Signal Processing for mmWave Communication for 5G and Beyond
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Module - 10
MIMO-OFDM beamforming
Lecture - 51
General OFDM

Welcome back, for Signal Processing for millimetre Wave Communication for 5G and beyond. So, today we will be covering the MIMO-OFDM concept. It is a module of MIMO-OFDM. We continue the general discussion on the OFDM part and then gradually move to in subsequent classes.

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What we will be covering today? We already covered some of the concept on the OFDM, but we will continue to do that.

Now, today we will be stressing more on the OFDM data detection and the CFO concept because that is one of the impairments which will be extremely useful ok continuing here. So, now you were discussing about the OFDM part, right and we try to cover what happens when I add cyclic prefix. So, if I add a cyclic prefix towards the length N and we have shown you what exactly happen.

So, what are the consequences that happen what are the consequences of adding CP and thereby discarding the same CP at the receiver. So, you have the data here. So, you have the data here. So, let us say I have a data S bar whose length is N cross 1 vector, ok.

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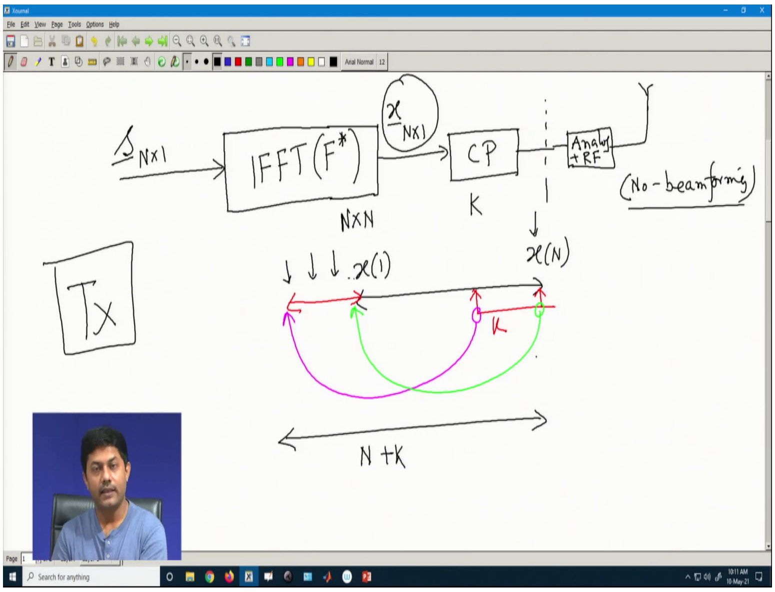

Now, this was the IFFT part. This was the IFFT part, right. Now, instead of writing IFFT, I will in bracket I may write it something like this. So, that is the operation naturally. So, just for your understanding.

So, here I am multiplying with the IFFT matrix, ok and its length is again N cross N. So, I send a information data which is N cross N, N cross 1 and then, that gets multiplied by a matrix which will be again a resultant of an N cross 1 vector. So, this vector was x vector whose length is N cross 1, right. And then what we did? We add CP cyclic prefix ok. So, what is meant by cyclic prefix? The last you know the last K number of data of this x vector will be piggy backed.

So, if this is your x vector say x 1 to x N, so this last k this last k length I will just piggy back here. Piggyback meaning this one will be here and this one will be here, ok. So, that is kind of a piggyback. Now, the complete length becomes N plus K, ok and this is my digital system completely. So, this is completely digital and this I will send it through whatever the frontend process. So, then there will be DAC, then there will be RF section and finally, it will go through one antenna. I am not I am not really bringing in beam forming right away, ok.

I am currently explaining the OFDM because unless you understand that part and then going back to the MIMO-OFDM thing unless you explain unless you understand that part beam forming has no impact on that. So, we will combine both of them together. So, it is essential that by die hard you understand the OFDM and MIMO OFDM. So, this will be your analog and RF section whatever is there, this is your analog plus RF part.

And finally it will be going through one single antenna. So, there is no beam forming. So, no beam forming here assumption currently no beam forming, ok. So, this is my transmitter side, this is how I will be doing it and I have well explained in the last classes why a CP is required because if you add CP, then only my effective channel will no longer be a toeplitz matrix. It becomes a cyclic circulant matrix provided you discard the CP again.

