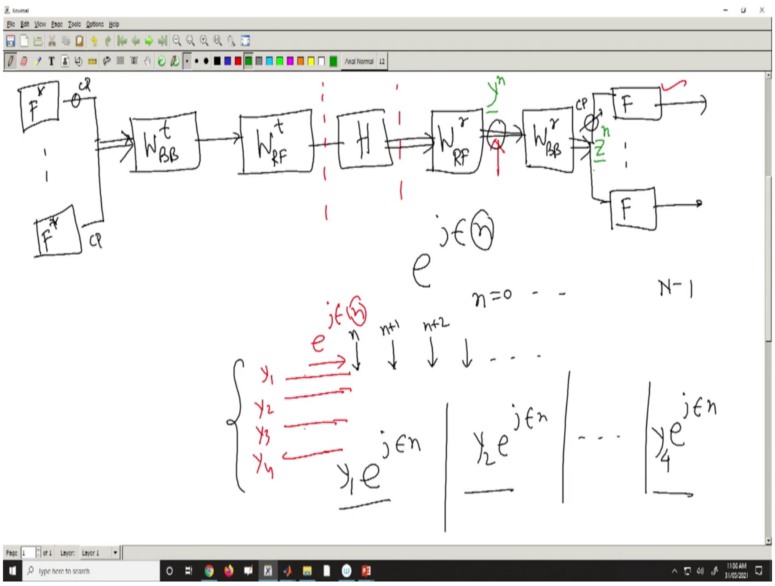
Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 12
Multi User Hybrid beam and Impairment and analysis
Lecture - 63
Multi User Hybrid beam and Impairment and analysis (part - 4)

Welcome to Signal Processing for millimetre Wave Communication for 5G and Beyond. So, today we will be continuing the part 4 of the impairments and thus multi user hybrid beam forming which we have already discussed little bit, but the focus of this work mainly would be on the impairments. So, today we will be continuing the CFO part, but in the MIMO context and then we will be moving to the hardware impairments, ok.

(Refer Slide Time: 01:05)



So, these are the things that I, which I will cover today. So, in the last times we left at basically at the MIMO part right MIMO hybrid beam forming part and then we said that how exactly my modelling would be ok. So, let me redraw the whole thing again.

So, this would be W BB t just get acquainted with all these diagram very much. So, that it will be I may be drawing multiple times, but then this will be H let me draw directly the H part and all. So, that it might be easier for you W RF r and W RF BB W BB r. Now, if it is in the OFDM context then we said that there will be FFT.

Now, I am assuming the CP is discarded meaning there is a CP discarded CP will be discarded here. So, I am not drawing that. So, this is just F and there will be FFT here and you will have the data here right, so this is how the data. Now, at different stages we would like to know where my CFO can be modelled and in the similar way here also in the transmitter side also I may be I may be drawing it.

So, you have the effect is i FFT here, ok. And of course, there is a CP here there is a CP here and that that part is anyway there and this is a vector; I think you may have known it is just the parallel line vector and this is also a vector parallel line vector which goes there ok. So now, we would like to know where exactly the CFO gets added here.

Now, as we said CFO always gets added just after your demodulation right, so after your RF. So, which means that my CF would be added somewhere here ok. And how the CF will be added? CF will be added in the time domain sequence, it never added in the spatial domain sequences right.

So, now if this is the vector, this is a vector line right this is not a single data line that is going, because it is a matrix right this is also a matrix, but that vector is in the spatial domain meaning the it depends on the number of antennas and how it goes. So, the CFO that will be added as we said it is always rupturing the time domain sequences, it never touch the spatial domain.

That means, e to the power j xi n e to the power j xi n that is how your CFO quantity will be added right. So, this n is not a spatial. Meaning, n does not vary over the antenna it varies over the time. So, it will start from n equal to 0 to you know as many as samples you would like to have it, so that is how it is.

So, I would say if I just magnify this part. So, if I say there are many lines here depends on the antenna the e to the power j will never be added vertically rather it will be added horizontally.

So, this is where your e to the power j omega n will be added ok. So, which means that if at a particular n-th, so try to understand that it at a particular n-th instance or n-th time instance because this is where your FFT is what is FFT; FFT is always done in the time domain or frequency domain, but it is on the samples right it is a; it is a; it is a; it is a time samples you take it and then on that you take the FFT, right.

