Signal Processing for mmWave Communication for 5G and Beyond
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Module - 11
Design parameter estimation
Lecture - 55
Design parameter estimation (part-1)

Welcome, welcome to Signal Processing for millimetre Wave Communication for 5 G and Beyond. So, today we will be covering, how do you estimate the Design parameters ok, this particular class.

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So, we have just started the design parameters, combiner, precoder whatever we have talked about; how do exactly determine them, today we will be talking more on that. So, in that context what I need to tell you is that, let me just go back to my diagram; I am not getting to the exact hardware diagram, rather it will be more on the mathematical diagrams.

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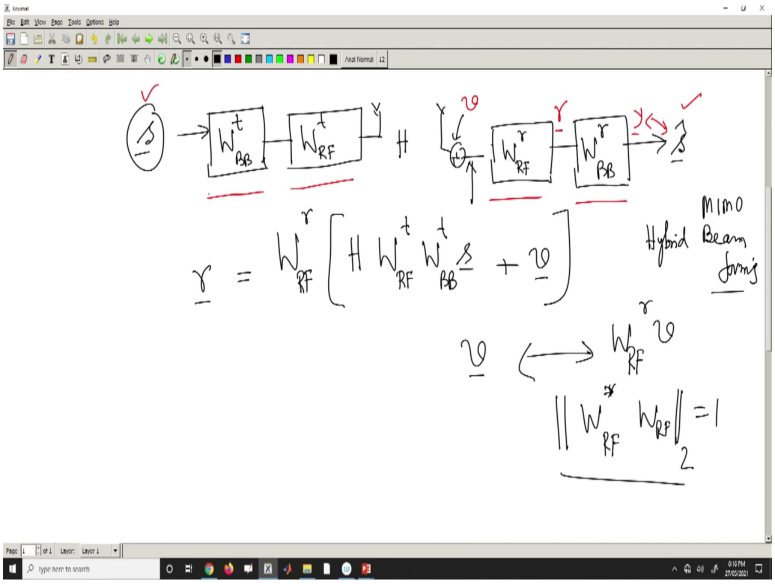

So, you have the vector s ok and then it will be going through the, we have the base band transmitter side precoder; then followed by it would be W t into RF side. So, BB meaning it is a base band or digital side. So, sometimes some paper you may they may call it a digital base band ok; I call it a base band, normal base band. And this RF meaning, it will be purely in the RF domain.

And then this is where the transmitter side and there will be multiple transmitters here and at the receiver side what you have is that, you will be multiplied multiplying the signal with W RF first; but with a r and then you have W BB r and this gives you what your estimated value of s, this is what we have done it, right. And in between there is a noise, so let me just correct this part.

So, when I say antenna, antenna need not be just the antenna, it is just a pictorial vision. So, there is a noise here, so that noise can come this side. So, this is what we have discussed it so far. And this is where your medium, wireless medium H. Now, this though it is a single antenna; it is shown as a single antenna, but it can be even a MIMO system. So, the point here is, this is MIMO beam forming; MIMO hybrid beam forming, not just the MIMO beam forming, ok. So, this is the MIMO hybrid beam forming.

So, we are sending, basically it is not just one data stream, rather it is a multiple data stream. Now, that can be OFDM symbols as we said; because as long as this the structure is important here, it does not really matter how s can come from, where it can come from; it can be just that directly a constellation or it can be just a OFDM symbols also. So, it does not matter. So, and that is of least interest to us, ok.

So, our interest now, how do you determine these four matrix, ok? So, this is the problem. Now, whenever you have. So, what is the goal? Goal is that whatever you have sent or intended to send, I want to get back that; going through all these you know this matrices and this noise v, let this be and then finally, I want to get back.

So, at every stage we have shown what the data model would be and if you remember, the different data model in the last various other classes, last classes; this I denote this as a r and this was this was r and this was y, you can say y and r can be same, that does not matter what variable we really took it here, so ok.

So, let us try to understand the data model here. So, at the receiver side, at the first level, at this level; so I am saying that let us not get into the digital or analog or RF. So, let us say this r, r vector. So, this would be your you know this is your, what can I say.

