Signal Processing for mmWave Communication for 5G and Beyond

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Module - 12
Parameter estimation and impairment
Lecture - 60
MU System

Welcome, welcome to Signal Processing for millimeter Wave Communication for 5G and beyond. So, today we will be talking about the module 12's lecture number 60 that is the Multi - User System or the MU system. So, in the MU system how beam forming can be used there and what are the you know consequences or what are the design challenges that will come into picture when you go for a MU system or a multi user system.

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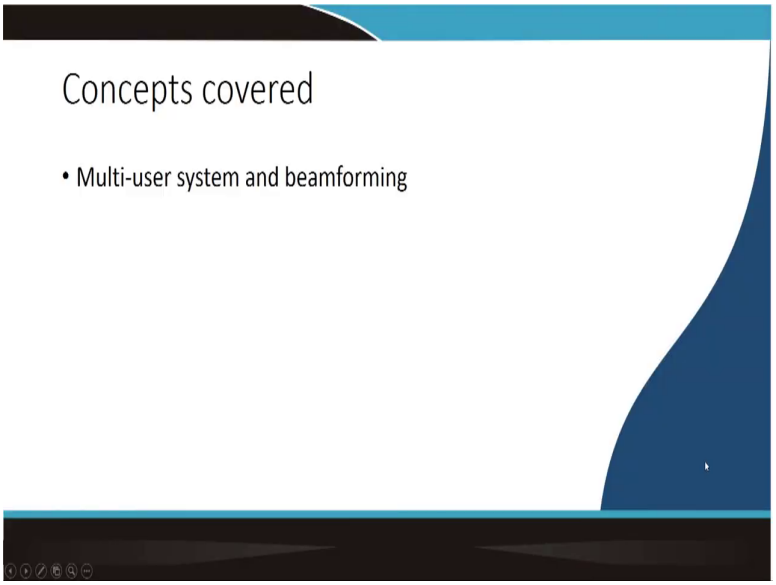

And the stuff which we will be covering is mainly the multi user system and beam forming. So, we will continue the discussion from our earlier classes where we have the single antenna case not single antenna, single user case meaning you have only one user one base station and one user it is like a point to point communication, but now we will be graduating to multiple user cases.

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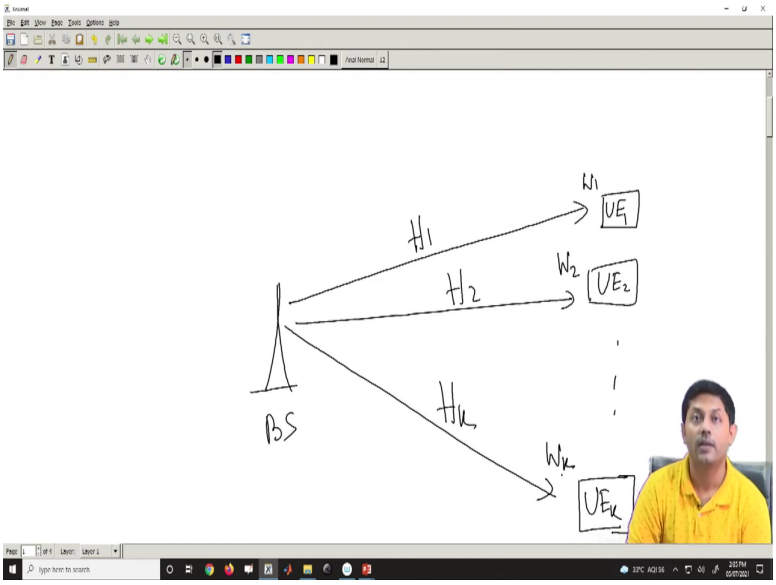

So, let me just define how system diagram could be. So, you have the say base station here you have the base station here, but what you have is you have multiple users here ok. So, you have user 1, user 2, probably I may have discussed it, but for the benefit of this stock we can.

So, let us I have let us say I have UE k number of users here. Now, in a normal MIMO use multi user case what will happen when there is no beam forming, I mean there is no

difference between that multi user MIMO and the beam forming except the fact that the beam forming will be entering that is the only part.

