Signal Processing for mmWave Communication for 5G and Beyond
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Module - 11
Design parameter estimation
Lecture - 59
Design parameter estimation (part-5)

Welcome to Signal Processing for millimetre Wave Communication for 5 G and Beyond. So, today we will be talking about the Design parameter estimation part 5 ok. And as we talked about it last class, it was mainly the LMMSE part of the cost function description ok.

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And naturally, this should be the things that will be covering again ok. So, this is where we left it in the last class right.

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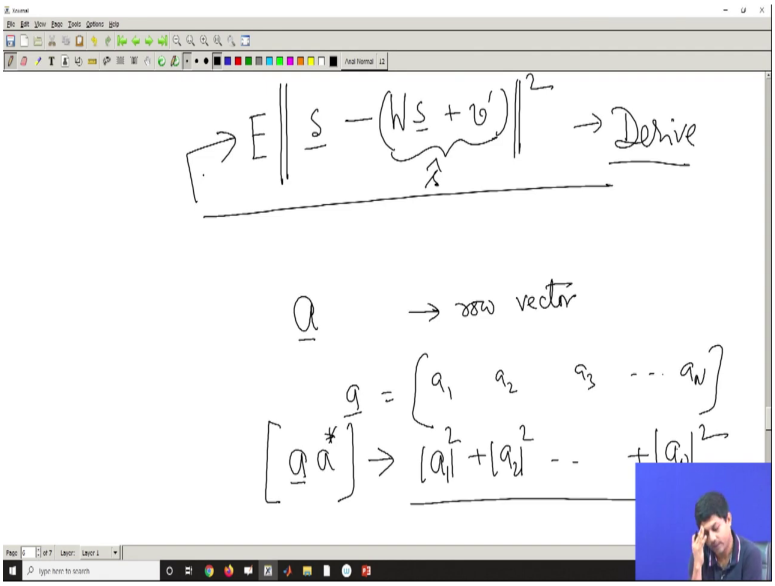

Where we said that this is my cost function. I would like to have it from the M S E point of view. So, this is s minus s cap, because if you look at what is s cap.

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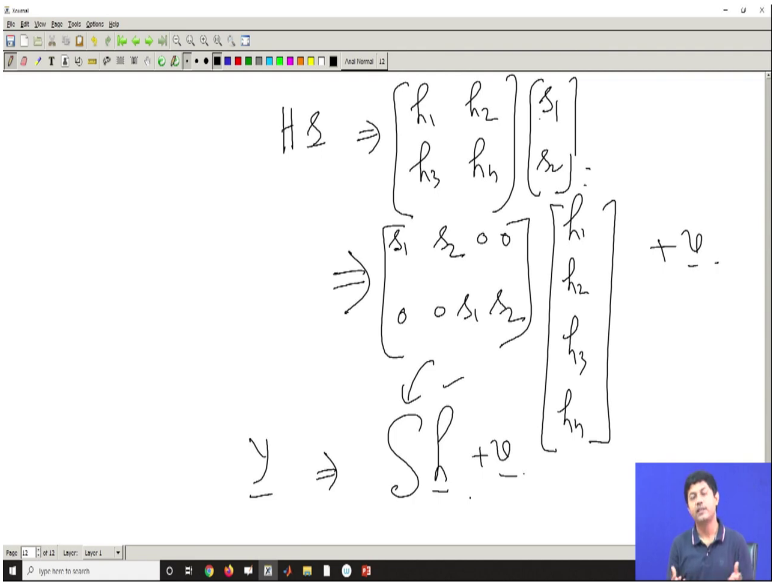

This is what is s cap right W s plus v dash and this is what I have written W s plus v dash. Now, let us expand it right. Now, there are two ways to do that either you see s s is a vector. So, either you can do it like this say for example, say a is a vector right and it is a; it is a; it is a row vector. So, what does it mean a cap is equal to say a 1 a 2 a 3 say a N ok.

