Signal Processing for mmWave Communication for 5G and Beyond
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Module - 10
MIMO-OFDM beamforming
Lecture - 52
OFDM Spectrum and CFO

Welcome back. Welcome back to Signal Processing for millimeter Wave Communication for 5G and Beyond. So, today we will be covering the MIMO-OFDM beamforming module which is module 10 and I will be covering some of the OFDM Spectrum and CFO part.

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So, the concepts covered concepts more on the OFDM with CFO and some of the OFDM spectrum concept.

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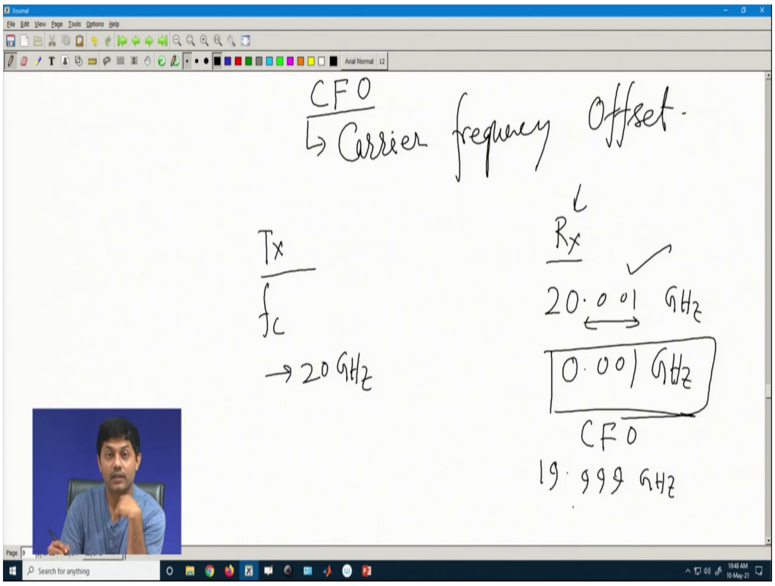

What is exactly a CFO is? So, CFO is nothing but the mismatch between the carrier frequency of transmitter side and the receiver side ok. So, let us understand what the impact would be right.

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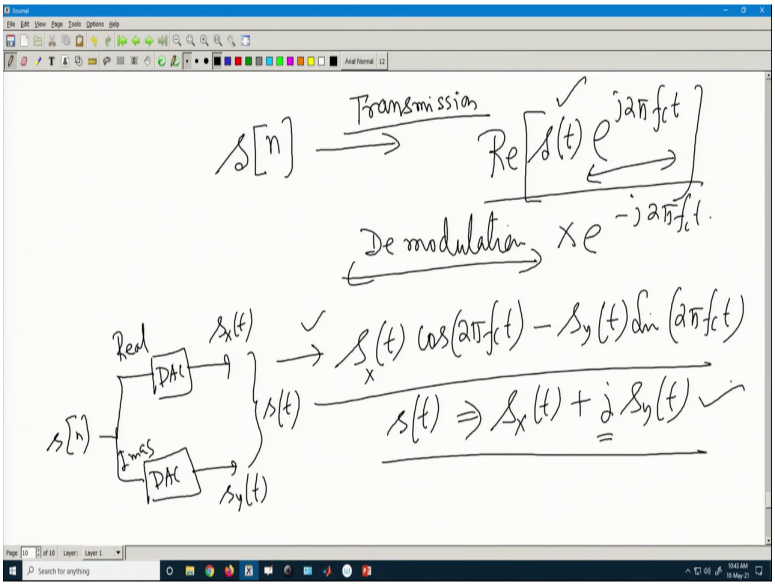

So, let us say let us say what happens when there is a CFO. Let us say I have let us say I have the signal called s n which is a digital signal ok and when I transmit it I cannot transmit a digital signal rather I transmit analoged envelope multiplied by a modulator.

So, transmission happens. So, how the transmission happen? I would say rather say so, this is your baseband equivalence of a e to the power j 2 pi f c t this is what happens right ok. Now, when I demodulate what happens? So, tentatively I demodulate this signal with e to the power minus j 2 pi f c t, this how the demodulation happens right when I do a demodulation right

So, exactly how a signal is? So, the transmission signal is something like s of t ok. If you do not if you are not comfortable with continuous signal you can also think of it as a as a continue as a baseband equivalent signal, but this is how it is. x t what is x x t? It is a complex. So, s t is a complex analog signal right.

I think if you remember some of the notes in the earlier classes your modulated signal is a complex not modulated your analog equivalent signal is actually a complex analog. Then you make it real using a modulate modulation and that real signal will go away. So, this will be cos omega c t or 2 pi f c t minus this will be sin 2 pi f c t. So, this is how typical you send. This is what you actually transmit. So, the same thing real part is nothing but the s x t.

