

Signal Processing for mmWave Communication for 5G and Beyond
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Module - 12
Multi User Hybrid beam and Impairment and analysis
Lecture - 62
Multi User Hybrid beam and Impairment and analysis (part - 3)

Welcome to Signal Processing for millimetre Wave Communication and for 5G and Beyond. So, today we will be talking about the part 3 part of the hybrid beam forming impairments, ok.

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Concepts Covered

- Impairment – CFO (OFDM)
- Impairment : hardware

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So, these are the things that we will be covering today, impairments CFOs the leftovers and will be introducing the impairment at the hardware. So, ok.

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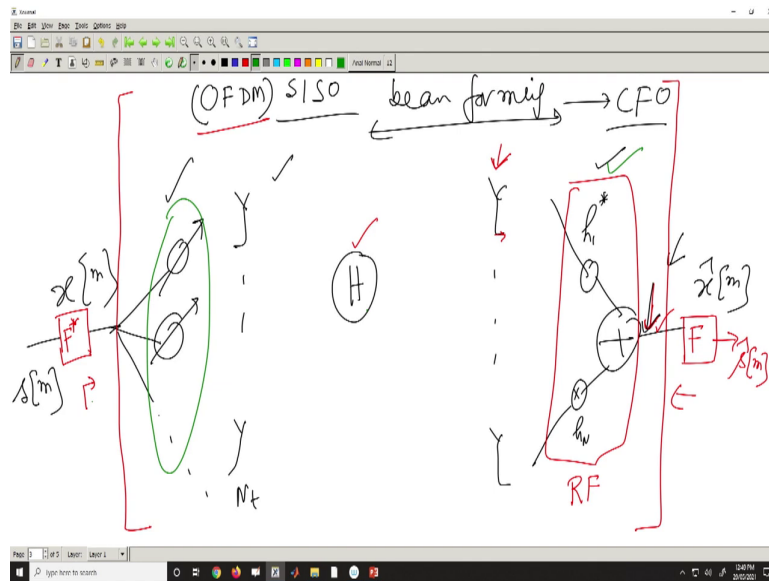
$$\underline{y} \Rightarrow \begin{bmatrix} 1 & & & 0 \\ & e^{j\epsilon} & & \\ & & \ddots & \\ 0 & & & e^{j(N-1)\epsilon} \end{bmatrix} \begin{bmatrix} y^o(0) \\ y^o(1) \\ \vdots \\ y^o(N-1) \end{bmatrix}$$

P $w/o \text{ (CFO)}$

$$\Rightarrow P [H_c \Sigma] + v \Rightarrow \boxed{P} H_c \Sigma + v$$

So, this is where we left it last class, where for a normal MIMO case this is how you model your CFO impairments, right. Now, what happens when I go to the hybrid case, hybrid MIMO case, hybrid beam forming MIMO case. So, does the whole thing changes? I would say nothing changes, ok.

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So, let us say I am just using a SISO beamforming, ok. So, what was the SISO beamforming case? In the SISO beam forming case if you know that would be easy to understand case and we will be having what happens when there is a CFO here, right.

So, SISO was very simple. So, there you have N_t number of antenna here, there is a channel right and then what you do here is basically MRC. So, you have this something h_N^* here, you just multiply this and then you do an addition, that is precisely the case, right. And there would be a phase shifter here, phase shifter and this is your data x data, right.

So, this a digital form in a digital form I am writing x_m . So, of course, internally there will be RF and all these things. So, here you have this $x_{cap m}$. So, that is basically your single

SISO beam forming case; this was the steering part, this was the beam forming parts, this is the combiner part and you get the data, right. So, that is we have explained this part.

Now, let us understand that, what if this x_m is a OFDM case. So, how exactly things move? So, if the x_m is a OFDM case, it is just that there will be an FFT there, right. So, the extra part that will be coming into picture is the FFT part, ok. So, instead of SISO beam forming case I would say, we should now make it OFDM, SISO beam forming case, right.

So, then there is an F coming into picture and I would write it would be s_m and naturally and c_p and cutting the c_p I am not drawing it, because that would be implicit. So, this is my FFT block and I will get $s_{cap} m$ that is the extra part for OFDM. So, extra this red coloured blocks extra part when I go for a OFDM SISO beam forming, right.

So, now where is the CFO business coming into picture? So, CFO will be coming when you are in the channel part right, because that is where the CFO enters here. So, at the you know, at the time domain part of my antenna. So, whenever I receive it, there will be a shifting, there will be a problem on that part.

