Signal Processing for mmWave Communication for 5G and Beyond
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Module - 11
Design parameter estimation
Lecture - 57
Design parameter estimation (part-3)

Welcome to Signal Processing for millimetre Wave Communication for 5 G and beyond. So, we will be continuing the Design Parameter Estimations this is the 3rd part of that and we will be talking about what are the different approximation and approaches one cost function to solve the NP hard problems. And again I am keep on repeating the statement again and again do not take this approximation as a global approximation because approximations are your own choice as long as you have your own defence ok.

If you can defend your approximation go ahead. So, I can have any ways of approximation, under that approximation things can be solved when something is very tougher to solve, you have to make this approximation and you have to go into such kind of sub optimal solution. So, whatever we have explained so far is a sub optimal way of solving that particular NP hard problem ok. So, next go back here.

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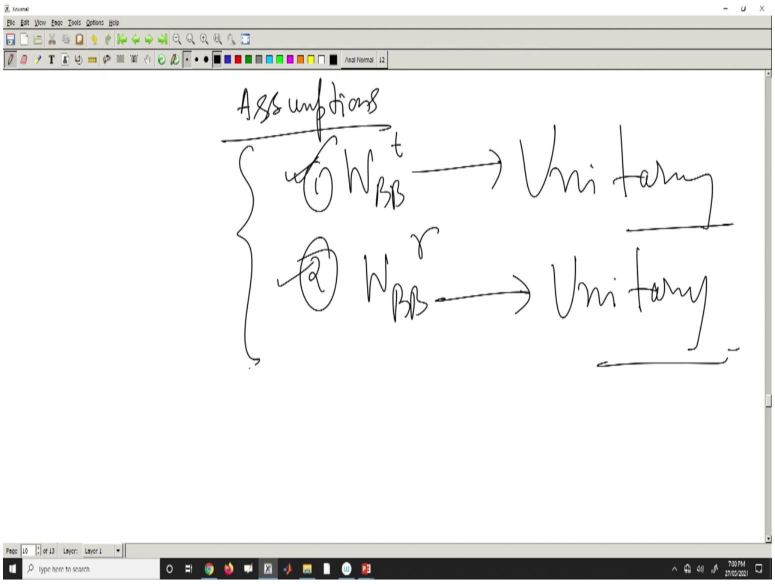

So, this is what our assumptions in the last class that we assume that what if we make the base band you know the base band combiners or equalizer whatever you call it; it is not a big issues as long as you understand what I try to get it here So, this is W BB t and W BB r I am now assuming what if it is unitary ok. So, what does it lead do, this is what it leads to ok.

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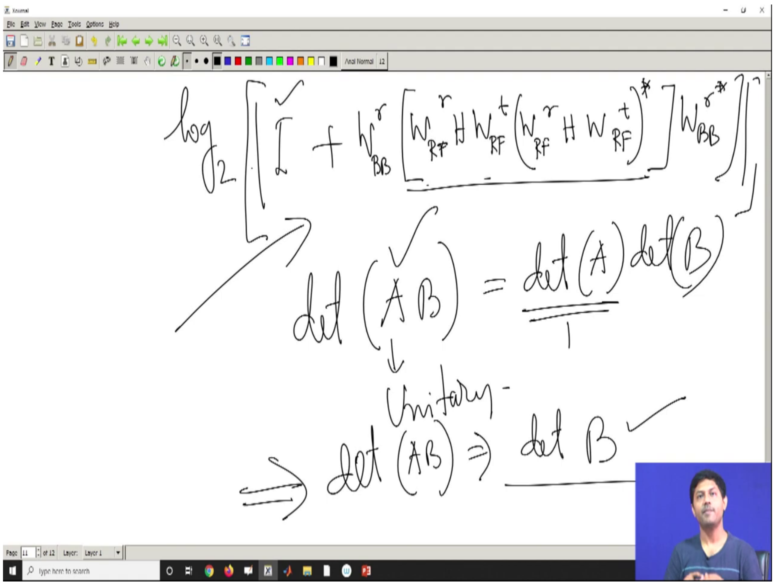

So, in that big gigantic equations I was showing it. So, what was there log base 2 determinant of sorry determinant sign is already there I plus let me not write everything there W RF sorry W BB [FL] this should be BB r and this whole thing W BB r star this was the equation we have written. And this in this case what is there the H W RF and all these things was there I mean let me write it also sorry let me just go back to my equation and see what is written here.