So, I would say s t is a analog complex signal which will be this plus j s y t. Now, this is just a notation ok because there will be two channel which will be going because I multiplied sin and cos. How do you ok, by the way how do you represent a complex analog signal? I mean it is very tricky right, I mean in digital you know. So, there is a real part there is an imaginary part of a complex number.

So, when I say 8 bit real number there is an 8 bit complex number as well 8 bit imaginary part as well. So, total 16 bit complex number, but what you do in a in analog form? Analog does not have any complex concept right. So, basically the way it is done, so, you have an s N right, it internally has a real part complex part.

So, the complex part or real part you send it through one DAC complex part also you send it through another DAC. So, this is your real part of s l this is your imaginary part of s l. Now, this is your s x t, this is your s y t. Now, you do not merge them together because we merge them the notion is lost. It is just that physically isolated this two signal comes into picture.

So, this whole thing I call it s t. So, I call it s x t plus j s y t j meaning it is it is a mathematical notation, but physically it will be isolated right. So, this is what I send it, but when I demodulate, I am suppose to demodulate using you know sorry using this part. This is a complex notation again in RF.

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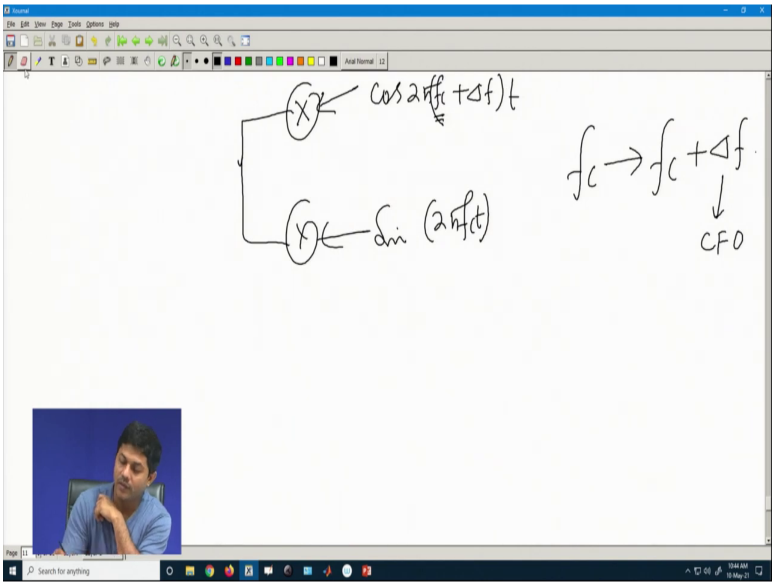

So, when I demodulate it will be like you multiply it with a cos 2 pi f c t right and then you multiply this one with sin 2 pi f c t, but that never happens right because as we said when you go to the r x side this f c may not be the exact f c that I want. So, this f c may be f c plus some delta f and that is precisely your CFO. Whatever we have discussed is 20.001.

Whatever I mean just number, but this is the mismatch between your t x and r x ok. So, now, what you multiply here is this quantity delta f t, here also. Probably I will remove them ok. This will be coming into picture ok. So, if you multiply it what will happen? Try it at your own end, I am not going to that level of details.

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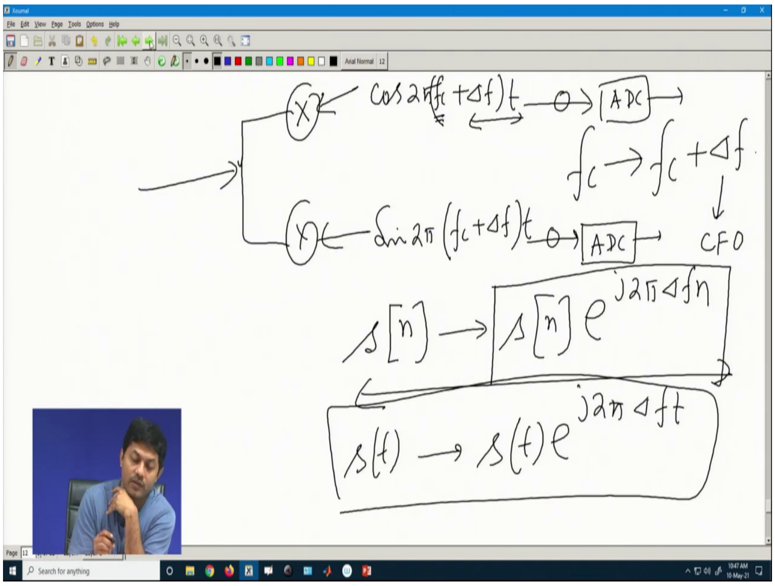

So, this one this whole thing you give it to them and you can try it what exactly happens there. You will see that and then you pass it through as if like you are taking it through ADC and all, pass it through ADC of course, filter and ADC of course, filter and I am not drawing the filter part LPF filter; obviously. So, there is this filter here also LPF filter. So, I am not using the channel right away because channel will come next.

