Signal Processing for mmWave Communication for 5G and Beyond
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Module - 12
Multi User Hybrid beam and Impairment and analysis
Lecture - 64
Multi User Hybrid beam and Impairment and analysis (part - 5)

Welcome to Signal Processing for millimetreWave Communication for 5G and beyond. So, today we will be taking the almost the last part almost the last part that is the Hardware Impairments part and it is little bit of the analytical issues how what are the different type of analysis that I have to adjust there.

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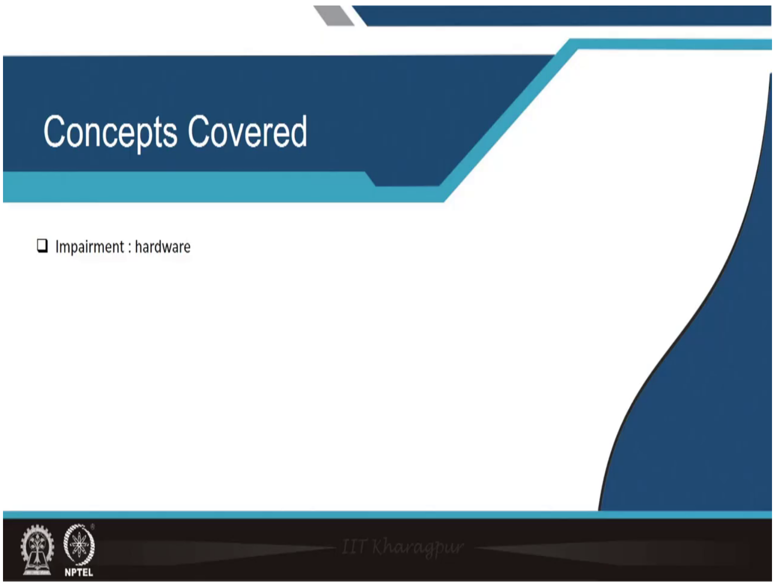

So, let us get into that this is the impairment parts coming from the hardware that comes into picture here.

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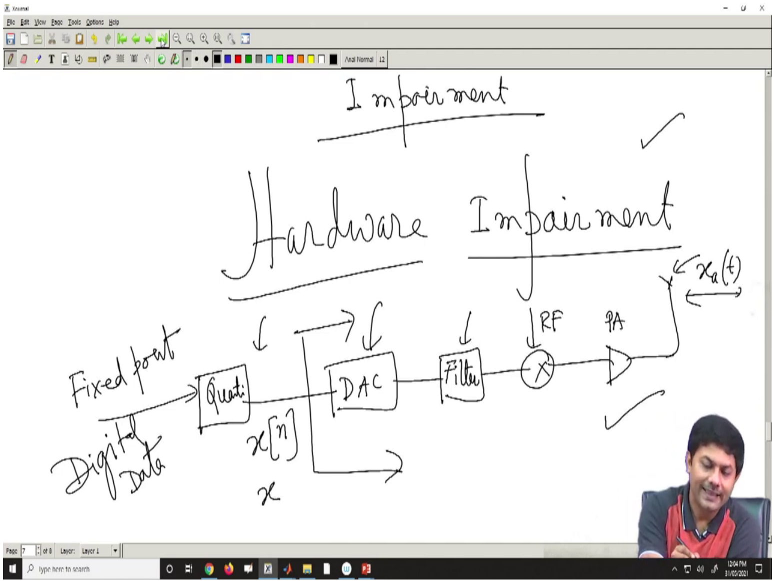

So, this is where we ended in the last class that how exactly my hardware impairment comes into picture. So, whatever I will be talking mainly that is mainly dependent on the quantization noise, but it slightly addresses the DAC filter and RF part, but I cannot claim that this particular model I am talking is really in addressing each and everything, but to a high level, yes to some good level this generic model whatever I am going to talk about can address that, but not specific ok. So, what exactly the point here, point is the following.

See here x t right. So, this is let us say this is what I am going to transmit from the antenna x t ok. Now the philosophy is the following philosophy is that the impairments can come to the x. So, what is what is the impairment in the x a t. So, what is x t x a t, it is a it is a RF signal of course, but we will be thinking everything from the base band point of view, because at the end of the day after your ADC at the receiver; obviously, you will get digital data only.