So, this would be W BB t W RF t, sorry opposite W RF t W BB t into s bar and then there is a channel; this is what you will be receiving at the primary antenna level and after that this is not r. So, this will be multiplied by your W RF r. Now, I was initially even you know not putting W RF here; you can put it, it really does not matter, because W RF as you know, it is just a phase shifter.

So, if it is a phase shifter multiplying v vector which is some sort of a cyclic symmetric and Gaussian probably, it really does not change its statistics. So, what I am saying is statistically v bar and W RF r bar W RF r v are all same; provided of course, provided this and this are, it is a 2 norm of this matrix is 1. So, you as long as you maintain that, it really does not matter.

So, that is why I loosely say, where that v, v should be inside, outside it does not matter as long as you maintain this. So, anyway as long as you understand how things will go on. So, this is my r, right. Now, next comes was y, y or s cap. So, I wrote y in the earlier case, because it was more of a general y. Now, let us say I am doing some sort of a estimation from the r.

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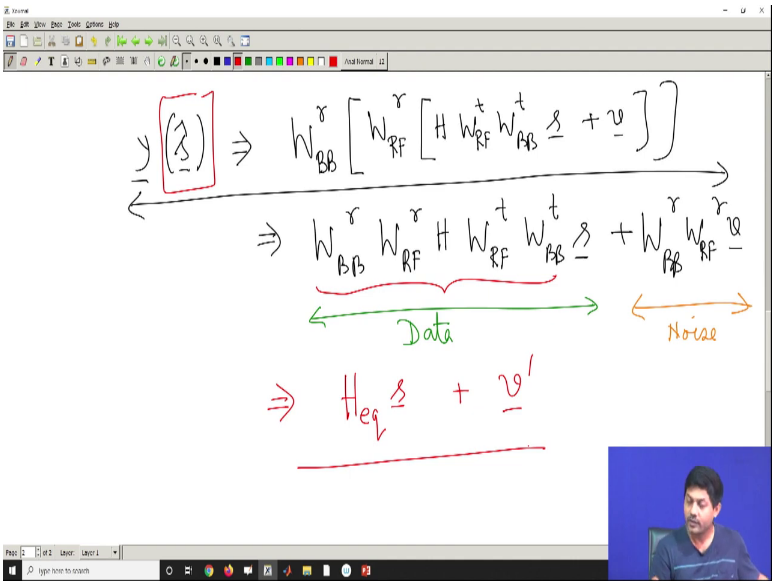

So, what can I say, in that case my y bar or you can say s cap; I prefer s cap, because that gives me more of a estimated value of s ok, this should be you multiply that BB at the r level and then W. Now, there are lot of terms that will be hectic terms that will come into picture.

Now, this I just copied from the other side, you see this whole thing will be part of it; H W RF W BB right, H W RF W BB, this is t, this is t, this is s bar, this is v bar and so and so forth. So, that is precisely what we finally want. I think this part we are tentatively tried to explain in the last few classes; but I had to reiterate, because this is our key equations which comes into picture.

Now, I am more interested here. So, whether it is a y bar or s cap it really does not matter. So, ultimately let us do some more surgery on this aspect. So, just I broke it down; this is one part plus W BB r W RF r into v bar. So, this is your, I just broke it into data as well as in the noise. So, this part will be your data part and this part will be the noise part ok; that is my noise, effective noise part, right.

Now you can say if this whole thing this particular part whatever I have written W BB r and W RF r ;if that fellows you know the two norm is again 1, then this whole thing, this whole quantity will be just like a one bundle. But that is ok, you can leave it separate as well, that is not an issue here. So, this is what it is.

Now, can I write some more much compact; you know some more compact way. Can I write this whole thing as H equivalence s bar plus whatever v dash, ok? Or probably I will put it in a different format; because as this is coming into picture here, so I will just put this as an H equivalent.