So, if it is an n-th sample. So, which means that n-th sample whatever it is let us say this is if it is y 1 this is y 2 and let us say this is y 3 this is y 4. And then each and every 4 lines will be equally impacted by e to the power j 1, so, that mean at this point if I say there is just at n-th point I sample it and let me put a different color here.

Let us say this is my n-th sample this is my n plus 1 this is my n plus 2 n plus 3 and so and so forth. So, this is say n-th this is n plus 1 sample this is n plus 2 sample. So, 4 lines I am sampling it multiple times right physics. So it is a, it is a spatially there are four points and I am sampling them at multiple times right.

So, at n if there is a CFO. So, at this point the CFO would be e to the power j sorry this is not omega this will be xi let us consistently maintain the notation. So, it will be e to the power e to the power epsilon n into y 1, similarly y 2 e to the power j epsilon n, similarly y 4 I have drawn 4 e to the power j epsilon n.

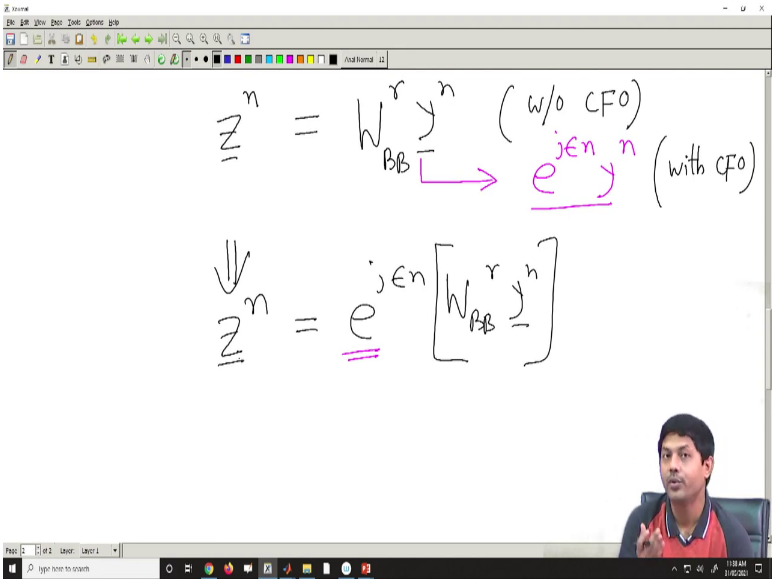
So, which means that, if I observe these 4 vectors at the n-th time they will be modified like this ok. So, what I try to give you a message is that the CFO will never be added in the spatial domain rather it will be added in the time domain ok.

So, taking a cue from that let us try to model where exactly my CFO would be positioned right. Let us say for our simplicity let us take a similar notation. So, that it will be easy for us to understand, let us say this particular vector spatial vector. So, spatially I am drawing y vector and this is my z vector spatially.

I put an n here to indicate that that is a vector which I am observing at the n-th time understand. So, that is important because it is an OFDM right; OFDM is not in the spatial domain of course, if it is MIMO it will be in spatial domain, but more than that it will be in the time domain.

So, let us say y n vector is my n-th time I am observing that vector and this is my z n is a n-th time observing the vector after my equalizer at the receiver. So, let us see what happens after that. So, which means what is the relationship between you know z n and y n, so let us understand that.

(Refer Slide Time: 07:42)



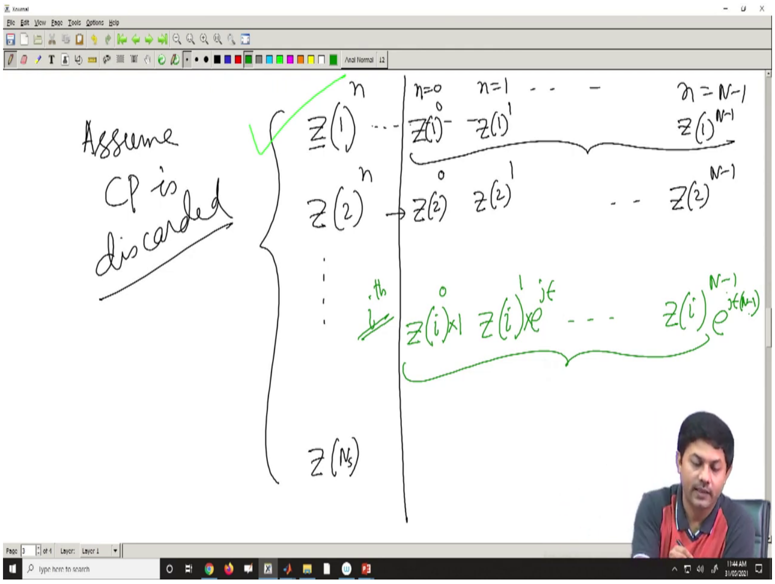
So, can I write it like that, this z vector n. This z vector n if there was no CFO it will be obviously, this part was there right there is an ambiguity right. Now, there is a CFO, so this is without CFO right, now if there is a CFO, what is the extra part? Because y n vector everybody will have the e to the power you know sigma n.