And so, the model that I have printed here is nothing to do with only t x it will also be coming from r x ok. So, let us only you know only model at the t x side. So, here as many as analog and RF blocks more number. So, now, the point here is that the kind of model there it is trying to do here, what gets impacted at x a t ok. See if x a t is a data now I think from the digital side. So, what I have transmitted is basically x n.

Now, let us not worry about n, n is more of a digital side a digital, but it is more of an nth time, but let it be, you can also just write x I do not mind it how you exactly model it. So, you can remove x or you can just keep x n also does not matter.

So, let us say it is x ok and it is assumed that most likely the time to time the error may not change much, but you can also make a time varying error as well. So, it is a very complicated models, but I am only talking of the simplistic model.

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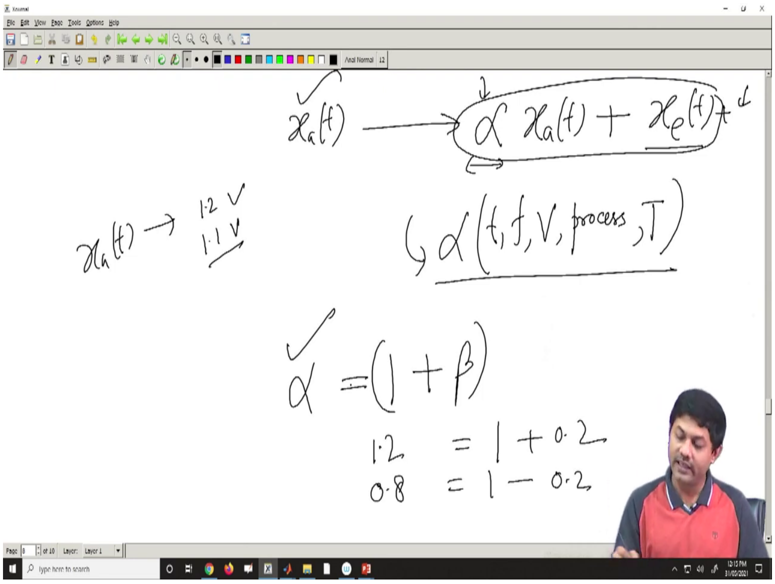

So, error is that your x a t when it is really getting transmitted it has two part when there is a problem, one part could be a multiplicative part another part could be an additive part ok. So, let us say I have an alpha that mean I am supposed to transmit x a t from my transmitter from my antenna, but instead because of this all non-linearity and impairments the amplitude is not exactly what I want.

Now, this alpha itself can be a very complicated model, now I have drawn a simple you know multiplicative; that itself may not be holding true because the way the non-linearity appears this alpha itself may be a function of time and frequency take my word. In fact, it is not just time and frequency it can even be a function of voltage, it can be a function of your process, you know what is called a process process is basically your silicon process, it can be also a function of temperature so many parameters can impact this alpha.

So, it is a very very dynamic and very nonlinear in nature I mean highly complicated model because if I really really take into account the power amplifier RF filter effects and all. So, as you know it is not that easy to model it ok, but if you get the information of how a non-linearity varies in the power amplifier or non-linearity appears in the RF circuit or non-linearity appears in the analog circuits, from the RF and analog designers and the antenna designers and if you get that value I mean in some sort of a mathematical equation you can put it and get that.

But, let us assume that I am making a making my life simpler by assuming that this is not a function of time frequency and voltage, but rather it just some sort of a constant, but in reality it can be extremely complicated and it is not very easy to even write a equation ok. Another form can be additive error ok. So, one can be you know can be a simple multiplicative error another can be a additive error.

Now, this additive error need not be a Gaussian function or need not be thermal noise, it can be thermal noise as well, but need not be because I am not talking of thermal noise in this case ok. If there is a thermal noise of course, there is another extra term that will be coming into picture, but here I am completely talking of a you know a simple kind of noise which appears due to the quantization can be. It can be appearing due to simple issues something like that I will tell you.

Let us say this how do I. So, this is the actual nonlinear model that comes into picture, now I can make the model even simpler. Now what I say this alpha n number if it is a real number I can always express it in terms of a plus something right a plus something right say let us say I am having 1.2 you can always express it 1 plus 0.2 right if it is say 0.8 I can always write 1 minus 0.2.

So, that mean any real number I can always express it as a 1 plus something some quantity alright. So, let us say this is beta some quantity I can always express it may be positive or negative so which means that if I just place it in the model.

