Signal Processing for mmWave Communication for 5G and Beyond
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Module - 10
MIMO-OFDM beamforming
Lecture - 53
MIMO-OFDM structure

Welcome, welcome to Signal Processing for millimetre Wave Communication for 5G and Beyond. So, today we will be covering the MIMO-OFDM part. So, last class we have covered the single-input single-output OFDM structure and also, we have explained little bit about the CFO part. So, today we will be graduating to the MIMO and then, we will be entering into the MIMO beamforming MIMO-OFDM beamforming. So, that is the final goal of this particular module, ok.

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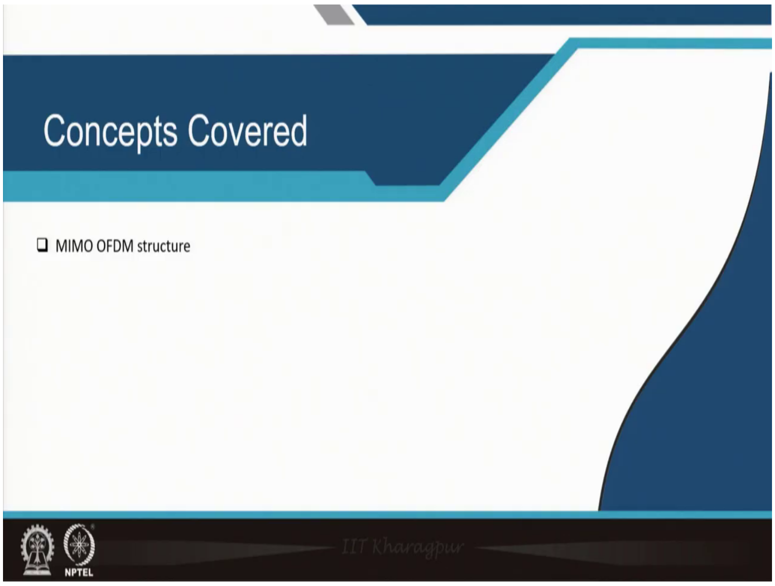

So, let us see that what we will be covering here. So, we will be just covering very simple thing that is called MIMO-OFDM structure. So, let us now go back to our structure again. So, we have already known what exactly OFDM is. So, now let us get into the a MIMO part here.

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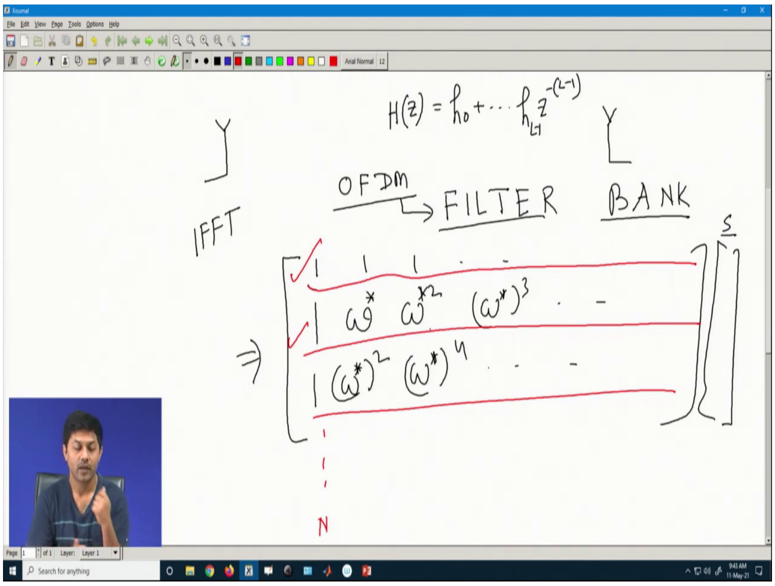

So, let us assume that for a single antenna case, this was the structure right. So, you have a FIR filter which was h 0 plus dot dot dot ok. So, that was the filter and finally, we have known how exactly an OFDM things work with CP addition CP deletion at the receiver and how the whole channel becomes a circular channel and what are the advantages and what is the effect of CFO. We have also discussed it and what are the different spectrum moment you do a FFT IFFT moment you do an IFFT, what is the time domain spectrum of it. Everything we have discussed.

So, in a very simple sense you can think of OFDM as a filter bank, ok. So, another view of OFDM is that it is basically a filter bank. OFDM is nothing but filter bank filter bank at the transmitter. Naturally there will be a counter filter bank at the receiver and that is nothing but the IFFT. So, if you have the IFFT matrix. So, this is your IFFT matrix.

So, this will be 1 1, then it will be 1 omega of course omega square, then omega star cube this is IIFT matrix, this will be omega you know omega star or omega star square you can say that way and then omega star 4 and so on so forth. So, you can think of this whole thing as a filter bank.

