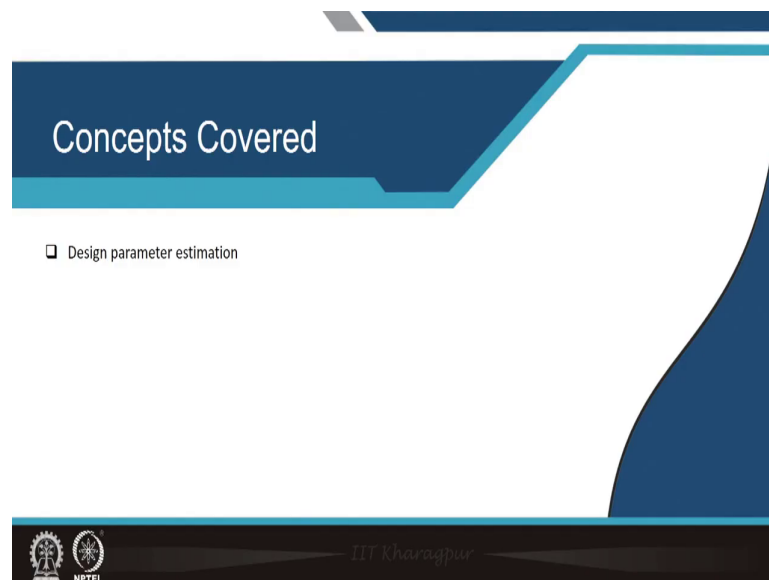


Signal Processing for mmWave Communication for 5G and Beyond
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Module - 11
Design parameter estimation
Lecture - 56
Design parameter estimation (part-2)

Welcome to Signal Processing for millimeter Wave Communication for 5G and Beyond. So, we will continue the Design parameter estimation this is the part 2 of that ok.

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And we would like to cover exactly the same thing, but we will it will be ok.

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$(H_{eq} H_{eq}^*) \rightarrow (W_{BB}^r W_{RF}^r H W_{RF}^t W_{BB}^t) x (W_{BB}^{t*} W_{RF}^{t*} H^* W_{RF}^{r*} W_{BB}^{r*})$
 $\|W_{RF}^t W_{BB}^t\|_2 = 1$
 $\arg \max C \approx \log_2 \left[\det \left(I + W_{BB}^r W_{RF}^r H W_{RF}^{r*} W_{BB}^t \right) \right]$
 $I = W_{BB}^t W_{BB}^{t*}$

So, we stopped it here right. So, this could be in the last class we have just explained this could be one of the problem set that we have just started it. Now, here also ok. So, this is one of the problem set that we have just left it in the last class. Now, we have said that if what if I make this kind of approximation this makes my you know life slightly simpler because that can just simply vanish some of the equations and you can solve it.

Now, does it solve? I mean is it a magic I mean does it solve every problem no I would say it is not a magic ok., Because it does not answer completely still I am left with two variables W_{BB}^r and W_{RF}^r ok. What next can I do some more because I am still I have reduced my complexity in my in my equations because from four matrix I have come down to two matrix, but it does not mean that I do not have to solve four matrices. It is just that at a time I have to take two matrix at a time.

So, this is one such approach. So, how do I solve even now this is still not done I have kind of you know not been able to solve it fully ok. Now, let us see can I take some more approaches to reduce further ok. Now, let say let us say now in this particular problem how many variables are there two variables are there right. Now, if it is a two variable I cannot solve it because it is not it I can solve it, but it is still NP hard, but it is a reduced complexity ok.

So, can I make one level of I mean I am just telling you how exactly a solution approach should be I am not telling you what is the solution what is the solution you can always get it in papers and you can always find lot of tools to exactly get that algorithms of you know stochastic descent or any other the kind of things, but what is the approach of solution. So, that is more important rather than exactly solving it; solving meaning you have the standard tools to solve it.

Now, in this case can I solve it in of course, this will be an iterative manner, but it is something like this.