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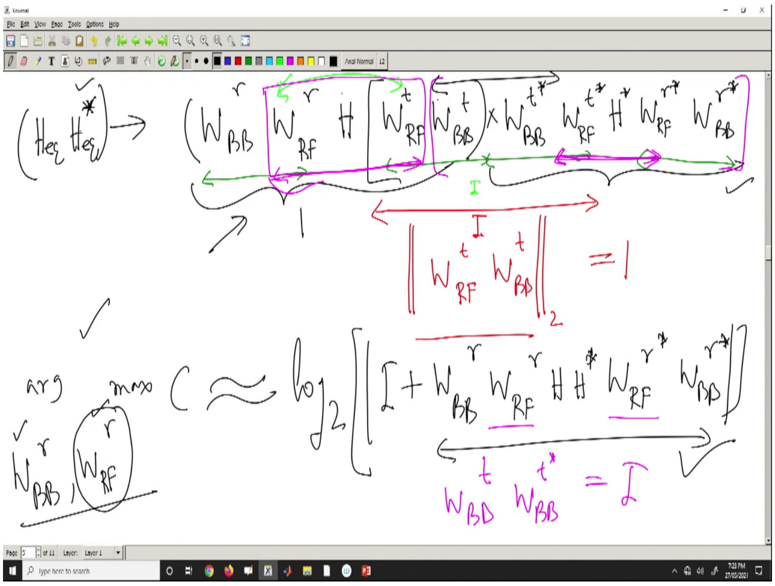

Yeah, this is the whole thing this whole thing what I am trying to say W RF H W RF t and all. So, now, this W BB t I am not writing it because I am assuming that has been assumed by unitary. So, the rest of the so, W RF r H W RF t this part and W RF t H. So, this will be let me not write it this way RF r H W RF t this whole thing I put it to conjugate is this fine yeah this is fine.

Because in between the other parts have been vanished already, now I am saying that from here if I assume W BB t is also this is a determinant. W BB r is also vanishing why because you know determinant of I do not know whether you know it or not, determinant of AB is equal to determinant of A into determinant of B this is a linear algebra result.

Now, what if A is a unitary matrix see if A is unitary what will happen. So, which means if determinant of AB would be unitary determinant of unitary is 1. So, 1 into determinant of B.

So, if one of them is unitary the determinant would be the rest of the component ok I apply this part here this particular corollary I apply it here ok. Now, I am still not done because I still cannot make the W BB r vanish. Why? Because there is an I there ok.

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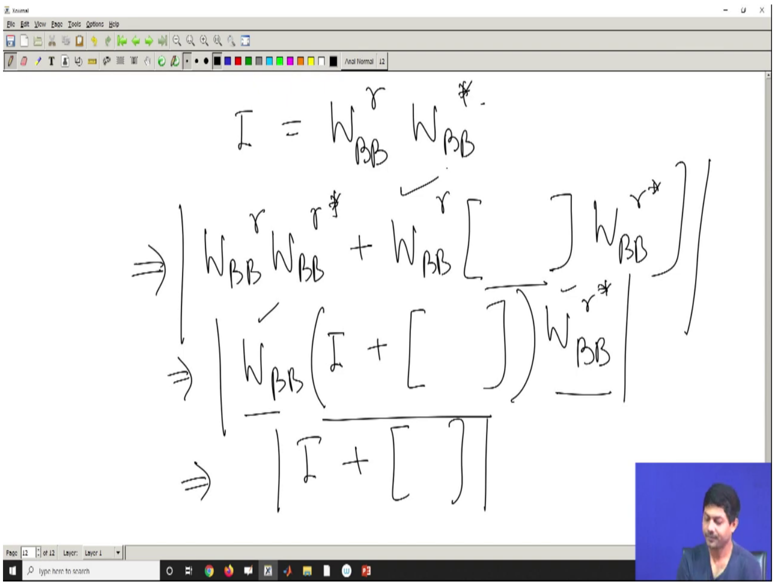

So, can I break this I. So, what if I break the I W r W BB r into W BB star. Now, then I am done why because then it is a determinant of W BB r W BB r star ok plus W BB r I am not writing the in between part for your easiness this in between part is nothing, but this part ok this is what it is, now this is the determinant ok.

So, can I write it like this determinant of W BB into I plus this part ok I can write it like that ok. Now, this is the determinant of one matrix, another matrix, another matrix three matrix. So, determinant of A B C is determinant of A into determinant of B determinant of C ok. So, this part will vanish this part will vanish after determinant.