What is filter bank? It means that series a parallelly there will be lot of filters and the data is going through each other. So, the data is your s vector ok. So, you can think of. So, this is one; this is one filter bank, this is the second filter bank, it is an FIR filter. You can think of whose coefficients are the following. It is the third filter bank and there will be n number of such filter banks. Let us see each individual one is a FIR filter whose coefficients are so and so you can think of it that way.

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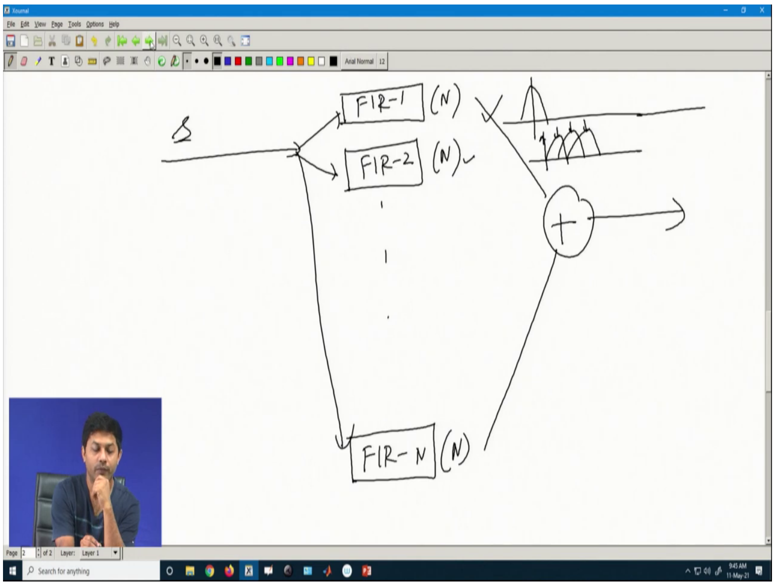

So, which means that your OFDM is nothing but so you have a data which is your s vector. So, there is a first filter, there is a second filter and there are nth filter nth FIR filter and each and every filter has a length N. This also has a length N, this also has a length N. So, the data goes everywhere and that is precisely I mean multiplication also shows the same thing.

So, here it shows precisely the same thing. So, this is the first filter first one and there is a second filter whose coefficients are written as I have written here and third filter and so on. So, these are the coefficient you can think of ok. So, like that it will be in N filter and finally, what you do you just merge them all together. So, this one will be having a spectrum like this.

Probably here this one will be having a spectrum like slightly shifted, not shifted like that here. Third one will be having whatever is like a shifted. The third one will be having a spectrum like this; fourth one will be having a spectrum like this and so on so forth. Like that there will be N number of such spectrum.

And we have explained that all the spectrums are overlapped. So, there is a counter filter bank at the receiver. They are nothing but the FFT filter and what the FFT filter will do? They are basically that filter is nothing, but a sampler ok. So, what it will do? It will sample the spectrum at a particular point and the output spectrum is a discretized filter discretized spectrum. So, we have explained all these things and we have also explained what is the effect of CFO. Now, we are graduating to the MIMO part. So, what happens when I go for the MIMO?.

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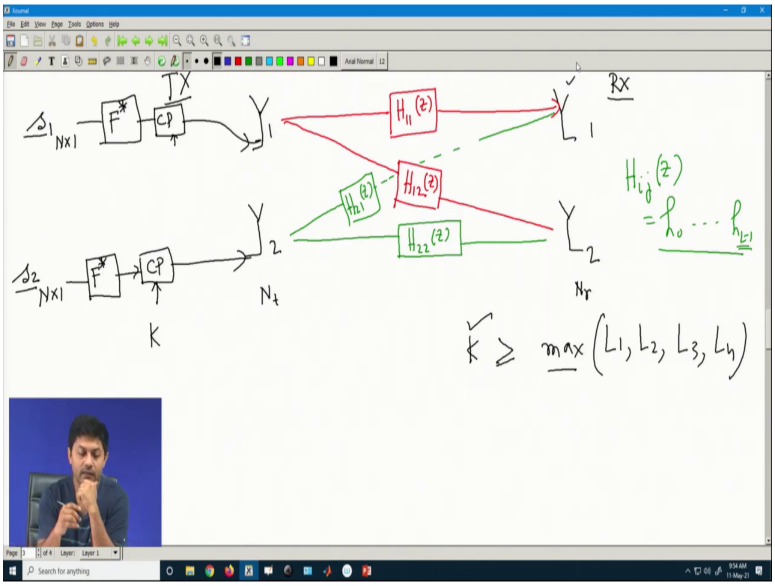

Now when you go for the MIMO, so let us take a bit of simple case. Let us say I have two antenna here. Now, all these things will be required when you go for the beamforming aspect ok. So, there will be MIMO beam MIMO-OFDM beamforming. So, let us say I have just taken a very simplistic case where you have a two antenna and two antenna at the input, two antenna at the output. So, this is my TX side and that is my RX side, ok.