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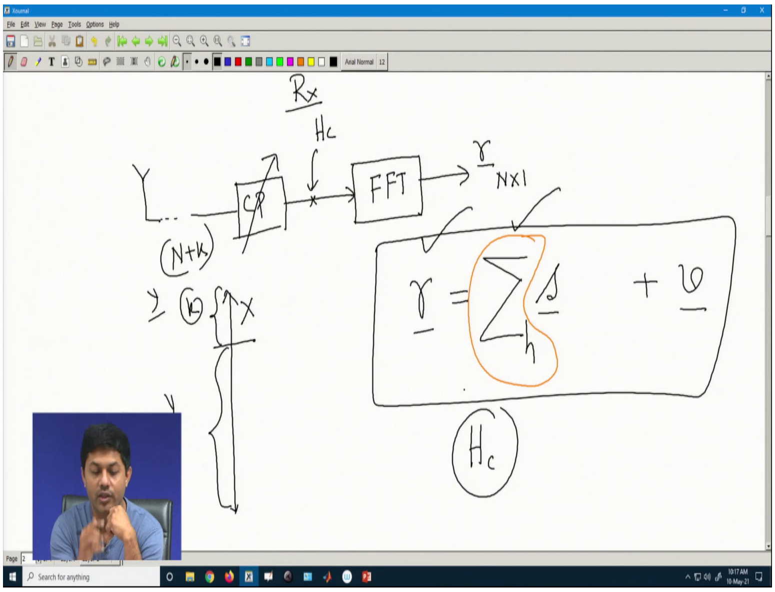

So, when I go to the R x side. So, this is my R x side. What I do here, I receive the data ok. So, what I send? I send a data which is x plus C P, this x vector plus C P vector. So, it is as if like I am serially sending. So, I am serially sending. There is no parallel sending right. So, a serially one by one. So, I will send this fellow first, this fellow second, this fellow third and so and so forth. Finally which was x n? That will be my N plus K th data serially that is being sent.

So, that is how I will be sending it right. So, this is at the receiver side. Receiver side it will go through all your RF chain and all. I am not really interested to draw the RF chain and analog chain. I am finally going to kind of digital side. So, this side will be the first task. So, I am assuming that I have done a DC conversion. So, what I will do here? I will put here the

CP discard. That mean N plus K number of you know data I probably would have received and I take the first.

So, if the length if you look at the last class, so this is your y bar and the first k data. I discard it. I just do not consider it, ok. Now, effectively I am taking only the N number of data. So, that is where I say C P discard, then here if I further do FFT, I will get back. I will get an r vector and that r vector I have shown you that is a N cross 1 length vector. So, this would be if you remember this was something like that I have written, right ok.

So, this was the summary of my received vector. So, what is the impact here? Impact here is that now this r individual r is a vector of course, but the individual line has no interference from the other data. So, that means what is the impact of a channel if the channel is a FIR filter, what is the impact of this FIR filter if the channel has multiple taps? The impact is that it will add an ISI right and that is quite natural see if we if I send it through a channel say h 0 plus h 1 z inverse.

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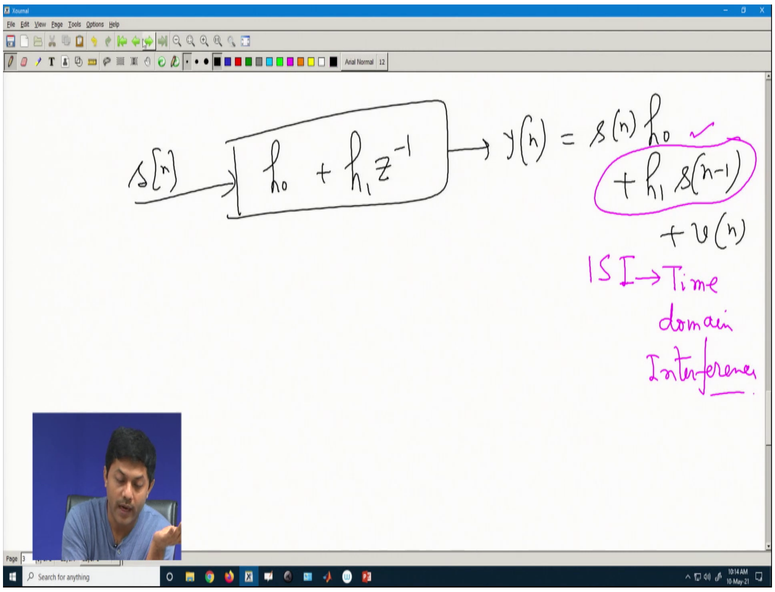

So, that is my channel let us assume and I send say s; s n. So, this is my y n. So, what is my y n? y n will be s n into h 0 plus h 1 s of n minus 1 right plus a noise. Now, this term is your interference term in time domain because this is the time domain activity and this is popularly known as your ISI Inter Symbol Interference because it is an interference from another symbol. So, it is inter symbol interference and this is completely in time domain because I am taking y n and all. So, this is a time domain interference, ok.