The e to the power sigma n will be equally impacting everybody because this whole vector will be modified to this whole vector will be modified to e to the power epsilon n into y n, this is how it would be right, that is the only modification. So, this will be with CFO this will be with CFO; that is the only change between y n to with CFO y n.

So, now what will be the my z n with the with CFO. So, the z n will be modified to that is the only thing this whole vector it will be just multiplied by this particular scalar term; this scalar term will be just added ok. The scalar term is it is a; it is a complex scalar terminal will be added it is not a vector ok.

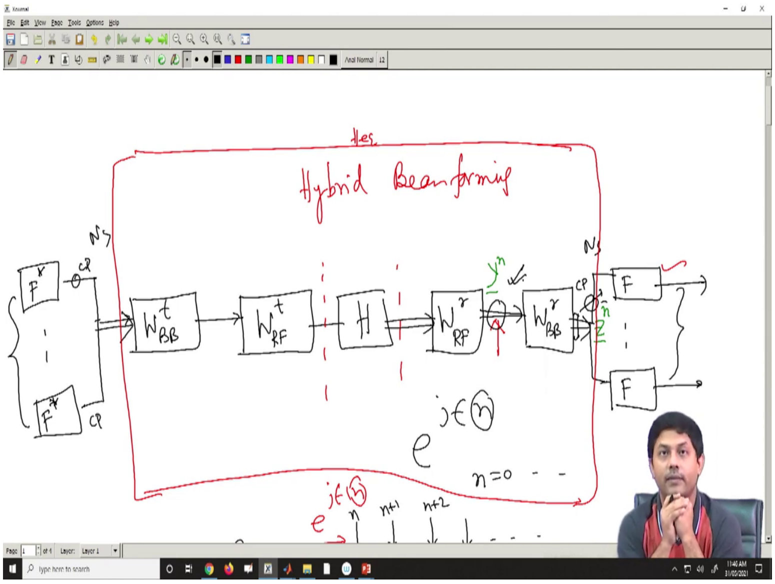
So, that is the beautiful part of it. So, this whole z inverse, so this will be this vector will be just multiplied. Now, think from the think from FFT point of view. So, what will happen because I have not still done the FFT, so how you model it? So, which means that if z n is the n-th one, so what does it mean?

(Refer Slide Time: 09:42)



So, it is like z vector ok probably it is the first spatial domain antenna, but at n n-th data z 2 n. So, what is this 1, 2, 3, 4.

(Refer Slide Time: 10:01)



It is the number of dimension or the number of antenna this particular fellow has vertically, right. Because this is the, these are all number of receive antenna or number of not receive antenna I should not say receive antenna, because receive antenna is different; it is more of a number of data streams ok.

See if it was let us say if it is N S probably this also I will keep it same just for our simplicity in the discussion. So, it is as if like it is a N S cross N S MIMO system right if you understand that is how it is. So, you can think of it that way. So now, this z say N S number of receive antenna I am just.

I mean there is no necessity that this z N S should be same at the receiver as well it can be different as well. I mean now by this time you may know it so, because it is like an n t cross n r antenna right. So, that n r so n r meaning it is not the exact physical antenna, but it is the number of data stream that by which I am referring.

So, let if I assume for simplicity it is the same as it is the same as this number ok. See if I assume this is same as this number how many number of data stream I have it. So, that many number of data might just keep it like that, but there is no such compulsion that this has to be the same ok; there is no such compulsion.

So, it can be N S this can be N S 1 also, because its it is like you are viewing the channel from here right this is your data. So, this whole thing will be now a black box to you for hybrid beam forming, this whole masala will be inside it this hybrid beam. So, this is your hybrid beam forming problem.

