Signal Processing for mmWave Communication for 5G and Beyond
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Module - 12
Parameter estimation and impairment
Lecture - 61
CFO and other impairment and their effects

Welcome to Signal Processing for millimetre Wave Communication for 5G and Beyond. So, today we will be talking about various impairments and their effects. So, we will start with CFO that is the Carrier Frequency Offset and essentially it means that we are in the OFDM system of beam forming, ok.

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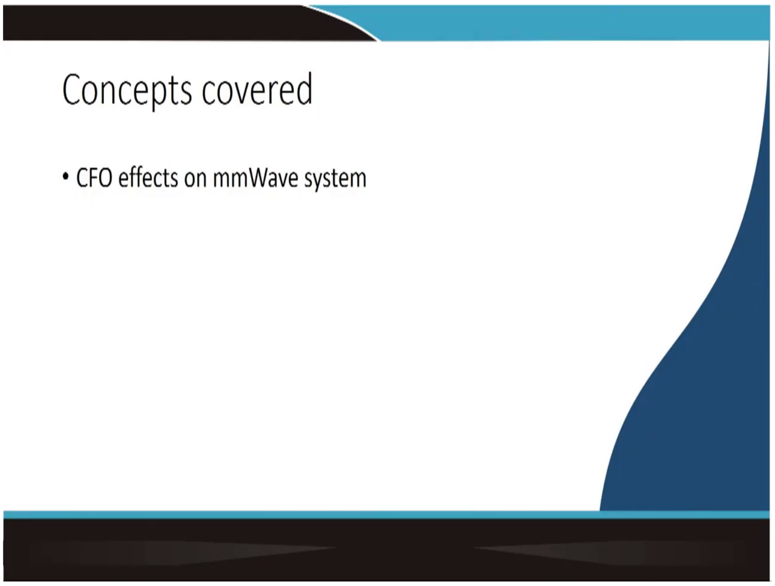

So, let us start it. So, the concept that we will be covering today the CFO effects on millimetre wave system.

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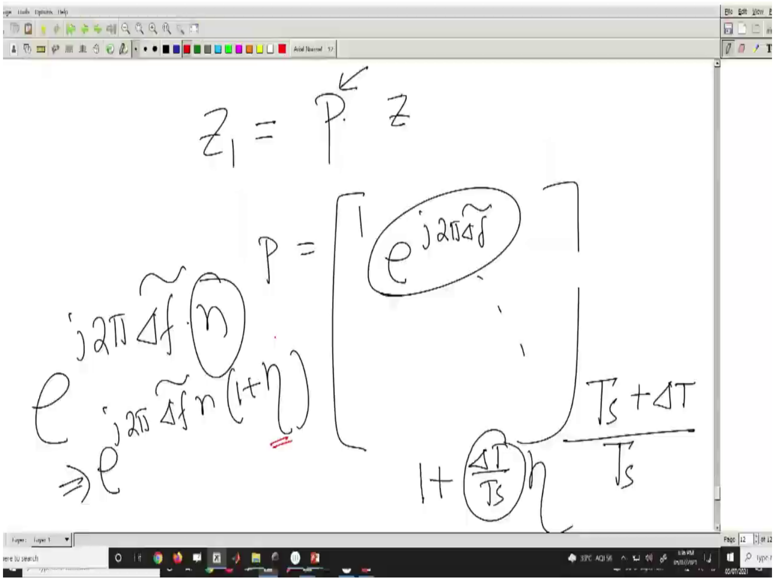

So if, so which means that in our system there will be lot of impairments ok. And what are the different impairments that we can think of; one impairment is that you have the channel impairment right channel itself is an impairment. Now, we know the channel so, we are not talking too much on the channel, but here the impairment meaning the residual effect.

So, what happens when there is a residual channel left which is the apart from your estimated one. Because if you can estimate it you are the king, if you can estimate you can always you know remove the effect of channel, but you never be able to do a proper estimation. So, there will be some residual.

So, we are talking of that kind of thing. So, mostly it will be residual, not the one which you really estimated. Along with it what are the other impairments that comes into picture is the carrier frequency offset very classic one. Again, I am only talking of residual ok, what is the other one you can have SFO Sampling Frequency Offset.

Again it is residual because I assume that you have done an estimation and you and you are trying to now. Here I am not going to really model any of these because, modelling of such parameters are well known ok. But what is my intention here is that how the impact your system probably not all of them in this particular course I can explain it, but at least some of them.

Another one which is nowadays coming up in a very big way called hardware impairments ok. So, if you have a channel impairment and there are many others. So, like for example, IQ imbalance everything is residual ok. Now, this hardware impairment is something that will be talking in details. Other impairments are very standard I mean it is very; it is not so different from a normal things that people have discussed. So, I will be discussing more on that more on that these two are ok.

But channel is so common channel impact and it is you know effect has been discussed in so many works so, probably I will skip that part and similarly IQ imbalance also. So, those carrier frequencies is not something new it is also there, but hardware impairment is something that I will talk more on that later, ok.

So, today I will be concentrating on the carrier frequency offset part and see how a residual effect is impacting the beam forming issues here ok. So, let us just get into it. So, let us get the beam forming in a MIMO system; MIMO OFDM system right. So, how we have done the OFDM system, there in a normal cache in a normal OFDM system with beam forming.

So, you have the IFFT here, you have a multiple such IFFT. So, that is your data ok then you have W BB here W B t W RF t send it through antenna multiple such antennas you receive it. Then you have W opposite effect, then you have the FFT ones ok.