So, that is what that is precisely what happens here. So, this ISI Inter Symbol Interference. So, it is a time domain interference ok. So, this is now you look this is what happens if you do not use the OFDM. Now, look at the case here am I getting any interference? Actually if you look at I am not getting any interference as if I am not getting any interference, ok.

And this channel is also like a square matrix, circular matrix everything and then I have done all this CP. I mean FFT, IFFT. So, you can think of this FFT is kind of equalizer and the IFFT whatever I have drawn in the transmitter side, you can think of this is something like a pre coder ok. Now, all things are fine, but what is the impact of r? What is the impact on the r vector? So, if you look at r vector, it is a diagonal matrix right. This is actually a diagonal matrix ok.

So, what is the r i? I think we are tried to give a notion of this r i.

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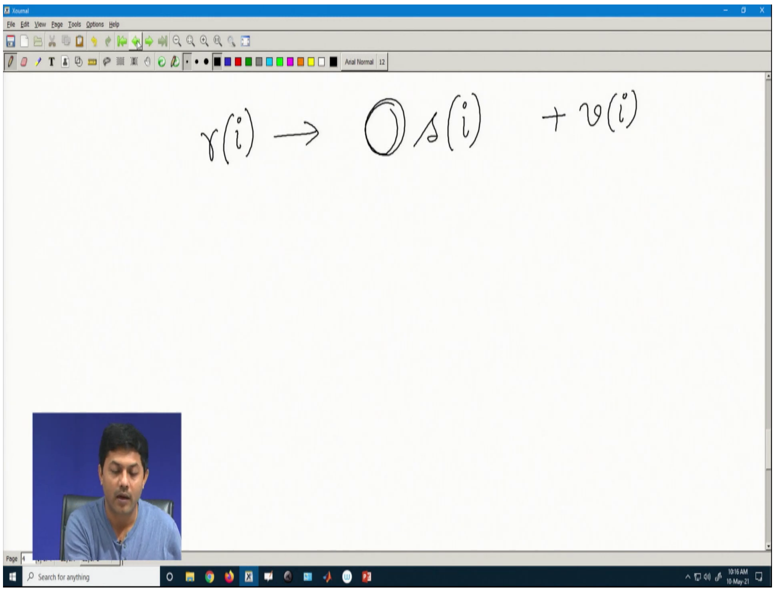

So, if I take r i, that means the ith you know the ith position r vector r i. So, this what it would be? I would say this is some s of i obviously, and noise is also the ith vectors, ith part of the ith element of the noise vector. And what about this one? This one what about that? So, that is nothing, but the ith diagonal element because this is a diagonal matrix right.

This sigma h what does it contain? It contains all the eigen values of your circulant channel matrix because at this stage if I discard the CP, my effective channel is a circulant channel otherwise not here. Here my channel is not a circular channel, here it is still a toeplitz matrix ok, but at this point, at this point, at this point, the channel becomes a circulant channel and it is a square matrix. Now, if it is a circular matrix I, obviously I can do an eigen value decomposition and I got sigma of h, ok.

Now, this is the impact. Now, this is basically a diagonal matrix having all the eigen values. Now, how do you get the eigen value? How do you know even the eigen values without I mean for that you have to know little bit of the channel, but let us assume I know the channel that mean my channel matrix, not matrix the channel taps I know h 0 h 1, this one I am talking this one maybe h 0 h 1 and so on so forth.

Those taps I know. So, obviously I know the h, I can construct h. So, sigma h I will explain that how you know exactly the eigen values of such circulant channel, ok. So, that is the second point, but my point here is that if it is a if it is a diagonal matrix, what is the r i? r i is let me remove this part, r i is nothing, but I would say probably I will use I should not use a sigma. Probably I will use e as the eigen values.