So, you really do not care what is the dimension inside it, ok. So, when are you in the OFDM, I am just looking at the baseband OFDM keeping the hybrid beam forming inside it. So, for me whole thing is like a equivalent channel if you honestly ask me. So, the whole thing like one box to me.

So, for me it is like a OFDMFFT IFFT and then FFT here. So, I am just thinking from the PFFT point of view ok. So now, how do I, so this is that particular case right. So, this is what the parallely these many number of data's are going at the n-th time. Now, what I am saying I am observing. So, let us say I am picking up this particular one ok.

Now observing N number of such capital N number of such data I am assuming that I have discarded my CP, so assume CP is discarded. So, here what does it mean assuming that I have already put this point this z n adjusting the CP part, CP I am not bringing it because CP does not add any value to you except making it a cyclic prefix it does not have any other problem ok.

So, assume CP is discarded. So, then what will happen this whole vector; now I am observing from parallely line. Parallely I am observing N S number of vector n time. So, it is like a N S cross n matrix observation right. So, let us say this is at the n-th time.

Now let us say this is n equal to 0 this is n equal to 1 and so and so forth. This is n equal to say capital N minus 1 right. So, what will be the case observation here? It is the Z 1 for 0 this will be Z 1 for 1 this should be Z 1 for N minus 1 so, N minus total n number of observations.

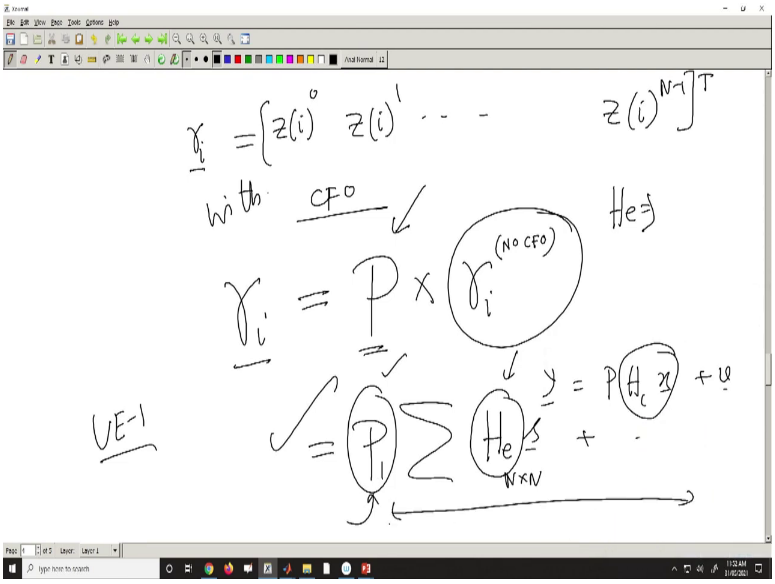
Similarly for this particular antenna or this particular should not say antenna data stream I will observe Z 2 for 0-th time Z 2 for second times and so and so forth. Z 2 for N minus 1 time this is how the OFDM comes into picture right. So, similarly, so total N S cross n. Now, when I say I am doing an OFDM. So, this part comes into picture right, so that particular vector will be coming into picture now, where is the CFO entering here.

So, now let us take only the first one or you can take any of them, say let us say I am taking the i-th one; let us generalized i-th one. So, how I will I; how will I even get it? So, let us take a general Z i for the first data for the first time data Z i for the second time data and so and so forth; Z i for the N minus 1, I am assuming you have discarded the CP. So, here 0 meaning after CP discarded; discard; discarding I am getting that 0.

So, that is the time. So, I think as long as long as you understand the basic part it should be fine. So, 0 does not mean the first sample, it simply means that after CP deduction or discard that data part ok. So, let us say I am taking this n vector right, now you tell me where exactly my CFO will be fitting?

Because CFO will not change vertically; how the CFO will change? It is CFO will change horizontally. So, can I say this Z i it should be multiplied by 1 this Z i multiplied by j epsilon. Now, this Z i the last part can I write it e to the power j epsilon N minus 1. Fair assumption because that is how my CP will be that is how my CFO will be added there as well ok.

(Refer Slide Time: 16:12)



So, let us say how then how the data model would be, let us say I put a different notation let us say that is r i, i meaning it is the i-th data stream. So, that is nothing but say Z i 0 Z i 1 and then Z i n minus 1 you can put a transpose also if I wish it is a column vector. So, that is my vector right. So, how can I write it? So, how do I write that vector?