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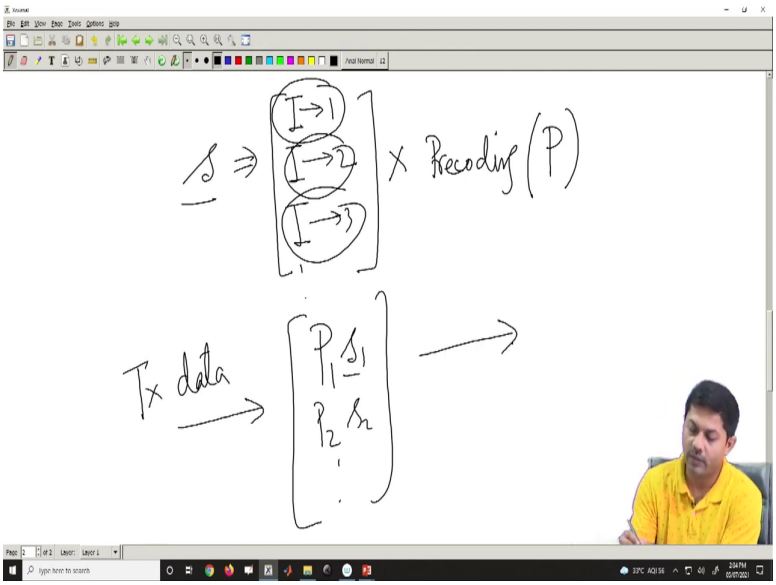

But if you have a multi user MIMO then you what you usually do usually in a mathematical sense you have a stream of data. Say let us say you have this s bar which can be a stream of data and that within the s bar you can say some data may be for user 1, some data for user 2, some data for user 3 and so on so forth. So, you have some. So, this block probably for the 1st user, this block probably for the 2nd user, 3rd users and so on.

Now, this particular data what you do usually we do a some sort of a pre coding right. So, you do a pre coding over that, what is the purpose of pre coding. The purpose of pre coding and it is mandatory ok, because unless you do a pre cording you really cannot do a multi user

MIMO very well. So, what you do there is that you have the precoder say I do some precoder design say P so, probably and that there will be multiple such pre coders right.

So, every individual data say let us this data, this data, this data can go through individual precoder and it will be something like a common. So, there will be P 1 say let us say s 1 blocks and then there is a P 2 s 2 block and so and so forth. So, this whole thing would be your you know the data that you would like to transfer. So, this is your T x data this is a typical way of going through it.

Now, when you receive it what will happen you have a multiple users you have this multiple users here and each and every user say I have a different channel. So, this has a H 1 channel, this has a say H 2 channel, this has a channel number here I am only talking in terms of the non you know non beam forming case, but the same concept can be you know expanded to the beam forming case as well.

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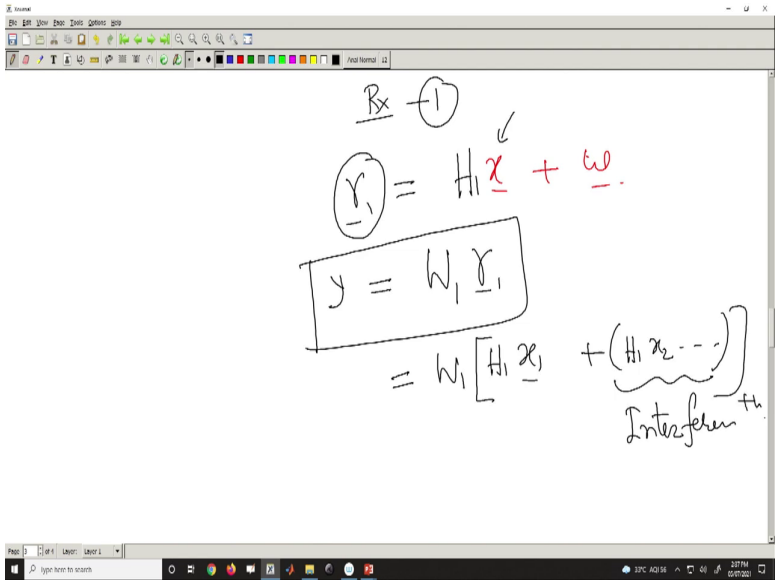

So, if you have multiple such channels what you usually do is that at the receiver. So, when you go to the receiver. So, let us say I am for the user 1. So, what it will receive, it will receive some vector r, which will be H 1 channel, but it will receive the complete data right it will receive the complete data whatever is explained here this whole data set will be you know will be there.