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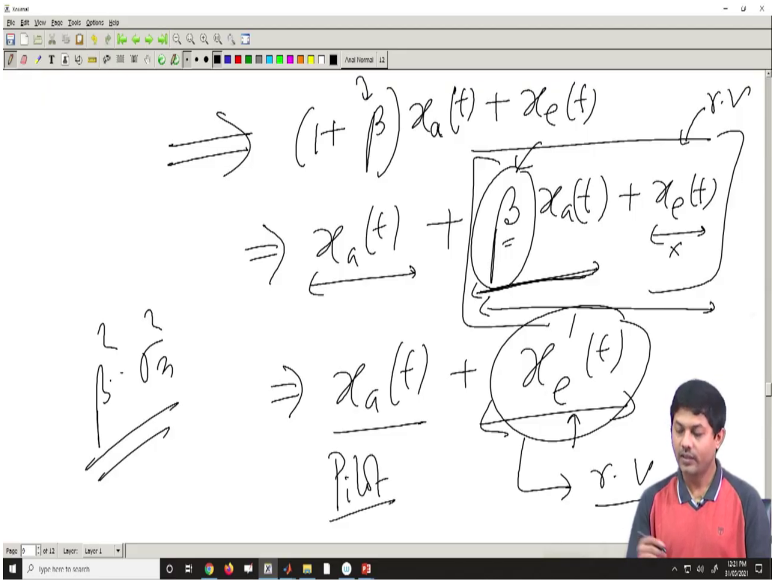

So, your actual non linearity will be always 1 plus say beta into x a t plus this noise which appears I tell you what is what can be the source of x e. So, this can be x a t beta x a t plus this x e t ok. So, can I write it like that, let us say x dash e t ok. So, what is this x dash e t, x dash e t can be a very simplified model of my nonlinear impairments. Now, why am I calling it as a hardware impairment because you know this who introduced this is beta, that is basically your CMOS circuits introduced the beta right because this alpha why this alpha appears.

Say for example, I want my x a t to have say RMS value if RMS value or it is a peak value 1.2 volt because there is a problem in my power amplifier or because there is a problem in my circuit I may not be exactly producing 1.2, but rather 1.1 volt right. So, I can model it as a either a multiplicative noise or may be some sort of a additive noise and so on so forth so; that means, I can have this model instead of say exactly 1.2, I have extra sum.

Now why this what exactly this beta into x a t is revealing, it is revealing that these impairments this is the total impairment right. This impairment can have two component one component has nothing to do with x a t that mean no matter what x a t came they may appear and that may be a random number also. So, this is a random variable it just keeps appearing, why. It can be a function of voltage just suddenly the voltage fluctuates or certainly temperature fluctuates or some other disturbance appear. So, they just gets extra voltage or extra noise may not be so, that has nothing to do with what you are transmitting.

And second component as a whole I can say this is your data dependent ok. Now, this data dependent components appear for example, if there is an RF circuit or if there is an analog circuit or if say for example, filter or a DAC depending on how much voltage you appear they may have a dependency on what exactly it is input voltage right.

So, this is an interesting point so; that means, this x dash e t this analog or rather I should say this error part has two component, one component is a dependent component another component is a data independent component ok. Now, again how much this independent components would contribute to, that is again depends on what exact circuit you cannot say this will be so and so.

It depends on the physical particular circuit which is present how much this beta would be again you cannot say anything. So, which means that this whole thing this x dash t whatever I have drawn this is a random variable.

So, even if you send as a pilot I am sending. So, that is a fixed data, but your x e dash is always a random variable ok so; that means, even if I send a pilot suppose this is a pilot that mean I know what I am sending, still it can have a random variable. Why it can have a random variable? This is beta and this x e t you really do not know their nature it is very difficult you have to get it from the CMOS circuit I mean it is a very extremely complicated nonlinear equation. But I really do not want to get into that. So, to me everything is just a random variable. So, can I model it like that right.

So, in a very simple you know in a very simplistic model without getting into the analog and RF nonlinear equations if you really want to get it that is well and good, but it, but its circuit dependent and you have to really get those numbers right. So, if you really do not want to get to the numbers the only way to model it is that.