So, this is basically the normal point ok. Now it is nothing to do with CFO or not CFO. So now, assume that there is a CFO. So, with CFO what will happen? The same r i vector same r i vector will be; now what should I say there is a P matrix that is coming into picture multiplied by this r i old one; old one meaning which was not if there was no CFO what would have been the case ok.

So that part, so what was the case that was mod of this part right I mean you can always take the CFO out and what should be the case and then that part basically. If you remember your you know OFDM part your y vector what was P H circular matrix then x plus v right. So, this part is when there is no CFO part. So, this part is mod of that when there is no CFO part.

So, ultimately what I am trying to say here is that here also you get a P matrix separated, so that is my whole discussion. So, even if I say my CFO is finally, getting modelled at this part here my CFO is coming just exactly just after my RF part and at the digital part; before the digital part it can still be modelled like a P ok.

Now, here how do I effectively combine the channel and all that is you have to go through this matrix multiplication and so on. That I mean numerically it is slightly difficult, but if you take in a you know computer you can see what will be my H effective and all these thing.

But finally, what I am trying to say is that finally, my P matrix is out always that is my first part of the story here. So, P matrix will be always the out part. Now this particular one r i old which is the; which is the case when there was no CFO or I should say old meaning probably I should have used it better notation. So, this is a case with no CFO.

Had there been no CFO, whatever the r i and if there is a CFO, what is the new r i? It is nothing but multiplication and this is precisely what the OFDM will also offer right and OFDM. Now, this particular r i will have it is own structure and so and so forth, this will be summation of your effective channel.

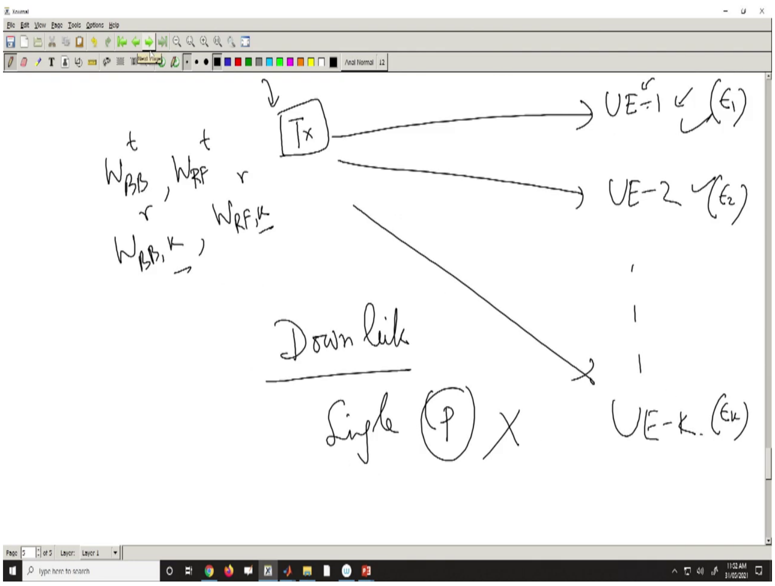
Now, this H is not the H I was talking of not this H not the exactly the antenna, this is the effective you know the square matrix and circular square matrix. So, this should be N cross N, I should call it some sort of a H e or effective H. And then data’s and all these things will be there and so and so forth, but this P will be final.

Now, this He it is complicated it is a circular channel matrix, but it is a function of all your W BB, W RF t n r and then actual you know actual millimetre wave channel that all combine. So, I mean that can be easily computed ok. Given a matrix that can be easily computed, but this is what that particular channel is.

So, but my whole point is that the P you can still take it out when you even go for the hybrid beam forming and that can be put here. But this is for a single user case ok and the whole scenario will change if there is a multi user case. In multi user case the scenario is completely different if it is a uplink ok.

This P will not be taken out it has to be within the P ok, probably as if the time permits I will can I can explain it here itself. So, quickly though it is not part of this course, but just as a basic understanding.

(Refer Slide Time: 20:42)



So, basically what I am trying to say is that if your say this is your Tx and you have multiple users say I have K number of users right. Now if it is a downlink whatever I have shown is perfectly fine.

So, that mean at every user level you can have your own P ok. So, if when you are in this user and you are analyzing the whole story that is fine, but say I am just making a uplink ok. If I make an uplink the whole scenario will be completely different ok then you may not have the same P. In fact, even in the down link also if I do a joint optimization.

