly with uniform distance I may get another what is this distance? This is your sampling time.

Because I am observing that right another path another channel path another channel path another channel path like that. So, there are L number of such paths existing right with the delay of T S, this is also a delay of T S, this is also a delay of T S. So, that mean the tau 1 whatever was earlier analog and R F.

Now, it is now a multiple of T S. So, T S, 2T S, 3T S like that see as if like that the channel is like all the reflectors and scatterers whatever was existing in the analog and RF cache that just simply like a it is like it is like it vanishes a new set of reflectors and scatterer appears, but they do not appear as if like that it is just like a feeling as if like that I am seeing a new set of reflectors and scatterer appear with a delay of exactly T S different different T S.

So, the with the one which is the shortest one path which is giving some value then the next shortest path will give you exactly T S delay, next shortest path will be giving exactly T S delay and so on. So, this is my h 0, this will be my h 1, this will be my h 2 this will be my h 3 and so on, this will be my h l minus 1. So, that is my new view of my channel.

So, you can feel the how different suddenly become it becomes when I digitize it. It simply does not give you a nice idea of your exact number of reflectors and scatterer completely that view is lost you cannot from a digital side. You really cannot guess how many reflectors or scatterer present. But, is it so? Can I not really guess it? Let us see that, ok.

I can guess, I can feel, but probably it may not be so accurate in number. So, exactly that is the point I am going to discuss it here. So, this is clear, right. So, you have L number of new channel appears or new you know new paths appear as if like that with a uniformly delay. This is my digital view of my channel ok. So, my channel will be something like this one like a some filter, some linear filters it will be, ok.

Now, let us first discuss what is this L, is it finite or it can be as high as possible or it has no limit? Let us understand that ok.

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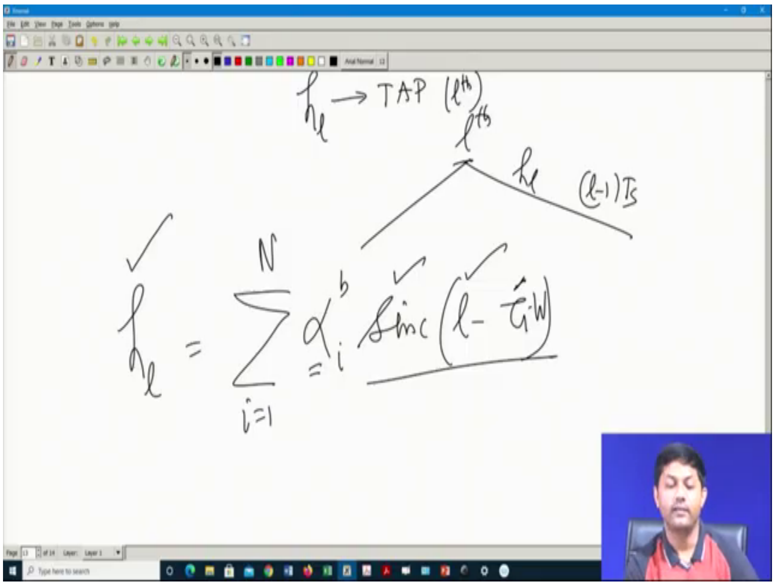

So, what exactly happening? So, let us say that lth channel, what is channel? It is the lth paths gain, ok. Now, in digital domain they call TAP. Now, probably from this time onward I will not call it channel rather I will call it the lth tap. So, it is a lth tap.

So, lth tap meaning it is some path it is a hypothetical path may not be a physical path hypothetically it is the lth path having a gain h l and having a delay of you know l minus 1 into T S it has to be an integer of T S right. So, that is the kind of a given assumptions is there ok it is as if like this is just like a hypothetically I am thinking. So, it is an lth tap ok.

So, what was the lth tap definition? Let us dig into it the equation. It is an interpolation of what of what you actually observed physically this was the question. So, what does it mean? It said that it says that the lth tap whichever I am calling it; is actually a summation of several sinc operation this is the sinc function several sinc will be there it is just the summation of them that is all it means ok appropriately sampled at some l tau W will be there of course, appropriately sampled at some integer point l.

So, let us understand let us completely understand what it means. So, it means that you can see the existence of the sinc will be happening somewhere around tau with an amplitude alpha i.

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So, which means that when I do a digitization whatever analog view of your channel was there say tau 1, tau 2 and tau 3, let us say this amplitude was alpha i 1 this amplitude was alpha 2 sorry, this is b of 1 let us say this had an it is analog view. Let us say this as alpha 3 b.

These are the three part of or the complex coefficient that we have seen in the analog channel this is the originally. So, this is the three original path that I will get it or original value that I will get it if it was an analog domain. So, I can clearly see how many reflectors are present. So, there is no ambiguity on that the only problem happens when it is the digital domain, ok.

Now, when I see in the digital domain the only thing I am seeing that around tau i I suddenly see a sinc around it ok. So, which means that there is a sinc some sinc is there. There is one more sinc ok and there will be further sinc that is what it means right alright this is what it gives us that around tau i I will now have a sinc.

Why this sinc? Was the sinc there in the analog domain? No, it only appears because I have done sampling see it is a interpolation, it is like a low pass filter. So, that is why the sinc appears here ok. So, because of the interpolation or the sampling the sinc appear. So, it is as if like that I am seeing it ok.