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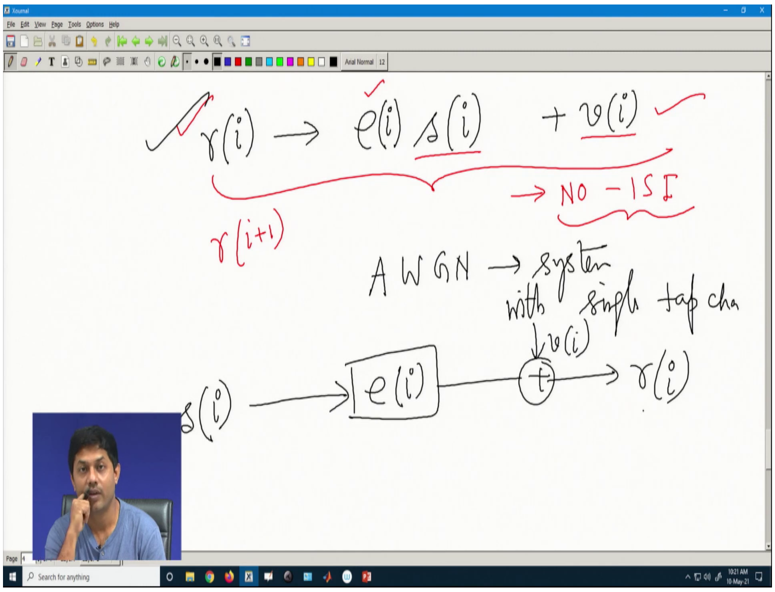

So, let us say e as an eigen values is the ith diagonal element of sigma h. So, the ith diagonal element it is a diagonal element, it will have N cross N matrix, it will have N number of diagonal elements. So, I am talking of the ith one. Now look at this equation. Does this equation contain any element other than s i? It does not right. So, that means the r i has no ISI I can say it has no ISI you can say.

So, that is the beauty part beautiful part of the OFDM. So, if you have a channel which has multiple taps and you use and you use the OFDM things, the only big advantage that you get is that you are free from the ISI suddenly, but does it mean that you improve your performance. Well, that is not the true things, ok. What you get the channel from a decoding point of view. Apparently, it becomes a ISI free channel. Suddenly you will see that ok I am seeing that channels are channels do not have anything.

So, how do you perceive such kind of system because you see every r i is independent from the previous one. Now, it is exactly not independent. I will explain that part given e i that mean if you know e i given e i, this whole system becomes an independent. Why? Because individual s i is independent from other s i individual v i if I assume it is a white noise, then every component of noise is also independent from others. Assume it is white noise for the time being.

Then also an e i is known so obviously r i has no correlation with r say r i plus say 1. It has no correlation provided noise is uncorrelated, data is uncorrelated and your eigen value of this you know that sigma h is known to you, then r i, r i plus 1 everything is like an independent kind of data stream. So, how do you how do you how do you how do you describe such kind of system? This is nothing, but a simple you know this is nothing, but a simple AWGN channel. AWGN system is simply AWGN system.

Just a normal noise is there v i is anyway known. So, you can say it is a simple AWGN with a simple one tap fading model one tap fading channel model. So, it is a AWGN system with single tap channel right. So, if somebody ask you how do you describe such system in a natural way? So, you can say it is as if like I am sending s i. It goes through a channel which has only one tap, nothing else is there and then you have this v i and you get your r i. That is all I can explain it, right.

And as everything r i, r i n plus 1 and so and so forth are all uncorrelated, provided noise is uncorrelated, data is uncorrelated and each of every e i is known to you, but e i's are actually correlated. I will show you that. So, that is what I say if you know e i, then r i, r i plus 1 is actually uncorrelated ok.