And once it is done through FFT then this part if you remember if it is a MIMO system will be detecting the system from vertically right, if this is a n length FFT, so I will be detecting vertically right and then we will be getting the data. So, that is a very standard way of doing it ok.

But where is the CFO coming into picture here? So, now this is a single antenna single user system so, which means the point to point communication. So, it is a P to P communication, it is a point to point communication. So, if it is a point to point communication. So, what happens when there is a problem between the or there is a mismatch between the transmitter and the receiver RF right, and how it actually enters into our system let us understand that ok.

So, if you go back to your retracing channel model so how it was. So, you have the h right, if you can remember. So, that was some alpha or some I mean some gain kind of things.

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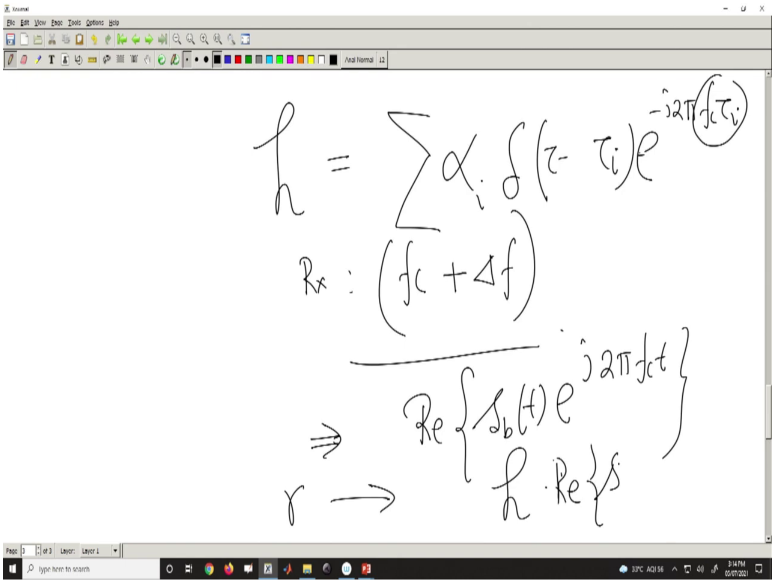

And then you have the Dirac function tau minus tau i and then there was an e to the power. So, this is a real part and if it is a base band it will be e to the power minus j 2 pi f c into you know tau i. So, that was the traditional case right. So, the problem happens when you have a mismatch at the receiver. So, which means that when I you know demodulate exactly what will happen.

So, when I demodulate it; apart from this f c there is one more term coming into picture. So, for example, f c at the receiver so when I am in the receiver I may not demodulate with exactly f c, but rather it will be some f c plus delta f ok. So, that is the problem.

So, now if I say go back to your you know basics again what was your transmitted data, your transmitted data was real s b t into e to the power j 2 pi f c t right this is how you transmit right and then there is a channel and so and so forth. Now, let us say that for our simplicity or for our calculation purpose I take the single tap channel ok. So, what is my received data, received data will be some h if it is just a simple multiplier and then this real part and all these things right, ok.

Let me just go to the other part.

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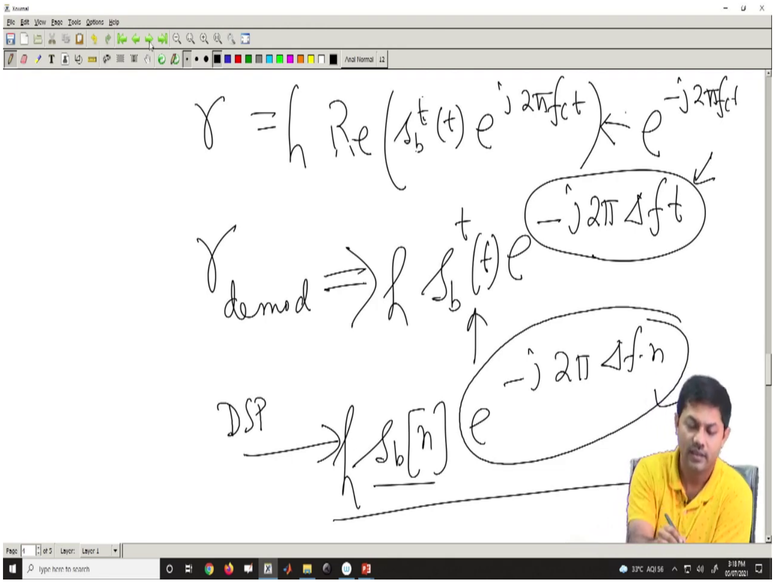

So, your received data will be something like h h into real of s b t of e to the power j 2 pi f c t right. Now, this is your received data now when you demodulate r demodulate. So, where exactly the CFO appears I am just trying to tell you. So, you multiply this part with e to the power minus j 2 pi f c t, but now instead of f c t you have to multiply f c t plus delta f c.

So, this whole thing now changes right. So, which means that in a complex system so, this would be s b t I am just saying that I demodulate enough. So, this residual part will be appearing right, otherwise I would have got s b t we are just multiplied it right. So, this is your concern.

So, which means that this part delta f part will enter into your system and it is actually a time domain part. So, now, you discretize it. So, in a very loose way of discretizing so, it will be you know h is there channel is there. So, let us say this is only channel. So, it will be s b n e to the power minus j 2 pi delta f into n; I am assuming some normalizations and so and so forth this is what your DSP wise or a ADC wise data.