So, naturally if this data, this data, ok. So, what is the spectrum? That spectrum would have been shifted right, because the effective part. So, if I do somewhere demodulation say somewhere here; so how exactly that appears to your case right, so that you need to understand that.

Now, where exactly the CFO appears? Is it really appearing here or is it really appearing here, you have to understand. So, look CFO appears unless and until. So, let say my t_x is perfect; so that means t_x is exactly giving my f_c , let us assume not much. So, it has sent a perfect spectrum, so no spectrum shift happens.

So, when the shift of spectrum happens is that, when you do a demodulation; unless you do a demodulation, you will not see the spectrum shift, am I correct. So, this demodulation is the one where your spectrum will be fast send otherwise you will never see the spectrum, ok.

So, that means this is where the case comes into picture; so that means unless I do my. So, let us say this part is my RF; will I see a spectrum shift after this adder? Naturally not, because I have not done the demodulation ok; this is a simple analog sorry, this is the simple RF level activity.

So, just the phase is shifted and there is an LNA gain adjustment; we have explained that right and then there is an RF adder. So, I have still have not seen my spectrum shift. Where will I see my spectrum shift? I will see my spectrum shift here, because that is after that what I do, I do a RF demodulation, right. If you remember what I have done, unless you multiplied e to the power minus $j \omega c t$, right.

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$$x_b(t-\tau) e^{j2\pi f t} \times e^{-j2\pi f t}$$

$$h = \left[\bar{a}_p H a_c^* \right] \rightarrow \text{scalar}$$

$$= \underbrace{\bar{a}_p H a_c}_{h_0} + \underbrace{\bar{a}_p H_1 a_c^* z^{-1}}_{h_1}$$

So, unless. So, you have this x_b if you remember in the last class I have done $e^{j 2\pi f_c t}$; unless I multiply with $e^{j 2\pi f_c t}$, I will not see any spectrum shift.

This is where the trouble starts, when I do a demodulation, ok. So, where I do the demodulation in this activity? I do a demodulation here, nowhere else; nowhere else I will do any demodulation right, am I making the points clear. So, this is where the trouble starts.

So, now is this channel getting affected because of my CFO? No, the channels will remain as it is; the CFO will appear here, because after that stage I actually start my demodulation. So, which means that, if exactly if I do OFDM; now from an OFDM point of view, so if I see the baseband channel.

My baseband channel will be coming from here to here right; this whole box is just my baseband. So, if my eye is here, suppose from here I am looking my channel, from here I am looking my channel what will I see? Will I see all these phase shifts and this antenna any of them? Nothing, I will not see anything. What will I see?

My that particular within that box my channel is nothing, but h is equal to, if it is just a single tap channel is equal to this vector, whatever this phase shift vector this one, ok. Let us call it a p vector, then there is a channel and this vector, this vector; because it combines it whatever it may be MRC or whatever, but let us call it combiner, this is how it is, right.

So, what it is? This is just in one tap, just one single complex number, just one single complex number scalar; that is my tap, because if I look at my tap from here and here, that is what exactly I will see.

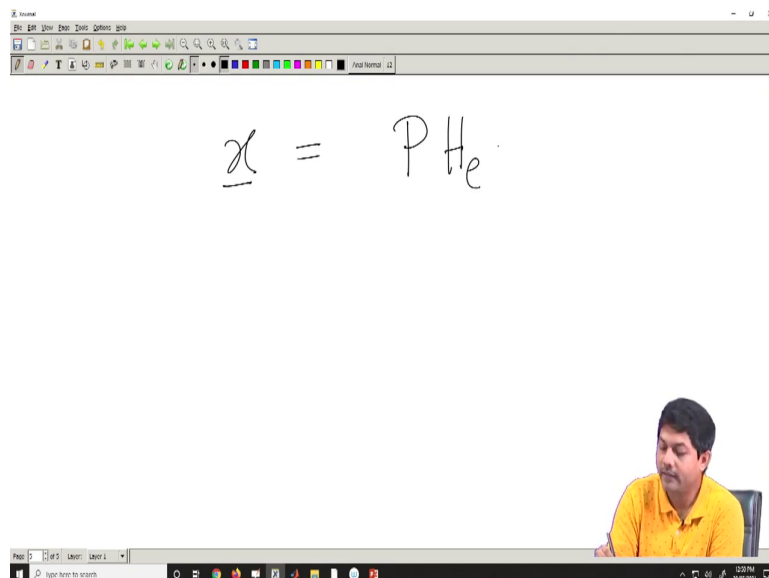
Now, if there are more number of taps, well this H will be multiple such H will be there, right. So, I mean I think that part also we have explained; it is a p there will be H a c star plus

a p H probably 1 a c star z inverse and so on. So, may be L number of. So, that is how the channel appears for me.