So, what will happen determinant of I. So, I comes back again, but now this quantity will be you know without W BB ok. So, that is precisely the point here. So, which means that now I am getting rid of or I got rid of from my capacity equation W BB part and W BB anything W BB t and W BB r both.

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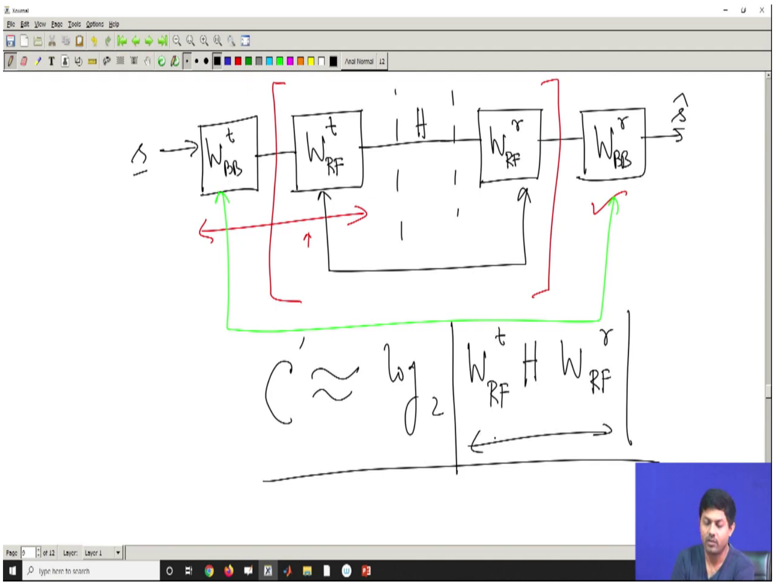

So; that means, what I was talking here is that, but now I got rid of my baseband part completely. So, that mean I have now a capacity where only the RF part is there provided that is two assumptions are made, these are all assumptions I can make my assumptions ok. So, that is another approach now still it has not answered this question how come certainly one of them. So, that is the simple question.

Now, this is ok so still there is an I here ok. So, here I make an approximation. So, I say that ok when I going to solve this I do not consider I because I is a mode of a constant so constant plus something. So, ultimately when I say log of constant plus determinant of constant plus something this I can be you know is you cannot ignore it, but in this particular case I am making I am sure that I is just removed from my case because this is the constant part for me.

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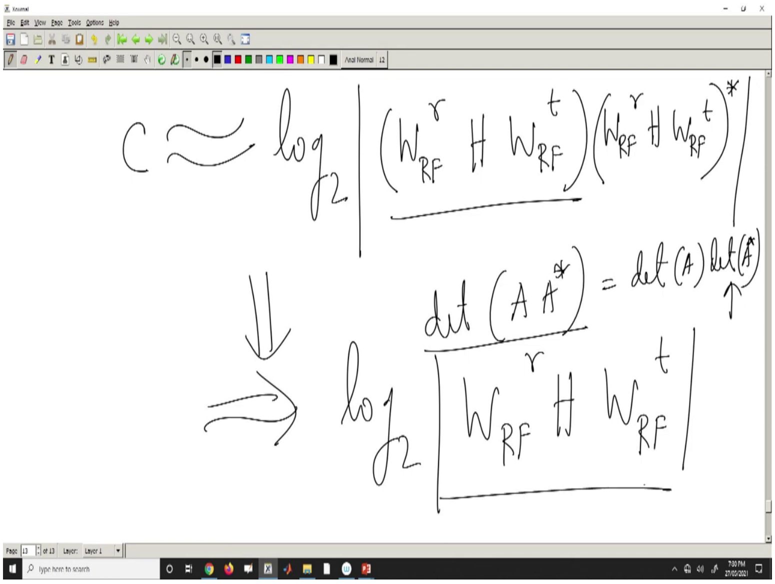

So, ultimately what I am left with is something like my capacity approximation of log base 2 I do not consider I because I is a constant, the introduction of I does not change my equations right, in the sense that it does not change my optimizations equations much though it is an approximation because it is just a constant addition ok. So, I just get rid of that. So, finally, what I get is W RF t H W RF r am I right, yeah sorry opposite this is r this is t and the same equations conjugate r H W RF t.

Now, I have some advantage, what is that determinant of A into determinant of A star this is like A into A star So, determinant of A into determinant of A star right determinant will not change. So, I can take only one of them right because this is same right determinant of A and determinant of A star does not change. So, I can just take one of them.