So, this is you can see this is a time domain component right. So, what is the impact of such things? If impact of such thing is that.

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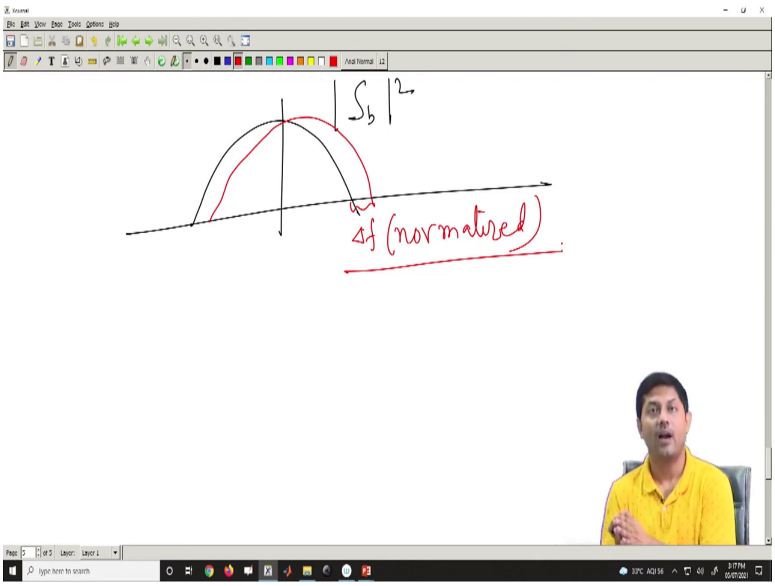

If your s b n has a spectrum this is your mod of s b right. So, what is the impact of time domain multiplication? It will be shifted, right. So, this will be simply shifted right because I am multiplying it in the time domain and you know if I multiply it in the time domain this will be shifted. So, that much is your delta f normalized with respect to the sampling and all this thing of course this is a normalized.

So, this is the impact of your CFO.

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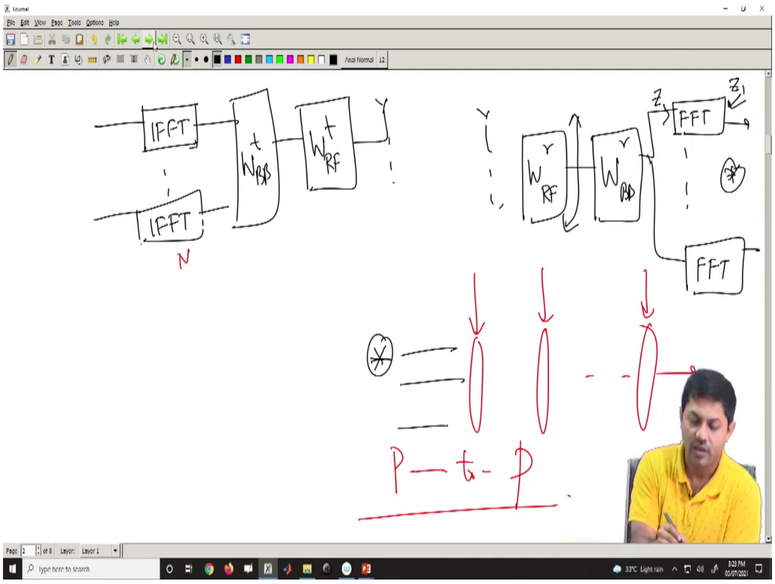

Now, how it impacts OFDM system. Now in our case we are not interested in a simple case we are interested in a more complex case. What is our case, our case is a beam forming case.

So, the point here is that how that gets impacted in our system right. So, where is the CFO coming into picture? Somewhere here, somewhere here because this is after that only you do our demodulation right, after W RF you do a demodulation. So, the point here is that how exactly this enters into my system ok.

Now when it is an FFT, if there is no beam forming or a normal case how it was impacting. It was entering you see instead of s b n it was like a one multiplication right. So, the same thing happens so which means that whatever you are supposed to receive it just multiplied by another time domain sequence right. Now, if I say this is my first block of my FFT in a beam forming right.

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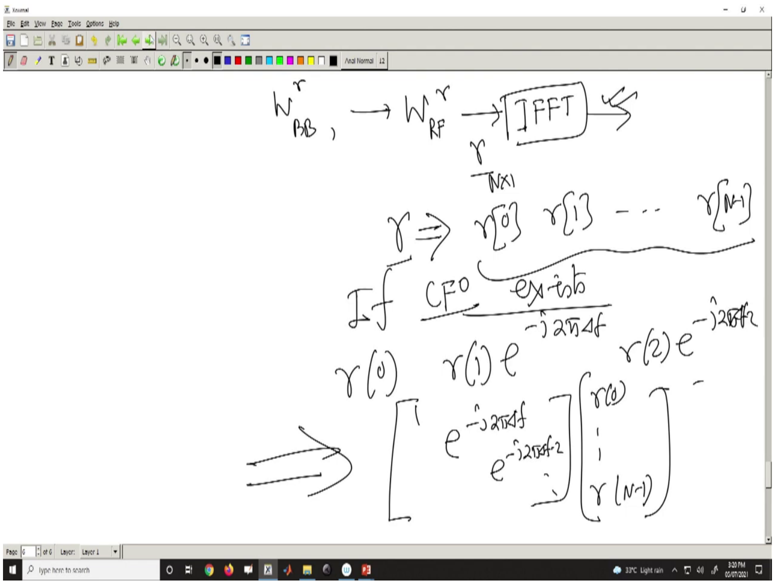

So, you have W BB r and then W BB W RF r and then you do an IFFT right of course, there is a CP removal I assume that that part is. So, silly I am not putting it here as a block diagram I assume the CP will be discarded. Now, here let us call that if there was no CP; if there was no sorry if there was no.