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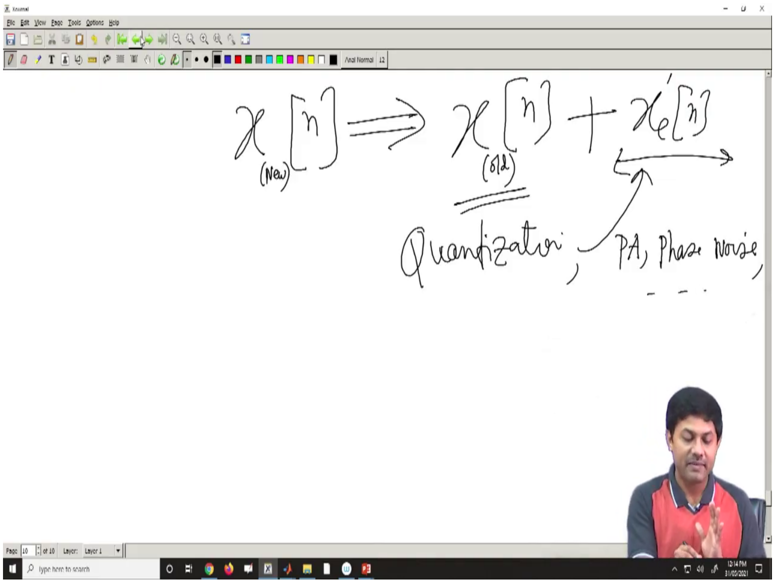

The one which now you go back to your base band so, the one which you are transmitting say base band way you are thinking it; it is nothing but so, the one which is transmitting. So, let us call it probably I will put a different notation here. So, let us call it new. So, let me just put a I put a bracket this is new; new meaning the one which I am really going which I am really transmitting.

So, basically this is the one which old meaning I am saying that the one which did not have any impairment plus dash or does not matter. So, so this is a very typical noise which can be additive modeled as an additive, but it has a dependency on this part. What is other philosophy, other philosophy is that if I have a quantization noise, can it model quantization noise? Yes, it can also model quantization noise. That is dependent on data? Yes, that is dependent on data, if the datas are high or it would be small depending on that quantization will appear there.

So, these are all can be modeled, all thing can models quantization, if you have a phase noise, power amplifier noise, phase noise and some other RF circuits noise all can be just modeled as a simple additive data dependent model and that is the interesting part of the hardware modeling. So, here the in here it avoids the exact model into that. So, you really do not have to know rather is I just model it, but it is again a it is very simple model ok. What are the dynamics that I have avoided, this dynamics this alpha I have assumed that alpha is a constant over time which may not be the case it is very complicated.

But if you really take into account that then you have to get the circuit model. So, this is my simple model. So, which means whatever I am transmitting that a within it there is another noise coming into picture. Now, let us get into my hybrid beam forming case. How I model this gentleman so, which means that you have the same s.

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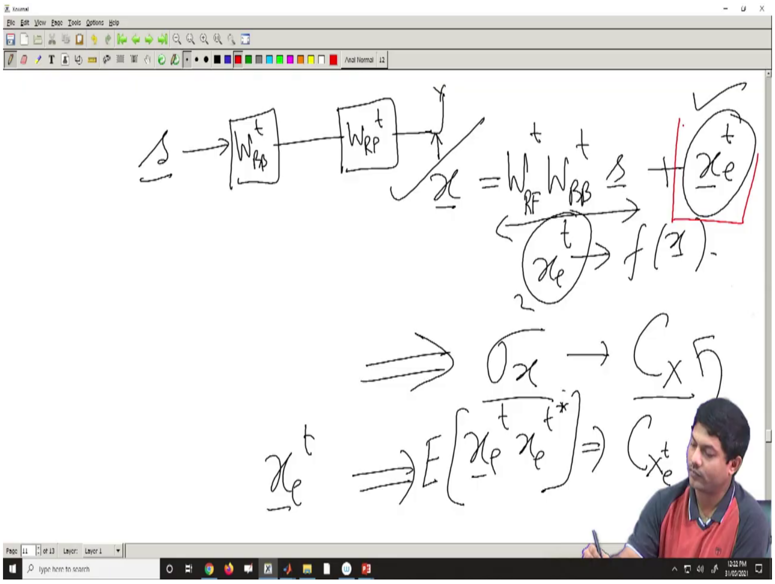

You have the same s vector ok and that has W BB t t and so and so forth, then there is an H now let me not introduce the H part this is how your transmission happens right ok so, which means that this is what you are going to transmit.