Now, what is a a star that is inner product what does it mean. It is nothing, but a 1 star plus a 2 star mod. Of course, if it is a complex number plus a N star right ok. Now, this is a scalar number right. Now, the same thing can also be expressed as.

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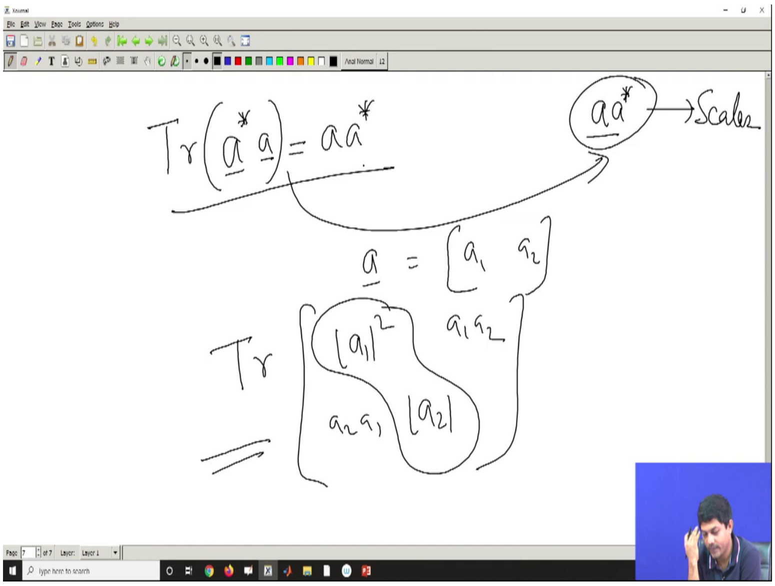

What if I write it this way you see the difference. Earlier I wrote a a star, but now I am writing this way a star a. Now, this is a outer product. Now, I can also. Now, this is inner product which is a scalar. Now, if I express it in the outer product that becomes a matrix. Now, how do I correlate between the two. The correlation is that this matrix if I take a trace of that; that means, if I take a trace of this matrix, this will be nothing, but your a star a whatever you get it from your a star same thing will be coming ok.

You can try it say let us say a bar is equal to a 1 a 2. We know what is a a star. So, what is a star a. It will be a 1 square a 2 square right. And then, there is a 1 a 2 a 2 a 1 right. Now, trace of that. Trace of is nothing, but this quantity it is addition right. So, trace of a star a is equal to a a star for a vector ok. So, now, the same thing I will also apply it here. So, this is a scalar quantity expectation of this, because it is a mod square right. So, that is a scalar.

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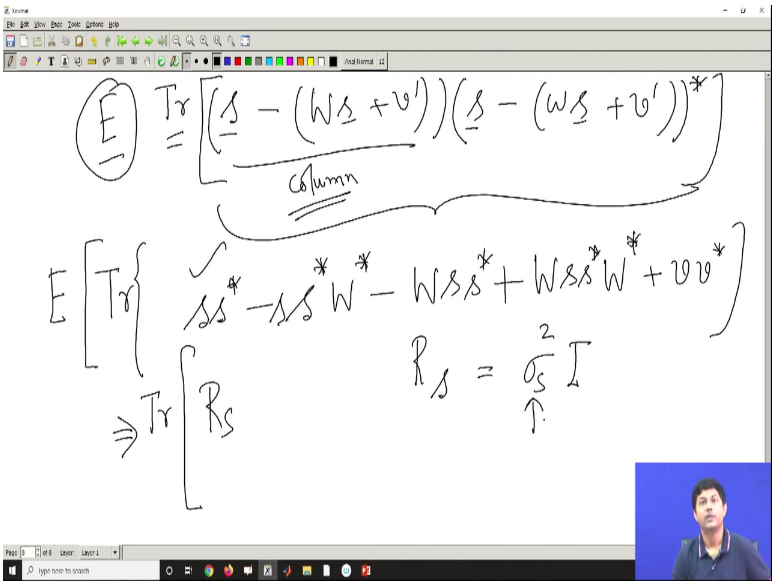

So, I can write it that particular quantity as trace of s minus W s bar this whole thing. Suppose, this is column vector this is column vector. Let us assume ok this is what will be coming. Now, let us not worry about the trace part, let us calculate the matrix part first and expectation part ok. So, what it would be. So, if I if I take that. So, what it will be. So, this would be s s star minus you can write it how it will be W sorry s star into W star like that it will be coming right.

