Signal Processing for mmWave Communication for 5G and Beyond
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Module - 11
Design parameter estimation
Lecture - 58
Design parameter estimation (part-4)

Welcome to Signal Processing for millimeter Wave Communication for 5G and beyond. So, this is the fourth part of the Design Parameter Estimation of the module 11, ok. So, today, we will continue to cover the design parameter estimation, but today we will be talking about one, we will be taking one example from LMMSE. So, which means the cost function will be MSE, ok because in the last class we have talked about how you can solve it for a maximization of the capacity.

Now, today we will be taking one example with the MSE MC minimization and see how we can solve it, but the fundamental philosophy would remain same ok.

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This is basically the again the design parameter estimation.

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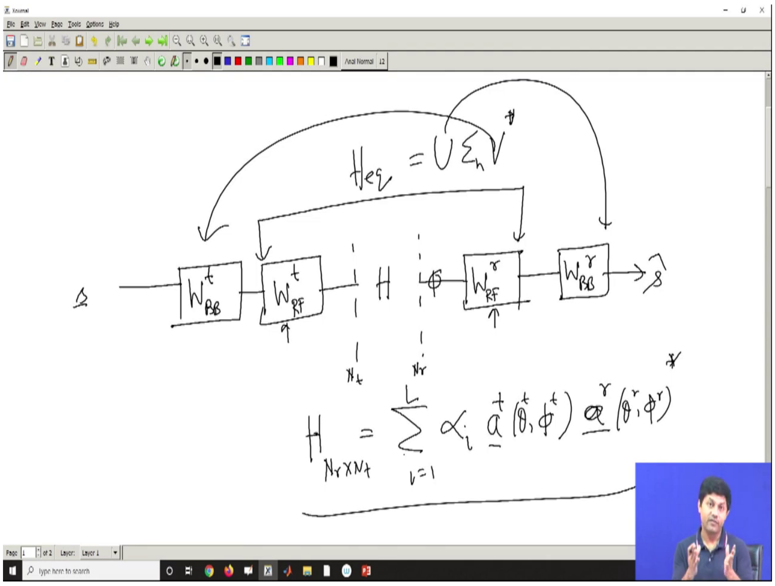

So, what we learnt last times is that I just redraw the, I just redraw everything so that it will be easy for you to recollect. So, let us say this is my s cap and this is where my date first W BB t that is the digital base band at the transmitter side. Then you have the W RF t and this is where your transmitter side and then there will be an H here which is the millimeter wave channel.

And, by the way just for your reference how the H would have been? See if there are say N t antenna, N r antenna so, this would be N t cross N r. I think we have talked enough about the dimension of it. Just a second and so, this should be slightly changed here this should be N t N r cross N t, ok. So, that I think so, this I would like to just remind you a vector from the transmitter side which can be a function of your theta and phi t, alright. And then you have this a r vector which is the array manifold vector from the receiver and the transmitter side from the receiver side.

So, this would be ok and this should be a star here so that it becomes a matrix and I can be 1 to if you have L number of path that is just a part there. I mean in case you forgotten just a reference. Because this is how the channel was r matrix, but the number of parameters would be much lesser, ok. So, and the at the receiver side you have the W RF r you have the W BB r and this is where you get your s cap, right.

This is what the fundamental structure of the complete hybrid beam forming and this part whatever is written as an RF it is an RF and whatever is written as a baseband that is digital part, ok. So, this is what we have learned it and few things or few question would have come from you that if I put W RF t and W RF r in the optimization framework.