So, let us assume I am in the downlink, I am designing a downlink let us even take that case also downlink. But if you look at the earlier classes for the hybrid beam forming when I was designing the parameters, what are the parameters that I was designing? So, this will be W BB t I was designing W RF t I was considering at an each and every users W BB for k, W RF k, k meaning the k-th user. So, it can be all the that mean all the parameters when I was designing at this level.

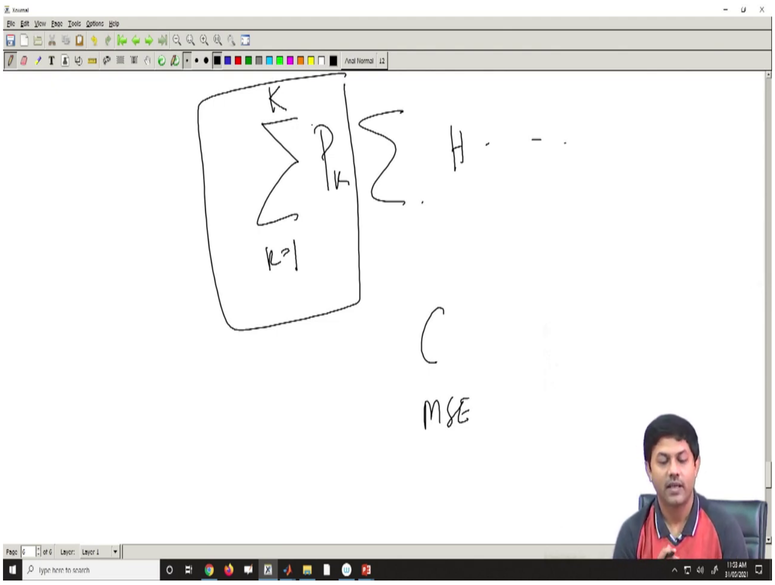
Then when I try to model it there you cannot just have single P that is wrong; you cannot have that, why? Because at each and every users the CFO can be different, why? Because CFO is arising out of what? It is a mismatch between your transmitter RF and your receiver RF frequency right.

And each and every users are all isolated users, they have physically dislocated existence right. If there are they are physically dislocated and electrically isolated this user 1 has it is own clock frequency this user 2 has its own clock frequency and user k also has its own clock frequency that mean every user has it is own clock frequency. So, it is unlikely that everybody will have the equal amount of mismatch with respect to the transmitter RF.

So, he may have a CFO epsilon 1, he may have a CFO epsilon 2, and he may have epsilon he may have a CFO epsilon k ok. That is the case then it is not a very simple task. So, what changes happen? What are the changes happen? The only changes that can happen from the user side there is no change.

So, this if it is a user 1 this equation is fine probably I will have a P 1, but when I see the same thing suppose I am trying to optimize it completely from the system prospective I am considering all the users then this equation is no longer valid. What is the extra change happen?

(Refer Slide Time: 24:00)

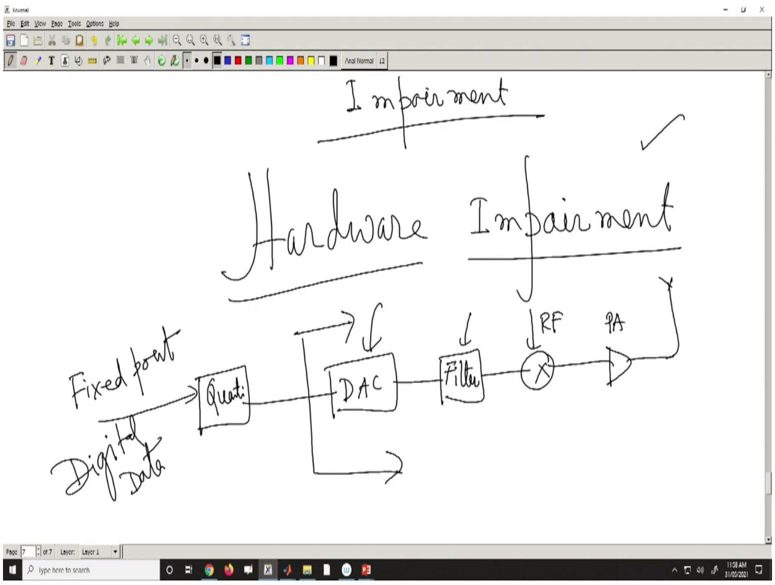


Then extra changes happen in the sense that instead of taking the P out I have to take the P i also inside it not P i I let me put a different notation P k; I should put it right P k and this H all these things will happen. So, then one extra summation whatever summation was there one extra summation has to appear. So, this will be k equal to 1 to say capital K, so probably this may enter here, rest of the things.