Now, s to v if I multiply and then take an expectation if they are independent, because it is a input data versus the noise. So, naturally things will not come into picture right ok. So, I will not consider anything between s and v ok. So, given that so, any multiplication between s and v dash will just no longer stay back there. So, other things I will consider it minus W s s star, then I just take this fellow of minus this will be plus W s s star and then W star.

I am taking little time because. So, that it will be easy for you to understand what I am doing it here. Now, again W s and v star. So, v star has a v component and s is independent between s and v. So, they will be again 0 if there is an if there is an expectation unless and until there is an expectation will not come.

So; obviously, there is an expectation; obviously, there is an expectation before it. So, I am not putting that. So, probably I will write a trace here and expectation. So, that part is given that part is given ok. So, this should be the case.

So, it will be W s W s s star and W star W s v will be gone case ok. Then, comes the v dash. Now, v dash to s again it will be 0 v dash to s, it will be 0 find it will be v dash to v dash right correct. So, this will be v 2 v dash something like that it will be coming into picture correct ok.

So, now, all these things will just come into picture here. So, now, if I take the expectation inside, what it would be that is the covariance matrix of my data. So, let us call it again trace is there put the expectation inside. So, let us call it R s, R s is you know it is very simple to get the R s. R s is nothing, but the covariance matrix of your data. So, if your data is a independent from stream to stream or data to data it is nothing, but a diagonal matrix right.

So, for example, s or R s it would be something like a sigma square s into an I; unless the datas are uncorrelated correlated. It will be the case. If it is uncorrelated data this is what it is zero mean uncorrelated. So, it is very easy to get. Now, this is your perspective density of your input data which you know, because this is what you are transmitting it. So, this part is known. So, I am not again putting it here, because it is just to ensure that that part is known to you ok.

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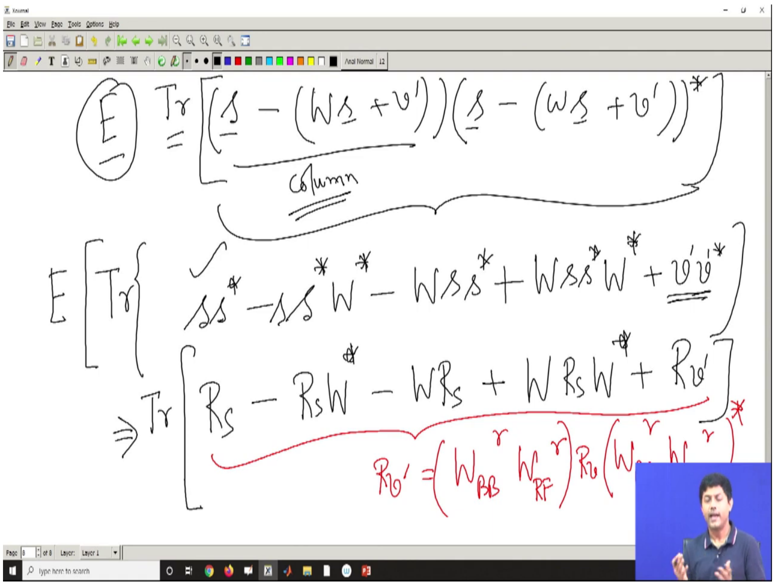

So, again this should be R s W star minus WR s plus WR s W star plus this is nothing, but covariance matrix of my noise and that is also again known. Now, what is this R v sorry it will be v dash. Now, R v dash is interesting point. What is R v dash? Look at your channel look at your noise which I call it as noise this part is your noise.