So, from here I can further approximate this whole thing 2 is not equal mind that I am not saying this is equal this is a; that means, solving determinant of A into A star is as if like determinant solving just A. So, that is what the point. This is what is coming, only one of them is coming into picture 2 does not come into picture why 2 should come here ok, because this whole thing is like a determinant of A into A star.

So, what does it mean determinant of A into determinant of A star, will this extra thing gives me anything extra because there are same variables are there just conjugate determinant of A determinant of A star all are the same right. So, I can just take only one of them this is good enough for me. So, this is what come.

So, if I make these two critical assumptions, now here I am not making the other assumption which I solve which I assume that W RR t and W BB t and their conjugate equal to I, that part I am not making an assumption. So, here it is totally different assumptions I am assumption that this is just a unitary only this part is unitary nothing else nothing else.

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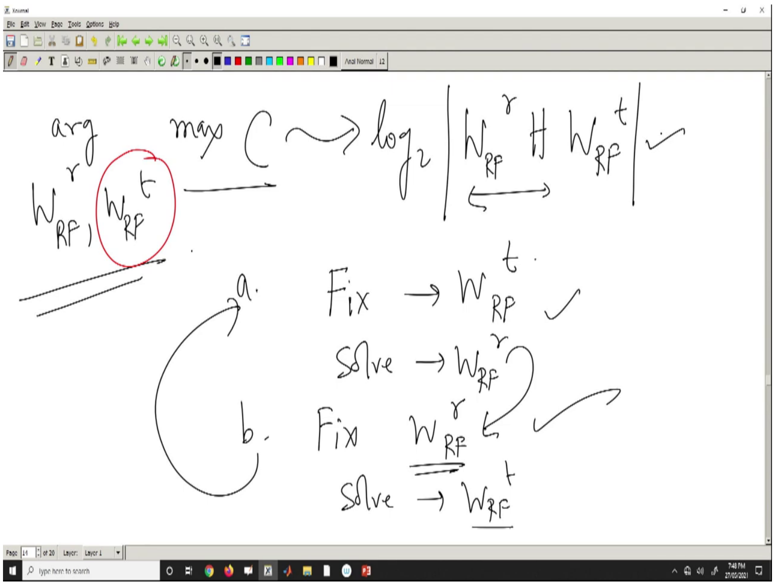

Now, I got 1. So, what is my final optimization equation C which is equal to log base 2 just determinant of you know W RF r H W RF t arg sorry this would be maximize here it will be arg maximize this C over W RF r and W RF t that is it. So, I have solved this equation I have not solved it. So, it is only a basic way of representing another it is another optimization framework just another optimization framework ok.

How to solve this still I am not done, take the other approach well, what is the other approach. Now here I have reduced from 4 matrix to 2 matrix. So, what is my approach step 1 what should I do, fix W RF you fix W RF ok W RF t fix you W RF, then what will happen, only one of them will be remaining that mean instead of solving this I am just left with W RF r. So, solve W RF r then what will happen, fix W RF r and then you solve W RF t.

Now, when I say fixed W RF it means that it is the solution that we get it from the first stage, take that solve it and then you go back here again you fix W RF. Now in this case fixed W RF is not something random it is the solution that we got it from the second stage and this whole iteration will keep on going keep on going you iterate till you converge in the sense that till you think that you are not getting any new W RF r at every iteration stage you said ok that is what my W. So, now, you got it, this one very simple approach ok.

Now, you got W RF r and W RF t ok, what about the other two equation, because now what I am saying is in this approach I got W RF t and W RF r, assuming W BB t and W BB r as an unitary framework or unitary matrix under that assumption this problem can be solved easily ok, this is a simple problem to solve ok. So, that I have solved it. Now, what about this other two ok either we can put it in a optimization framework or there is a another interesting way of solving it ok.