So, this whole thing just one number, one scalar; this is like a one coefficient for me, it is still an FIR channel. In case there are multiple taps come into picture, just that H remains H would change. Here H is for the first instants, next H is the delayed instants, next H will be the delayed instants and so and so forth; but that is how things move on, right.

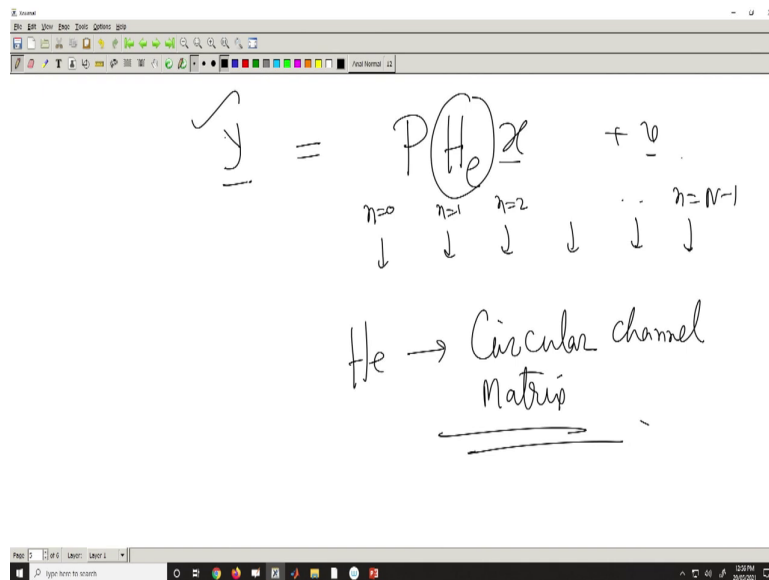
So, my point here is that, it will not appear here; the p will not appear there. The p will appear where? The p will appear unless and until you do a demodulation here, ok. So, which means that, if I say what my effective channel is what my ultimate p would be; so it is this, it is this data or rather this data x; x data ok, that would be my P.

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Now, I write it H effective let me just complete it and this will be, probably I will put a different notation, so that it would not confuse you y bar.

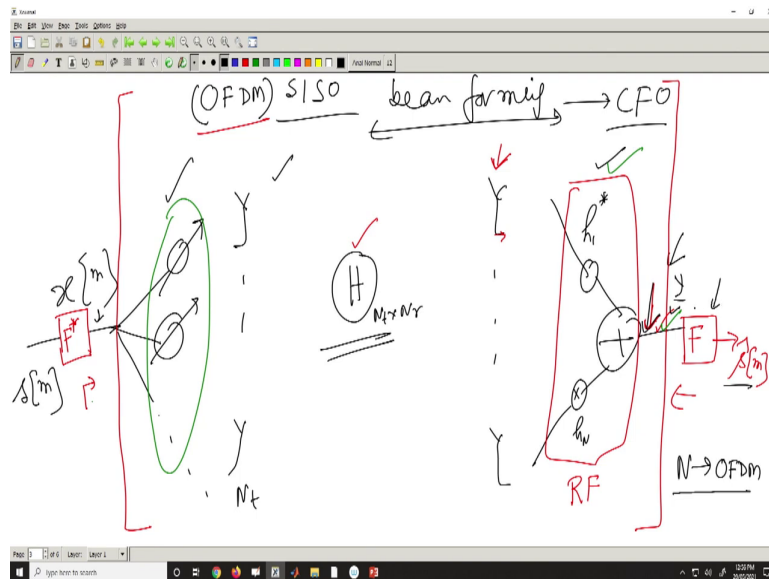
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The image shows a whiteboard with a handwritten equation and a note. The equation is $\underline{y} = P \underline{H}_e \underline{x} + \underline{v}$. Below the equation, the elements of the vector \underline{H}_e are labeled as $n=0, n=1, n=2, \dots, n=N-1$, with arrows pointing down from each label. Below the equation, the text $\underline{H}_e \rightarrow$ Circular channel Matrix is written, with the words "Circular channel Matrix" underlined.

And let me change my notation here as well; let me call it, because it was in the context of no OFDM.

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So, now, as I am doing an OFDM, let us call this vector as \bar{y} . So, this is my \bar{y} , $P H$ and then it would be \bar{x} right; because this is what I have transmitted, this is what I have transmitted \bar{x} , \bar{x} plus v whatever.