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At first: \checkmark Fix $\rightarrow W_{RF}^r \rightarrow$ start with random matrix

arg max $C \rightarrow \log_2 \left[I + W_{AB}^r W_{RF}^r H H^* W_{RF}^r W_{AB}^r \right]$

W_{AB}^r

Simple problem

So, can I say I fix W_{RF} sorry ok can I fix W_{RF} in this case. What does it mean? It means that this is an iterative optimization problem. Now, I am I make one step ahead to solve it. So, what I do I said that ok I take W_{RF} , I fix it what does it mean by fixing it you take some random matrix there ok take some random values.

So, what does it mean you know what is W_{RF} ; what is the W_{RF} ? W_{RF} is basically your steering matrices right you assume some steering matrix some initial. So, you start with a you fix it some start with some random number keeping that structure same keeping the way the you know AOA and AOD equations are coming into picture. So, those kind of things of course, should be maintained you start with that.

So, now again I further divide the problem or rather I just say that in this case I fix this part I fix it I start with some initial known value. Now, what is the known value if you do not know

anything put a random number with some exponential because the structure of WRF is an exponential matrix right it is like each and every component is e to the power something. So, that is how the AOA angle of angle of departure equations enter there right.

So, if that is the case it should be some exponential some exponential start with some if WRF is say n cross n matrix start with some n cross and random values, but each and every value is e to the power j theta format start with that I do not mind it ok. So, you fix this then finally, what happens?

Finally, equation does not change equation does not change, but the only thing that has changed is W BB r probably I put a different number W RF r and they remain same as it is put this I put a different color code. So, that it will be easy for you to understand W BB r ok.

Now, what is my what is my optimization now? You maximize this quantity, but argument is just WBB . Yes, is it now an NP hard problem probably no. Now, it has been well reduced, but here catch is it is for a fixed W RF . So, it is does not solve the problem, but it reduces the complexity. So, that mean whatever W BB r you get that is for the particular W RF you have assume, but that may not be an optimal W RF right.

So, what is the next algorithmic level? So, this is my so, the so, if you see how what I am doing is it is a cost function having four matrix and making some approximation and assumption and I am trying to reduce the number of complexities as much as I can ok. This is something that I can I can think of ok.

Now, you can also think of your own assumptions and start doing optimization no worry because this is all n p hard problem all depends on how what is the approach that you fix it ok. So, this is one more approach I fix this W BB r ok. So, now, what will happen? This WBB r what you get now this is a simpler problem. So, this is a simple problem to solve it.

Now, simple does not mean that computationally it is simple only that is what I am saying, but still it has to go through iteration. So, still it does not give you the close form expression it may not give you. So, you have to still solve it using stochastic descent or any other Newton

Raphson's or any other method that that is there. I am not getting into those kind of issue because that is more of an optimization discussion.

So, what exact method of solving that is up to you; you can even take a genetic algorithm you can take any other grid or non grid algorithm whatever approaches, but this is the formulation of my optimization problem. So, this is a simple problem ok. Now, but the problem here is this W BB r is only optimized for a fixed W RF.

Now, how to it is not optimal right because this is just one part of the story. So, can I, I have to I have to now go to the second level; second level is this is my first level. So, first at first I would say at first you fix WRF further reduce the complexity.

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At second: Fix W_{BB}^r (soln. of Prev. stage) ✓

arg max $C \Rightarrow \log_2 [I + \dots]$

W_{RF}^r

First stage $\rightarrow W_{BB}^r$

Second stage $\rightarrow W_{RF}^r$

Then what you do second what you did. So, fix the other part other matrices $W R W BB r$. Now, in the first case I said $WRF r$ you start with some random matrix because that is the only starting point you can have. If you have if you have nothing you can start with something start with some random matrix with that structure maintained then you get $W BB r$.

Then what I go to the second one; second one I say that now just the opposite one fix $W BB r$ and then you solve for the second one capacity equation $\log_2 I$ plus determinant of same equation the equation does not change equation remain same, but in this case in the color coding $W BB$ would be black here and $W RF$ will be the magenta here because $W BB$ is a fixed one.