So, what you get is that this is your s n, this complex number, two things you can combine as a complex number. You are supposed to get s n, but instead if there exist a delta f, what you get is the following. This part will get it and that is natural. So, this delta f 2 pi delta f t will pop up f different.

So, this is what is coming ok. So, s N will be multiplied by s N e to the power j 2 pi delta f t. Now, t will not be there of course, because I have digitized it. So, it will be more on n. So, that is the nature of that. So, in analog form s t you are supposed to get it, but actually you will get it s t e to the power j 2 pi delta f of t is what is usually get it something like that ok.

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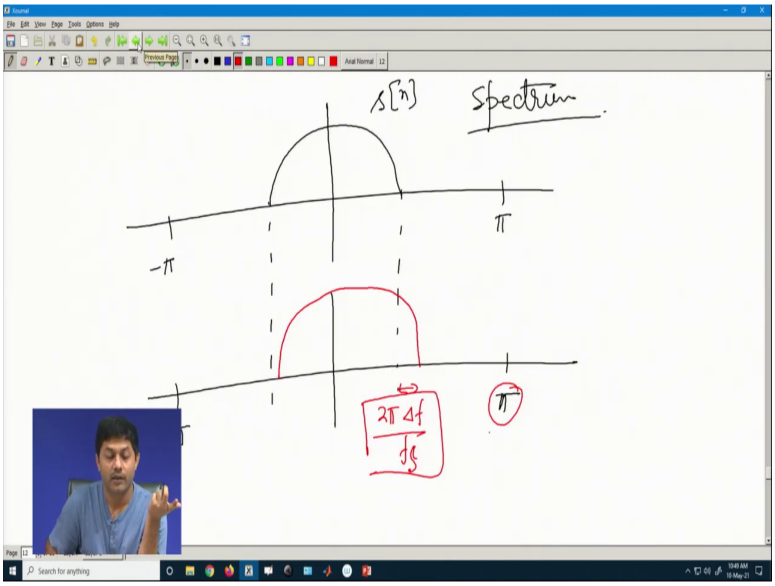

Now, that is what is the consequence of this. So, consequence of this is that if s t or s N let us get into the digital domain. If the s n which is a baseband signal right, if it has a spectrum, so, let me just put it little better, let us say this is the spectrum I am supposed to get from for s n is a spectrum.

So, what is the consequence of this kind of data? The consequence is that or this kind of data. Consequence is that it will be the spectrum will be shifted right because there is a n here. So, it will be shifted. How much it will be shifted? It its delta f normalized to pi. So, this will be pi 2 minus pi ok.

The spectrum which was supposed to be here will be either positive or negative depending on what is the delta f is it is a positive value or a negative value. You can have delta f negative. How? If you make it here, so, here its delta f is positive because it is more than that. What if it is 99.999 gigahertz? So, it is negative. So, that is the point here ok. So, now, what will happen? The same spectrum may be slightly shifted ok.

So, the amount of shift this amount of the same thing I have not drawn to the scale. This depends on delta f. It will not be delta f because this is pi. So, it will be delta f 2 pi delta f by normalized whatever your sampling rate. So, I should say f s by 2 or f s. So, that is the kind of amount of shift that will happen. So, my point here is that CFO will make a shift to the spectrum ok. It is a very important aspect in the OFDM concept or in the OFDM context.

So, how this spectrum shift impacts my activity in OFDM? So, now, let us go back to our OFDM spectrum. Now, this is my general impact of CFO. Now, what happens when I go for a you know go for a OFDM kind of things? Now, look at the OFDM ok. Now, first you have to understand what is the OFDM spectrum then you have to understand what is the impact of OFDM spectrum on what is the impact of CFO on the OFDM spectrum. So, let us understand the OFDM spectrum first ok.