So, let us call that as r or maybe say probably I call it x data. So, probably I will just say this is your x vector ok. So, this will be plus some noise will be coming there. Now, what will happen now you do some sort of a equalizer I mean usually that is the choice here. So, which means that once you receive it each and every user will be going through a individual equalizer.

So, which means at the user level you will have a equalizer W 1 W 2 similarly you have the equalizer W k. So, what is the purpose of this? The purpose is that once you receive it then you just send an equalizer use I mean you can pass it through an equalizer. So, what will happen? So, let us call it y. So, y would be some equalizer W 1 into r. So, that is typically the case. So, let us call it r 1 vector, but does it complete everything.

So, point here is that what is so special about the multi user, multi user is that this particular r will be having an x which will contain the data meant for user 1 as well as the data meant for other users and that is a trouble ok. So, which means that this particular W 1 will be containing probably H 1 with the I mean with the what I should say the intended data. So, let us call it x 1 part plus it will have H 1 x 2 other data part also right and this is your interference plus the noise, ok.

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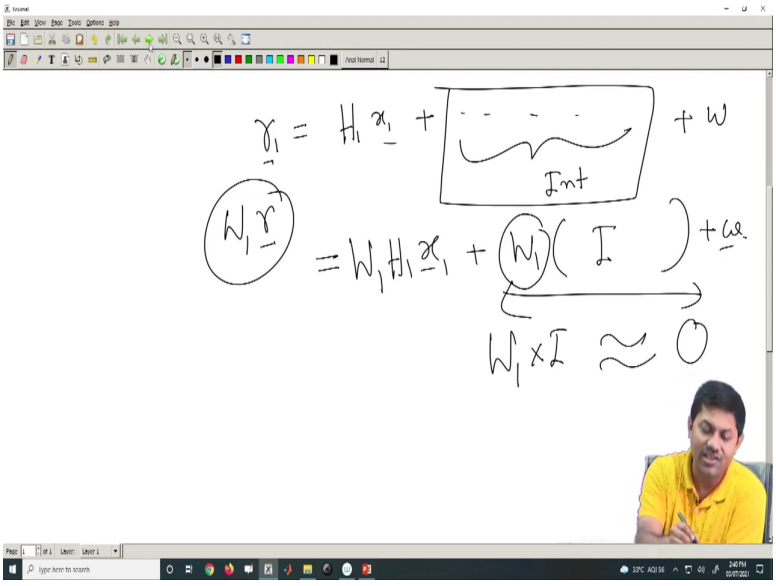

So, this r 1 whatever r 1 I just explained that will contain the intended data H 1 x 1 plus it will have the interference data plus w and this is the problem of every multi user system. So, it is meant for one data, but it will other user data will also appear ok, that mean your W when I multiply it should be such that it nullifies this complete interference it should be logically right, because that is the purpose of my equalizer ok.

Now, this W 1 whatever this interference is will be you know I am sorry this would be w noise part. So, this whole part should be nullified and that is one of the worst part ok. Now, how do you do that there are theories and all, that mean this W 1 has to be you know null it has to reach in the it has to be in the null space of the interference and so and so forth; that means, you can say this W 1 or this equalizer multiplied by this side this should be some sort of a null matrix or a 0 matrix.

Then only this interference can be now there are many theories to do that I am not getting into that ok. But this is the fundamental part of every multi user communication. So, this is the difference between a normal MIMO and a multi user MIMO system, where you know where specifically a part of it a part of the data will also be from the other users.

Now, let us go going into let us go back to my beam forming case what happens to the beam forming. Now, beam forming has an advantage ok in the beam forming the extra part that will be coming is the in the multi user beam forming the extra part that will be coming is the addition of beam forming part ok.

Now, beam forming inherently it has a certain advantage for example, if you know I think we may have discussed the MIMO case where if you have a multiple such you know RF you can create that many number of beams right. So, which means that if my system is capable of a MIMO system I can create multiple beams right.