But, here what is that fix I said $W BB r$ you fix it and then you what you do then you maximize this argument over $W RF r$ fine, but now what is this fixed value? Now, this fixed value is not random anymore. This is the solution this is the solution you get it from this step. It is an optimization problem you solve this optimization problem you get $W BB r$ you get something use that solution of I would say previous stage here you are not fixing it you fix it, but it is not any random value it is the solution of my previous stage what you have got it then you get $W RF r$.

Now, this $WRF r$ is for this particular $WBB r$ then now can you realize it what I am trying to do here it is more of an iteration. So, stage is that then you again go back to the stage number one, but in this now case $WRF r$ when I say start with random and that will not hold anymore because from the second stage you have got some $W RF r$. So, you go to the first stage and put that as your initial value solve for $W BB r$ again and this whole process will just keep on going.

So, I would say you have this first stage you get $W BB r$, then you go to the second stage you get $W BB W RF$ sorry $W RF r$ and then what you have to do you have to again go back here then again come back here and this whole process will keep on going keep on going ok.

Then what will happen at some point of time they will converge; obviously, because you see this W RF I started with initially some junk value because it is a random then I get some W BB then from that W BB use that W BB as my initial stage for the second one and then solve for W RF and then again go back to the first stage and take that W RF what we got it from the second stage put that as an initial value in my first stage and so, and so, forth ok.

So, this is an iterative one how will it converge we do not know whether it will converge or not, but you try give a try depending on how the matrix really will it give me the optimal value again that is not a very guaranteed thing optimal in what sense can I get the best capacity ever there is no such guarantee. Why? Because fundamentally what you are doing you are making approximation under this approximation yes probably you get some capacity.

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Handwritten notes on a whiteboard showing the capacity formula and an optimization problem:

$$C = \log_2 \left[I + P_0^{-1} (H)^* H \right] \quad \text{--- } \textcircled{3}$$

$$H = W_{BB}^r W_{RF}^r H W_{RF}^t W_{BB}^t$$

Optimization problem (labeled ①):

$$\begin{aligned} & \arg \max_C \\ & W_{BB}^r, W_{RF}^r, W_{RF}^t, W_{BB}^t \\ & \text{s.t. } \|W_{RF}^r W_{BB}^t\|_2 \leq P \end{aligned}$$

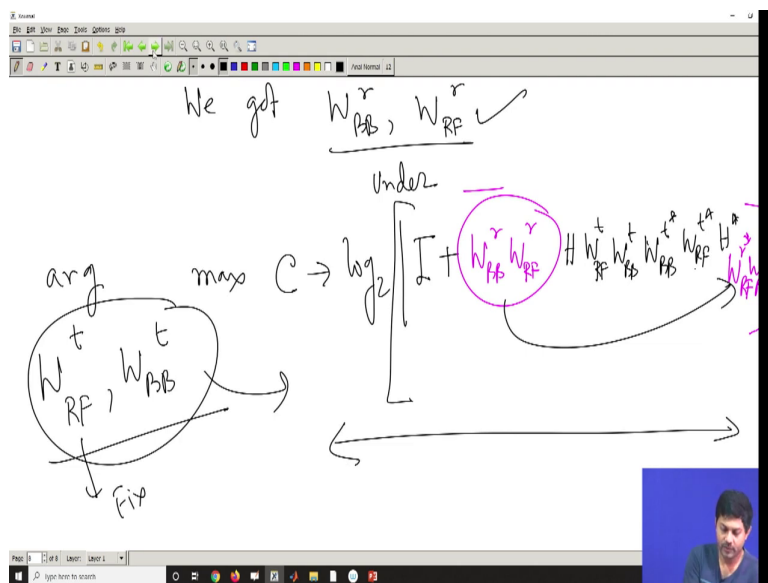
NP-hard

But, the best way to get it is that solve whole thing together, but that you cannot do because it is an NP hard problem and non convex NP hard problem that you cannot solve it so, easily. So, you have to make all these [FL] you have to make all these approximation and try to get as much as good capacity as possible that is it, but I am still not done.

Why I am still not done? Because I have only got this two ok what about the other W RF t and W BB t that I have not solved it. I have not solved it yet right this is just a you know just kind of a assumption say ok because using that assumptions I have solved it, but I have not solved the other two ok because I have only got two things.