So, when I say lth tap, it is this is my view. This is my tau domain, actually this is how the envelope the continuous envelope would have been had I consider the analog this is how it is like. Now, I am now sampling it, right. I am sampling it at l equal to 0 l equal to 1, 2, 3 like that l equal to 0, 1, 2, 3 like that sampling ok.

So, can I sample before it? No, right because if it is a tau 1 I have to sample always like. So, say let us say I am sampling uniformly at this point at this point I am sampling. Next, I am sampling somewhere here, next uniformly sampling somewhere here, next uniformly sampling somewhere here, next uniformly sampling somewhere here and so and so forth.

So, this I call it l equal to 0, this I call it l equal to 1, this I call it l equal to 2, l equal to 3 and so on and so forth. That is my l where I am keeping my sampling right that is an integer point.

So, which means that when I put l equal to 0, it means that the value would have been somewhere here, ok. l equal to 1 the value of the tap's will be this.

So, when I sample it here, so, you see there will be a component from the red also now. It will also the simple side it infinitely goes, right. So, I have kept it slightly wide. So, you will not feel that there is a mixture, but actually there is a mix. Had this tau 1 and tau 2 been little closer what would have happened say something like that.

You have one tau, you have one sinc like this, you have one more sinc like that. So, this is my tau 1 delay, this is my tau 2 delay and I am sampling like this one sample here like this I am sampling uniformly sampling this envelope. So, the guy which is here is a mix of this black one and this red fellow is this component. The guy which is here is a mix of red one and that black one.

If I have some more sinc if I have some more sinc everybody is actually contributing either small or less it depends on how they are spread across right see if they are widely spread and if I sample it very fast what will I get? I may get less impact. But, if I see tau 1, tau 2 are very closely spaced and my sampling is also very in the order of that, then I will see this kind of behavior.

So, it is a mix and match now. In the analog it was very pure, it was absolutely pure. The coefficient is coming from that particular reflector, but now when I digitize this when I say h 0 that first tap that first my virtual reflector it is not just coming from one reflector, it is whatever reflectors physically present everybody is now contributing to that fellow right because this is what my equation is saying.

This everybody's contribution for the lth itself and the diagram is also very clear here say for example, I am here third one. You see black is also contributing blue is sorry, red is also contributing. So, the value could be somewhere like this. I will get this value because everybody is contributing somewhere here you think. Blue is also contributing red is also contributing this is a mix.

Probably at this point black is also contributing, probably it is not so significant, but technically it is contributing. So, it means that any tap any digital tap's whatever channel I am giving it every analog or RF component is now contributing, ok and why I am seeing intermediate values because the sinc pulses are there. So, if I just see what can happen my view?

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If I see if I just see that what will happen? I will see as if like I will get one data here I will get some data here I will get some data here, I will get some data here, ok. Here I may get some good data here, again some small data here and there again nearby I get some large data, again I get some small data and die down.

Look at that now that mean if there is a reflector present in the vicinity of that point I may get a larger value, else I will get a small small values. Now, this is a guess now you can interpolate you can apply some sort of a intelligence there and guess where the actual reflectors physically present, ok.

So, that mean if your sampling speed is very high compared to your tau 1 and tau 2, probably you have a chance that the larger values are actually the physical reflectors or the physical scatterer. The smaller ones probably it is just the tail of the sinc pulses. It is just coming because there are tail there, ok.

Ok, we will stop it here and we will again explain more on that in the subsequent classes regarding the channel views and so on and so forth. So, we still have not got the delay spread.

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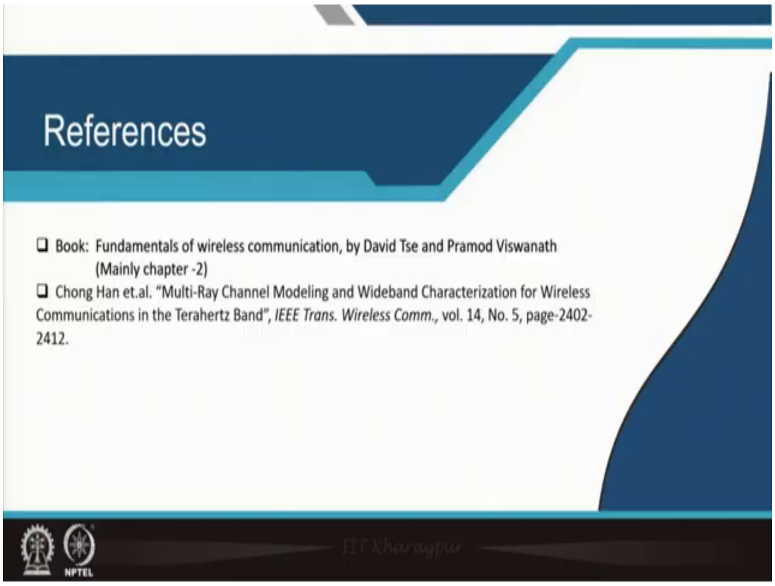

So, these are the references Fundamentals of the wireless communication by David Tse and Pramod Viswanath, the same books I am following it and this paper that is being followed.

Thank you.