Now, this is a time domain sequence; which means that I am observing in different different time instances, not one single case. So, this is my n equal to 0, this is my n equal to 1, this is my n equal to 2 dot dot dot this is n equal to N ; say let us say my OFDM is N length OFDM, so I am observing that. Now, what is this H effective, that is a circular matrix. So, this H effective is also circular channel, circular channel matrix. Now, be very careful here, channel matrix here; it is a circular channel matrix, now this H effective.

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The diagram shows a circulator matrix H_e and its corresponding antenna-to-antenna matrix H .

The circulator matrix H_e is a circulant matrix with elements h_0, h_1, \dots, h_{L-1} in the first row and h_{L-1}, h_0, \dots, h_1 in the last row. The matrix H is a 2x2 block matrix with elements a_p, H_1, a_r, H_2 .

Who are the taps, that mean that H effective will be some you know h_0 , then it will be $0 \dots h_{L-1}$, then h_1, h_0 ; then finally, it will be h_0, h_1, h_{L-1} .

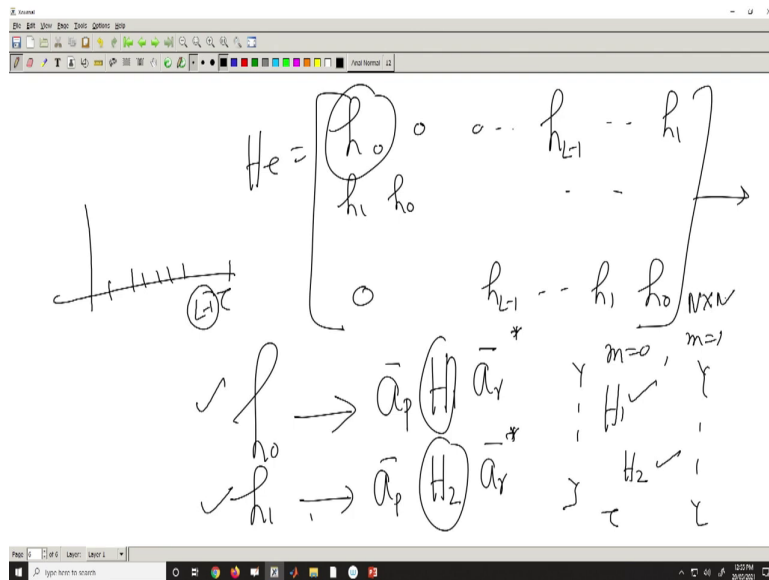
It is a circular matrix. Now, who are this h_0 , who is this h_1 ? So, what is this h_0 ? What is this h_1 ? That is this part, this is my h_0 ; this part is my h_1 ok, not the channel matrix between this. This is just the antenna to antenna, that is only a physical level; but when I look at from a baseband level, I will not see this H , because there is a phase shifter and there is a combiner, effectively I will see just one complex number, right.

So, here I would say this is nothing, but a phase shifter H_1 something like that; then a p H_2 a r star. Who are this H_1 and H_2 ? This H_1 and H_2 is you can think of this is something like a

matrix of, it is a matrix obviously; but I see at a different tau domain, tau meaning delay domain.

So, that means these are the antennas at a particular; now I am thinking of a tau domain, right. If you remember your earlier channel modeling case, what is your H 1; for a matrix case, what will be your H 1?

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So, it is when I sample say at a particular tau of say it will be something like this at m equal to 0, that is my channel.

Now, when I sample m equal to 1, my channel it is in the tau domain; of course, in the tau domain I am trying, the channels delay domains things will be slightly, it will be different. So, that is called your H 2, it is not Doppler, it is not time of sampling, it is time of my delay

domain. So, this is my H_1 , H_2 and because this that is how you get your FIR filter right, how do you get your FIR filter. So, that part is your H_2 and so and so forth.