Now, now you can realize what I am going to do it. Now, just look at this equation or rather I would say this equations yes this equations also fine this equation here because that is that is the main part. Here I have put W BB W RF I have solved it ok. You can now ask yourself what should I do next because out of this four matrices now I got W BB r and W RF r ok, W BB r and W RF r I have got it. What should be my next stage well simple stage in the next one. So, I can say. So, I have solved W BB r and W BB W RF r.

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So, we got $W_{BB}^r W_{RF}^r$, but we still have not got the W_{BB}^t and W_{RF}^t . So, we still need to get now this we got it under what assumption under assumption that that whole you know that whatever I have written that whole thing is the transmitter side matrix multiplication their conjugate and that will be I that was the assumption here.

So, now what is my next job? Next job is that I go back to this equation go back to this equation. Now, here this $W_{BB}^r W_{RF}^r$ this whole thing is no longer there it is there, but it is known to me then you put whatever you can have it let us put a different color code H ok. And then W_{RF}^t and $W_{BB}^t W_{RF}^t W_{BB}^t H^H$ and then W_{RF}^t sorry this part what I am trying to say this whole equation I am just trying to write it here $W_{RF}^t W_{BB}^t W_{BB}^t W_{RF}^t$ sorry H is ok. So, just a second ok.

Now, this would be W_{BB}^t star W_{RF}^t star H star and whatever this part is also there ok. So, let me write even that for your confusion removal I can say W let me put a different color code W_{RF}^r star W_{BB}^r star ok. Now, this part is known to me ok. So, can I write it simply maximize it now who are the argument W_{RF}^t W_{BB}^t .

This is the second next stage of the my problem. So, I first use that particular constant and solved it I got something then again I do that again I got something you know how to solve this problem. Now, again you have two matrices what you should do again you start with some initial value of one of them on probably W_{RF} you can fix it fix this first fix this fellow first solve for W_{BB}^t and then the similar way you have done W_{BB}^r and W_{RF} .

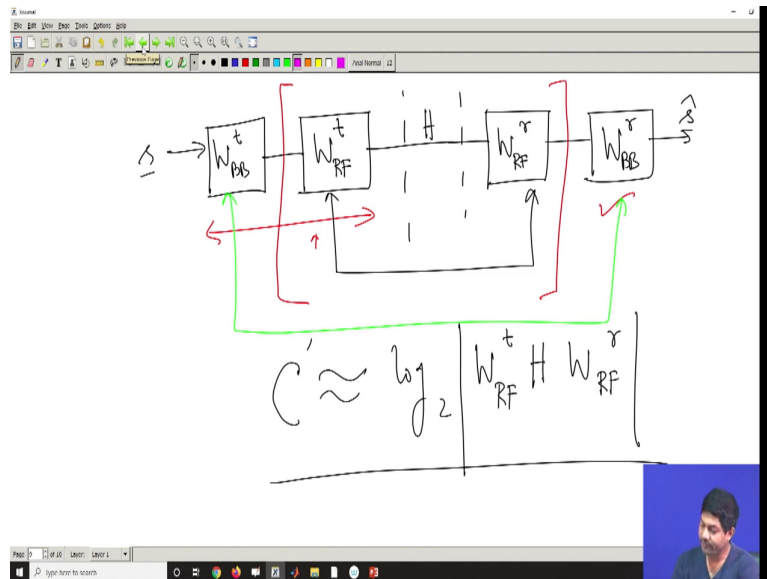
So, this is kind of a iterative environment. Now, is it a very best way of solving it well I am not saying that is the best way of solving it is one of the way of solving it. Is there any other way? Hundreds of ways you can look at so, many published paper where people are making several levels of optimizations with different type of assumptions ok. In fact, the paper I am looking at I am slightly deviating from that particular work this is more of a simplistic way of proceeding it. So, this is one of the way of solving it ok.

Now, I am not showing you the exact optimization method that is standard method again I repeat that, but breaking the problem into a simpler problems is what we look for in a particular physical layer signal processing aspect always ok. So, this particular one let us get into some other ways of solving. So, this is one way of solving it where I have assumed that some of them are you know I and some of them are not I.