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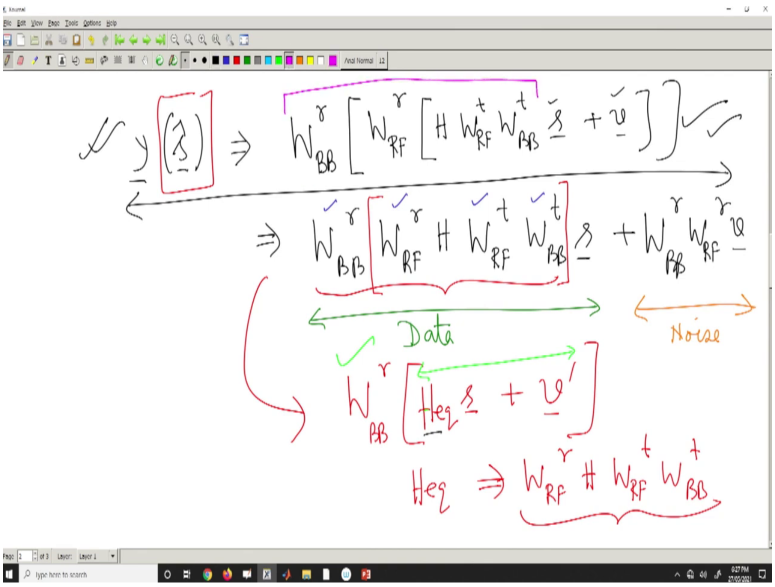

Let us say I put this part as an H equivalence, let us say this one, ok. So, what does other part is doing? So, this will be H equivalence s bar and this W RF r into v; let us call that as a v dash. Can I write it like this? Let me put it this way, this s cap; this whole equation, can I write it like that, where you can say H equivalence is in this particular case it will be W RF r H W RF t, there will be lot of bunch of you know matrices that come into picture.

So, let us, can I write it like that? Yes. Now if you look at this particular equation, you know how to develop W BB r; I think if it is a LMMC or any other things or you can even call you can include W BB also inside the H equivalent and call it. So, it does not really matter, how exactly you express the whole equation in what format.

So, ultimately at the end of the day what I am trying to, what I am trying to do here; what I am trying to do here is to estimate this, this, this and this, this four parameters. So, now, let us get into the optimization problem; but how do you do that? Because you cannot do such things, unless there is an optimization framework; that unless there is an optimization equation, it is very difficult to design the parameters. So, what are the optimization framework?

Like you require a first, you require a cost function. So, what is the cost function? It could be MSE, it could be you know it could be capacity, it could be SNR, it could be BER, it could be anything there are there are laundry list of cost function that you can take it. Now, at this stage, I would like to take two cost function and try to formulate a problem set, such that that can be a guiding force for solving this design parameters. So, let us take one such example ok. Let us say my cost function in this particular case.

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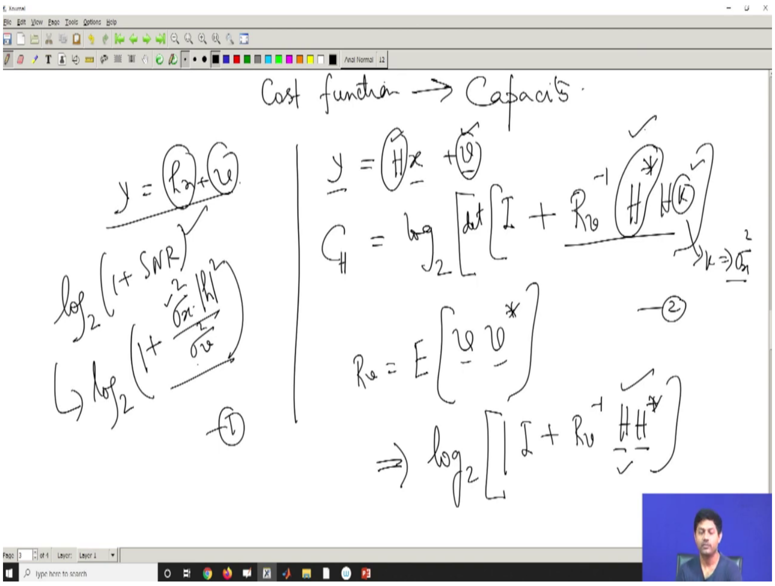

So, let us say my cost function is the capacity. What does it mean? It means that I would like to; I would like to design this W BB, W RF, W RF r and W BB r in such a way that at the receiver, I maximize my received capacity that is my design goal, ok. So, what does it mean? I need an equation and from there I just want to, you know I just want to know what are the what are the different aspects of this particular one that comes into picture here, ok.

So, let us see what is the equation from where actually now, this is that particular equation; this is what I have designed it, this is my ultimate goal x cap equal to so and so. That means, this x cap is my final design, final parameter of interest from which I want to take out them.