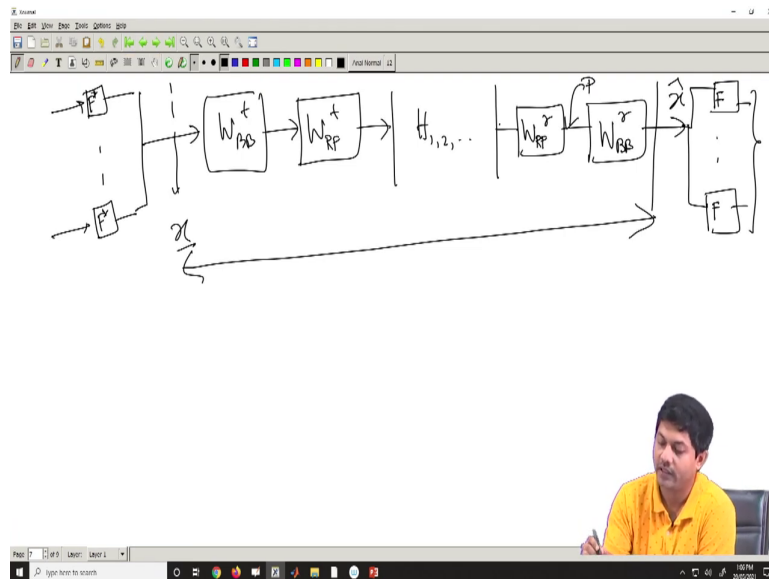
And then if you have L number of delay spread; suppose your delay spread itself is say from here to here such that you get L number of or L number of samples you get it. So, that is precisely the channel type. So, this is a matrix this is a matrix which I get in a different different delay domain sampling.

So, these are my h_0 , these are my h_1 ; never think this whole matrix is H_e is it has anything to do with that antenna to antenna. Because if you notice its dimension is N cross N very careful here; whereas, when you look at this gentleman, the dimension is say N_t cross N_r which is a physical antenna.

So, what is N ? N is the length of your OFDM. So, N is your OFDM length does it have anything to do with antenna nothing it has nothing to do with your antenna ok; whether antenna has hundred or hundred, N remains it N can be thousand also, ok. So, this is your time domain samples that mean I am taking N number of time domain samples or N length OFDM rather.

So, this is where the modeling comes into picture here. So, this is the this is where it fits here ok. Now what if I change the model from single SISO to you know multiple cases? How does the thing change, where exactly the things will be appearing. So, let us understand that, ok.

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So, now let us say you have the WBB t, then I am talking of the MIMO OFDM case where exactly my CFO appears WRF t, this is your I would say multi-level this would be your FFT data, ok. And this is where your channel; now this may be not one channel; 1, 2 depends on how the delay spread works there.

And then you have this is WRF r and this would be WBB r and this is your. So, if this is x vector, this is your first x cap vector; after that again you have to do IFFT sorry FFT and this is your next level of data. And you know how to decode that right; I mean you have to introduce all these things. So, basically this is your data part, this is your total data part, this is your data modeling part.

Now, let us understand in this case where exactly your shear flow gets introduced, ok. Where it will be introduced? Will it be introduced here or will it be introduced here, it is natural

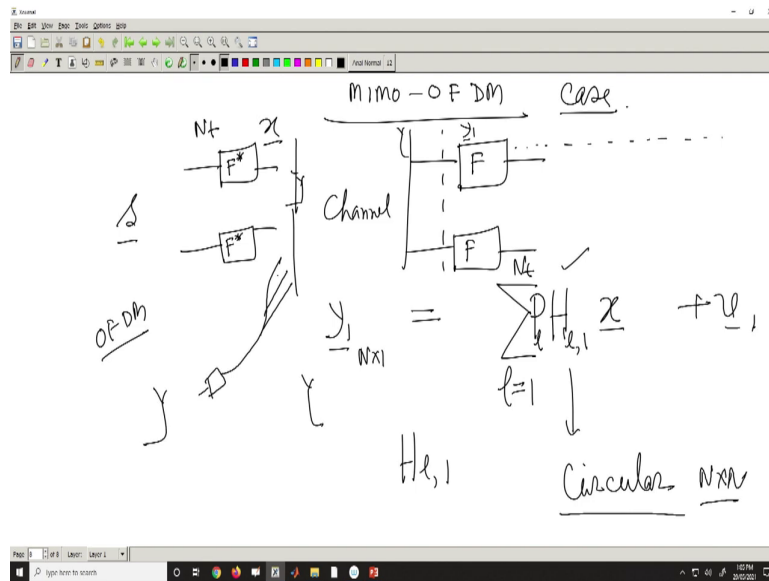
right; because this is still in the RF domain, unless I go past the RF, I will not have, I will not have any CFO there, ok. So, that CFO will be introduced there, is just that point. Now, how do you model it? That is a very complicated model, ok.

So, now if you look at the, now let us draw an analogy here in this case. So, how exactly my complete modeling here happen? So, this is the effective channel, where my P has entered and from there I get into my data model, right. So, what gets impacted? So, now, you have to also find out some sort of a effective channel and that is it, right.

So, now to model this part, how you should proceed it, ok? Now, to do that, you have to go back to your MIMO OFDM case; because this is the MIMO hybrid case, MIMO hybrid beam forming case. Now, you have to model or to understand this modelling, you have to go back to your MIMO, normal MIMO OFDM case, ok.