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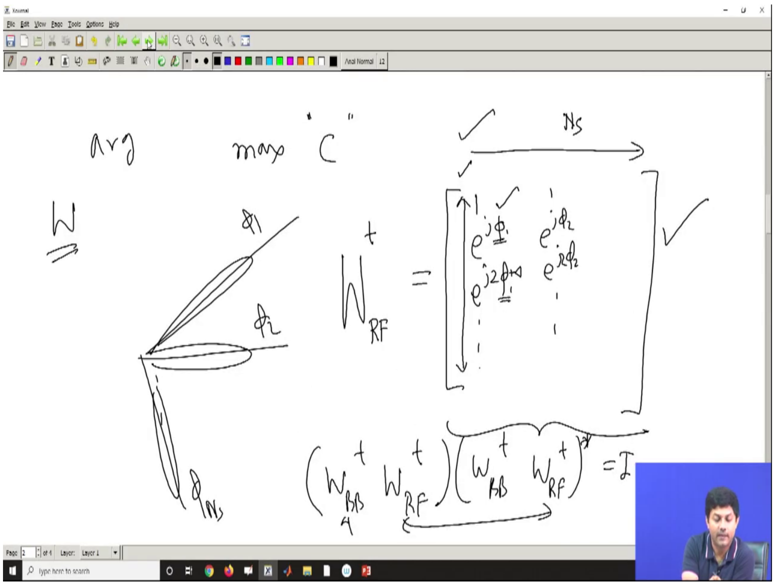

So, if you saw in the last class that we are basically maximizing the capacity part, right. So, if it was a capacity right and then we are putting all the arguments over all these W's right, but you may have come across this question that well, if you put some variable into optimization framework you have no control over how it picks up the values, right. Say for example, W RF t what is W RF t? W RF t was the phase shifter matrix right, ok.

So, if you have N S number of data and N t number of transmitting data you have phase shifter. So, N S cross N t number of phase shifter data that will be there, but the point here is that that W RF has a certain structure, ok. Yesterday I was trying to say that you have to maintain that particular property otherwise there is no point of beam forming, right. I mean the beams can be all scattered.

So, if you carefully notice what I am trying to do here the each and every say column is the phase shifter matrix is the phase shifter column, right. So, it if is the if there are say N s number of say here I am saying that there are N S number of data that is being fed and you have N t number of antenna that is that will be fed later. So, this particular column or row depending on how you structure the W RF one of let us say I am making my column as my phase shifter row.

So, this phase shifter row will be what they are nothing, but the if the form is like e to the power you know j phi, then e to the power j 2 phi and so and so forth, right. So, that is the form we are talking about phi meaning it is the angle not just the azimuth angle I am talking of as a whole theta and phi both together. So, this part I am talking ok it is not just the azimuth angle, ok.

So, now, the question that would like to answer here is if I put this whole W RF t into my optimization framework, this structure would be completely ruptured because you will have no control over how these values will be picked up. You can say well, you can say W RF all these constraint you can put anyway W RF into W BB we put this constraint right, if you remember and Frobenius norm of this fellow not Frobenius norm we have written this one right in one case.

You remember this W RF t this was I in the case number one when you did the optimization here and in the second optimization of course, W BB and those things are coming into picture, but W RF we have not put it any restriction. So, it is as if like you are leaving W RF t and W RF r to a to an open point. So, now, if it is an open optimization point so, W RF t can take any values any value it can take any complex value.

So, this particular structure like e 1 e to the power j phi to the power j 2 phi this particular structure need not be maintained, right. So, in that case how you can proceed? So, how you ensure that this whole thing is actually maintaining it is level of it is structural level. So, the only way or maybe one of the ways is that I mean probably it is a very simplistic way, see how W RF is calculated. W RF is calculated based on your angular angle of departure, right.

So, it is the angle of departure and then what is what should be your rotation, right. So, you have, so angle of departure determines what should be the, I think we have talked about it enough in last few classes. So, angle of departure they predominantly determine what should be my steering.

So, this W RF is a steering matrix. Now, the point here is that if I know my angle of departure, why should I make my W RF t in the optimization framework because I know the structure, right? Because I if I know how much I need to steer why should I you know enter into my optimization framework to redesign my W RF t, there is no point.

I mean it does not it does not tally with whatever we have discussed, right because our discussion clearly says W RF is a steering matrix and if I know how much to steer you can easily create your W RF t, there is no point. So, there is no point that you should bring W RF t in your optimization framework there is no point. So, then why should I bring my W RF t into optimization problem because if I know phi; that means, if I know how much to steer how do you know? That is nothing, but your angle of departure.