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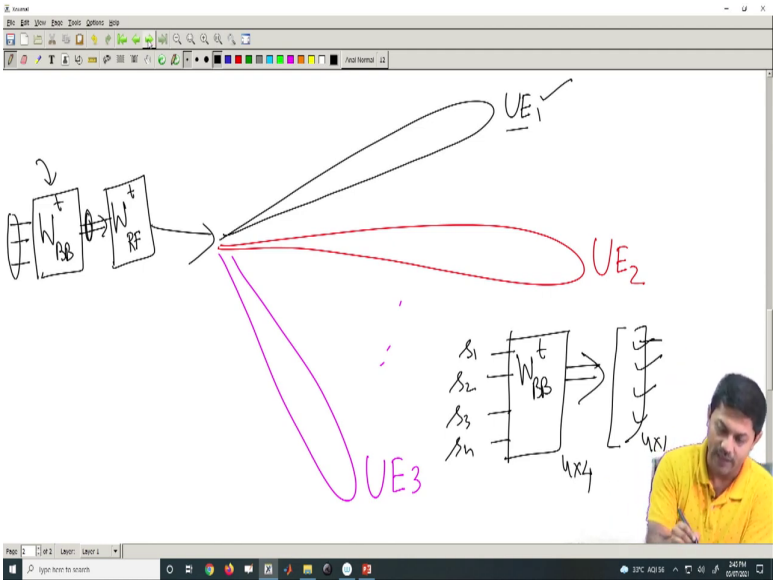

So, probably this is one beam, this is another beam, this could be another beams and so and so forth right. So, this is my transmitter this would be my user 1, this would be my user 2 and this would be my user 3 ok and so and so forth as well. So, when it is a beam for multi user beam I have the liberty to send data to individual user, but there is a catch here the catch here is that, the system has a problem, what is the problem?

Problem is that if you think of such scenarios it does not really you know does not really I mean you cannot get rid of the data meant for u 2 and u 3. For example, ultimately if you notice how beam forming is done. So, you have the you have the data here stream of data right, then it will go through W baseband transmitter, then it will go through W RF a transmitter and it will just piggyback it right.

And based on if I have a multiple such you know multiple RF chains I can create as many that many number of beams. Now, the point here is that because the data whichever you are seeing it goes through W BB. So, it means that if say I have say 10 users and I have 10 data here, but once it goes to the W BB, W BB is a matrix right if it goes through the W BB the output here it contains the data from every users right.

So, for example, I have 3 users or rather 4 users. So, I have four data stream let us say I am taking say one data stream meant for one user right. So, this is my x 1 this is my x 2 this is or rather I will put a different notation s 1, this is my s 2, this is my s 3, this is my s 4, but once it goes through W BB that is a matrix.

So, let us say it is 4 cross 4 simple ok. So, the output is a vector, but now do you think the first element of that 4 cross 1 vector contains only data corresponding to UE 1? No, right, because it is a matrix. If it is a matrix the first element will be a function of linear combination of s 1 to s 4 second will also be s 1 to s 4 and so and so forth right.

So, which means that the data stream now contains data from each and every user ok now you do beam forming after that it does not really matter. Now, you can do beam forming after that and you can create say force you know split of the data that is all fine. But bottom of the story is that each and every user will be having data for all the other users. So, beam forming only you know it helps to localize my power, but as far as the data splitting is concerned it is really not helping.

So, we are coming back. So, this is also called a multi user you know hybrid beam forming system. So, where I am serving multiple users at the same time, but the data that I am you know pumping it is once it goes through W BB it just mixed up. So, now, I have the same problem, which I just discussed which means that once I am at a particular user I also have to the I mean I get the interference from all of them.

Now, given that how do I design my beam forming parameters; such that it minimizes my interferences because that is the ultimate goal of every multi user system ok. Now, let us that

is I am just giving you a basic framework how it can be done, but exact solution probably it may take long time to even find it, because that is like a one big pay person it explains a lot. I am just giving you the optimization framework rather which will actually tell you how to you know how to do the beam forming; such that this kind of multi user scenarios can be tackled.

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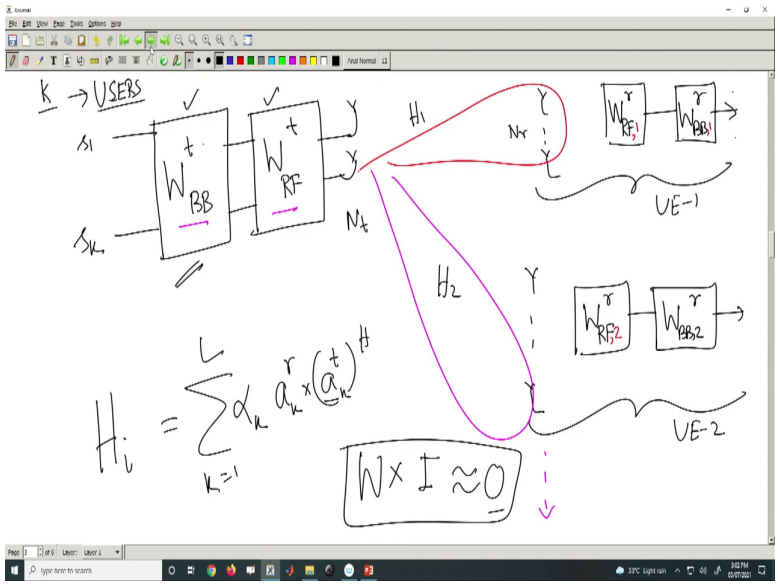