So here, so, let us call that r vector ok. So, what is this r vector this r vector will be an n length vector right because I am observing n time sequences. So, this is a r 1 or r 0 r 1 stigma right.

Now, if there is a CFO what will happen, what was the answer for CFO modelling. On the time domain sequence you just multiply to the power minus j 2 pi f RF 2 j 2 pi delta f into n. So, just you multiply it here. So, if CFO exists what will be the case? So, it will be r 0 r 1 e to the power minus j 2 pi, you know delta f if it is r 2 e to the power Z 2 pi delta f into 2 and so and so forth right.

So finally, I can express this whole thing as a matrix; a diagonal matrix where it is 1 e to the power minus j 2 pi f delta f e to the power minus j 2 pi delta f into 2 and so on so forth. And the traditional r what it should have been there is like for example, r 0 to r N minus 1, right.

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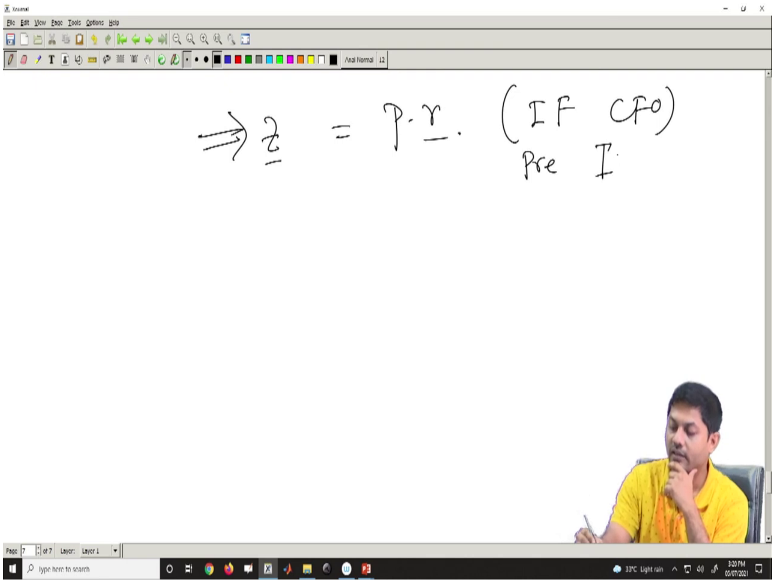

So, that mean after all these here the signal will be nothing but let us call it Z it will be some P matrix multiplied by the r vector after my you know FFT after it is a pre FFT if there is a if CFO exist if the receiver. So, this is a pre I, this is a pre FFT data it is not IFFT this is pre FFT.

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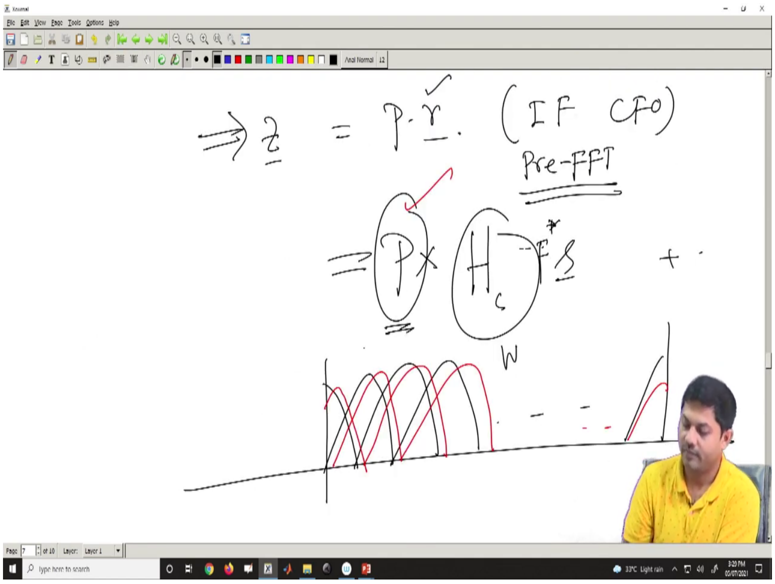

So, this is the pre FFT data. So that means, before the FFT if there exist a CFO if it was r, now what is r that you know that I do not have to explain it right. So, this will be multiplied by a channel matrix H, but this is a circular channel matrix this is a OFDM right and that multiplied by other datas will also be there and so and so forth.

Now, what does the H c contain? H c contain will it is a function of all your W BB W RF blah, blah, blah what is so ever was there everything will be just getting into that. So, that is the composite you know the circular channel that was coming plus s vector plus term. So, this is the point here.

Now we would like to have the residual effect right and what is the effect of such things. Now if it is an OFDM you know what is the effect, because it is just a band shift will happen for a OFDM the spectrum will look like this right, in normal case. But if there exist a P what will happen, this spectrum will be slightly shifted ok, just shifted. How much amount? It depends on how much the delta this is what we have seen it, right.