Now, what is the big deal about it? I decide just an extension if you understand how the first one works, this will be just a cakewalk. So, you can think of. So, this is the first antenna, this is my second antenna, this is my first antenna at the receiver and this is my second antenna at the receiver. So, you can generalize.

So, how you can perceive the this channel model because when you say OFDM, then naturally I would not consider channel to be a single type channel ok because if it is a single type channel, there is no point of going for OFDM. I mean you can anyway you do not have any ISI.

So, OFDM will not do anything extra. There may be spectral efficiency, but that is the except that you I mean there is no; there is no ISI removal for that because this is a single type channel. There is no ISI anyway. So, but here the actual benefit of OFDM is basically coming from the fact that it has an ISI and you are not removing the ISI, but doing some you know internal processing like IFFT and FFT, you are squeezing the you are removing the ISI not removing. I should not say the removing the ISI.

I would say that you are making the channels or the system as if like the whole system is like a ISI free system as if like that, ok. That is the advantage. So, so the channels I have to consider them as a FIR filter. So, that means when I say I am taking a MIMO, it is just that I am taking each and individual link channel antenna to antenna link all in the digital domain. See antenna to antenna link as a FIR filter. So, this will be my FIR filter number. I call it H 1, it is an FIR filter. So, which means that it will have l number of taps or whatever.

Similarly, I will have this maybe I call it H 12. Now, this H 11 it is all z transformation. I can say because it may have multiple taps just the way we express a filter, right and this probably

H 21 this is the channel and finally, this will be another channel z is there not to worry about that. So, each an individual. So, I can say H i j z will have it will have N number of taps.

Now, does it assume that each and every link from antenna to antenna is having equal number of taps. No, there is no such things. I can have any number of taps. There is no such thing. In fact, if you understand the OFDM really does not care about that how many these taps are there, frequency domain taps are there. Really, it really does not matter.

Though I have not discussed the channel estimation part really. In this context of OFDM if the time permit probably at the end when you go for the parameter estimation in the context of beamforming, I will definitely touch upon that how exactly the channel estimation for the OFDM system is done. So, there you will see that I really do not care about H 0 H l1.

What matters is the eigen values because at the end of the day is the eigen values that gives me everything, right. So, it really does not matter. So, really does not matter what exactly the length of this channel is. What matters is the eigen values anyway. So, let us assume that this is what my complete system and I would like to have the OFDM system here.

So, how do what exactly I need to do here is that. So, this is my first in individual transmit antennas. I am making it equal antennas as a transmitter and receiver, but nothing stops you for from generalizing it. So, you can have N t number of antenna here, you can have N r number of antenna here, but these are not beamforming antenna. Mind it they are just data antennas. I have not introduced the beamforming, ok. So, this is just the data antenna.

So, what I will do here, you can let us take a two for simplicity. Then in the same equation, I can generalize it too. So, let us say so the way the structure would be each an individual transmit antenna will be you can think of it like a one OFDM system. So, what you do here, I can say this is my s vector which I was transmitting it earlier. Now, instead of s vector, I call it s 1 vector because that is its a n length total n length vector. So, this will be n cross 1 length vector.

This individually I will send it through on IFFT, then there is a CP addition and then I will send it to the antenna, ok. So, that is the; that is the part. So, it is like a every antenna is self-sufficient from an OFDM structure point of view. So, it is as if like you do not consider the R x part just consider the T x part. Every antenna will be feeding their own individual data n length data.

So, one each and every transmit antenna is like a one OFDM length. So, here similarly I will have s 2 vector which is also an N cross 1 vector. I will have IFFT, then I will have CP here, I will feed this ok. See if I have if you generalize it, obviously that many number of structures, so that is the only change in the transmitter side ok.

So, this is the extra part. So, now every antenna so if you notice how one wave how a non-OFDM MIMO works is that at a time every antenna will be delivering one in delivering one individual data. So, here also it is happening the same way, but now there is a catch. The catch here is that earlier case when you just consider the MIMO plane MIMO, it was more on the spatial domain. I mean you are at a particular time your channel is structured, but now that is not the case because every antenna is getting a serial data of n plus k length data, ok.