So, if I know it then do it. Suppose this is my first one I know for the second one say j phi 2 e to the power j phi 2 2 and so on and so forth. I know how much to steer well then this will be. So, that mean if there are N S number of data I know for each and every N-th stream what are the different beam? Suppose, this is my phi 1 steering, this is my phi 2 steering and this is my phi N S steering and that is predominantly determined by angle of departure.

So, the point is totally different point is that yes, you still need to put it into optimization in spite of the fact that you may need the you may have the knowledge of your angle of departure. Now, why, the first reason is that what we missed it or what we have not put too much effort into the discussion is the noise part, right because if you look at the data model the data model was a additive data model, right. So, there is a noise here.

So, there is a noise here let me put it a little there is a noise here right and that is an adjective noise here right. So, if there is a additive noise then what will happen then how do you ensure that whatever you think that this is where your steering should be how do you ensure that if you exactly steer in that direction your capacity will be maximum there is no guarantee right because there is a noise in the system.

And, the noise can ensure that whatever you have estimated as your angle of arrival or angle of departure that may not be a correct estimation or a correct values that may be slightly different, right. So, the noise is making sure that whatever exact W RF t you are thinking may not be exactly that may be slightly different from an optimal you know MSE or a optimal cost point to few because the noise will be slightly you know deflect your data.

So, you think that may be at 30 degree I need to put the beam but, there is a noise there and the noise said hey, do not put it in the 30 degree probably, if you put it into 31 degree that may be statistically correct. Because the noise can you know slightly disrupt your beams S NR and all these things obviously, right, because that is the purpose.

So, the noise plays a role there and if you predominantly notice here this two framework whatever I have explained it last week they are actually Bayesian framework why it is a Bayesian framework? Because I have brought into the capacity in both the frameworks right whatever assumption I have made that is not an issue, but the capacity is a it is a statistical quantity, right. So, I have considered the noise statistics there. So, it is more of a Bayesian framework that I have the cost function is a Bayesian cost function, right.

What if I would have chosen a least square kind of? Least square is not a statistical quantity, it is not right. It is just like a x minus x mod square in that case probably you really do not have to put W RF into your equation because if you know your some estimated value of your angle of departure you reconstruct that. So, this point needs to be very carefully thought of. So, this is one of the reason you still have to bring W RF into your equation and try to get an optimal value.

Now, the question is that if I put it into the optimal values or optimal you know optimal frame optimization framework does not it rupture the structure because if you look at the way the structure is it is W 1, W 2W 1, 3W 1 and so on and so forth, right. Well, it may hold it that particular property most likely it may hold it if not it may create slight changes, ok.

Instead of 2 dot del 2 f 2 phi 1 probably it can add a slight delta there it depends on the optimization. Why does that? Because it tries to understand if I add extra value into any of this quantity does it optimize that particular cost function, ok and if it feels it can slightly change it may not be just integer multiple of your phi 1, probably it can slightly disturb that value based on how that cost function is projected.

So, the question is that in general if I do not get into the optimization of W RF you can leave that there is nothing harm. So, you can create your W RF if I know my angle of departure and angle of arrival. So, you can create a W RF t and W RF r, then you can just you know if I know W RF t and W RF r then you can always take the second approach which I talked about in the last class where W BB t and W BB r can be easily can be easily found out by a simple SVD decomposition.

In fact, that could have been the best and simple choice of creating my whole hybrid programming structure you do not have to really get into any optimizations at alright. Because if you know W RF t W RF r you can always take the yesterdays I mean the earlier classes approaches, right you can just take the complete you know this one this whole thing you can take it like a one bundle call it H eq do an SVD done.

Then put the V and U you can just put this fellow here, put this fellow here that is it W BB t and W BB r are available, no optimizations ok that is could have been the easiest choice. Well, that is one approach, but that may not give you the best result all the time because your W RF is constructed out of the estimated angle of departure which may not give you the best value, ok. So, that is the motivation why I need to still bring the W RF into my optimization, ok. Another point few more points ok, this is one point.