Now, again if you recollect how MIMO OFDM cases CFO have been done; MIMO OFDM has been done, so how CFO enters there, ok. So, let us try to understand that aspect, once that is done, then this will be just a cakewalk, ok.

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So, in the MIMO case, plane MIMO case, plane MIMO OFDM case, ok. So, let us say I have N_t number of data, where each of them will go through FFT, each of them will go through FFT, then it will be a channel, ok. Now, there will be a multiple such channels; I should not write it is a channel; because the in OFDM case, the channel is an N cross N channels, it is basically unless you sample n times, the channel will not even appear there, ok. So, this is a channel I would just write a normal case.

And then you have this F here, you have this F here; I am not drawing the $c p$ part, that I assume that it is known to you. So, at this level what it would be, right? So, let us say this is my y_1 vector. What is y_1 vector? y_1 vector is basically a data which I am, which I am getting it after n number of serial samplings; so it is a time domain data, not the special

domain data, that mean it is not a data across the antenna, rather it is a data within an antenna if I sample at multiple times, ok.

So, your y_1 vector it is the first antennas data, but I sample n times. So, it is my n time domain data. So, obviously, the length should be $N \times 1$; what should it be? If you remember the data was, if it was a MIMO data, it was some sort of a . So, if this is my s vector, this would be my x vector.

So, what it would be? It will be the x vector, then this is my channel; the channel would be something like, it is a let us call it l equal to 1, say if it is 1, it is $1 \times \bar{r}$ right, that was the case. I think that we have explained in the context of MIMO OFDM; because that was what is this H_{l1} ? What is this H_{l1} ? It is the channel, it is a time domain channel $N \times N$ channel; it is like a SISO channel between the l th transmit antenna and the first receive antenna.

So, if you have one antenna here and one antenna here; in the OFDM case how the channel is, in OFDM case channels are all circular channels after the cp cancellation at the receiver even if it is a single antenna, because I am receiving at multiplex. So, if I take n number of samples, time domain sample; even if it is a single antenna case, your channel still becomes a matrix, right.

But it is a circular channel matrix and it is a circular square channel matrix. So, this H_{l1} is the basically the l th antenna transmit antenna to the first receive antenna channel, if I am in a OFDM mode. So, it will be also. So, each and everybody is a circular channel, circular square, obviously $N \times N$ channel.

So, this is what my data model is, what my data model was right for the first question. Now, where the CFO appears here? The CFO appears, because it is an antenna to antenna right and let us assume that every you know every, it is a receiver, right. So, if it is a single receiver, what does it mean by what does it mean by single receiver? Meaning it is the same IC die, where you are feeding all the antenna which is the same clock sources.

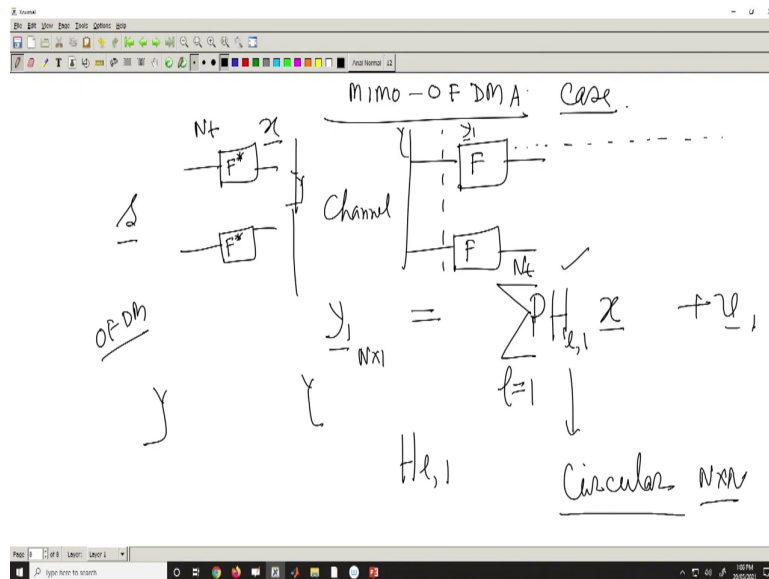
So, the amount of CFO seen by every antenna maybe or tentatively may be the same ok; because the clock source is the same. Had it been a multi user, then of course it is a separate story; but it is a single user case, but it is a MIMO case. So, that mean every antennas demodulation clock is the same frequency; but that may be a skewed or that may be slightly ruptured compared to what I exactly and that is where the CFO comes.