Now, from this particular equation whatever I have written, what is the capacity of this equation? Now, it is a linear equation right; this part is the noise, this part is the data. So, can I get the capacity equation for such things, ok? Yes, so that is very simple job.

So, let us for example, if some of you do not know how do you get it. So, let us say I have a simple equation; y is equal to H into x plus w x plus v. So, let us say, ok. So, this is my simple equation. So, this is a matrix. So, what is the capacity equation of it? So, capacity will be log base 2 ok determinant of; I, I is basically the identity matrix plus. For a single case what was that capacity?

So, let us for example, y is equal to H x plus v; it was not a vector, it was just a single scalar case. So, what was the capacity equation? The capacity equation was log base 2 1 plus SNR, right. So, that is comes around one log base 2 1 plus; SNR is what? It could be sigma square x into sigma square h divided by sigma square v, averagation, right.

So, this is my signal part, this is my noise part. What is the signal part? Average signal power, spectral density; sigma square x into sigma, sigma square x is the you can say that is the variance of that x, sigma square h is the variance of h and sigma square v is the variance of. So, that is the very standard equation.

Now, when I extend this whole thing to a matrix, what is the case? So, you can just correlate the whole thing, you can just create an analogy of it. So, what it comes around; it will be something like you know. So, in case if it is a v generalize it. So, let us say R v inverse is the covariance matrix of it, where R v is equal to expectation of v v star.

So, if you have this kind of equation; then it will be H star into H, that is what it is, now that will be some factor k comes into picture, that is basically the power adjustment factor those kind of. So, I am not bringing the k part; because I am assuming that the whole k can be you know engulfed in my whole H and all these things.

So, this is my this is my actual equations here; R inverse H star H into H star into H into k, k is nothing but it can be power or it can be transmit power those kind of restriction come into picture. So, this is my, in this particular case what will happen if x is a thing; then this k is nothing, but just sigma square v, ok. So, in our case it can be as simple as sigma square x in this case, it can be. So, anyway, so this is what the equation, right.

Now, if you look at this equation and this equation, there is no difference, except that if it is a vector and if it is a scalar. So, if it is a vector you can say R inverse v is nothing but sigma square v; H star H is nothing but and of course, there is an expectation here. So, if this H is a constant, this is what comes; if that H is not a constant then of course, H star H will be coming seen.

Similarly, in this particular case, if H is unknown, it is not a random variable; instead of sigma square h, what will be the case? Instead of sigma square which you have to say, it is nothing but mod of h star right, if h is a known. So, that mean the capacity for a known component of channel, not like a statistical capacity over all the realization of h.

So, it is your choice. So, this can be capacitive for a given channel. So, this is the parameter, right. So, this the same. So, this equation and this equations are exactly the same, provide unless and until there is a dimensional problem. So, they are basically. So, now, I extend this equation to for my data model, this is my data model, right. So, you just correlate that.

So, finally, what I need? I need H, I need a v. So, in this equation who is H, equivalent H; not this one, this was mod of my. So, here this is my equivalent H; that H whatever, not this H I am talking of with respect to that equivalent H, this H. And who is the v? v is this whole thing, this is my v, total v, that is all so; that means this is also simple a linear equation and I would like to have my capacity here. So, let us say. So, now, I am trying to understand what is my capacity here?

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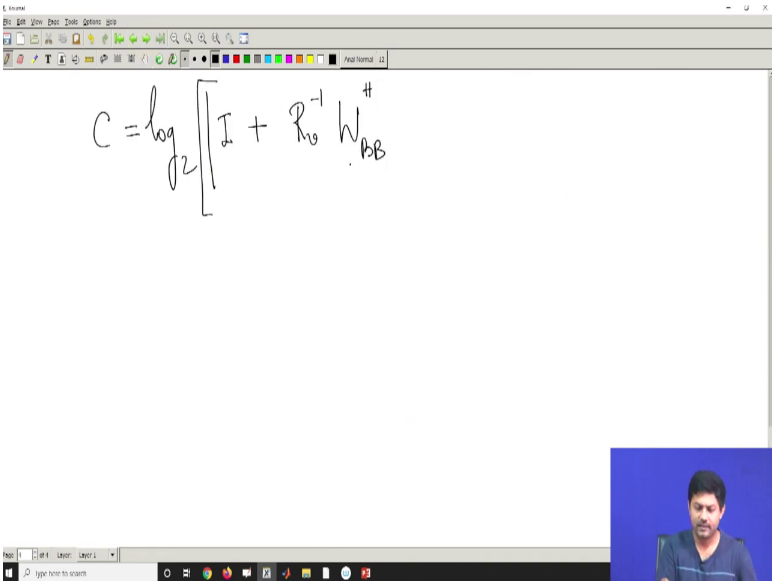