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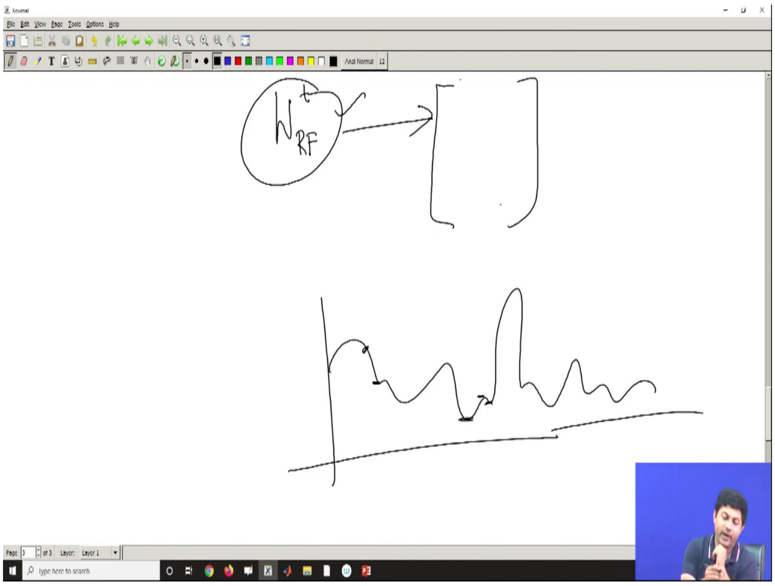

A second point is that when we start when we say that ok use because if you look at in the first approach what I talked about yesterday where we say that it is a iterative solution. Like for example, you first assume W RF t then go for W BB t and so and so forth, right, so those are iterative. So, when you assume a particular structure of W RF t what you start with? Last time you said that it from optimization context that you start with some random number well you can do that, ok.

Nothing harm in that, but the point here is that that does not mean that you get the best one out best out of it why? Because the cost function is not a convex function ok it the cost function itself is a non convex function. So, which means that the cost function can be something like that ok if that is the case you do not know where you start with suppose you start from here, then you may get stuck here suppose you start somewhere here you start here.

So, if you start W RF t in a random manner you do not know where the function would be stuck there, right and you have to be satisfied because the projection matrix or ST based decision algorithm will always get into a local minimum, ok. Then can I make my W RF t a better choice? Well, still it does not guarantee that will give it will give you a global kind of minima that guarantee that it never guarantees, but what you can do is that you can assume your W RF t or you start your W RF t from the structure that I just explained, ok.

If you know your d you construct your W RF t. So, that could have been a minimum that could have been a initial choice for your W RF, right. Similarly, W RF r ok now if you know your channel you can always reconstruct your because it is kind of an MRC, right. I mean we you have enough explain. So, W RF t, W RF r, you can easily reconstruct given your channel information because channel has the AOD and AOF information.

So, if you know your channel you can always you know you can always get into that. So, this part can be easily reconstructed here. So, you can always create your you know you can always start W RF in your optimization because last time we were talking of initializing the optimization from some value of W RF t, if you remember for the first case. So, what should you start with? You can start random or you can start with the that particular construction.

You start whatever AOD you have it, then reconstruct it as per the structure and then that would be your first choice. So, what the optimization will do is that if the optimization framework feels that that structure itself is you know kind of close to optimization or close to the good value, then it would not change it much, it would not disturb it much which is ok right that is precisely what you want right, ok.

A third point because I am only talking why you need to bring W RF t into your equation third point, not every time you may be getting your angle of departure or angle of arrival. So, accurately it may happen that you just get H just your H. Now, what is H? H is a summation of your you know summation of this point H is a summation of how do you know this L?

L is the number of paths that it has trouble I mean number of path that it has trouble in a digital sense, ok because you are in a digital data not in analog a data ok this is a digital channel model I mean you are perceiving it from a digital end. So, it is not the exact L, but you know the sink tail how many L it can have that. So, that L how do you determine that? It is it is not a very easy task to do it, right.

So, if it is an LOS channel probably L can be just one, if it is not then determine that L itself is very difficult. So, in that case of course, that is an i here, sorry. So, in that case getting the you know angle of departure, angle of arrival for individual i path may not be a very easy task to have it, right. If it is an LOS, fine, you can just have one, you can just have one angle of arrival and angle of departure.