So, finally, this x vector right what this x vector will be your W BB t sorry W RF t and s vector right this is what you are going to transmit it. But now I am saying that he introduce this vector as well let us call it x t ok. Now, this is a function of ok function of my x because that is a data dependent one right. So, which takes into account all my you know all this [FL].

Now how do I get the if it is a dependent can I make even simpler. So, dependent does not answer my questions, what answers my question is that well, it is dependent, but can I model it statistically because we are statistical people right we understand statistics and problems. So, whenever you have you know this random variable the first question that comes into picture what is the probability, what is the distribution of it, ok. Now exact distribution you can assume it to be Gaussian and it is very tough to even prove the that could be Gaussian because they are all like all these quantizations, RF impairment all this comes into picture, there is no guarantee that they will be Gaussian, but in a very simple term we are engineers right.

So, we can in a very simple term you can assume to be Gaussian you can, but do not ask me question that is it really Gaussian that you have to put a measurement and so that this is really coming into a Gaussian because this is not a thermal noise ok. This is a hardware impairment noise, there is no guarantee that it will be Gaussian ok, but you can model it as a Gaussian, for system design that is the only option you have right ok.

So, you can assume it, but most importantly as I know this is dependent on x. So, can I have can I leverage that statement? Yes, so, to do that what we do is here, if x has a power spectral density sigma square x ok. So, what is power spectral density, power spectral density is you know what is power spectral density. So, that is the what power second per hertz right.

So, if sigma square has a power spectral density say sigma square x it can even be you know a dependent variable, it can even be I mean if it is an x vector there is no guarantee that each and every individual points becomes code I mean independent. So, in that case sigma square may not exist, but rather you may have a covariance matrix like C x you can have it. The point here is that what is the covariance matrix of x e t? That is the point I am trying to answer.

So, the covariance matrix because it is a function of x so, there is certain relationship between the covariance matrix of x e t. So, if x e t has a covariance matrix which is expectation of x e t into x e t star let us call it C x e t and there is a certain relationship between that is what I am trying to say ok. So, what is that relationship?

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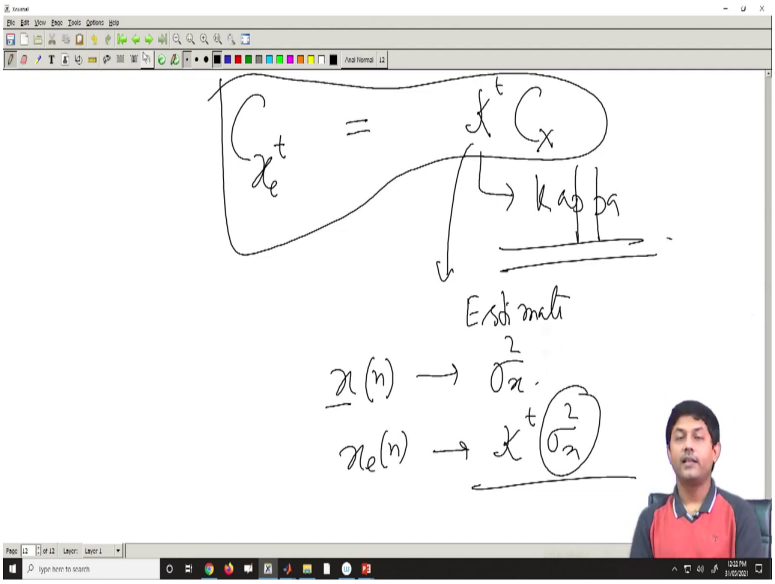

So, it is called C x and I would add a new variable called kappa ok, some kappa will be added now how do you figure out this value kappa, you can have it measurement based you can estimate that ok. So, which means that the relationship between these two is nothing but that sometime people use a square also, but it again it is a constant whether it is a take a square or anything does not matter. So, some constant you can multiply it always ok and you can estimate this. So, this can be estimated.

So, which means if x say one of the x is independent and has a power spectral density x e then x e n will be some kappa, now I am not really that is how it is ok. So, that says that it is actually a correlated data, but correlated even if it is correlated, but it is variance has a dependency on the; that means, if the sigma square is low this will also be low if it is very high it will also be high.

So, there is such some sort of a relationship because of that beta part this part this beta part. Now, I cannot say that I mean you can feel that part. So, if this is what if you can ignore this part so, that is nothing but variance is what.