So, if it was a you know, if it is a P, P appears there. Where the P would have appeared? So, P would have appeared, in the single case the P would have appeared here, just before the matrix. Now, is this P different for different l? Probably yes, probably no. So, I can liberally write l here; so that means for different antenna to a single receiver, I may have a different way.

But practically it may not be possible; because your transmitter though they have multiple antennas, they have the same clock frequency ok, because the source of every antennas clock probably the same. So, the same one is fed everywhere. So, there is a less chance that every antenna will get different clock frequency, ok.

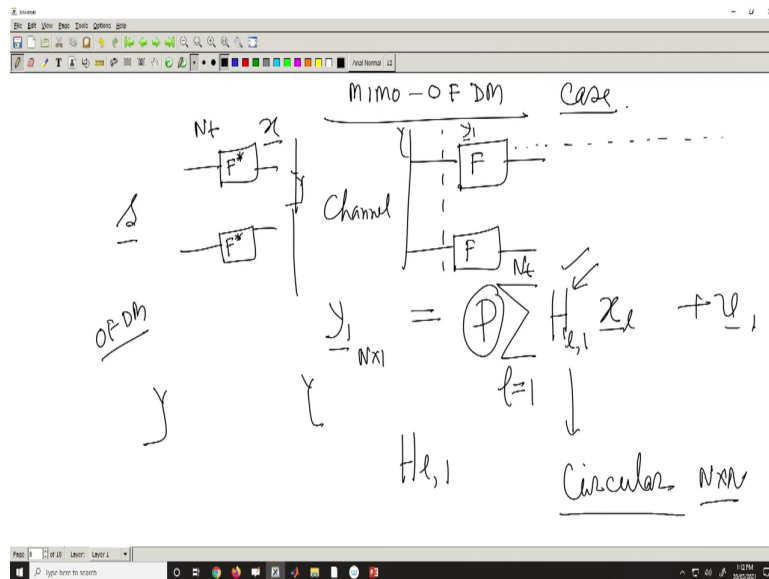
So, it is the. So, it is there is no possibility that, you will have a different P. So, I remove the P here; you understand the logic here, this is very important to understand. So, you should not have a different P matrix, should not have a different P matrix even if the transmitting antennas are different; because every transmitting antenna are from the same source of the clock, because it is the same user here.

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But, if it is an OFDMA, suppose I just make it MIMO OFDMA; the whole scenario will change, you may have a different phase, because you know what. So, everywhere that source, I mean it is a different different user, they may have a different different one. But I am not using a OFDMA, so I am using a plane OFDM. So, in this case my P will be same. So, what does it mean? It means that I can take the P out, because it is a common P.

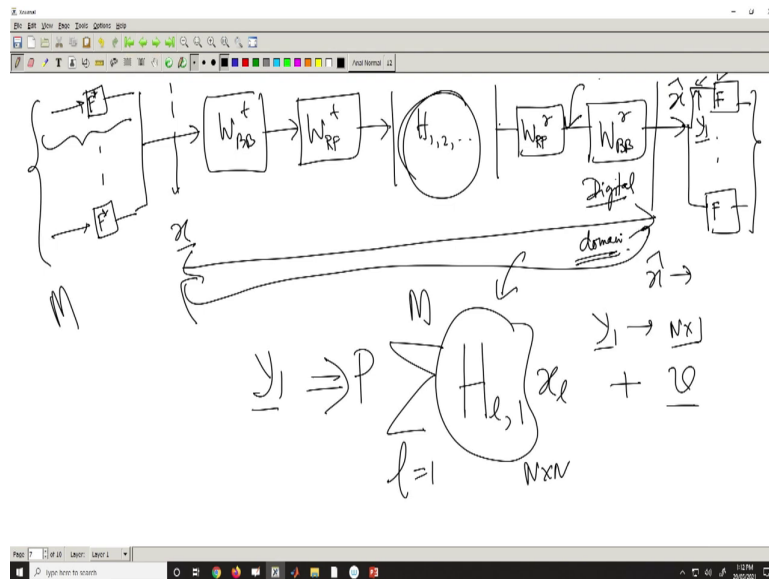
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So, I will not draw the P inside it, ok. So, I will draw my P outside and that is precisely my data model. So, at the end of the day, you need this matrix P and this is my.

Now, let us go back here, now let us go back here. So, how I would do my channel model when I go for the MIMO OFDM. So, again I repeat, where introduced? I introduce my P here; I should not write P, this is my data, right. So, till here can I write my data model right? Because we do not know where the exactly the P and all this coming into picture there, right.