Now, do not jump into a conclusion that your R v will be a diagonal matrix. It is no longer the case, because this part is there. Had it been purely v, then R v would have been diagonal matrix, but there is a there is something beyond it right. So, W BB r and W BB r will be there here. So, that would completely make it different. So, this R v dash is slightly different ok.

So, this should be W BB r W BB sorry W RF W RF r. And the actual R v which can be a diagonal matrix depending on whether it is a noise is a coloured noise or a independent noise. If it is a white noise; obviously, R v will be a diagonal matrix. If it is a colour noise, then R v will be some other matrix it just a matrix ok. And then, this should be W BB r W RF r Hermitian. So, that is the structure of R v dash.

Now here, you see that everything is known to you alright. Here, if you look at this is my cost function; what I am trying to say that is where your cost function comes. Now, let us say R s I. Now, I make slightly slight changes here, let us say my R s is more of a diagonal matrix. So, what it comes to you?

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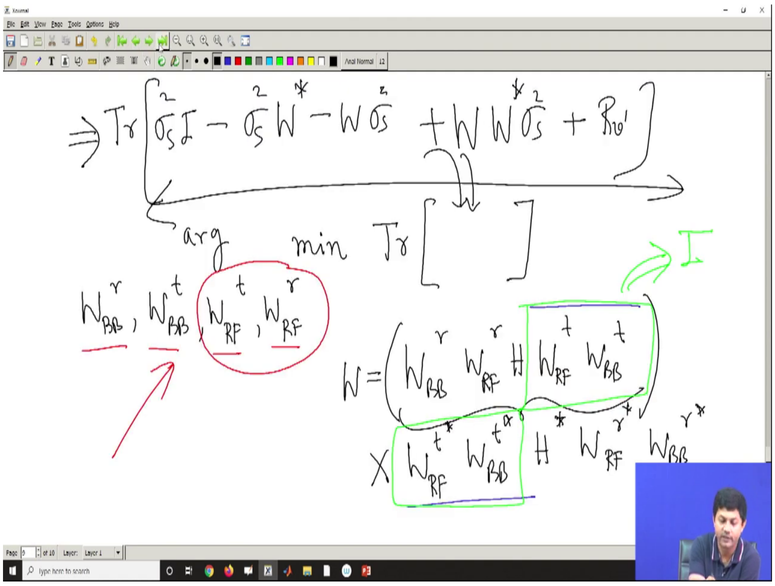

So, this should be something like you know trace of sigma square s I then, what was that minus R s W star minus sigma square s W star minus W sigma square s anyway plus W W star into sigma square s plus this particular part R v dash that is not a (Refer Time: 11:07).

Now, what are these W's? W is the this quantity W is this quantity. It contains channel and all the other part there. Now, again the optimization techniques it is your choice how you want to do the optimization technique, but what is more important is that the cost functions are given in this particular case ok.

Now, let us get into the cost function. So, this is what my cost function comes into picture this is my cost function this is what I am trying to tell you that. So, trace of this. So, this whole quantity trace of. I am not rewriting it. So, this is this whole cost function comes into picture.

Now, I would like to minimize it over all my argument W BB r, W BB t, W RF t and W RF sorry r ok this four matrix I wrote it here. And based on this four matrix, this whole trace can be minimized it here. Now, here the tricks comes. How you play your tricks is completely up to you ok. So, let us say I just give you one example. Let us say how you can play your tricks. So, what was your W? That is W BB r is W BB W RF let us say this is what my tricks is W BB r W RF r H W RF t W BB t right this is what your W standard and classic one.

Can I make some assumptions here, similar one ok? If you remember what kind of assumption we made in the last times. First approach if you look at, we said what if I make this fellow. I remember our first approach. First approach was that what if I take W RF t and W BB t that I take it as a I ok not exactly, because it is not exactly that I, because there is a Hermitian. So, multiplied by. Let me just put everything there. W RF t star W BB t star H star W RF r star and W BB r star.