Suppose, x e t is not there or x e t may be very little. So, what is the variance of beta into x a t? It is nothing but beta square into sigma square x right the variance of this whole quantity this whatever is written. If this part is ignorable then this will be beta square into sigma square x so; that means, there is a certain relationship between the variance and all so this is what is been captured here.

So; that means, the covariance matrix of your noise and covariance matrix of your data can be related with this quantity ok, but what is more important is that your data model is slightly now changed, this is x e t comes into picture ok. So, now, what happens to the receiver thing ok.

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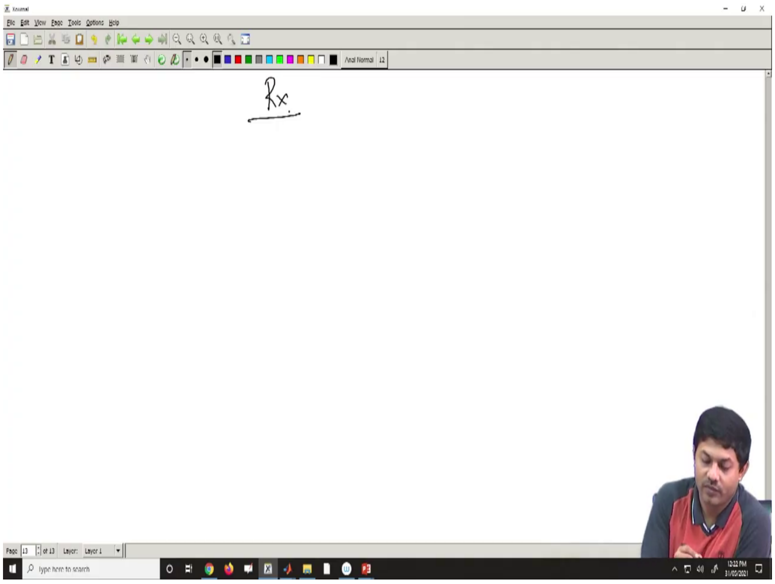

At the receiver so we have nicely captured the transmitted data. So, this is your extra part that comes into picture ok.

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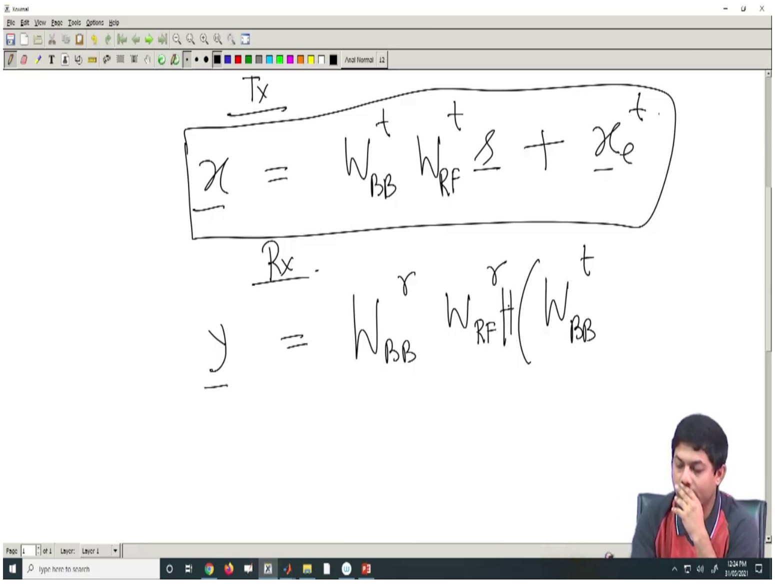

So, now in the T x side what is my data model? My T x side was actual data model was W BB t W RF t into s bar right original data model, but now as we said I have to add a hardware impairments. So, this is my new data model let us change the R x part what will happen ok. Now so what is my RF, originally what it was? Originally it was W BB r W RF r right multiplied by W BB t there is an H here correct.