But, if not then you have such multiple angle of departure angle parable because each and every path can have your own arrival and departure path. So, determining that itself is another task, ok, so it is not a very easy task. So, that complexity is again coming into picture right in the H. So, you can estimate this H in a simple way. So, what you get finally, is that you just get a matrix ok, but clarification on L may or may not be there. If you have it well and good then you can reconstruct your W RF very easily in that case, ok, so this is what it is done, ok.

Now, today I will be starting off another cost function and probably that would be the end of this particular module.

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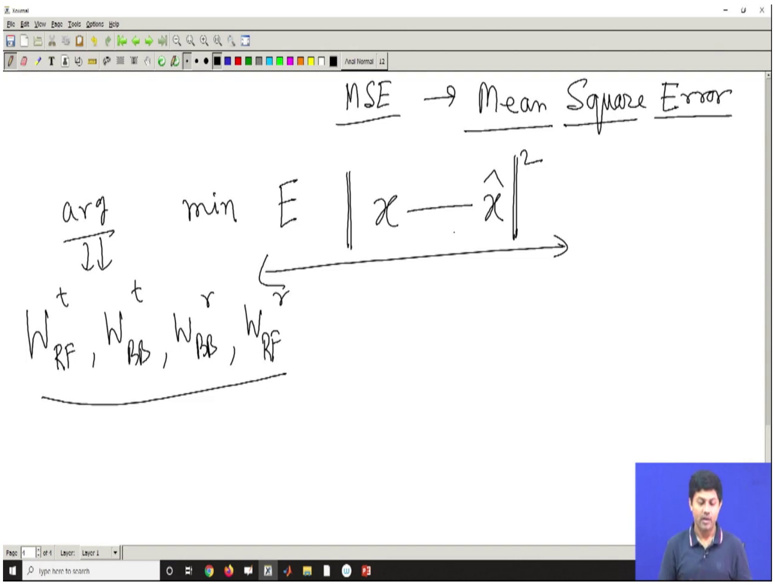

So, the cost function that I will be talking of is the MSE mean square error, again this is a Bayesian cost function ok, mean square error. So, what is this mean square error? Mean square error is how do you define a mean square error? Suppose, you have a variable x and that particular variable has an estimate x cap. So, mean square error meaning it is the square error between the two and there is a mean over that mean meaning expected value, so statistical mean.

So, this MSE is also a statistical quantity right, ok. So, now, this is the fundamental part of your MSE now. How I can create or how I can fit my problem set whatever we have discussed into this particular cost functions? We have discussed the capacity part, right and definitely I would like to have it maximizing or minimizing this quantity. Naturally I would want my x cap to be such that this is much closer to x that is an error, right. It is a if you notice what is the full form of it is a mean square error.

So, which means I would always love to have the error as minimum as possible. So, instead of a maximization I would definitely want a minimization of this quantity over the argument. Now, who are the arguments that we have to see based on the problem set. Now, in our hybrid beam forming case we will see who are the argument that is obviously, for us we know who are the arguments I mean because we are the designer. So, we know who would be the argument here.

Obviously, this four matrix are our argument. Now, we have to now fit the cost function into our equation into our data model. So, now let us try to do that ok.

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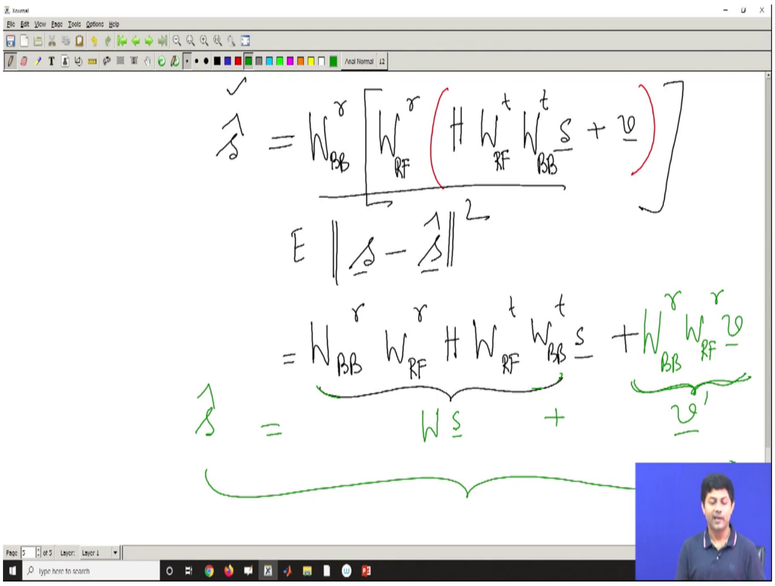