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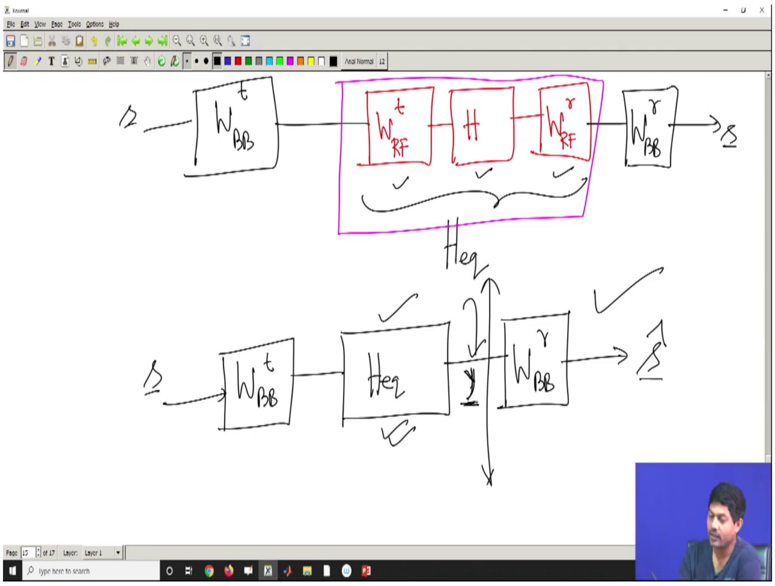

So, let us discuss that ok. So, let me draw it for your convenience this is the case and let me put a different color and I will tell you the reason W RF this is t, this is H put a different color because now I am going to solve for the W BB part base band part or the transmitter and receiver this is what I am going to solve now.

So, what I am going to solve these two black boxes. So, the red boxes I have solved it, how do I solve, what is the assumption W BB t W BB r areunitary that is my only assumption. So, I have solved it I have solved this and I have solved good. So, here all these cases I know it is assumed that I know my channel ok that is the key assumption. I am not getting into the channel estimation algorithm, in the next last topics last module probably I will try to have a channel estimation algorithm which is nothing, but your angle of arrival estimation angle of departure estimations algorithms ok.

So, now this part is done this part is perfectly done ok. Now, this is this red boxes are solved. Now, I want to solve W BB t and r. So, either I can put it in my normal you know normal optimization framework and then again that same way assume now W RF and W RF are known then you fix W RF t BB r to some unitary value then solve for W BB t then again and this iteration. So, one way of solving it, but this is the iterative and who wants iterative, everybody wants some you know closed form solution.

Now, this particular approach can give you a closed form solution for W BB t and W BB r and I tell you that. Now, if you remember the way we have solved the precoder problems, SVD based precoder do you remember, I do not know if you remember or if you do not remember I again reiterate the problem.

Let us assume this is my equivalent channel ok. So, what does it mean? So, this is my let us assume this is my equivalent channel let us assume. So, how do you view this system, you view this system something like that. So, this is a precoder digital precoder where you are sending an s vector and then it goes through equivalent channel called H e q then your job is to design equalizer such that you get back your s cap ok.

So, what is the approach what is the good approach by which you can design W BB t and W BB r such that they maintain the unitary case ok. So, I may have discussed it you can check your previous classes, but again in this context let me just give you another flavour. So, now, you know H e q that is known to you because you know W RF t because you have estimated, W RF r you have estimated and H channel is also let us assumed it is known to you. So, whole thing is known to you. So, now, this whole matrix is known to you ok.

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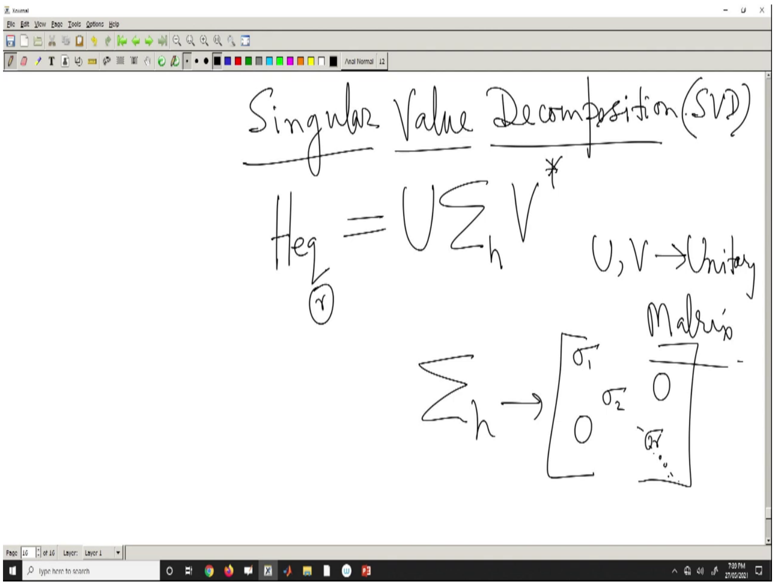

So, can I do an SVD singular value decomposition ok of what; of H e q what is that. Let us assume this is something like that because if a matrix is given you can easily know its SVD there are algorithms by which you can decompose into algorithm. Now, what is this U and V? U and V are unitary matrix. There are both the unitary matrix as per the definition of SVD and what is sigma H, sigma H is the diagonal matrix having you know singular values rest of them are 0's ok.