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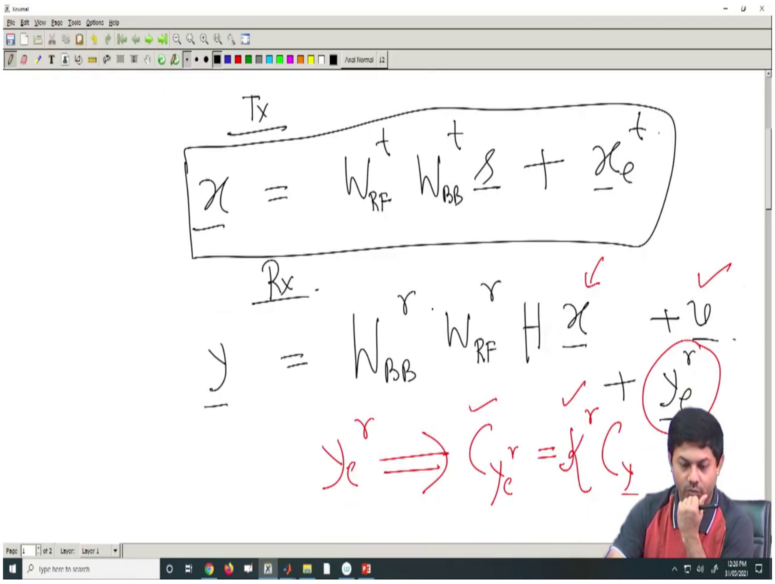

Let me put it in this format H into x bar plus v bar this is the original one earlier, but now what will happen now you will have y e this extra part will now come into picture ok. So, that is the hardware impairment and this y e covariance matrix because this may be you can you cannot say that this is a independent noise that is never guaranteed because y itself is not independent.

So, the covariance matrix of it let us call it C y e r will be sum kappa and C y this is what is coming. So, you can find out C y it is easy to get it because you have the equation easy to get it. So, from there you have to just multiply with an extra number you get the covariance matrix for y here. So, that is the noise part. So, never think that is.

So, now, you can realize it this is very different from your a WGN noise. Because a WGN noise may be a white noise it can be white noise it can be colored noise, but it is a WGN if I say it is a already a white noise. So, that mean it has a it has a covalence matrix which is a diagonal matrix, but y e r need not be ok. So, that is the point how to, now what is the change data model because I have not put the y here. So, let us let us put it back here. So, let us see what happens.

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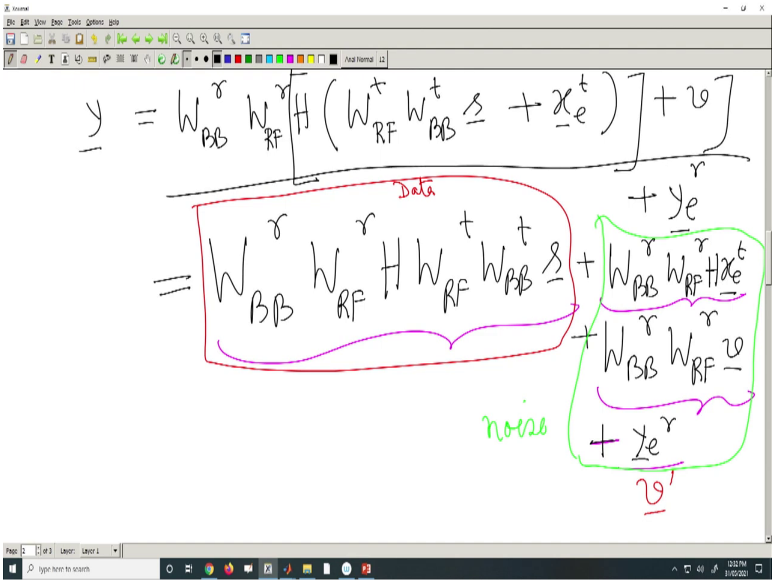

So, your y would be W BB r W RF r ok let us see what I have written this is H, H will be there and your sorry W RF t yeah. So, here slight mistakes this will be W BB W RF ok yeah W RF W BB alright. This will be W RF W BB, this is t this is t s bar. So, this is the original case, but now that is one more x e t bar is coming plus v will also be appearing. So, here this slight mistakes ok.

So, this was the case yeah. So, this will be the v this is the original one plus y e vector this was the vector. So, this is the now new data model. So, what will be the case W BB r W RF r

H break it W RF t W BB t s bar ok plus this whole thing now you multiply W BB r W RF r H x e t bar correct plus sorry this H will be then this H will be slightly up ok.

So, now I break it here W BB r W RF r v now you see this remains same there is no change, this remain same ok plus this is newly added plus y e r this is also newly added so many parameters now newly added ok. Now this part as it is. So, this is your data part, can I write this whole thing is your noise part? Now this is your noise part now ok. Now what extra change, what is the covariance matrix and also, but that is analytical part you require.