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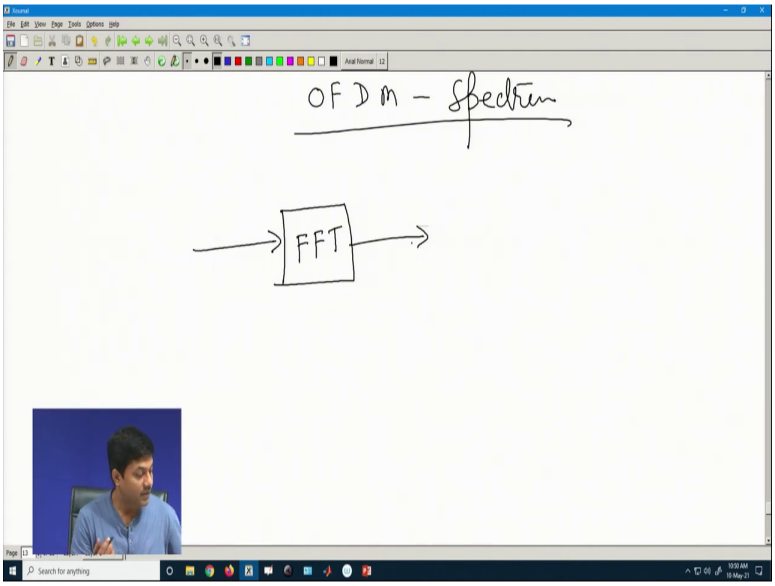

Now, what exactly the OFDM spectrum would be? So, if you look at the receiver after the CP discard the receiver will finally, give it to the FFT right ok is that that is how it is right or if you look at even the transmitter side let us understand the transmitter side that will be easier for us.

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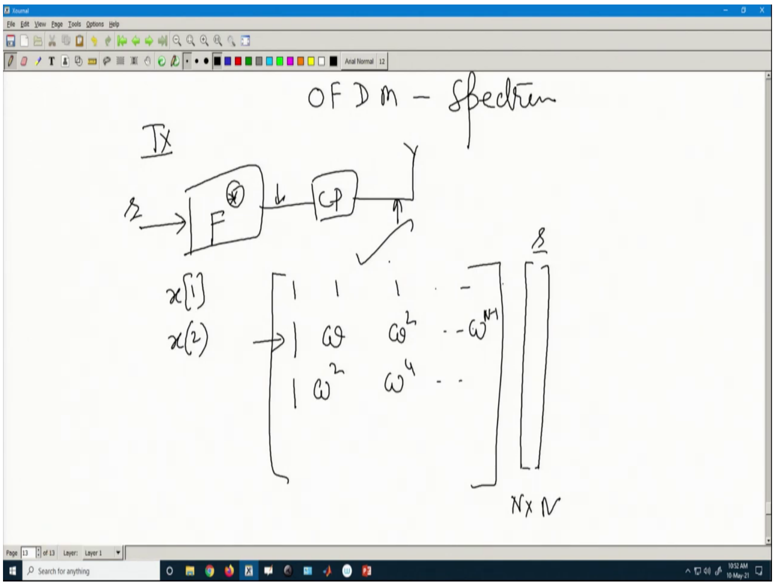

So, let us say I am in the transmitter side because spectrum is more shift at that point and then receiver is more of a sampling point. So, you have an IFFT then you have a CP addition and then you basically send it right. So, how exactly aspect? So, this is here your analog and RF will be there. So, here it will be a discrete spectrum because you are in the digital domain and what is the spectrum of such kind of thing. So, if you notice what exactly an FFT?

So, let us understand the structure. It is 1 1 1 dot dot then 1 omega omega square omega to the power n minus 1 because if it is a n length then 1 omega square omega to the power 4 and so on so forth. And then what how do you how do you visualize this concept? So, if you have a data here, so, suppose I am giving my data here s vector. So, this is my s vector right and this is forget about CP.

For understanding spectrum, CP does not give you an impact because this is a time domain you know addition. So, let us say the x. So, the what is x 1? x 1 is nothing but multiply this one the first row multiplied by this s vector right. What is the second one? Multiplied by the first, second row multiplied by the s vector. So, how do you what exactly this means? So, how do you get a spectrum of it?

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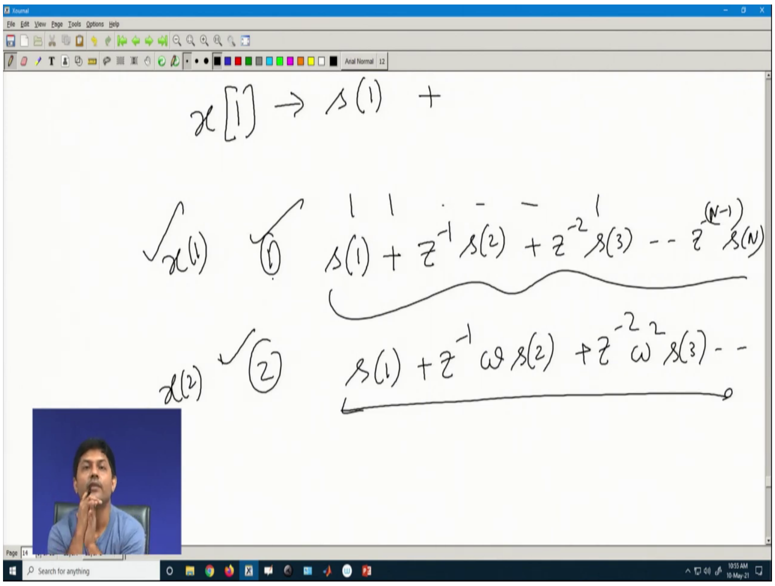