If it is a full rank matrix then you will have up to every up to every diagonal elements will be non-zero, if it is not a full rank matrix it depends where what is the rank.So, if the rank of this matrix is r you will have maximum r number of positive value rest of them diagonal element will be 0. So, that is ok. So, that is singular value decomposition problem. So, it is you can do an singular value decomposition.Now, if this is the model what is the this is so, it is as if like this is your ultimate goal. So, this is your you know this is what you are receiving. So, let us call it y. So, what is your y?

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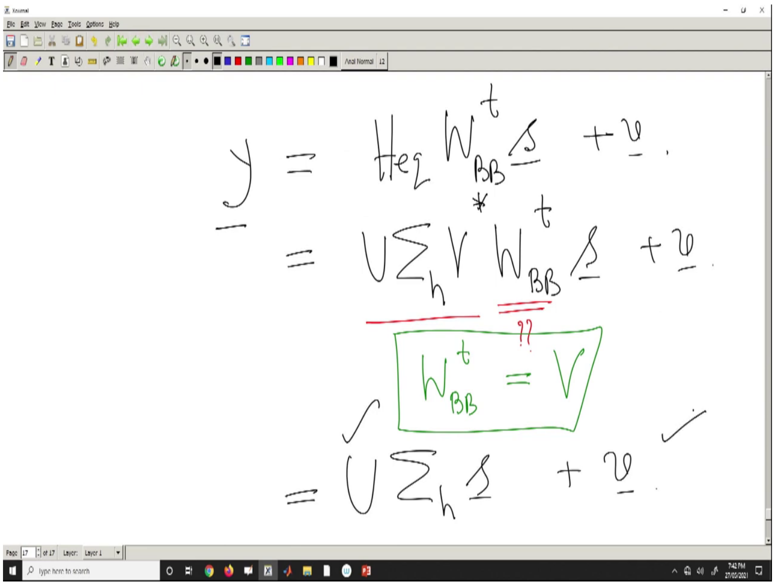

Let the noise be there ok, y meaning this part I am talking this point if I say this point is y what is there, because I am yet to design my W BB r you just break it down U ok right ok.

So, what should be my natural choice of W BB then? What do I want, I want to reduce the complexity of my system right. So, what, now there is a theoretical analysis you can also say what can maximize your capacity and all, but in a very simple lemon term what is your natural choice of W BB if my channel is broken down like that; obviously, you do not want v to come into picture right in your equation.

So, what if I take my W BB t is nothing, but this V, what will happen simply this W BB take make W BB t if I assume it to be V this whole received y will be, it will not have any V into its equation. So, directly I will get it correct. Now, this U part is left, now what should be my natural choice of W BB r such that if somebody gives me this kind of equation I can have very easy way of, you know easy way of decoding what would be the best choice.

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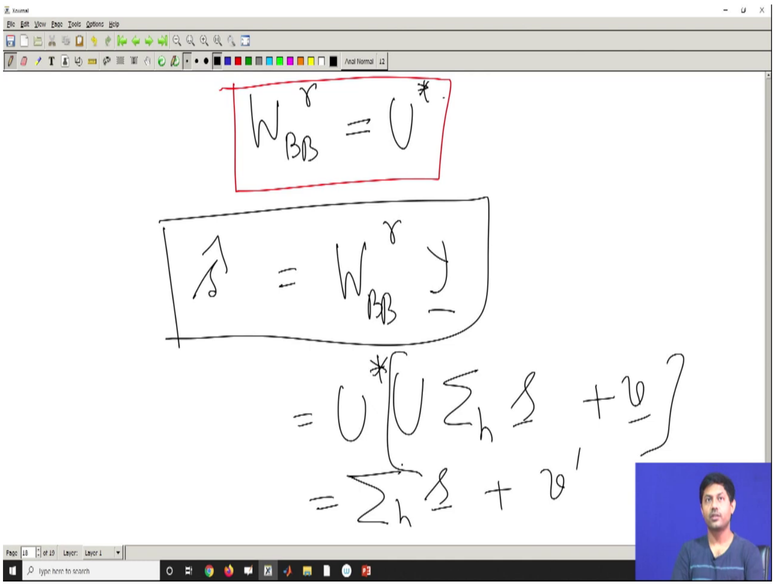

If I make this choice W BB r is nothing, but U star how would it be. So, if it is this how that would be. So, can I say in that case my s star s cap that is what it is, it is nothing, but W BB into your y bar right. This is how in a linear system the equalizer will be developed like that.