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So, because this is my digital domain. So, can I write my data till that point, let after that let us see what happens, ok. So, let us say I am getting my data at that point, ok. Now, if it was a single case what will happen, the P appears here, right. Now, what is the data model here, ok? Let us call that, this is the total is \bar{x} . So, let us call this vector; if it is a n length, now this x is basically a spatial domain observation. So, this x whatever I have written it is spatial domain, right.

So, which means that it depends on how many you know antennas are there, physical antenna. But, now I am talking it is a OFDM; so it is my, I am not only considering a spatial domain sample, I am also considering a time domain samples as well.

Now, let us assume that, that is the reason I say another vector \bar{y}_1 . So, this \bar{y}_1 is a vector which is a $N \times 1$ vector, that is just appearing before my Fourier transformation,

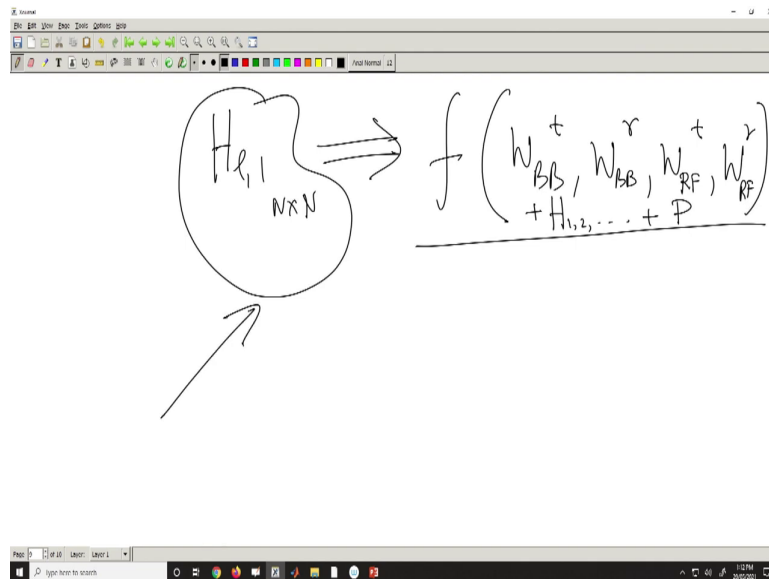
the FFT transformation here, ok. Now, what is this \bar{y}_l then? Let us understand that, ok. Now, that will also have some effective channel obviously, now whatever it we will come to that point, ok.

That will have some effective channel. Let us say this is your \bar{s} or $x \times x$ data, different x data right, there is a small channel here, so ok yeah. So, what will happen? This \bar{s} probably I will use a different notation here; x_l basically the data at the l th side, l th type of antenna, this l equal to 1 to if you have M number of data stream, this will be now. What is this? Now, this I know, I can just draw an analogy and create my data model here; but what I have not explained explicitly is that, what is this channel now, ok.

Now, this is a combined effect of all these points here. How you model this particular gentleman in your system, ok? So, this is the, because this H_l is a square matrix; but if you look at this complete hybrid beam forming, who is square here, nobody is square matrix, right. So, why suddenly this $N \times N$ appears in your system; how exactly this $N \times N$ matrix come into picture, this is a circular channel matrix, right.

At the end of the day; that mean it is a combined effect of all of them and that gets put here. So, what exactly this channel? So, that mean this H_l in.

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There will be x here, this $H_{l,1}$, which is N cross 1 , N cross N matrix; it is a matrix which will be a function of $W_{BB}^t, W_{BB}^r, W_{RF}^t$ and W_{RF}^r plus some other variables. Who are they? H , H is not that H ; I am talking of the H between this one, this one, the actual millimetre wave channels, ok. I call it $1, 2$ dot dot dot dot; because that have a different values at different different delay spread part, right.

So, now in the next class we will try to give a highlight of how exactly this channel be constructed in our case. And we will also show you, is the position of P will be here or it will be inside it that will also show. So, I would say this will be also a function of your P matrix, P meanings your CFO matrix.

So, we will construct this $H_{l,1}$ minus 1 , $H_{l,1}$ comma 1 N cross N this matrix part ok; that is a complicated equation, we will try to get into that and see how that particular channel can be

constructed with the CFO model. But this is just the motivation that, how exactly my CFO will enter and is it right way to model the CFO here or it should be somewhere else and so on so forth, ok.

So, with this I conclude the session and in the next class, we will be talking about more on the CFO part and of course, the hardware impairment part, though I have not touched upon it, but this will be of course carried forward and hardware impairment parts will come into picture here, ok.

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So, that is it from and this is the reference; but this reference is only talking about the CFO modeling part and hardware impairment part, but at least from the CFO modeling part this can be looked at.

Thank you.