So, what was our s cap? Our final s cap was W BB r ok and let me just this is my data, this would be my W RF W BB t, this is what my transmitter side, this is my channel side, right. And this is what I got as a input data as a noise data at the transmitter at the receiver, sorry and then you have put a different color here W RF r ok, this is what is coming. So, this is my data model final data mode. So, this is what I want s cap.

So, what I want? I want my s minus s cap in this particular case to minimum. I want this to be calculated, right, it is not a very difficult task ok. So, let us try to do that what it means. So, let us break it down, ok. I am just breaking down this equation and what is the philosophy? If you have noticed it the philosophy is that this will be W RF t then you have this W BB t s cap this whole thing I have put it here, ok and probably I will put a different notation and let me not mess up with the notation s bar vector plus the noise part I put a different color.

So, that it will be explicitly saying that that is the noise part. Let us call it v dash, ok, let us call it W just for your reference, ok. So, can I say my effective data is this? This is my effective data model like s cap that is what I want to have it s cap is equal to W s plus V s I mean you know what is W. W is this whatever I have written as a this bracket this whole thing this whole thing this whole thing and this part is your v dash total v dash, ok. So, this should be, so I have not still built the MS MSE this is the data model, ok. Now, it will be very easy to build the data model. So, I give a brief about it and then probably in the next class I will go for details on that.

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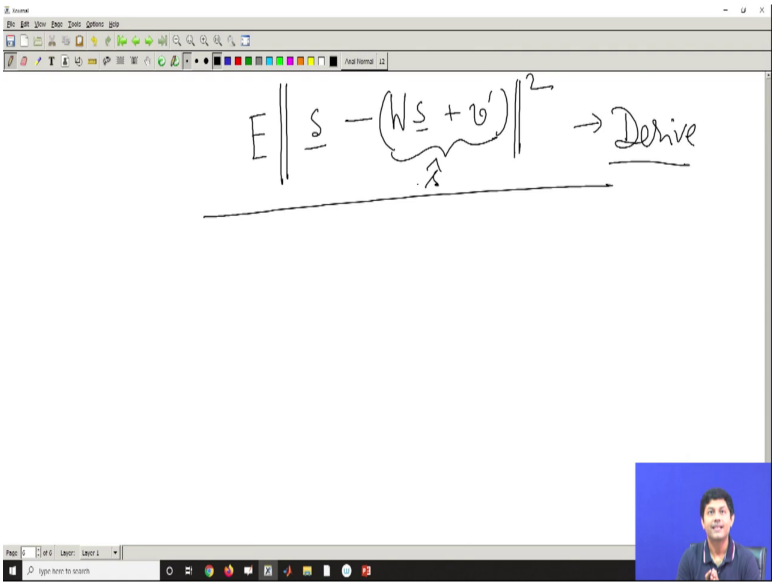

So, let me just start that and then I will leave at this point. So, your MSE would be m it would be S bar minus your H of S cap right this v dash. This is what your S cap, this is your S cap, right alright, not this W this should be W. So, this is the quantity that I would like to estimate not estimate first let me derive that let us derive it and then you know this w is a function of all these four matrices, but let us derive this. Because that is the cost function right finally, that is what will be optimized.

So, if I know what is the cost function I can get into my optimization framework ok. So, in the next class, I will derive this equation and see how I can fit it into my optimization network, ok. So, with this I will go back to my, go back to my teaching here again.

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So, today we have just covered and in the next class I will be talking about the LMMSE actual derivation, ok.

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This is the reference. So, this particular paper I am referring it, but I am not picking up everything from there. So, you may see slightly different from that because that paper talks about many things which probably you may not need it right away, ok.

Thank you.