So, what is what we assumed in the first optimization in our case earlier slightly extended. So, we extended that what if this and this together this whole thing. So, how do I put it. Let me put a colour code here, this one whole thing what if I assume it to be I. This was our first case. If you look at our earlier classes, this was the first approach that we did there also right.

Now, what I made it here. So, I just remove my transmitter side parameter, then I was left with my receiver side parameter W BB and W RF. How did we solve that optimization? We have explained it; that is also kind of an iterative solution right you assume W RF. So, that mean this whole cost function this complete optimization problem will be now, devoid of this two quantity if I make these two assumptions. And then, you can solve it using W BB r W BB t ok.

Then, you come back to the first case and solve for W RF t and W RF r that is one approach. What was our second approach? Second approach was what if the digital side I take it off that mean, I just consider only my initial part transmitter R F part and receiver R F part that also you can do it.

So, there are many ways you can approach and break this big problem into smallers problem, but this is the overall you know this is the overall way the things can be performed, because that is the approaches you should take it and what exact approaches you take it that is up to you. There are hundreds of works how you can approach it totally different ok.

So, which means that I can solve it. So, my pointer is that you decide your cost function ok. So, in this particular course, I have taken only two cost function; one was the maximizing the capacity and second approach was the minimizing the M S E ok. If it is minimizing the M S E, it will be slightly easier problem, because it does not involve the log parameters and other parameters so rigorously.

Now, in this particular case also, you can also put a constant. See, this whatever I have written is not a constraint optimization. What does it mean; it means that is this optimization problem a complete one. So, there is a problem with this approach. What is the problem what is the problem with this what is the problem with this approach?

If you just leave it to the optimization tool. Now, if I assume this is I, there is no problem. This is an unconstraint problem and you can solve it ok. Sometimes, you can also put a constraint saying that this whole thing this W BB R F W BB t whatever I have written instead of assuming it to be I can I assume it to be some other value? So, instead of I, can I some write it something like that.

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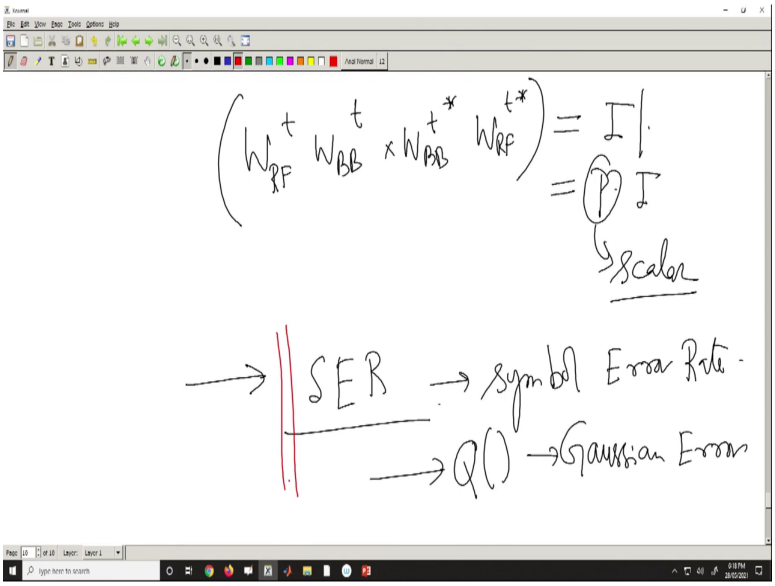

So, it should be like a W RF t W RF t W BB t and then, multiplied by W BB t Hermitian W RF t Hermitian; that is precisely what I have done right W BB t Hermitian ok. So, opposite anyway so, this whole quantity there is a slight yeah.