And how do I now this fine. So, we are not interested this because this is known fact right, we are interested to do analysis, because that is our another goal right. So, can I create an analytical model, I am not going to do an analysis because that itself is another you know another course it could be; how to do an analysis.

But can I create a framework again, that which can you know which can help you to understand a analysis better or which can help you to do analysis better of course, ok. And that is this is my exact focus of this because, this particular module is mainly the impairment and its implications ok.

So, now, implication is well understood, but what happens if there is a residual effect. We know how do I do the analysis part right, how mathematically I model it ok. So, that is what we are trying to understand here.

Now, in this particular case what is my IFFT data here this data, this is the one I am talking of right, this is the data. I am talking of this particular one. So, let us call that; let us say this is, this was the W or the Z and this is say Z 1 vector the after the FFT operation. Now, let us say how do I analyze the spectrum of Z 1 in the presence of FFT, can I do an analytical thing? Yes, and that is what the focus is.

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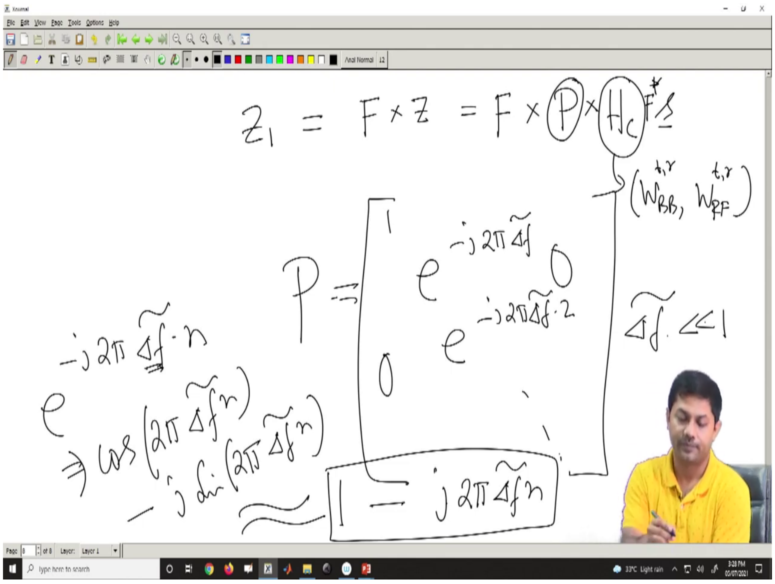

So, what is Z 1, Z 1 I just explained this is nothing but the FFT matrix multiplied by my Z and what is Z, FFT matrix multiplied by my P matrix multiplied by my circular channel matrix plus my s; s whatever s was there so that part of the s right. Rest of the things I can model it as my channel part right. Now inside it this will be a circular channel, how do you find the coefficient and all that I think is part of my MIMO OFDM things ok.

Now, this H c will be a function of all my parameter W BB and W RF, receiver and transmitter receiver and transmitter that is all as simple as that ok. So, that we have kind of understood it. Now, I am interested to understand the effect or the rather I would say the so that will be f here of course, F star because if there is a IFFT. So obviously, there will be some F star here because I have done an IFFT right ok.

So now, such kind of P will completely block your analysis part you cannot go anything beyond this point. So, can I do a simplistic approach by which I can create an equation which can help me to do some sort of an analysis? What is the analysis? For example, I would like to know in the presence of so and so residual CFO what would be the impact on its capacity.

I would like to know in the presence of so and so, residual CFO I would like to know the what is the; what is the BR or a SER or MSE of my particular receiver I would like to know. So, for that I need to create a data model, but this P if it is sitting like that it will never get it will never give you anything, right. It is a very complicated to you know come to a very closed form solution or so. So, can I create such kind of data model which makes my life simpler? Yes the answer is here.

So, look at the speed and this is the particular focus of this talk here it is not just the impairment, but it is impact from an analytical point of view. So, let us say that this is how my systems are right delta f. Now let us say this is basically the residual part these are all 0 diagonal 2 pi del f into 2 and so and so forth, right.

Now, what if this delta f is very very small, I should not say less than that it is small or rather I would say very very small quantity what will happen. It is the residual what does it mean by residual. I assume that I have done an estimation of CFO with this model, but still there is something left and would like to do an analysis and I just live at a point where you will be comfortable in doing an analysis. That is the intention here right.

So, how see if it is a residual, so it can be assumed to be very very small. So, what will happen? So, let us say. So, e to the power, so it is in the form of exponential right. So, some j 2 pi del f say n and del f is very small. So, it will be cos 2 pi j sin 2 pi del f n right. Now, if delta is very very small, so what can I approximate this function to.

It will be 1 minus j sin is sin theta if it is small it will be almost a theta. So, this will be j 2 pi del f n. Now this is a very beautiful approximation which means that this P now you think of this P. So, how each and every element if I just replace it by 1 minus this quantity how would the P look like?