Now, another point that I would like to make it clear the CP length has to be same, ok. The individual channel length can be different. Somebody have a l 1 channel length, l 2 channel length, l 3 and l 4 does not matter, but the CP length has to be same. So, which means that if the CP length is k, this k must be greater or equal to your max of channel length. Suppose there are in this particular diagram, there are four channel blocks are there, right. Four channel FIR filters are there and assume that everybody has their own channel length tap length, ok.

So, whoever is the max I need to pick up k max of all of them. So, k is uniform, the k is not uniform, then whole things will collapse ok. As I said I really do not care about the channel length. So, I will always pick up a number which I would feel that ok my channel lengths are all within that. So, as long as that part is ensured, these things will work ok. So, this is how the transmitter slide is. Now let us go to the receiver side what happens, ok.

So, let us go back here. So, here just think of the first antenna ok. Just think of the first antenna. So, the first antenna receives the data from the first received antenna takes the data from first transmit antenna and the second transmit antenna, right ok. So, you can say that this R x 1, the first antenna of the R x if it takes N plus K length data, so what will happen if it is an N plus K length data.

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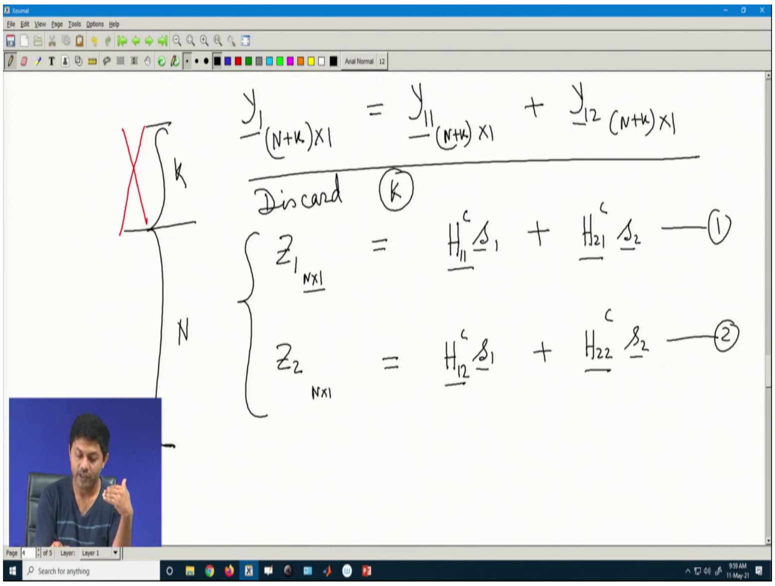

So, I call it say Y 1 bar see if it is say N plus k cross 1. So, what will happen it is the data which is coming from the first which is due to the first antenna. So, there is an N plus k cross 1 and there is a what is Y 11? Y 11 is basically received data from the first antenna, Y 12 is the received data from the second antenna. So, it is when you say Y 11 you just think that the second antenna second transmit antenna is not present. It is like the H, it is a solely responsible by the H 11 kind of things, ok.

So, that is the only thing. Now you can imagine what can happen it just that it is instead of one OFDM received data, I will have two OFDM received data. That is all. Rest is same. Again what you do? You discard k, the first k not the last k. So, if you have Y 1, let us say this is my k and this is my N, this is the total N plus k data. You discard this part just the way we have done it in the OFDM. Also, you take the last N data. So, what will happen, then I will get one what is the effective data then.

See it is as if like it is as if it is to summation, right. So, the first summation and there is a second summation. The first summation is an individual OFDM, second summation is another individual OFDM. So, that is all. So, if I discard it, so let us call it H 11 c, it will be circular channel right and it is completely s 1 vector. This is precisely what we have done for the single antenna single antenna and single received antenna case. So, this will be what was the antenna, this will be H 21. So, this will be H 21.

Now, I put c because just to ensure that it is a circular channel because it will be circular channel once I discard the N. So, this will be your s 2, bar ok. Now what you call it? So, this will be Y 11 or rather Z 11. Let us call it just for a simplicity Z 1 which is a N cross 1 length data. Now this is precisely the case. So, this is my equation say let us say 1. Now, in the earlier case if it was a pure single antenna case OFDM, this part was not there. That is the only thing otherwise it is just another term will be added, ok.

Similarly, for this is my case number 1. Similarly for a second antenna, second receive antenna this one ok what will happen? So, let us call it Z 2 N cross 1, right. So, this will be let us see what else comes for the second antenna. It will be H 12 and H 22. So, this will be H 12, but it is the s 1 data, right. This will be H 22, this is my equation number 2. So, this is a pre FFT data. I received it that is it. I am done.

So, now these are the two data that I have received it, ok after the CP discussion. So, in this diagram when I go for the receiver, what I need to do, I received individual data at every receive antenna and then, I discard the CP. So, which means that in this diagram let us remove this part, ok.