So, the covariance so, if it is a noise I can always write it this whole thing I can say this is nothing but let us call it v dash this whole three component I add them up and call it v dash ok.

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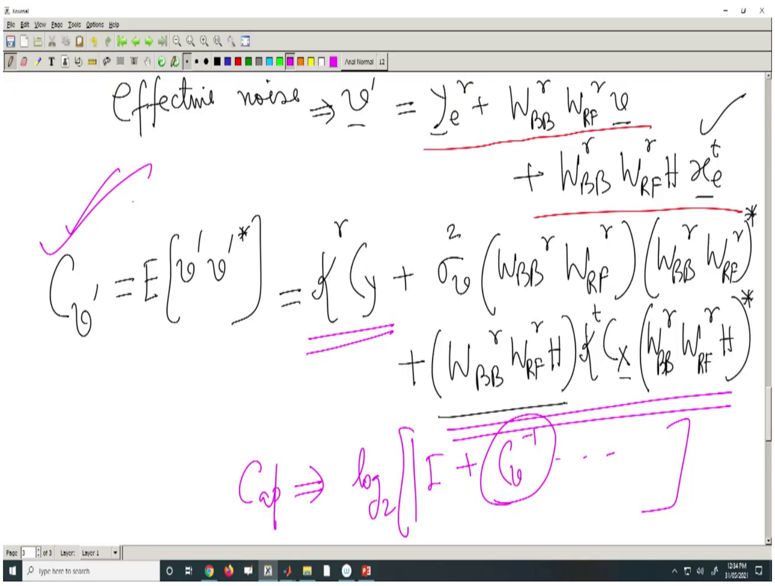

So, what is your effective noise now? Effective noise v dash vector is the other three. So, that is the y bar e r plus W BB r W RF r v dash this that is the original part plus this component W BB RF H x e t ok. So, that this whole things will be just appearing there as well ok, W BB W RF and r W BB W RF H x e t vector so, that is this is r, this is r, this is r. So, this is your new equation new channel equation new noise equation now you see how complicated it is this whole thing has changed. Now what is the covariance matrix of it you have to you have to calculate.

So, if I say what is the covariance matrix of it how do I get, expectation of your v dash v dash star ok that would be kappa C y this is r. Now, what is this C y, if somebody ask you what is the C y that is easy to get it right because you know your original. So, this is your y.

So, when you calculate your original C y there you do not consider y e r and from this equation from this whole thing you can get your covariance matrix right. So, that C y you can easily get it plus if this is the noise. So, if I assume noise is a colored noise. So, it will be sigma square v, but it will be W BB r and W RF r W BB r W RF r hermitian right plus what will be the case.

W BB r W RF r H multiplied by this ones, this one is say we have already assumed this is a kappa t into C of x that we have already defined it now this rest of the term this term that. So, you see how gigantic this covariance matrix of my new noise. So, this part is extra and this whole part is extra, because of that rest of the things remain same.

So, now, the question could be how do I then do optimization, nothing has changed except the fact that the effective noise covariance has changed. So, if you can remember your equation there if you say I just for your kind information if you say my capacity.

If you remember the cost function was capacity and then there was a log base 2 and then determinant of I plus covariance matrix of v inverse and something was there. So, this C inverse whatever was earlier there instead of that this is the new C inverse instead of C v inverse now it will be C v dash inverse that is the only change we will change anything else nothing will change. So, only that part will be changing everything would remain same ok.

So, with this I just end the hardware impairment modeling yeah it is very simple in this case I made it simple, but it may not be that way of simple modeling there are complicated modellings available, but just for a start point this could be one of the good start ok.

So, with this I conclude the session and hopefully you understand the complete stuff. So, what we started with? We started with a 6 gigahertz channel then we went into millimetre wave. Then we went into terahertz and finally, there are different hybrid modeling, hybrid channel, hybrid beam forming and all these things we have just come across and then some of the impairments MIMO waved hybrid beam forming those kind of things we have just come across. So, with this I conclude the session here and so the concept that we covered today hardware impairments.

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And again the reference is the same whichever I showed it in the last class that is a, but it is not nothing to do with the millimetre wave, but it gives you a fair idea about the hardware impairment and CF impairments effect.

Thank you.