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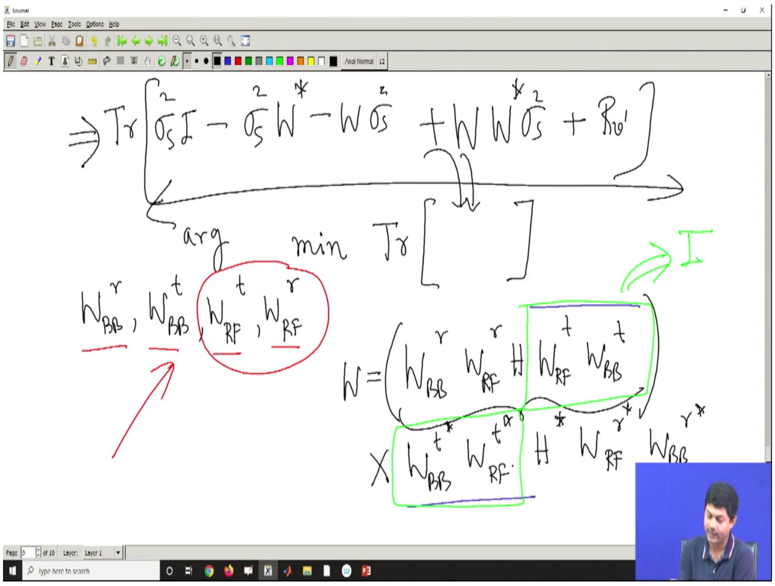

So, slight changes here, this should be the first one BB will come first RF will be second case ok. Now, this I was assuming in my first case I ok. You need not to assume it to be I. Why need not assume to be I? Because you can have your own power right. So, you can say this whole thing trace of it of course, trace of it need not to be.

You can say I meaning if it is a trace, it could be just like a some number right instead of I number. If you do not have a trace. Suppose, I do not have a trace there that is ok you need not to have a like a trace here. You can also assume it to be some constant say P multiplied by I. P is just a scalar number.

But it does not really matter whether you assume it to be I or P into I. So, what is this P; P is mode of a transmit power kind of thing? So, if it is I, the assumption here is the transmit power is some sort of a unit power. So, it could be 1 milliwatt or some unit power.

But if you are little matured enough to consider non-unit power, you can just write a P. I mean it does not change the optimization framework, because P is just some known quantity, but it is a scalar quantity, but fundamentally it is the same there is no difference of what we said yesterday.

So, this is the approach that people take it. So, you first take this transmitter side parameters of solve it for the digital side, then bring back the transmission. I think you have explained it how you have solved it, but overall the approach might be same ok.

So, that is it. So, now, there are other cost function. I am not getting into those cost function, because that might be too many things, because you have hundreds of papers and there are different type of cost. There is another cost function which may be very tough, but I do not know if you can really if you are really realize it such kind of cost function.

Say what if I take S E R Symbol Error Rate. But probably this is a one of the most gigantic cost function, because that involves that involves Gaussian Q function ok. So, this is called Gaussian error function. It involves that if the noise is a Gaussian noise.

Now, handling such kind of function itself is very tough. So, this is not a very I would say this is not a very usual or this is not a very good cost function to proceed it. See, this MSE and the capacity is I mean are two most prominent cost function ok.

So, with this I try to conclude the this module 11 with the parameter estimation things. What we have not touched upon is the channel parameter estimation. Now, what we have estimated is basically the design parameter estimation. So, how do you design those parameters ok. But using the design parameters, how do you estimate the channel that we have not talked about.

But as the scope is also not limited and. Scope is limited in our case that we cannot go into the channel estimation part. But I can just brief it how it can be also done for example, in a normal you know in a normal MIMO case, what exactly you do.