Now, this will be a very gigantic equation, ok. Now, these are all optimization problem and whenever you face such kind of gigantic equation; you have to at least write down the equation and see what happens, ok. So, let us see that. So, for our case who should, what should be my capacity equation? So, naturally this will be my log base 2 and this determinant, [FL] I just put a bar here.

So, that that just and you can imagine the dimension, it has to be n s what whatever your final received and received antenna part there, that is the only thing. So, now there is a factor, I am not getting into that transmit power factor. So, it will be more on the R, let me just write it down, ok.

So, who else is there? W BB. So, there is an H part, it is basically you can see that, it is H Hermitian, right. So, what is the H here? In our case, who is the H? H is this whole now, now you will be tired of writing this. So, let me just not write everything there, because it might be too much tiring.

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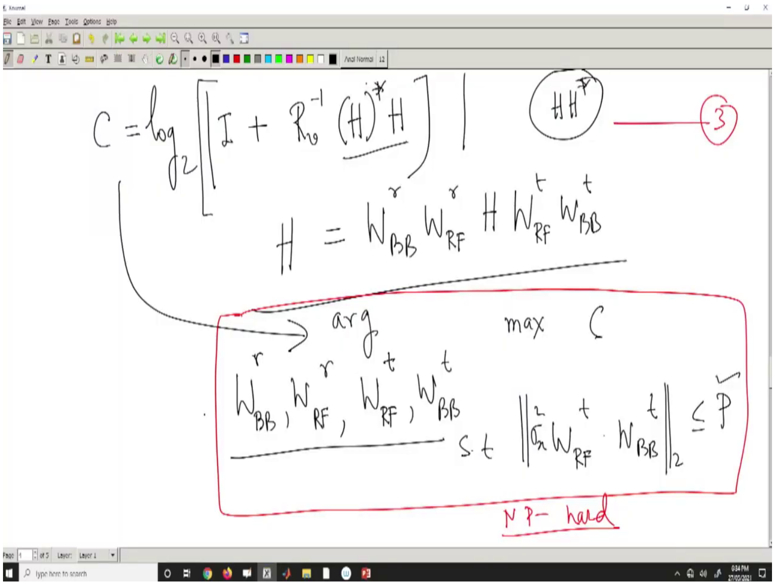

So, let me just remove some of the things. So, that it will be easy for you to understand; because otherwise this whole page will be written with this equation, because that is not the intention to, intention is not to scared you, rather intention is to make the things simpler, ok.

So, that is basically the case here, right. So, who is the H here? So, let us understand that part, ok. Who is the H here? H is this; W BB, W RF, actual H, W RF; H meaning this is that millimetre wave channel part W RF, W BB. And I think W BB, W RF r, r, this was the H, then W notation wise W RF, W BB, W RF, W BB, this is t, this is t. So, that is the point here right, it is a gigantic equation whatever comes.

And you can imagine how big the equation would be. So, anyway, so I am not getting into that; but sometime this equation can be H, H star or sometimes this equation can be HH star also depending on dimension, I think that part you must be knowing it.

Because when I say this y equal to H x plus v H star H this is also valid; log base 2 determinant of I plus R inverse H H star, provided it all depends on the dimension. So, if it is m cross n and if the m is smaller, depending on that how this particular H; sometimes H H star or H star H depending on the dimension, such that H star H is always occupying the lesser dimension, right.

For example if it is an m cross n and m is less than n, then this is valid if m is greater than n, then this is valid. So, I think that part is given, ok. So, whatever, so either it can be H star H or H H star depending on the dimension. So, typically I do not write that, but now this is your cost function, right.