So, when you try to get a spectrum of such things, so, it is as if like the x 1. How do you how do you get the x 1? So, it is as if like I am multiplying s 1 because I am trying to do spectrum things right. So, what is 1 1 1 1 1 1? What does it mean? It is as if like it is a it is a its a FIR filter having n length, it is an n length FIR filter having coefficient 1 that is how I will give it right, so, which means that this is a filter. This is a filter. This one, again a filter.

It is a frequency domain to time domain conversion, but that is actually kind of a, I can think of it is like a filter right. So, for what does it mean? So, it is like a is like a n length filter having coefficient 1 1 1 1 1. Second one is a filter having coefficient 1 omega square and so on and so forth that is that is precisely what I say ok.

So, how you how do you get a spectrum of such x 1? So, the x 1 is a signal which is a time domain signal. So, it is as if like it is as if like it is s 1 when I try to get the spectrum of it plus Z inverse s 2 plus s 3 and so on and so forth Z to the power minus N minus 1 s N. So, this is how when I try to get a spectrum of it right. So, now, this is for the 1st case. For the 2nd case the same thing Z inverse omega s 2 plus Z inverse 2 omega square s 3 and so and so forth ok.

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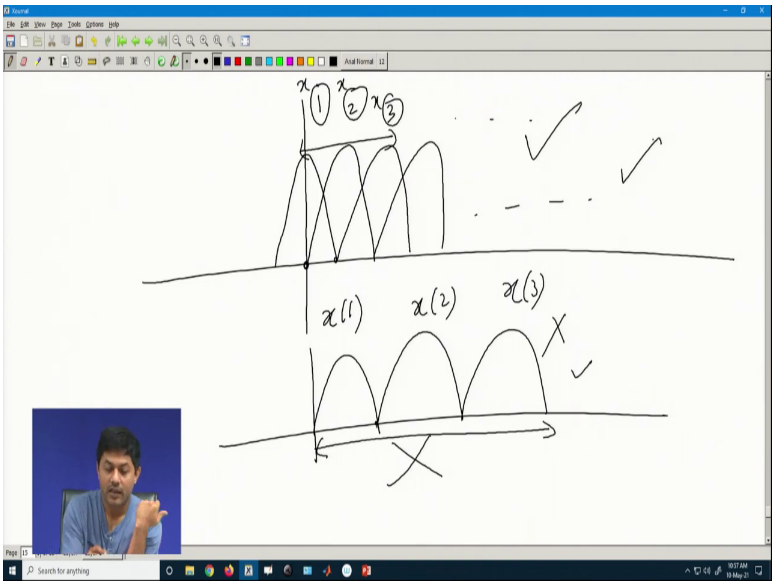

Now, if the spectrum of the first case is something like that if I plot it between pi to minus pi, the 1st case ok. The second case if you take a MATLAB and try to plot it the 2nd data stream which is corresponding to x 1 x 2. So, that mean x 1 is a data stream, x 2 is a delta stream and so on.

Now, I am trying to understand what is the spectrum of x 1, what is the spectrum of x 2, spectrum mind it. I mean it is a time domain signal, I want to know the spectrum of x 2 itself. So, that is how the relationship right. So, if you take a MATLAB or any other tool or try to plot that concept you will see that the second one will give me a spectrum like that. This is for the second one. The third one will give me a spectrum like that and so and so forth. So, this is how the spectrum will look like.

So, you can see that the spectrum of OFDM individual lines individual data if this is an n length thing the individual one will give me a small small spectrum and they are all overlapped spectrum. So, that mean if I send x 1 to x n individual x 1, x 2 blah blah and then x n every one is individually a small what shifted spectrum they will contain. So, that is your So, this is the spectrum for x 1, this is the spectrum for x 2, this is the spectrum for x 3 and so on and so forth, this is what will be happening.

So, interesting part is that they all start from the middle point. So, it is not this is not the case. So, that mean this is x 1, this is x 2, this is x 3, this is not the case, this is that case overlapped spectrum. So, this is one of the advantage of the gain OFDM. Why? Because if there are say n equal to 3, so, you see how much spectrum I require versus if it is a OFDM you see that this is not an overlap is actually overlapped spectrum.

See if it is 3, you can see it is much more squeezed. So, what I can say? So, this is giving you some sort of a spectral efficiency because you do not really need n into you know total individual spectrum rather you require much smaller spectrum ok. This is one of the good advantage of my OFDM system that it always gives you some sort of a spectral benefit ok.