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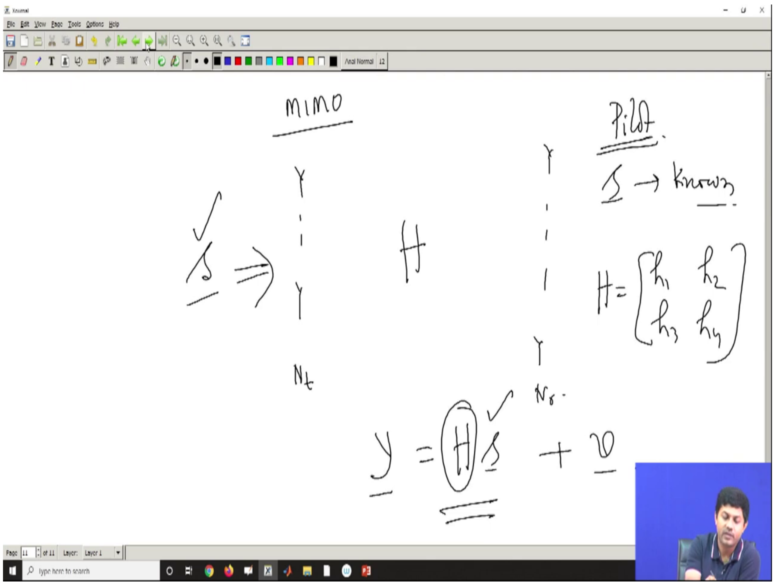

So, for example, I have a basic MIMO ok. So, because if I explain it for MIMO. I just give a touch upon so that it can be extended to the larger hybrid beam forming case as well. See, if it is a MIMO case, you have this N t antenna here N r antenna here right. So, what is the data model; if this is a channel H into s plus v that is a standard data model right see if I feed s here ok.

Now, from there you can estimate your s; that is natural you have many estimator, like you have an LMMSE estimator. You can directly do an ML detection whatever your choice that is not an issue, but how do you get your H then ok. So, what you do? Instead of sending unknown s, you send a pilot; pilot meaning you just send a known signal; that means, in your case s is known ok.

If the s is known, can I manipulate this quantity? That mean if H is say I have a h 1 h 2 h 3 h 4; just four quantity let us assume that. So, and s is just two quantity. So, can I manipulate in such a way that I put s in the left side and H in the right side. So, what I am mean to say is that.

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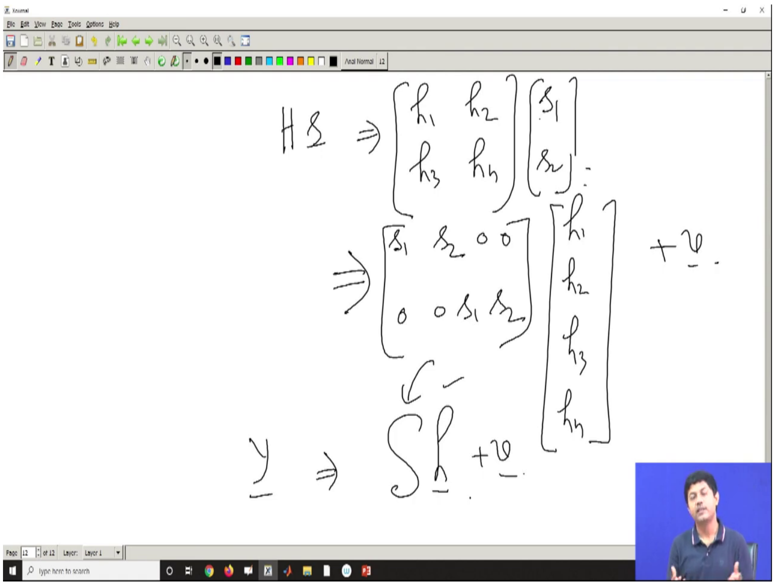

Instead of saying say H into s bar which is equal to say just take that example. Let us see it is a single tap channel and here, you have a s 1 you have a s 2 ok. So, can I write it in a form of a matrix where h 1 h 2 h 3 h 4 which where this comes as a vector and then you have a normal noise, but what should be the case here right.