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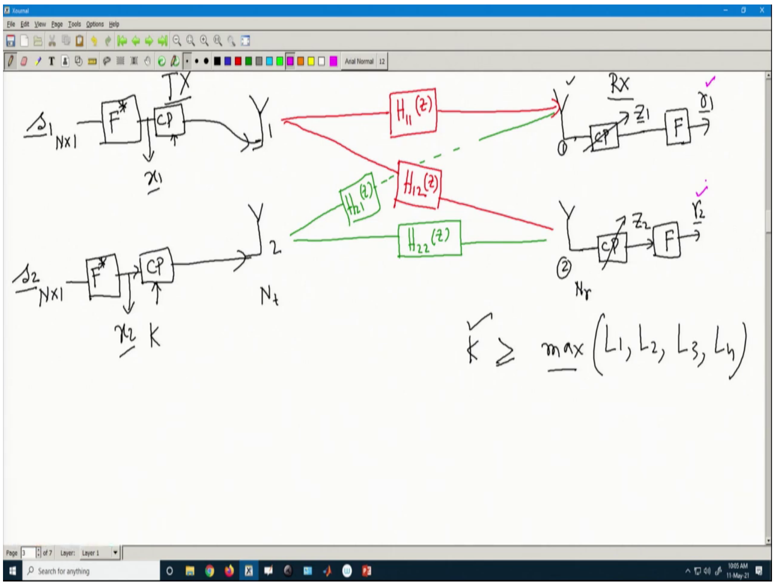

So, this is my antenna number 1; this is my antenna number 2. So, the first thing I do here at individual is I discard the CP here. Also, I discard the CP. What I received Z 1 vector, here I receive Z 2 vector, then what I will do is, I do the FFT. FFT I do, FFT operation here. Also, I do FFT operation. So, I multiplied with the F matrix I should naturally guess R 1 vector here R 2 vector.

Now, let us see what is exactly R 1 and R 2 vector, so that is the point. So, I have just done CP discard. Then what I am proposing is that now as the structure suggests that everything is a circular channel matrix. I should just do an FFT. So, now we do an FFT. That means, every Z 1 every Z 2 will now go through an individual FFT. So, what will happen?

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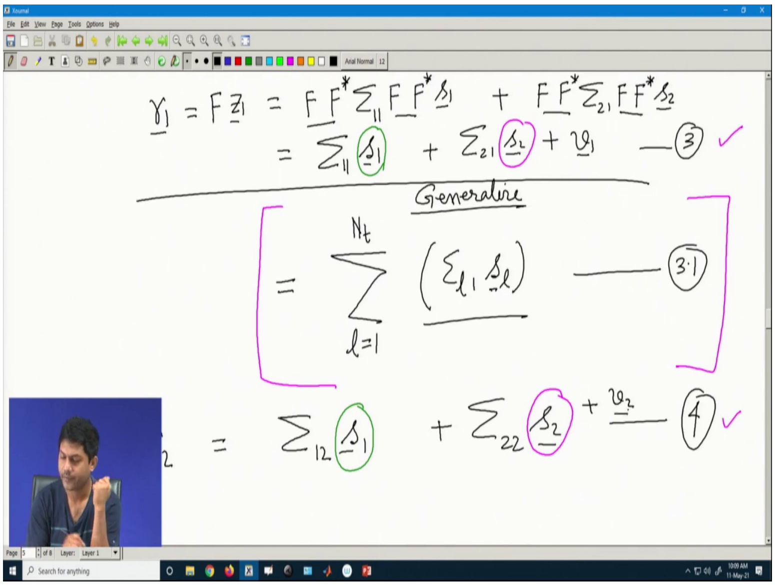

So, let us call it r 1 vector which will be F into Z 1 vector. So, now it will be F H 11 is there ok. So, let us break it F star sigma 11. Let us call it ok f ok and here yeah. So, just a second. I call it X. let me call it X, so that it will not create any confusion. This is what a transmitted vector is.

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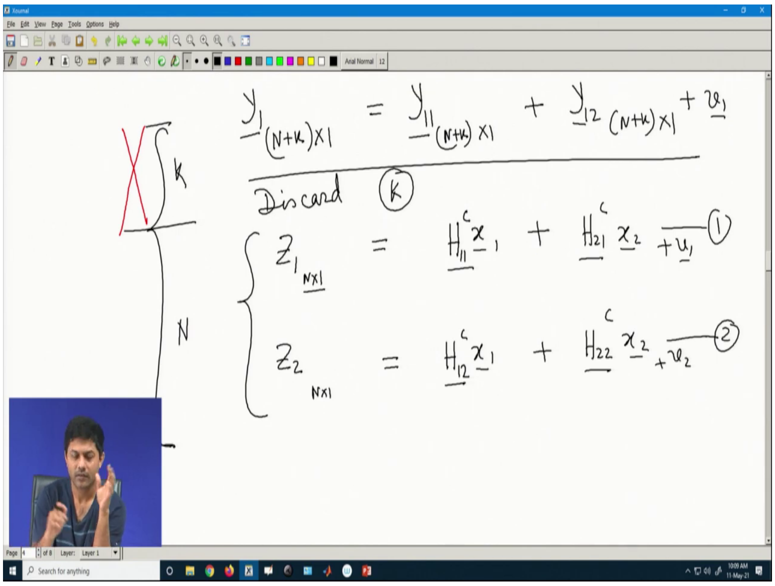