So, let us do that ok. So, let us just generate the system model for it and the system model can be say for example, I have k number of users let me just generalize it. So, that it will be easy for you to. So, k number of users. So, let us say I have data stream here and I have a data stream s k right, now this I call it W BB W BB t ok. Now let us see how I can do the same problem, how I can solve the same problem here W RF.

But, now you have every user. So, let us say for the first user. So, how now there will be antennas here right. So, it can be N t number of antenna and let us say I have every user and

every user has N r number of antenna it can be heterogeneous also like N r 1 and N r 2 and so on so, but just for our simplicity let us assume all the users are having the same number of antennas. So, what is the first thing so, like each and every users will be having the same set of blocks.

So, you will also have a W RF, but it is at the receiver side then you will have you know W BB at the receiver side and so forth right. This is how for a single user case now, this is one users. Similarly, I will have here also the same thing W RF r, now let me put a different slightly different notation here let us call it comma 1, just to indicate that because for every users that W RF and W BB has to be different right. So, you will have a different, different W BB and W RF. So, this is BB 2 r and so on. So, this is for user 2 and so on and so forth right.

So; that means, the same data probably I can create a beam here, I may create a beam here, but it is only for channelizing my energy, but the actual inherent problem of multi user is not really going away. So, it is still there. So, that mean even in you channelize the beam the data's for user 2 will also appear in user 1. So, the problem remains the same I mean it is the same as your normal multi user system problems ok. So, now, how do you optimize such kind of things, ok?

Now, you have a multiple such you know k number of such users that is coming into picture. So, let us just create a formulation because I am only interested to know you know the formulation that can you know that can solve my problems ok. So, so let us say the actual channel matrix whatever I will have it let us say this is my channel matrix H 1 now, this is my channel matrix H 2 and so on so forth.

Now, what is this channel matrix, this channel matrix is nothing but your summation of you know I mean that is the same as what we have discussed earlier alpha by this let it be k just for your notational simplicity, then your you know array manifold vector of receiver and all these things right multiplied by I think I have discussed enough about that. So, those things some k is there, some k is there because there are k number of paths can be there in individual.

So, that is the that part is there I mean. So, that is the same chance I mean every individual beam will have their same channel model as we have just discussed so far for the millimeter wave, but now so that part is clear. So, now, how do I modulate. So, let us say whenever you are given such kind of problem I think we have discussed it that you have to first come up with a cost function.

So, you first so, what are the things that you need to find out, you need to find out W BB, you need to find out W RF, at the same time every individual user for every user you have to have the W RF and W BB. So, there is so many parameters you finally, have to find out right.

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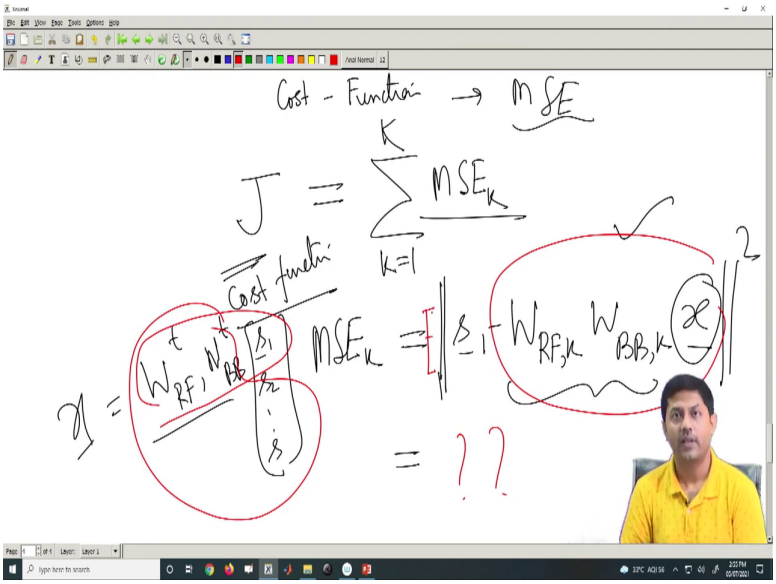