So, this happens when you have this is also filter band ok, but this is orthogonal filter band this one. But this one if you look at this is exactly not an orthogonal, but it will be orthogonal once you sample at the FFT point. So, this is what the analog. So, by the way this x 1 I am drawing it more conceptually from the digital point of view but this is how the spectrum will be looking, this is a digital spectrum ok, but this is how it will be looking at is a time domain signal ok. So, that is the spectrum I will be taking it. Now, let us understand what happens when I go to the receiver side.

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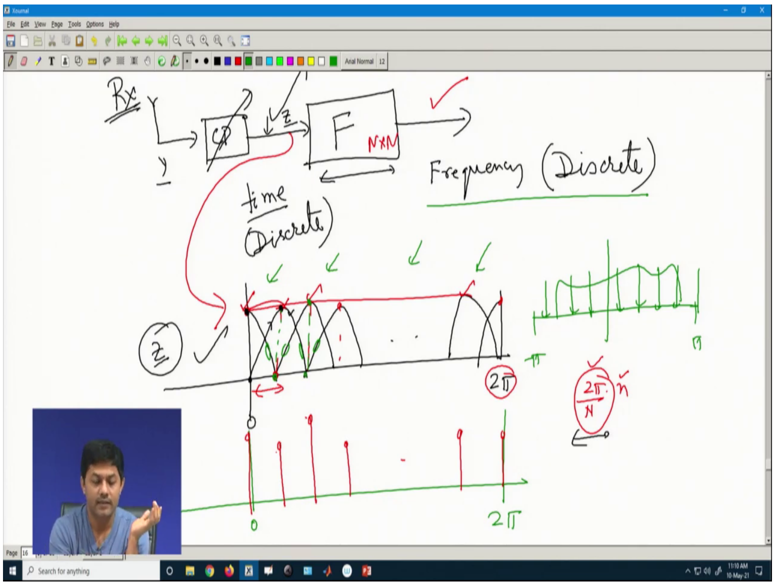

Now, receiver side what exactly does? It does an exactly opposite thing. So, you have an F matrix. So, what does that F matrix? What is the output of F matrix? Actually if you look at the output f matrix this is a frequency domain. The output is a frequency domain data. This is a time domain data at the receiver side. I do FFT right. At the receiver I have to do FFT for OFDM. So, the input of my FFT is a time domain signal, output is a frequency domain signal. So, this is discrete. This is also discrete.

The spectrum of this fellow is the one which I just explained whatever. So, this will be 0 to 2 pi you can give it like that ok. In fact, there is a slight mistake here. It will be it will be here yeah, if it is 0 to 2 pi yeah. This is the spectrum at this point that is the time domain signal and that is the spectrum. But what is the spectrum after F ok? That is the spectrum after F is completely a sampled version.

Because what does the FFT do? FFT will discretize in the frequency domain. So, that mean it has to do a sampling in the frequency domain. So, what is the spectrum that FFT module sees? So, that mean I am already saying this is the frequency domain signal that mean the data that I will get is actually frequency domain data, but it is a discrete frequency domain data. So, how does the spectrum look like?

So, if I plot the spectrum the same thing 0 to 2 pi. What the FFT will do? We will sample these points that is all the FFT will do that is what that that is the job of FFT right. The job of FFT is that if you give a continuous discrete continuous digital spectrum the FFT will just discretize the spectrum nothing else it will do. So, if I have say spectrum like that 0 to pi to minus pi, it will just discretize it. That mean just it is a sampler in frequency domain. So, that is all it is.

So, now, if the spectrum whatever you are seeing it the blue colored spectrum which is the input to the FFT the output of the FFT it will just do a sampling with an equal spacing right. So, the this will be the first sample then what it will do? It will exactly do a sample at the midpoint of this point. So, that mean this is the peak point it will do. Again it will sample at this point, again it will sample that is how the because if you look at if it start from here and there are n there are 2 pi total range and total range 2 pi meaning individual gap.

I can say it is 2 pi by hence individual gap is there and I am saying that FFT will do a uniform sampling n number of uniform sampling because the length of FFT is still n. So, what is the sampling gap? It is 2 pi by N that is all it is right. So, individual gap, so, 2 pi by N into n is what your multiple thing. So, the individual gap is to say at the rate of 2 pi n at the rate of gap of 2 pi n it will keep sampling. So, the first sampling will be here. So, I can put an n. So, that it just increase the sample.

Second sampling will be at 2 pi by N which is nothing but this point next will be 2 pi by N into 2 which will be this point. Now, this will be uneven. I am not saying that they are all equal that need not happen because that depends on what is your what is the length of your s N. Suppose s N comes from all QPSK probably they are all equal because the value will be equal.