So, can I say this will be my s 1 this will be my s 2 0 and 0 ok then, it would be 0 and 0 s 1 and s 2 can you write it like that. So, can I say this whole thing is something like a capital S h vector plus v; your y remains the same right. So, now, this is as if like that is a channel this is your data. So, these are.

So, now, I am not getting into the estimation framework, because the purpose is different. So, to estimate the channel, you can approach this kind of things. So, it means that here what you can do.

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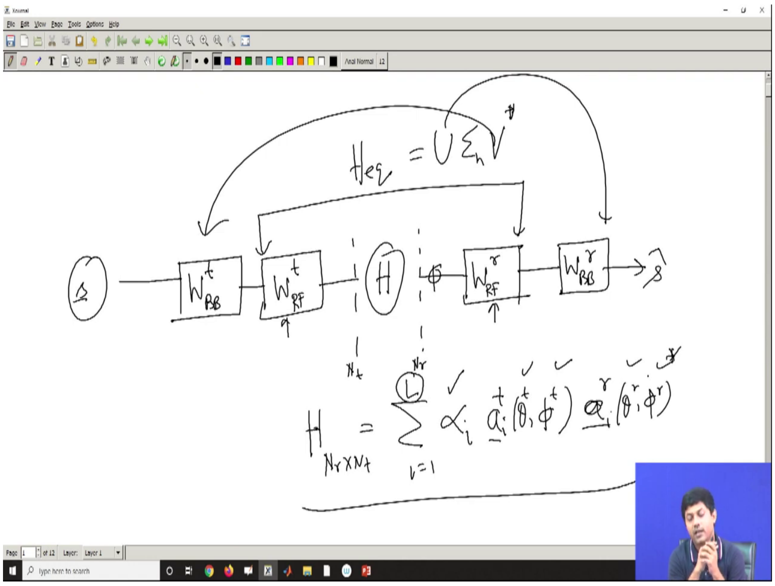

Here, what you can do? Instead of s, you take the s as a pilot take this as a pilot, then manipulate this H in such a way that H becomes like a vector appearing ok rest of the things can be done; that is the point I am trying to say here ok. So, you can estimate your channel easily. At least, you can do the channel estimation simply.

But that is not a very good way of doing it, because if you have N t cross N or say 100 cross 100 then; obviously, you cannot do the channel estimation there. So, probably you require less number of variable, where you just have a alpha theta phi t theta r phi r; only few amount only few number of variables will be good enough to estimate H.

So, that comes into picture when you have a angular parabola and there are certain algorithms. There are plenty of algorithms which can do this A O D and A O A estimation so, but this is this I am not scoping it in, but this particular courses. So, as so, in this particular case, I am assuming my channel is always known to us ok. So, given a channel, what are the different design parameters I am going to estimate it ok.

So, I hope basic flavours of my design parameter estimation; one is the LMMSE based, another one is the MSE based another is a capacitor based and these two popular methods are mostly used.

So, with this I conclude this particular session and in the next class, I will be talking the multi user MIMO hybrid beam forming ok. So; that means so far, I have not used multiple users only a single user has been adopted here. So, what will be the scenarios when it becomes a multi user case and that would be the next modules activity along with the impairments.

So, far I have not introduced impairment something like a C F impairment something like hardware impairments; how they can be model into the system and how that complicates the whole stuff there. Probably I will get a. I will give you a glimpse of some of those signal passing aspect there in the next modules.

So, with this I would go back to my content here.

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So, we had the design parameter estimation and the reference is again this particular paper. So, where it talks about the MMSE case and MSE cast case cost function. You can find details of the algorithms there. I am not putting the exact algorithms, because that is a very standard iterative algorithm. Mostly, it will be the steepest decision and those kind of thing.

So, what is more focused or what is more important is that given a problem, how do you break it into a smaller problem, because that is important. If you know how to break it into smaller problem and then exact solution are standard methods ok. So, with this I conclude this session.