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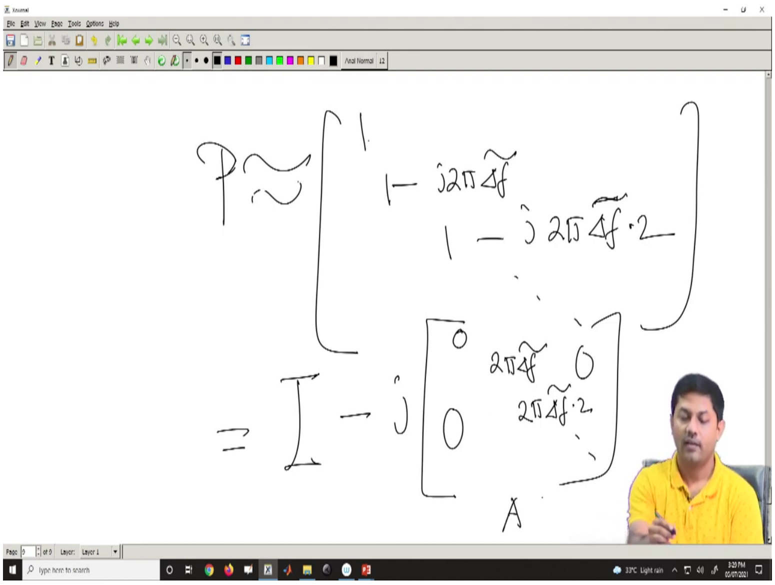

The P will look like the first element will be something like that right 1 then 1 minus j 2 pi del f residual then 1 minus j 2 pi del f residual multiplied by 2 and so and so forth. So, what can I say? This will be I right minus j, another matrix what is the matrix; 0 first element, next element is 2 pi delta f, next element is 2 pi del f into 2 and so and so forth rest is 0. Let us call it A matrix and I know what is the A. So, how can I you know; how can I create my data model again?

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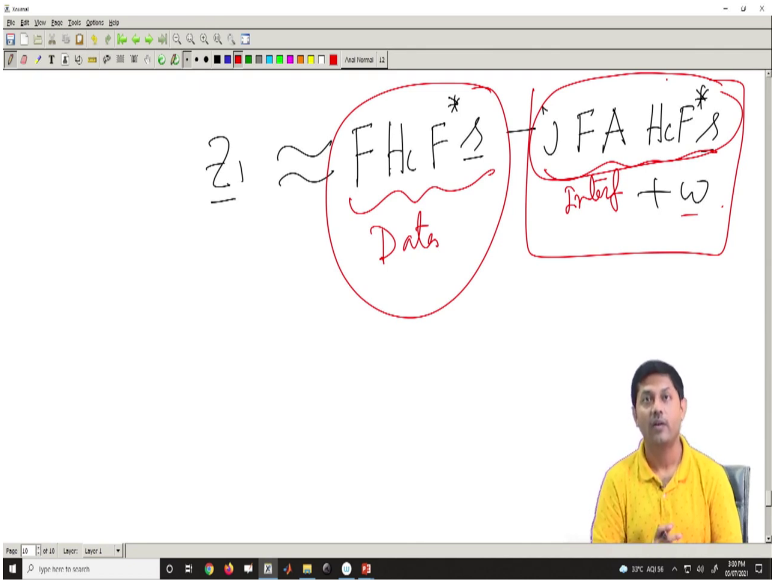

So, that mean my Z 1 vector can be approximated to F because it is a P now I can replace the P right, this P this P I plus something right. So, this will be F H c F star into s vector minus j F and that particular a matrix let there be whatever s vector plus w.

Now, look at the beautiful part of it. Beautiful part of is that I separated that as a data part, I separated as an interference part is an interference part or sometimes people call it ICL inter carrier interference right. So, whatever there is an interference part and this is the noise part right.

Now, what can I say; this is my data which is a pure matrix form, this is my interference with a pure matrix form and there is a noise. So, can I find say SNR of this data, yes you can. Can I find the SER or symbol error rate of the system? Very much, because now this whole thing I can model it as one unit of you know data or a noise part, another part is the data and there are classic analysis for doing that. So, this is what my interest is.

So, I would like to show you that when you go for analysis this can really go for a, this can really help you for doing lot of analysis because I have just broken down into a simpler system. But now this is a beam forming part right. You do not separate that from beam forming or an or a OFDM system, because it is a OFDM system the effect of normal OFDM will also be there. So, whatever is there it is also be there. But this is kind of you know modification will make your life similar ok. So, this is the one impairment that comes into picture here, ok, got it.

So, now another impairment probably I will just touch upon, but hardware impairment will be discussed later how the hardware impairment comes into picture.

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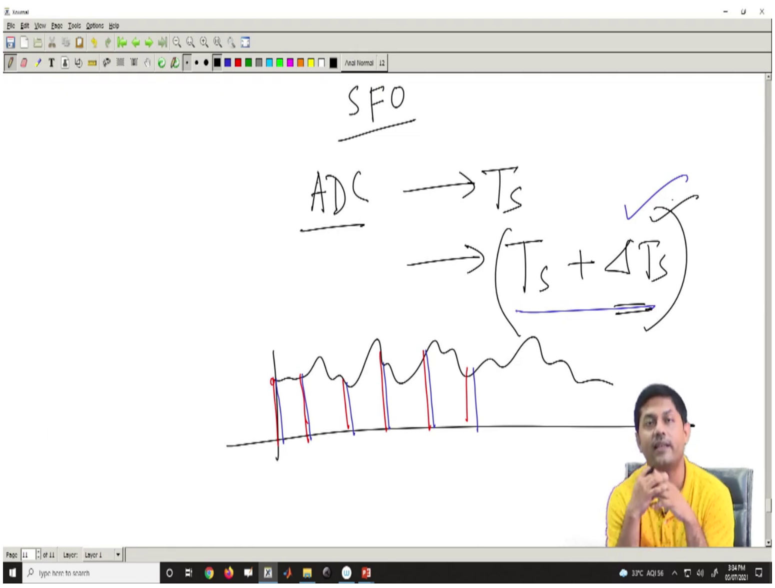

I would like to have the SFO impairment sample of Sampling Frequency Offset. So, how exactly it impacts your system and in the beam forming context, how exactly again I would like to you know model the part there ok. Now what is your sampling frequency offset ok? Let us try to understand that.