If the s N comes from M-QAM say 16-QAM then obviously, this length will never be equal ok and also there is a channel part that I will come to that later. So, like that. So, this point will be one more sample here. This point will be one more sample. So, this is the you know spectrum of my FFT. Now, what is the impact? So, this is a very standard kind of a discussion. Now, what happens when you have a CFO? You have to understand that ok.

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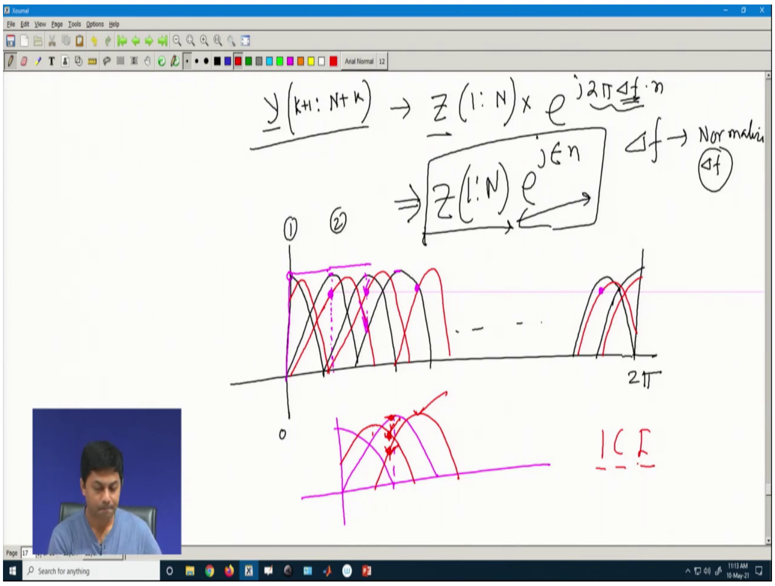

So, this x vector, so, received vector Y or I would say Y vector ok this Y vector, I have received it. So, if it is say now this Y vector is mainly here. This was my CP cancellation. So, this is my Y vector ok or rather I would say yeah that was my Y vector, this was more of a discarded Y this after CP discussion.

So, I will say this will be k plus 1. I have discarded my CP. So, let us call it for our time being let us call it Z, Z 1 to N, this N length data. So, that is the data. This Z is basically here this point I am talking. It is Z vector ok. If there is a CFO is the time domain sequence plane time domain sequence right.

If there is a CFO what is the impact? If this is a CFO, so, how a CFO will impact here? So, this Z is a vector. This will be multiplied by another vector ok where you know this part will be present or rather this part will be present that whole. So, that mean with every time domain sequences I multiply you know individual data here. What is that yeah?

So, it will be e to the power j 2 pi some normalized delta f into n. Delta f meaning it is a delta pi will say it s a normalized delta because I am I have digitized this normalized delta f. So, which will be a delta f by f s what is the sampling it that that one. So, it is normalized. So, the point here is that let us call this epsilon just for your simplicity epsilon into n let us call it like that ok that is it, that is the CFO point.

Now, how would the spectrum will look like because I have multiplied j epsilon n right? So, if this whole fellows spectrum is this gentleman that is the spectrum of your Z bar right and then I multiply e to the power j 2. What will happen? This whole spectrum will be shifted right or left depending on how exactly delta f is. So, it will be shifted. So, that this spectrum will be now will be shifted.

So, let me put the original one, this was the original ok. So, this is between 0 to 2 pi. This is for the 1st case, 2nd case and so and so forth individual sub carrier spacing. So, what is the impact of CFO? It will be shifted, everything is shifted ok. So, what is impact now? Impact is that the FFT module the FFT module, he does not do that he does not know that spectrum has been shifted.

He sees 0 to 2 pi. Now, whether within its spectrum is shifted or not shifted does not care. He will sample starting from 0 and then uniformly at a space of 2 pi n he will keep sampling it. He does not know CFO, he has no concept of CFO, it is just a spectrum sampler. If the spectrum itself is like a now you visualize that spectrum is now shifted like a red, now that is your new spectrum. Will the FFT is still sample at exactly midpoint here?

No. Because he does not know spectrum has been shifted here. What will? He will continue to sample starting from probably I will I put it starting from here. From here he will now go to this black colors wherever he was used to sample. Now, he will sample it here; that means, this red part this is the new spectrum red, but instead of sampling the peak part of the red he will sample slightly before.

Then for the second one also he will sample it peak here. So, we will sample it here. So, these are my sampling point, these are my sampling point. So, what was the advantage if I sample exactly at midpoint? The advantage was that if I sample at midpoint, suppose I sample first here will I get any anything from the second? It is an overlapped spectrum mind it, but FFT after FFT I do not see any ISI.