What is X 1 and X 2? So, basically this one, this one is your X 1 vector, this one is your X 2 vector. So, that makes the life slightly easier. So, this will be F star into S 1 vector plus F F star. Let us call it sigma 21. So, let us call it sigma 21 F F star s 2 vector, ok. So, make sure that it is X 1; X 1 is the one which is transmitted. This is the IFFT data, it is not S 1. So, what it would be? This will be gone, this will be gone, this will be gone, this will be gone right. So, it will have a sigma 11 S 1 vector plus sigma 21 S 2 vector. That is the only thing.

So, let us call it equation number 3, ok. Now, you see that it is a summation of two individuals, you know two individual summations ok. Number of now if I generalize it, so from here itself I can generalize the transmit sectors. Let us I have a N t number of transmit. So, what can I write this equation number 3? Can I write it like that summation of sigma say l, ok? 1 s of l let us say l is equal to 1 to N t. Do you agree with this?

Let us say this is 3.1. Just a generalization. Suppose I have instead of two or transmit antenna, I have N t number of antenna transmit antenna. So, I will have N t number of such sigma right. So, that is precisely what I have done. You have N t number of such sigma and then, you have N t number of such data vectors also present, right. So, that is the generalization that is, ok.

Now, similarly I will have R 2 also right. In the case R 2 also let us take second case as well. Now, this is a special case. I would say this is a general case, 3 is the special case of the channel. So, I just put it in a bracket and for the R 2 vector I just make an extension H 1 to H 2. So, what will happen sigma H 12 S 1 vector plus sigma 22 S 2 vector, ok. Now, this is precisely what happens when you go for a data. So, let us call it equation number 4. Can I generalize it? Of course, you know how to generalize it.

Now, can I generalize the R vector also? Of course, because this is only for R 1 and R 2. If I assume there are only two received data, can I generalize it? from the receive antenna point of view, yes you can.

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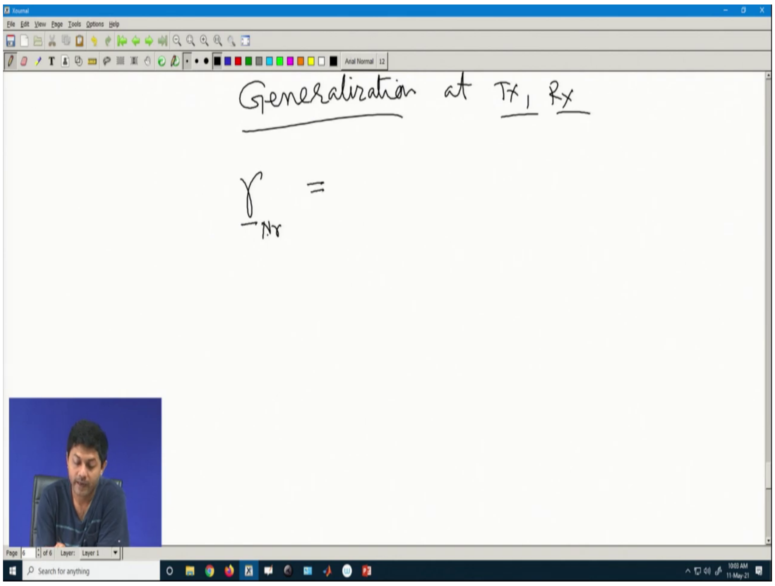

So, that means even more generalization at the receiver side also S N at T X R X both. So, what is that? Simply I am assuming now there are r N r number of received data. So, you have N r number of received data N r indexed total data. So, let us call it even general 1.