Now, I say W BB r is nothing, but U star U sigma h s bar plus v bar right. So, what does it mean, it means sigma h s bar plus v dash; v dash why write v dash, because that does not change the statistics of it if it is surplus symmetric Gaussian noise. So, now, this is very simple because this sigma h is now a diagonal matrix.

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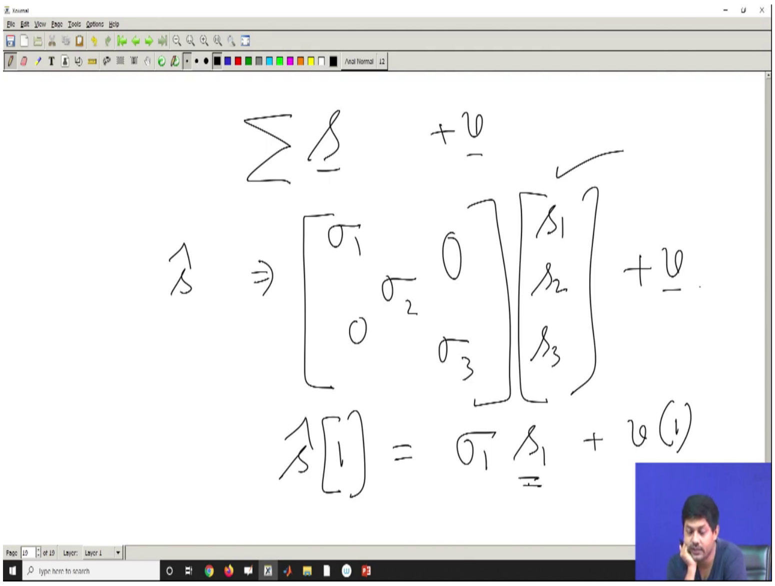

So, what does it mean, it means if it is a diagonal matrix I can easily decode. So, let say this is like something like this sigma 1, sigma 2 let us take an simple example sigma 3 this is 0 0 and this s 1 is s 1, s 2 take a simple case plus v bar take a simple case, how easy it is to decode s 1. Very simple, why because what if this.

Let us say this is s cap. So, I can just decode it individually say let us say I take the first one what it is? It is nothing, but sigma 1 s 1 plus v 1 who is not known, only s 1 is not known. So, it is a simple a WGN equation right.

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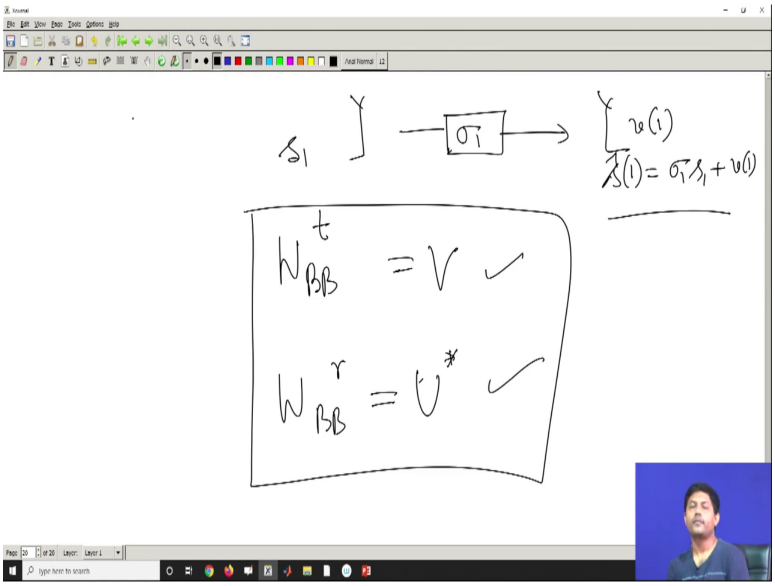

It is as if like I have a you know single antenna single antenna my channel is just a simple sigma 1 and I am sending s 1 and I am having a noise v 1 hence my output receive data is sigma 1 s 1 plus v 1 is not it like that is not it like that like right.

So, I do not have any matrix because if you notice the wave OFDM has been introduced by me simple way of decoding so; that means, that makes my whole decoding also simple ok. So, what does it mean, it means if I take my W BB t and W BB r as U star and V. Who is U star and V?