Now, I what I want? I would like to optimize this cost function. So, what I mean? I want to maximize this cost function, this cost function. Over whom, who are my parameters, over which I would like to optimize it; this should be the case, this is BB r, this is RF r and then this should be let us say RF t, W BB t transmitter side ok, all this four parameter and you can maximize this equation, ok.

Can I have some constraint? Yes, you can also have some constraint. What is the constraint? You can say my transmitter side constraint can be there. So, for example, sometimes you can write such that W RF t and W BB t 2 norm of this matrix can be less or equal to set some transmit power, say some P; you can add also sigma square x as well does not matter, so you can add the constraint also.

Now this is, this part, this part is the optimization part, ok. So, this is your cost function. Who is C? C is that equation, equation number 3. In this case either H H star or H star H. Now, who is H? H is this whole gigantic equations, ok. Now, this is not a very easy task to do;

because the first of all this problem itself is a non-convex problem and it is an NP hard problem, ok.

So, you cannot simply get a closed form expression for such kind of problems, ok. Now, this is one set of issues, ok. So, what does it mean? It means that, I have to now start doing some sort of a approximation ok; because this is not a very close form solution, you can look at this.

So, that means I will never get any of this matrices with some equations, very difficult to get it because the way this problem has been formulated, this is an optimization problem. And you cannot go further, unless you use some sort of a an approximation to get some you know close form solution.

Now, such problem, now there are hundreds of papers available works available in various journals, where people are making several approximation to solve this problem. So, this is one such optimization framework. So, in this optimization now who is the cost function? Cost function is the maximization of my capacity. So, that is the first case.

Now in subsequent classes, I will take another cost function; that is basically your MSC, that can also be a cost function, now this is one of the cost function in this case. Now, let us try to make some sort of an approximation and try to see or try to give an example of what sort of approximation I should do, such that this kind of problem can be easily solved, ok. So, let us try to get into that kind of things.

Now, when you ask this kind of questions, the only problem is that, this kind of questions can be answered by the following approximation; this is the approximation first, the first approximation is that, what if I, ok. Now, you see the H star H or H H star if I just expand it. So, let us say I take the H H star, let us take one part of the story, ok. So, let us say I take one part of the story.

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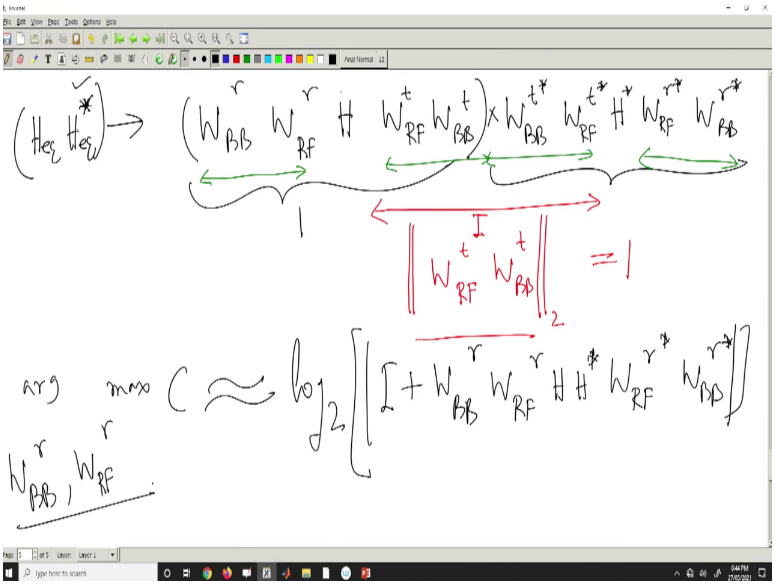

So, let us say H H star, just take any example. So, who is that H H star? Let us take one, let us expand it and see what type of approximation I can do, such that I can attempt to solve this optimization problem, ok. So, let us take W BB RF, W RF BB, ok.

So, W BB, W RF H is the RF and BB, ok. So, this is one ok and this is my H; meaning I am not talking of this H, I am talking of that equivalent H. So, probably if it confuses you, I can write H equivalent. This is Hermitian, sign is just a Hermitian; sometime people write it like a capital H, I put a star there, ok.