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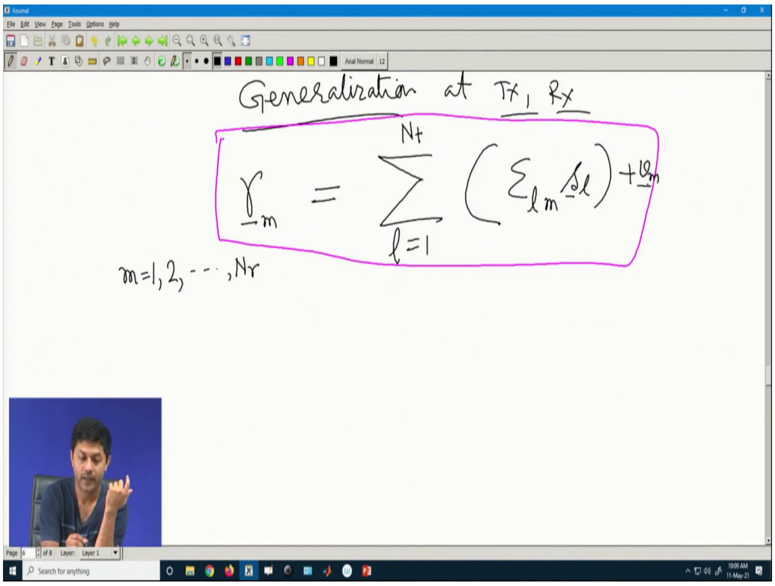

So, let us call it say r m. I mean mth you know the mth received data where m starting from N r any one of them. So, how do I generalize it? It will be summation as usual 1 to N t in the case, then sigma. So, what was the case here l 1 here. So, this will be l and this will be m, ok and then S l that is all. That is the generalized data data model. If I say that is my generalized data model generalized received data model at the mth received antenna, but that is ok. You can think of whatever way you can have.

So, now I am not putting any restriction on that. Now, let us get into the. So, this is the main part of my OFDM MIMO-OFDM structure. So, as long as you get your data. So, that means at each an individual received antenna, so at the end of all this operation what r 1 or r 2 gives me is a summation of multiple OFDM structure. That is all after the removal of your CP and after doing FFT, that is the data that I will receive it here, ok.

So, this is the main part of the I mean this is the main part of the OFDM structural point of view, but what I have not yet done is that how do I decode it. Can I decode r 1 and r 2 individually? Let us discuss that part ok. So, I got the data that is fine. So, let us take a simple case, our simple data model ok instead of generalized model. Let us take the this particular model first one.

So, equation number 3, let us take the equation number 3 and equation number 4, ok. So, this one just two antenna cases. Now every individual data every individual received antenna, they have got their data model. After FFT can I decode individually? That is the point notice that equation 3 and 4. Both of them contain S 1. It also contains S 1, it also contains S 2, it also contains S 2, right.

So, if that is the case how do I individually decode r 1 and r 2? So, that was the case for single wave antenna OFDM case SISO OFDM case, then you just receive a data, then what will happen you take the vector, take the individual point I mean the say ith element of r vector and I can decode it like a normal AWGN channel.

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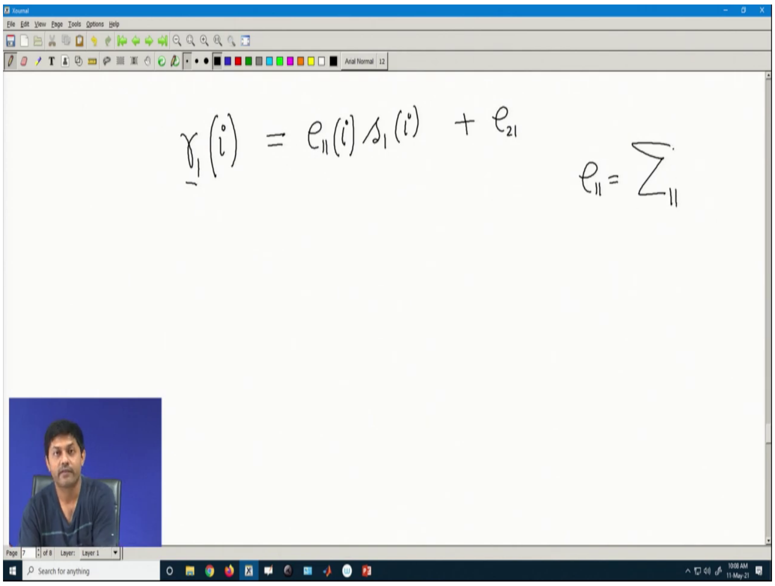

But this is not the case. If you take the ith let us take the; let us take the ith say r 1 vectors ith data, what it will contain? It will contain some. So, this was say sigma 1 1. So, it is the ith diagonal element of sigma 1 1, ok. So, let us call that e 1 1 ok. It is the ith diagonal element of e 1. That is the eigen value of that matrix plus what plus s 1 vectors ith element plus what else is there. S 2 or sigma 2 1 e 2 1.