That is the that is the SVD of my effective channel. What is effective channel, effective channel is the this gentleman this the one which is shown in the red color that is my effective channel ok. So, as all three boxes are known red colored. So, I can have effect equal I mean effective channel and I can get it is SVD decomposition.

And if I simply take these two quantities does it violate my unitary assumption? No it does not violate my unitary assumptions, because V is a unitary, U is a unitary so; obviously, W BB t and W BB r is also unitary ok. Now, that is very good way of solving the problem.

So, if I make these two them these two of them as unitary matrix I can very easily get the solution for at least these two matrixes, but of course, W RF and W RF r and W RF t has the same level of difficulty in as in the first approaches which where I have you know solved it slightly iteratively, but here at least for W BB and W BB t and r I do not have to go for any iterative solutions I have to go for a closed form solution and that is the beauty of this particular approach ok.

So, these are there are many ways you can solve it. So, these are the two approaches I have shown it so far. So, in the first approaches I have solved transmitter I have made it transmitter side I have made it unitary; that means, the total transmitter side combiners digital as well as analogue combiners when it is multiplied with its Hermitian that I make it I and then we solved one approaches. In the second approach what we solved is that W BB and W BB t n r; that means, the digital side of my receiver and transmitter those two things I assumed them to be unitary.

Then I solved for the just the combiner at the RF level transmitter and digital level, transmitter and receiver level once I solve it, but I cannot get away with that too because that is still iterative, but that approaches has made my life slightly simpler because there if you look at the cost function; the cost function has been considerably simplified ok. It is considerably simplified in the sense that it is not only just two matrixes that I need to deal with at the same time I have to deal with a very limited number of equations component because I made further level of approximations thereafter and that has made the whole thing simpler.

But, at least this much complexity I still have to bear it in this particular approaches and then I say W BB and W RF W BB t and W BB r unitary and just a simple precoder based term SVD based solutions I can take up, but that is a closed form solution. Now, this is two level of approximations I have shown and using these two level of approximations I have solved the four matrixes that arise in the hybrid beam forming cases.

Now, in the whole cases what is the cost function that I have minimized or maximized, the cost function I have optimized is the capacity, but can I take some other cost function; obviously, what if I take a cost function like MSE mean square error. If I take a mean square error kind of problem sorry cost function how would it translate to my solutions ok.

So, in the next classes I will show you what if I take the cost function mean square error and what will be my approaches to solve those four matrixes ok. So, with this I conclude today's class where the cost function is taken as a capacity and with two examples of approximation, how we have approached our optimization problems and; obviously, in the next class I will take that MSC as a cost function and we will see what is the approach to solve that problem ok.

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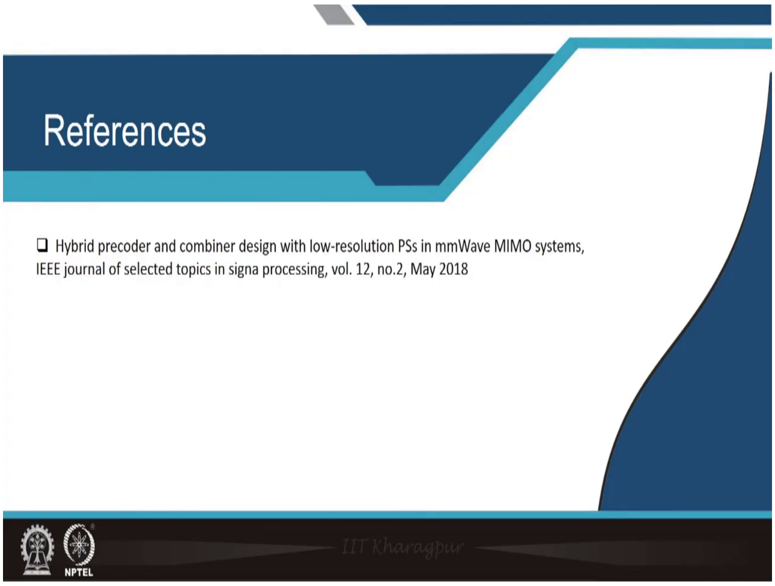

So, we have covered we have covered most of the cases of the design parameter using and the paper remains same, but as I said two approaches I have said only one approach the last approach is explained in this paper and the first approaches is partially explained from this paper, but you can make your own extension.

Thank you and we will see in the next class.