So, let us formulate the problem because I am only interested in formulating the problem not exactly the solving the problem, because solving the problem is a very standard methods is a simple you know optimization methods, but formulating the problem is this. So, now, when

you are given such kind of issues you first formulate a optimization problem. So, what is the optimization of, what basis on what is the cost function on what basis this parameters can be found out.

So, let us say my cost function for this case and you have hundreds of such cost function is the standard MSE, usually we love MSE because that is a that is a kind of a two norm and it is very convenient to solve first ok, because moment you have a norm two norm you can do lot of things like you can do lot of matrix manipulation simple matrix manipulation right. So, that is why norm is a very two norm is a very simple way to go for every optimization problem.

So, we take MSE as our cost function. So, let us say how do I then solve it. So, now, for that mean I would like to have all this system all this parameter to be estimated in such a way the overall MSE of every individual you know individual users can be minimized. Now, if I just I want to minimize for user 1 and solve this problem, which is very good, but the only problem is that you get a W BB and W RF which is only meant for user 1, but that is not the case right, because you have a common W BB and common W RF.

So, now, then it has to be some sort of a joint optimization. So, now, I make the statement slightly changeable I said that I would like to have a system in such a way that the MSE at every individual user the summation or the total MSE if I call it that should be minimized for all these parameters. So, instead of just considering MSE for a particular user I call it I would like to minimize the complete MSE of my all user and that I call it J now this is my cost function ok.

Now, let us find out what is the MSE k, I think if you notice if you have gone through your earlier notes and all it is very simple to find out the MSE for individual right. So, what is the MSE? MSE for the kth user, what should it be case the case, now look it is the W RF and W BB, which is coming into picture, but at the same time at the same time it will be the interference from the other data as well right.

So, let us keeping that in mind so, which means that it is something to do with W RF k then W BB it is not k. So, this should be slightly changed k W BB k. So, that is your common you know W then the data, which is meant for you know meant for that particular user let us call it say x 1 bar now mod of this should be minimized ok.

Now, instead of x 1 bar I can just generalize it slightly because I may not get exactly x 1 bar I we will get the whole x first and then I am trying to minimize it. Now, this is for my this is my this is not the MSE by the way this would be slightly modified, it will be x 1 vector this is the case right x 1 is the vector meant for the user 1 and that is only. So, this is your complete MSE formula right now you can get it right, I mean it is not so tough to get it, now you know what is x right.

So, x vector will have all your W RF t W BB t and it has the x 1 vector, x 2 vector, all these things right. So, that has that instead of x 1 I should say it is s 1 sorry because that is the x s 2 vector dot dot dot s n vector. So, I will make a slight change here. So, this should be s because that is the original data ok.

So, which means that from here you will have two component, one part corresponds to the this part another part corresponds to this part, which is the interference part, but that is ok, I mean I am not worried about it that, but whatever comes something will be coming I mean you know right I mean how to solve this problem. Now, if I take an expectation take an expectation over and above you know how to solve that problem, not solve you know how what is the equation of it right I mean it is not.

So, straight forward it is not so difficult to get it, it is straight forward ok, Because we in our earlier classes I have done it for a normal MIMO system how to get an MSE formula for a normal you know vector and a MIMO system. So, probably this could be one of the assignment or an exercise from your side what will be the exact formula for it, you just take it and you can find it out. So, I am not getting into the exact final formula of it, but let me just try to you know try to get into that.