So, now this multiplied by what should I write here, opposite right. So, this W BB t; what should I write, some star here, then W RF, this would be t star here, H star here, W RF r star here, W BB r star here; you can see this, this is the second part of the story, right. So, this I make it.

Now, what is the approximation I can take it? Now, this BB are basically digital predecoder right and this RF is the RF precorder; neither it is in the transmit side or it is in the receiver side, ok. So, can I make the first approximation, ok? So, the way I can solve it is that, first I can solve; what if I assume that I design this parameter in such a way, this whole thing just becomes I, are you getting my point?

This is an approximation by the way, I am not saying that is how the problem should be solved; because actual problem is this problem, whatever I have boxed in the red colour and this you cannot solve it ok, it is an NP hard problem. So, you can solve it by using some sort of an approximation method or you can just simply do an iterative algorithm.

So, there are lot of iterative algorithms available. So, like for example, stiffness (Refer Time: 29:12), projected (Refer time: 29:13) of course, it will be projected (Refer Time: 29:15), Newton Raphson method, many and you can solve it and it may converge, you can get some iterative solution after long time.

Now, the only problem here is that, this is very difficult to handle it. Why? Because you are handling with four matrices and handling four matrices are really tough problems; you cannot just like that you cannot solve it, ok. So, people never attempt to go into this brute force method.

So, let us do an approximation. Now, what is the first approximation? First approximation what if I design my W RF, W BB t that particular you know that branch, in such a way there you know; I am not saying that W RF, W BB should be I, it is the W BB and W RF. So, in a simple sense W RF t and W BB t it is an approximation; it is my wish how I do the approximation, nobody tells me by the way, ok.

There is no hard and fast rule that, this is how you should proceed; if you because how to solve this problem, there could be some more than maybe probably 100- 200 I (Refer Time:

30:27) journals available or other journals available. So, everybody makes their own level of approximation, ok. So, it is very difficult to you know come up with one unified solution frameworks, ok.

So, I just take one example and just give you a flavor of what can be the way to wave forward; because this is not nothing to do with hybrid beam forming, I mean this kind of problem you will encounter in any you know any physical layer problems. So, you have 4, 5 matrices given and over the 4, 5 matrices you have to solve those problems.

Now, what I am making the assumption is that, I am making the assumption is that, in the transmitter side whatever my precoders digital as well as my RF precoder, their two norm is 1, that is precisely what I am assuming ok, that is specially what I am.

In fact, I am not even assuming exactly that I am going little forward; I am saying that this quantity whatever I have written and it is and it is conjugate matrix is actually I ok, does it buy me anything, ok. If you think very carefully look at this equation whatever I have written.

You see this particular part and this particular part the receiver part right; this is W BB r W B W RF r; this particular part and this particular part is in the transmitter part. So, what is the philosophy here? Philosophy here is that, out of this four matrices can I shut off some of them; it is like you have four problems to deal with, you just somehow shut off two of them and deal with another two as simple as that, ok.

Now, there are many ways to do that, this is one such way. What is that way? That way is that out of this four, you just say ok I do not want to consider my t x side at all at this stage, I just want to solve my r x side. So, can I make this assumption that, this multiplied by that would be simply your I.

Well, if you make this assumption, your life is easier. So, that mean my cost function whatever cost function I have written; that is gets common, see this was my cost function, where it has all the four matrices. Now, I am saying hey, I am assuming this four this particular one W RF BB t and all they are I. So, my cost function can be approximated log base 2 determinant I plus opposite W BB r W RF r H H star; because it is gone right, the other part is gone RF r star W BB r star.

So, this is mind it, this is not an universal solution. So, you cannot question me is it the best solution; no, I am not claiming, it is the best this is one of the solution out of hundreds of ok, this one. So, now what I do, maximize this part over the argument. How many arguments are there now? I have just two argument left, you see how beautiful you have done, ok.So, today I stop this class here. In the next class, I will see what are the other type of approximation I can do and try to combine more and more stuff here, ok.

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So, with this I conclude the, I mean this is a very long discussion.

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And the reference I am taking this particular paper, which is a 2018 IEEE paper and that particular paper I have; there are many papers, in fact 2020, 2021 also there are many reference. But this is one of the very preliminary work, where how they have shown how exactly solution; but there are many so, and there are depending on the paper, the approximation levels will also be different.

Thank you for this particular class.