Well can I have some other flavours by which I can solve some more problems yes nothing stops you from doing that. Can I mix and match say for example, this is a particular approach whatever I have looked at that approach solves what did it solve it is basically fixing the transmitter side right it fixes transmitter side solve the receiver side receive RF as well as the base band part.

There are works where it mix approach. What does the mix approach meaning? It means that can I have base band of TX and RX both solved and then go to the RF. So, what does it mean?.

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It means that you have this W BB t I am only telling you the approaches that people take it differently W RF t then I am not drawing the antennas and all. So, this is my transmitter side and at the receiver side. So, you can imagine that this here is the channel all these things W RF r and W BB r and then here you have s cap right this is your. So, this is so, what is my approaches in the first case. The first case approaches was that this one I made it vanish then I solved this first ok then I come back here and solve the first one.

So, basically I am vanishing one side transmitter side whole thing there is another approach. What is the other approach? Other approach is that other approach is that can I make can I

only solve these two first then solve these two what should I write it, can I solve the RF combiner part and then solve the digital combiner part ok.

So, what is the second approach? So, so far whatever I have explained in that work it was a transmitter side approach that mean I first solve I make the transmitter side parameter disappear go to the receiver side solve it whatever optimal way may not it is sub optimal everybody knows it.

Then I come back to my transmitter side using this parameter and say hey I solve the transmitter parameter one approach and this is what I have explained it. Now, I am going to say what is my second approach? Second approach is that can I solve a mixed meaning some part of transmitter side, some part of receiver side and then go back to my transmitter side go back to my receiver side again.

So, my first approach is that can I now can I now solve only the RF part in the transmitter and receiver pipe that also I can do ok that also I can do using some smaller approximations and so and so forth. Yes I can do that definitely I can do that ok. So, here is the approaches so; that means, first instead of going to the transmitter and receiver I mean only the transmitter making the transmitter vanishing. So, I go back to my earlier equation and say hey can I make this fellow can I make this. So, that mean instead of assuming this as an I let it be I do not make big claims about that. So, let that be that is ok.

But, what I am more interested is that $W_{RF,r}$ and $W_{RF,t}$. So, this is what my interest is I am not interested in rest of the things got my point. So, what I am trying to do here. So, it means can I just look at the boundary of my data here. So, it is as if like as if like I am only considering my data from here to here are you getting my point ok. So, that is the first thing.

Can I just consider only this much rest of the things I simply do not you know do not care about these two things I really do not care about ok. So, whatever this quantity it is? So, can I make another approach say. So, what was my capacity equation capacity equations you know what is the capacity equations and so and so forth that would be there ok.

So, can I say this is a quantity let us call it C dash can I optimize this quantity first because I do not have any other things ok. So, can I say I first solve $W_{RF} t H W_{RF} r$ ok. So, that is my first rational or first way of solving. Now, is it a capacity equation no this is absolutely not a capacity equation this is some equations where it is trying to maximize that particular. What is the rationale behind it? Why should I even do? That let us go back ok.

So, what I am trying to solve here. So, basically I am trying to solve only the RF part as we said; that means, this part exactly the this part ok. What happens to the rest of the part rest of the part I just ignore it with some approximation I said ok what if I just say that some of them are I so and so forth all these things happen. So, just the way we have taken it off I mean this is not a capacity equation, but this is only just focusing only that part ok.

So, can I solve only a small fraction part of it small fraction part of it ok. So that means, from this whole equation can I just grab a part of it and say that ok just what if I take the log I plus so, and so, only that particular one. What is the rationale behind it? Rationale behind it is that can I ignore this for the time being is it as if like I am not looking at W_{BB} . So, does it mean what does it mean it means in my whole transmission this $W_{BB} t$ and $W_{BB} r$ is not present get my point.

So, I am just sitting at the RF level I am just sitting at the RF level. So, I am not saying this is a capacity equation at the RF level this is a quantity ok this, but a particular quantity which comes into picture and I say that just I look at that particular one and I see can I maximize that particular data ok.