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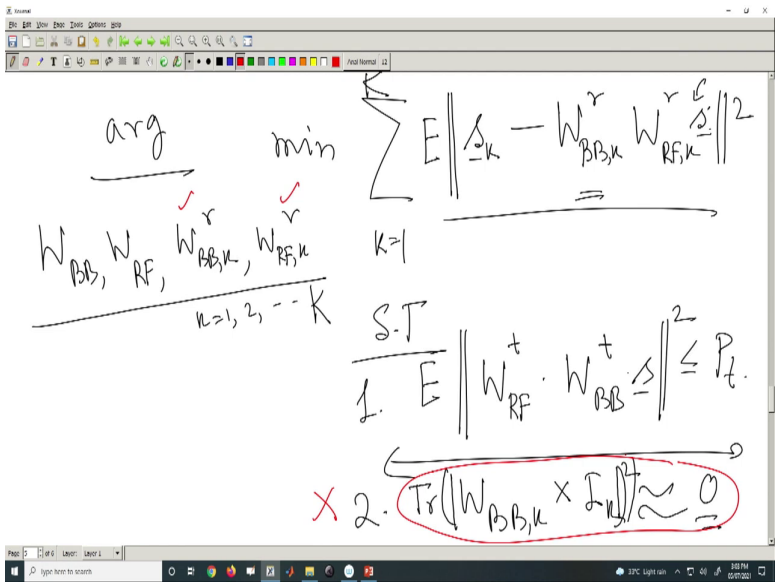

So, finally, what I will get into that is that, I would like to minimize this quantity from k small k equal to 1 to capital K ok. This quantity as such expectation I leave the exact equation for you to solve s k vector minus W RF k W slight mistakes W BB k W RF k x bar mod square probably I will correct the same problem here sometimes this is BB this is RF sorry. So, this would be BB, this would be RF anyway so, as long as you understand what happened it is.

Now, this is your cost function, now you need to minimize this whole summation. So, minimize this whole summation over who are the argument, the argument if you check it; obviously, the argument will be W BB W RF, where it will enter, it will enter here in the x vector right, then you have all the individual you know W BB and W RF and for k is equal to small k equal to capital K. So, you can see this, but this is not enough right because you have to put some constraint. So, let us put some constraint what are the constraint.

So, first constrain is that I would like to put a constraint on my I know transmit power. So, who are the transmit powers ok fine. So, you know what is that how to calculate the transmit power right. So, this would this is the transmit power W BB t into your s vector this should be the s vector yeah. So, mod of that you can put an expectation I do not mind this should be some you can put some power constraint you can put some more constraint ok that is all fine that is ok and apart from that this is the first constraint.

Apart from that you can have some other constraint as well, but usually you cannot solve this problem. So, easily ok though I say that it is a matrix and all actually it is a matrix problem you see this expectation and all these things will be coming as a matrix format right, but you cannot solve it just like that I think in the single case I have shown you how to solve this problem with some sort of an approximation ideas who are the approximate ideas.

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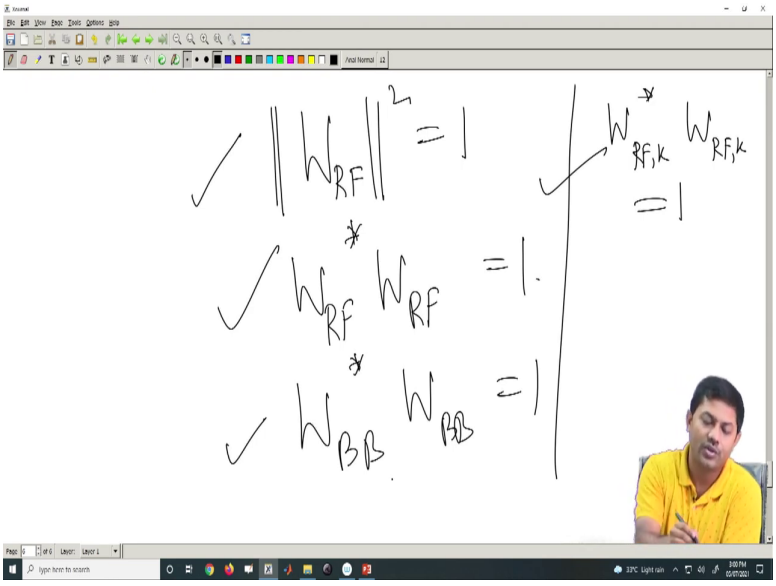

One of the approximate idea was that this one should be 1 right. So, which means that W RF star W RF should be 1. So, are ok. Similarly, big if this is there you can also extend it to individual user as well, though this is not a constraint, but this is some sort of a you know you can make such loose assumption because this problem is tough to solve.