So that means, the neighbouring spectrum do not appear, the neighbouring spectrums do not appear to me. That mean if I sample it here the neighbouring spectrum is this one, he does not impact me because whenever I sample it here the second spectrum has a value 0. If I sample it exactly here this spectrum and let me put a different color. This spectrum and this spectrum both of them has value 0.

If I sample it here this spectrum and this spectrum has value 0 at my sampling point. So, what does it mean? Though they are overlapped spectrum, but the point when I am sampling it here both of the overlapped spectrum has value 0 there. So, I do not feel that there is an eyesight that is precisely reflected in my time domain equations right that is the reason it is all reflected in my time domain equations. These are all reflected in my time domain equations.

Here you see there is no ISI. The presence of ISI is not there because the overlapped spectrum do not create any trouble after I do FFT. But, if I do not do FFT the spectrum is still overlapped it is as if like you are still in a ISI mode. So, unless you do an FFT there is no point that you can get rid of this spectrum, FFT spectrum, CFO spectrum ok. So, now, this is a problem.

So that means, if there is no CFO, if I spectrum at peak points I do not see any ISI though the spectrum is overlapped because the other points are actually meeting at a point 0. But, now the question is this is not the case when there is a CFO because if I sample it here if I sample it here you see just take the red.

So, what will happen? Will I really get any individual like any non overlapping spectrum? Yes, I will not get it. I will get an overlapping spectrum because if I sample it say here. So, I get a value from here I also get a value from here. If I sample it here probably I have not drawn to the scale. So, this is coming to the scale properly. So, if I sample it here this fellow I am getting it ok, I am getting it here.

So, which means that if there is a CFO because I am still sampling at the black points you know black points peak. I am still getting I will now get a CFO because if I shift it like this was my original one. Let me just this is my original one where this one is not getting in trouble. Now, this is my new one, this was the mid from here I am getting a second one. Now, what will happen if I sample it here as if like I am sample it here?

Now, I will see the red one here I mean red one here and the red one here not this other one. So, if I sample it here as usual I will get a value from the first one I will also get a value from second because there is a point here. So, this point and this point I am supposed to only get this, but I am getting a component from the second one as well ok. So, this is your ISI. So, if I have CFO the ISI removal is lost now.

So, now I will start seeing all these interferences, but are these interferences time domain or in frequency domain? If you notice they are actually from frequency domain they are coming. So, that mean from a side lobe this interference is coming. So, this is known as ICI, inter carrier interference, it is not a time domain interference. So, I cannot call it an ISI rather I call it a ICI because it is a interference from a side by spectrum. It is a frequency domain interference ok.

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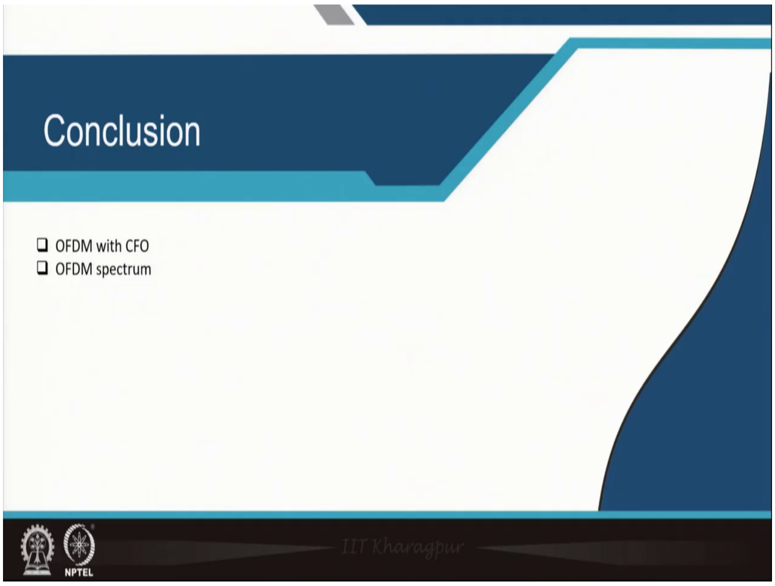

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So, with this impact ok. So, reference again the same book which I referred it chapter 3, chapter 4, you can just read it, but you know what these concepts are more of a DSP concept in. I mean explicitly you will not get it everywhere kind of concept come from multi domains like DSP and all.

So, it is very tough to say this is a particular point where you exactly see how a spectrum is really coming into picture, but this is because this is more of a DSP concept. So, I should have written a DSP book, but again in any DSP book nobody will explicitly specify how a CFO works. So, I have just kept it like that so, but point is you can read some of the concept from any of the DSP books like Proakis or any other things.

Thank you. So, with this I conclude the session.