Sampling frequency offset is the same that in a sense that in your ADC your sampling time is always T s right [FL]. But you will never be doing sampling at T s right because your clock frequency may not be exactly matching right. So, you will be doing it at T s plus delta T s right.

So, what is the impact of this delta T s? Either you will be over sampling it or you will be doing an under sampling it right. So, for example, if you have a data like that and you are supposed to sample exactly at this point, this point, this point, this point and so and so forth equally spaced T s. Instead of that what you will be doing is probably will be sampling slightly off, slightly off, slightly off, slightly off and so and so forth right ok.

Now, you may say that: hey, what is the problem, so that the problem here is that this T s plus delta T s can be larger or can be smaller right. If it is larger what will happen. If it is larger it means that if you are supposed to get say for long time say 1000 data and your T s is larger you may not get 1000 data, you may get 900 say 90 data, because your T s is larger right.

So, that is called a under sampling. So, you have missed a data. Or instead of 1000 if your T s is smaller than your normal T s your actual T s instead of 1000 you make a 1005 samples right both are dangerous. So, neither you should get 1005 sample nor you should get 909 samples right.

Because you know you are supposed to get only 1000 data, otherwise you cannot do a detection because your number of data that you are sending is maybe 1000 for example. So, both the cases are dangerous. So, this has to be compensated right.

So, where it gets reflected ok? Now, it is not so easy to model it ok. So, usually what happens, there are classic way of I mean estimation is ok, I am not getting into the estimation. How do you get rid of that problem? So, first you have to estimate it right and you have to first know; what is the data model that is coming into picture right.

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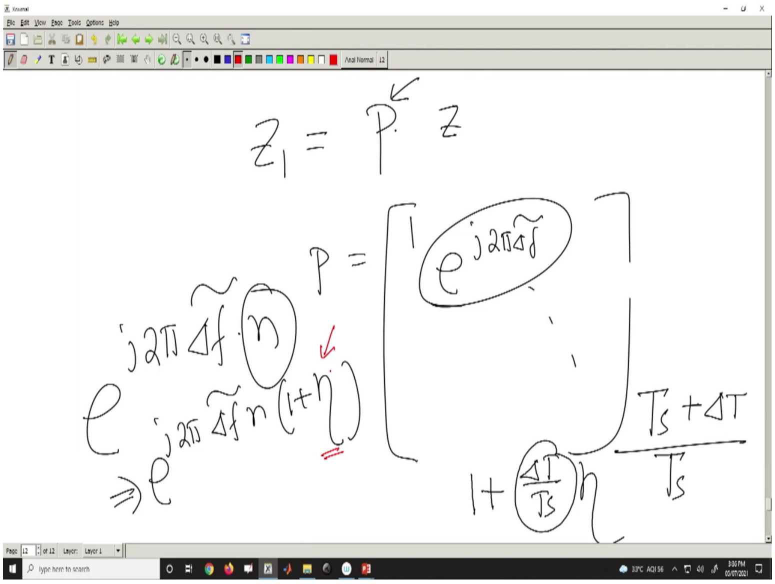

In the first data model that is coming into picture is that that Z 1, whatever we have written right that P into you know your Z it pops up very well in P right. Because if you look at P what is the P, P is the it contains all your sampling points OFDM part sorry CFO part right should mean 1 e to the power you know j 2 pi del f tilde and so and so forth right this is what it is.

So, what happens is that. So, which means that I can say this components are nothing but e to the power j 2 pi del f into n that n-th part right. Now, n is what n is your sampling time assuming there is a T. Now if you are sampling say n, sorry you are sampling at T s plus delta T this is your actual sampling time right. So, how do you do a normalization?

So, you usually divide it by T s and you say 1 if it is actually T s it will be exactly 1 into n or it will be some delta T by T s. So, let us say this I call it eta. So, here this whole thing wherever there is a you know n it will be n plus n into 1 plus this eta factor will be coming into picture, this factor. So, this factor represents that you have a you know SFO problem ok.

Now, this is the only one part of the story, it does not complete the complete part of the story. Complete part of the story is that even if you estimate it you can only you know you only know the this particular variable, but you have to readjust yours you have to readjust your samples because you have done a wrong sampling.