So, e is what e is a vector containing all the diagonal element of sigma. So, basically what I am trying to say e 1 1 is nothing but this sigma 1 1. That is the matrix right but it is a diagonal matrix. So, the all diagonal components.

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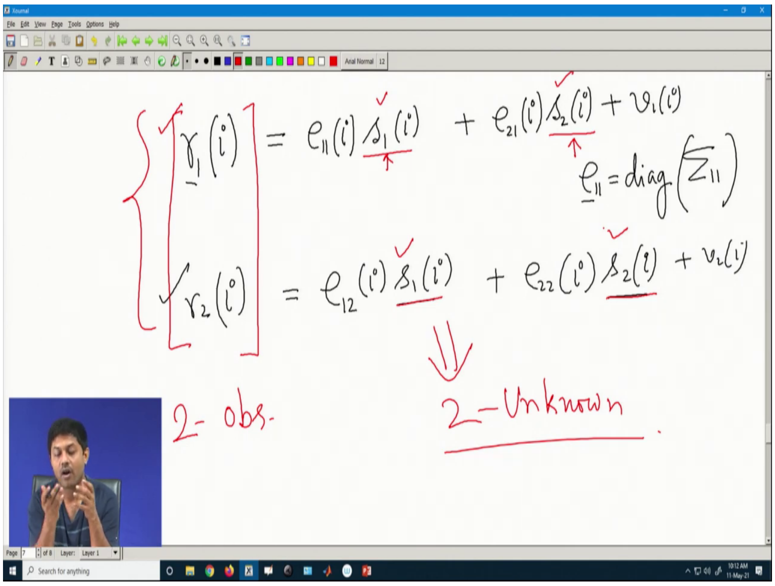

So, if I write it put it this way probably, I put a different notation. It is a diagonal element of all this sigma 1 1 matrix. So, that is e i e 1 1 vector. It is a vector right e 1 1. This is a vector. I should write it is a vector that contains all the diagonal element of that matrix e 1 1. Let us assume that it will be easier to understand.

So, here also similar notation, but now this is I have not seen I have not used the noise here. You can use noise that is not a big deal. So, here also you can should have a noise vector I mean I have not conveniently used this noise part, because that is not our goal to I am not doing an analysis, but let us ok you can have a noise vector just for your completeness. I do not mind it, ok. So, you can have a noise everywhere, you can have a noise yeah you can have noise yeah.

So, that is not our, so just for your understanding I just put the noise here yeah. So, this is the face now that mean at my receive antenna, the ith element is having this part and this part. So, now there is an ISI again. Now, this ISI is not exactly I should not say it is an ISI rather this ISI enters symbol, but it is not from the same antenna, but from a different antenna ok and at the same time.

So, this is an interference for r 11 or if I take S 2 as my data, then S 1 as my interference or if you say no, there is no interference because both of them are important to me. Then it is as if I am again coming back to the same scenario earlier that when I was dealing with the ISI channel that I have one observation from where I have to determine to unknown, ok.

So, now this whole MIMO-OFDM will stop me from doing an individual decoding that I did it for the OFDM because now I cannot take the ith element of my vector and decode for S 1 and S 2 and so on so forth. Similarly, if you go to r 2, I am just motivating you r 2. This will be the similar situation similar situation that when I take the r 2, this also has the same problem both of them r 2.

So, I cannot take individual point and go for a decoding ok. This is not possible. I should put a different colour of line because both of them, but have you noticed one thing here. Probably that is the motivation for my next class. Motivation is that if I observe both of them together instead of taking one individual, let us take them adds together. That means, I am stacking them together that mean both the equation if I consider it together as both of them have the same variables of interest now in a better position.

Now, I have two observations, but two unknowns can I decode it. So, here if I take them together instead of thinking ith element of individual data, I take ith element of all of them and stack them together ok. So, two observations here I would say and here two unknown now in a better position. So, if I take both of them together I can decode this fellow properly instead of individually. If I take the why is it happening because both the equation have variable of interest, same both of them have the same variable of interest.

So, why do not you merge both of them together rather why did not you consider them as a vector, the observation vector and let us see if I consider whole thing together as a vector what will be the magic that is coming into picture, ok. So, that is where the extra part that MIMO-OFDM will offer. That is extra part and we will see in the next class how we solve this equation from a linear algebra point of view, ok.

So, with this I conclude the session and in the next class I will be talking in the decoding part of the MIMO-OFDM ok. So, with this I go back here.

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So, conclusion is that we just covered the structure of the MIMO-OFDM how the structural it can be presented.

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I think that should be make it clear. References are again these are all very standard part. I do not find any one point references, but IEEE explorer if you go and type MIMO-OFDM thousands of thousands of good preferences will be there, but so I put IEEE explorer as one of the rich source of contents instead of just saying one content.

Thank you.