So, this will be now coming into picture extra P k will come into picture because for every users it will be different. So, when I was trying to estimate the capacity then MSE all these things they are extra this P k will enter there because each and every user will have their own

CFO. Anyway that is not scoped in this course the how multiple CFO appears in multiple users.

(Refer Slide Time: 25:13)



So, this is just a single case scenario that is what we wanted to find. Now next point or next impairments that we have it is the hardware impairment ok. So, let me just introduce all these impairment first hardware impairments and CFO impairment then we will try to do some basic analytical idea.

Though this course does not give you a very detailed analytical idea, because analytical idea will be it is very case dependent and it involves a very rigorous mathematics and some other mathematical tool for example, random matrix theories and so and so forth. So, I am not getting into that details, but probably I would give you a of glimpse of what are the because somebody may try to do PhD or MS thesis or Master Thesis.

So, probably analysis is required, but I will only introduce you how exactly you can analyze stuff not in details, but at least a framework later on of course, end of this course part but let me first introduce all these impairments.

So, we have just introduced the impairment called CFO, a second impairment that appears here is the hardware impairment ok. There are many other impairment; for example, I can have a SFO like Sampling Frequency Offset, and channel estimation error; mainly these are the force part, but let me just predominantly whatever is there I will just try to get. So, next impairment is your hardware impairments ok.

And let us understand how it appears. Say for example, why hardware impairments appears? So, it can appear from your digital side, it can appear from your analog side, it can appear from your RF side ok. Now this hardware impairment it is actually a very big topics ok. And it is very difficult to model everything in one shot. For example, you cannot say suppose what is my transmitter change say let us take a simple case.

You have a DAC here, you have lot of analog filters here, filter here maybe analog filter, then you have an RF here again you may have a power amplifier and then you have an antenna right. So, where exactly your impairment enters and before DAC also there is another level of impairments that is your quantization because your data is a fixed pointed data right.

So, this is your fixed pointed data. So, this is your fixed pointed digital data; fixed point digital data. So, here you have a of course, so here you have a quantization noise here. So, DAC will have it is own analog issues there will be a phase issues, filter will also introduce some because none of them is perfect right. I mean errors will be more once you get into the RF and analog mode.

In digital its very much predictable data right, because you know how much fixed point that you have done and how much error or quantization was that you can have it. But when you get into the analog and RF part it is you cannot precisely say this much error has happened rather you can kind of you can say bound you can bound out this much error can be there. I mean the and the models of those errors are very much non-linear in nature.

For example, power amplifier errors are extremely non-linear in nature, RF impairments filter. So, they are very very difficult way of I mean it is not a very straight forward mathematics or equation where you can just say ok you multiply this part with so and so and you get a; it is not that easy ok its really really tough job ok.

So, modelling the complete thing is really difficult thing ok, but we will try to give a glimpse. Like can I model in a very simplistic manner putting aside several other stuff. So, is there a philosophy by which I can model this whole impairment in a very very simplistic way? And the answer is, yes.

And probably in the next class I will introduce that part how exactly a harder impairments can be introduced in this case. And we will see in the hybrid beam forming case also how exactly that comes into picture, but it is a very very simplistic model, take my word ok probably it is a level one simplistic model. But it does not answer each and every model that exists there, it is a very simplistic model. And for this course I will introduce that part ok. So, with this I conclude this particular more details of the hardware impairment part ok.

(Refer Slide Time: 30:26)

So, we covered the CFO with the basic MIMO and then just started the impairments in the hardware.

(Refer Slide Time: 30:34)

This is the reference. So, these references has nothing to do with the hybrid beam forming, but this talks about how a CFO and hardware impairment can impact a system and its analysis some of the analytical part; I was talking right.

So, that analytics analysis is very tough job and it requires a separate mathematical tool called random matrix theory. So, this paper is talking about that. So, it is nothing to do with hybrid beam forming, but you can extend it to hybrid beam forming case.

Thank you.