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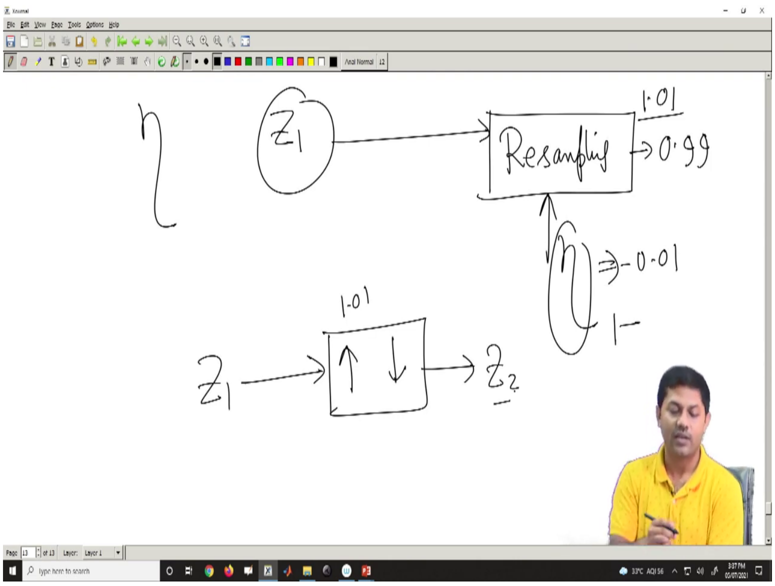

So, usually the process is the following, the process is you estimate your you know you estimate your this factor whatever this factor is, and then you get your Z 1 then you do a resampling part here, is called a fractional sampling or resampling; by the factor by this factor.

See if this factor is say 0.01, so basically what I am trying to do is that I am resampling it by a factor of 1.01, something like that, if this zeta value is 1. So, I am resampling it. If the zeta value is minus 0.1, so I need to I need to do a sampling by 0.99.

So, which means that this Z 1 has to do either up sample by a factor of say 1 0 1 in case this value is so and so or a down sample depending on whether it is a positive value or negative value. And then you have to get a new Z 2. This is the way the SF is calculated. So in summary, so, when you go for such kind of scenarios. So, basically you have to readjust your sampling then only you can you know remove the effect of your SFO otherwise you really cannot do anything ok.

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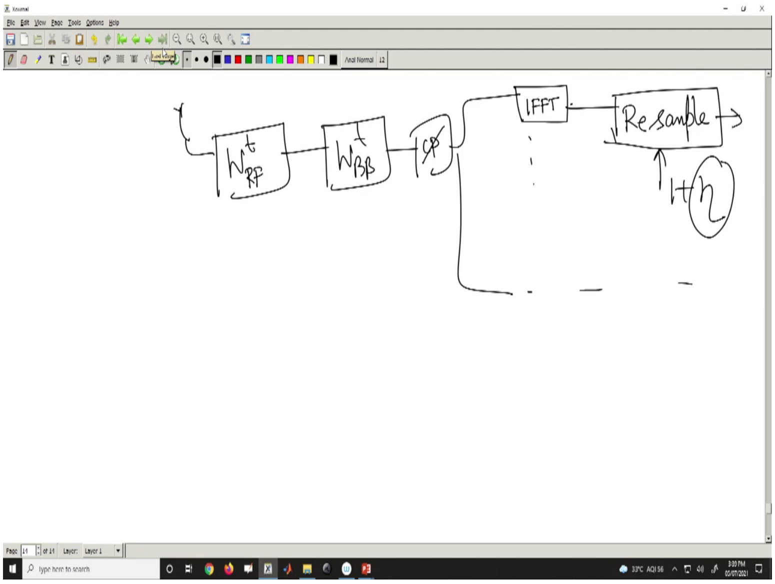

So, which means that after all your I am only writing the receiver part this is your W you know RF t part this is your W BB t part then you do a CP cancellation, then you do you know IFFT ok. And if there are multiple such things you can have multiple IFFT and then finally, you have to do a resampling here, resampling it. Let me just resample by the factor 1 plus zeta.

Now, this you have to; now this is b for each and every stream and then you do that. So, this is how SFO can be tackled or SFO can be managed there ok. And, but this part has to be estimated ok. How do you estimate it? Again it is a model, you can go to this model, this data model where this is popping up right this factor is popping out in your P.

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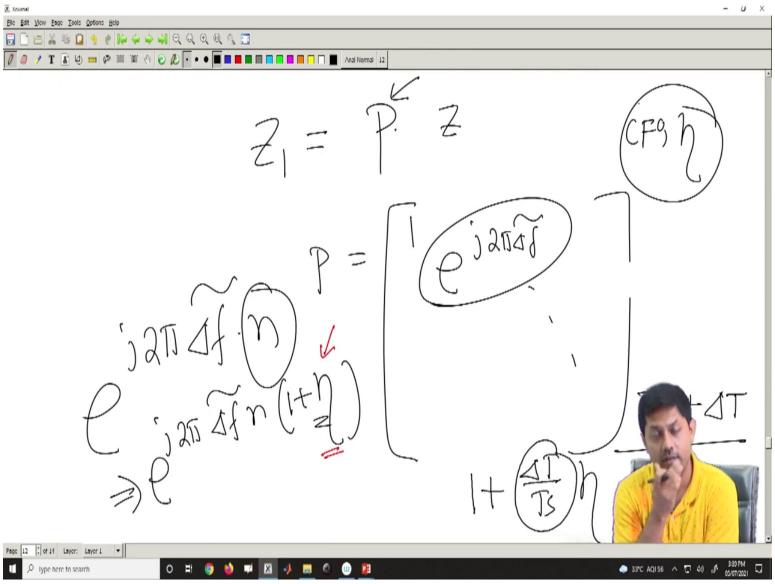

So, you do a CFO as well as this joint estimation ok. So, I am not getting into the exact estimation theory that takes another course itself, but I am just only giving this. If you can estimate it you can really correct the issue here ok.

So, with this I stop it the SFO part also. So, today we covered only CFO and SFO, next consecutive classes probably more and more impairments will be covered and their impacts will be looked into it ok, so.

Thank you.