Now, go back here go back here this quantity whatever I have written and this quantity whatever I have written here are what are just nothing, but the conjugate of each other. Now, from this equation solving one is the same as the solving other because this is just the conjugate part there is no.

So, what are the assumption that I am making now which makes my life slightly simpler. First assumption in this particular approach is that $W_{BB} t W_{BB} t^*$ as an I, just the way I have

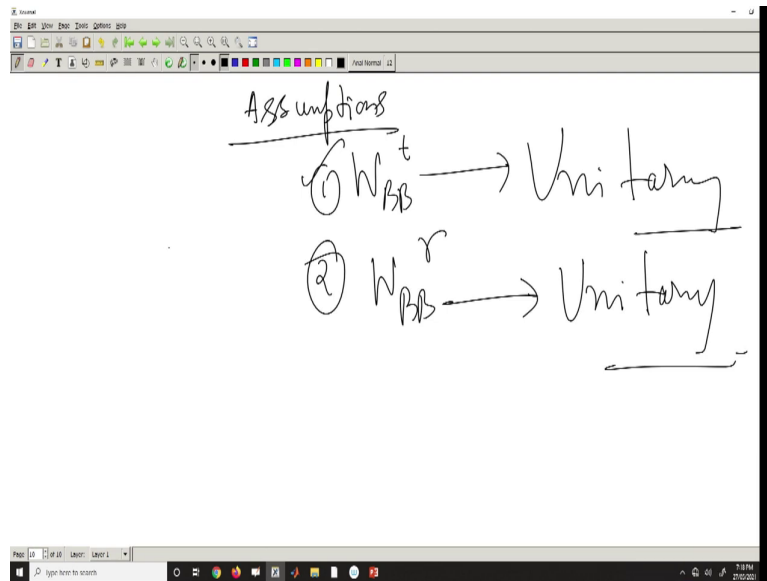
done it in the other case see as I said sky is the limit about your approximation as long as you have a reason to do that.

Now, in the second approach whatever I am explaining is that I am just assuming what if I assume W_{BB}^t and W_{BB}^{t*} as an I just the way I have done it in the first case. I have assumed the whole in between part make it I and I am assuming a if this is I what should be my solution and I will get some sub optimal solution.

Now, in this particular approach I am saying that what if this W_{BB}^t ; that means, that transmitter side base band matrix W_{BB}^t that is not I, but its conjugate if I multiply that becomes an I ok. So, what does this kind of W_{BB} refer to? This W_{BB} is nothing, but a unitary matrix so; that means, in my second approach what I am explaining is that what if I assume W_{BB} is an unitary matrix, how that makes my life simpler I mean does it buy me anything ok.

First so; that means, I make two approaches in this particular case. So, rational behind it is that what if I make the assumptions.

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What is the assumption here? My W_{BB}^t is a unitary matrix I can make an assumption I make an assumption my W_{BB}^t is an unitary matrix, under that scenario I want to solve my capacity ok approximation first assumption.

Second assumption W_{BB}^r is also I am assume unitary assumptions ok. So, how does it translate to, what does it translate to it means this middle part will vanish this W_{BB}^r and W_{BB}^r will also vanish from the equation original capacity equation y and that I will explain it in the next class. Why W_{BB}^r also would vanish from my equation if I assume W_{BB} is also a unitary matrix ok. So, with this I finish this particular one and I will likely take it for the next level of equations ok.

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Conclusion

- Design parameter estimation

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The slide features a dark blue header with the word 'Conclusion' in white. Below the header, there is a single bullet point with a square icon and the text 'Design parameter estimation'. At the bottom of the slide, there are two logos on the left: the IIT Kharagpur logo and the NPTEL logo. To the right of the logos, the text 'IIT Kharagpur' is displayed in a light blue font.

So, with this I stop. In the next class I would ok.

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References

- Hybrid precoder and combiner design with low-resolution PSs in mmWave MIMO systems, IEEE journal of selected topics in signal processing, vol. 12, no.2, May 2018

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And this is the paper, but exactly this is only a reference paper. Whatever I am talking of a part of it is there, but not every part at least the first part is not there some part of the first part is there. Second part is tentatively would be here ok.

Thank you.