If somebody gives you this problem this is the cost function and this is the only criteria and you solve it this is not a very easy way to solve it ok. So, to solve it what you do you put some additional restriction which can make your life simpler ok. And these are the additional restriction put some additional restriction then we will see that it is very easy to solve it ok. I am not going into the exact solution because solutions are dry you can always look at the paper and see what solution they have taken it, but this is the more or less the approach where even for a single case or a multi user case people have solved it and this is also the way to solve it ok.

So, in summary so, how do you solve a multi user you know multi user scenario is that the. So, what is the ultimate goal, goal is to ensure that your MSE is minimized right finally, and when you try to minimize my try to minimize your MSE two things happen you minimize the MSE at the same time you have to minimize your interference because that was the crack of the problem for multi user system.

Why? Because every user every user will be receiving his data as well as some other users data because of this problem, because of this W BB where it mixes all the data and send it to everybody ok. So, that problem inherent, that is the multi user problem. So, unlike in a multi user system where I specifically design my W BB such that you know it minimizes the interference part, here a slightly different approach could have been taken because here W BB is a common for all of them. So, what I do here is that we in general try to minimize the MSE.

Now, what is MSE Mean Square Error right, now if there is a if you say I want to minimize MSE a natural tendency of the optimization formula is that will try to minimize the interference as well not guaranteed, but it will try to minimize it. So, here I do not put an additional restriction saying that this much interference I would like to for example, in the

multi user MIMO you know normal MIMO case what we what extra additional restriction we put.

We said that your equalizer should be such that equalizer multiplied by your you know interference should be almost a 0 matrix that was a restriction in a normal traditional multi user case, it is a very tough to solve such kind of problem, but people do that. But here I just slightly deviate it, but nothing stops you even from that kind of restriction also. For example, here you can put the restriction, here you can put one more additional restriction here that W BB k multiplied by calculate the interference from here interference on the I kth user should also be some sort of a 0 matrix.

Or you can say the trace of this mod square should be a 0 you can put such constraint, but the point is that it will be very complicated to come up with, because the reason is that you have too many parameters to solve right, [FL] you have each and f for each and every user you have 2, 2 matrixes you have to solve. So, when you have too many parameters to come you really do not need this I mean you if you do it is there it is good, but you can still live without it ok.

So, to counter the problem is that you just in general say I want to minimize my MSE. So, minimize MSE inherently ensures that this part has to be as minimal as possible or the interference part ok. Now this is a good way of you know formulating the problem and I just leave it here because formulation of a formal problem is more important that exactly solving it in an optimization framework, this is the classic way of representing a system ok.

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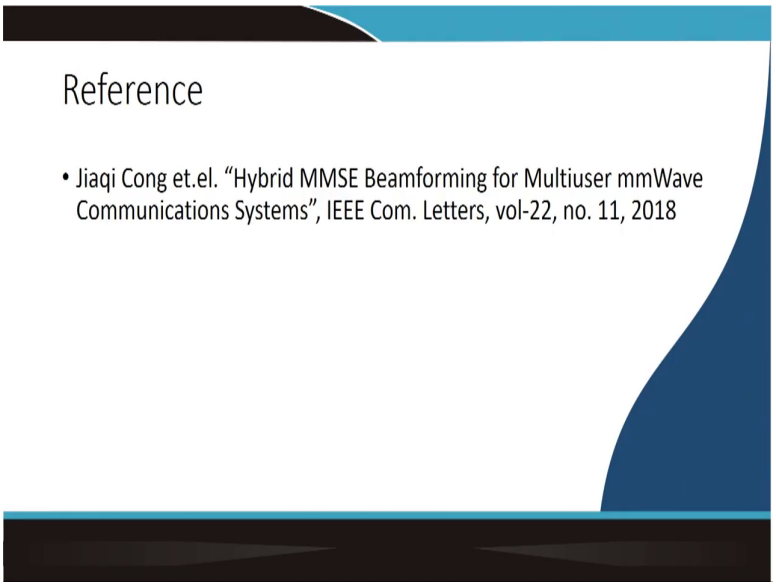

So, probably I will go back to my system here, and I will tell you what are the things that, we have covered today. It and this is a very classic reference and the beautiful part of this reference is that this is the one which talks about the MSE based multi user system. There are lot of works where people take a different cost function and it is a very complicated one ok this is a very very simple cosmic later. So, 5 days later please go through it and you will see or you can understand very simplistic part of the multi user beam forming using a MSE.

Thank you.