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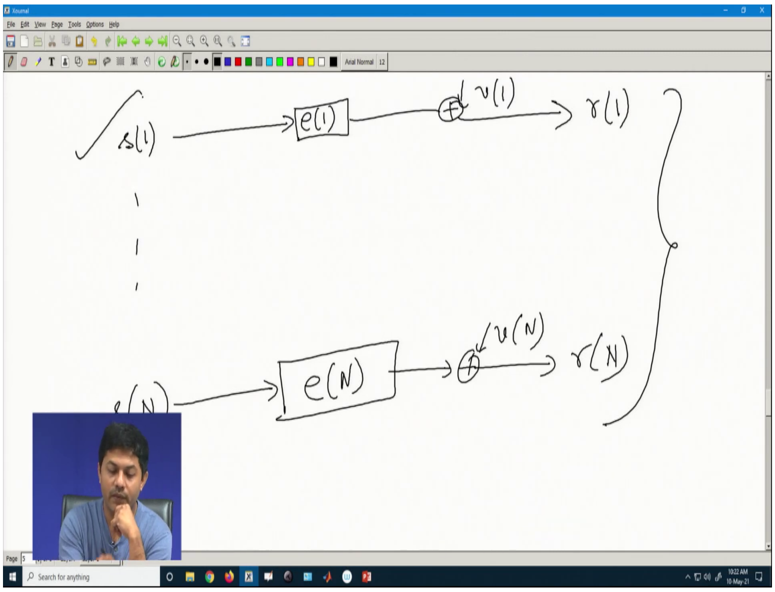

So, how do you perceive this system? You can perceive this system as if like it is like a parallel N number of AWGN channel. So, it is like I am sending s 1. I am sending s N and this goes through a single tap channel called e 1. This goes to single tap channel called e N, then its individual noise term comes into picture, individual noise terms comes into picture and I get it is like a parallel AWGN noise AWGN channels. So, this is the beautiful part of you know OFDM. So, it has no ISI because everything is like a parallel independent channel provided e 1 to e N are known to you, ok.

So, that is the basic part of the concept. So, now I will give a small brief of what have what is the I mean is it all good like suddenly I move to I mean all 4G 5G everything is moving to OFDM. Obviously, the advantage is that this is one of the advantages that your decoding complexity will be low point because if you look at if somebody gives you this kind of data model say s i is equal to e i, sorry.

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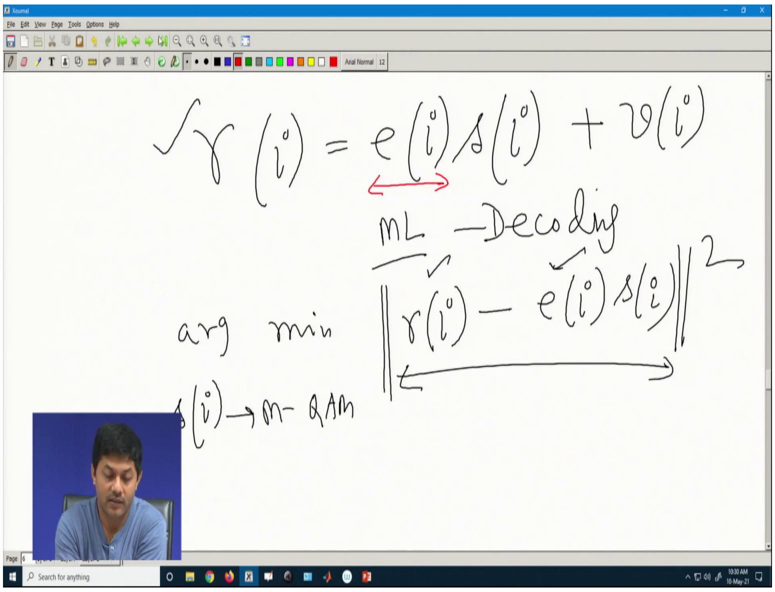

So, this will be r i v i, this is a simple data model and N number of such data model comes how do I decode it. I can decode individually. That means N number of times I decode it, but I do not have to think of other data stream is as if like I have received r 1 to r N and I individually take it and I individually decode it, ok. So, simple decoding no equalizer nothing, no complex inversion, nothing you see. I have never done any inversion in this whole scheme.

So, how do I do a decoding? Simple decoding you can do ML Decoding it is your choice. You can do ML coding. How do I do ML decoding? Well, you can just you know divide this is known to you and this is the variable. Minimize this over all your constellation of s i. So, this may be from say M QAM you know your constellation, search which were which constellation makes this quantity lesser, pick up that constellation that is your ML decoding, right.

So, you take this is known to you, this one is known to you, this is a variable because you do not know what exactly your s i is. So, you search it. So, this could be one search or you can even do LMMSE because this is a single tapped, right. How do you do LMMSE? You divide you can create your own equalizer or even in a simple least square also, you can do. How do you do a least square kind of decoding here.

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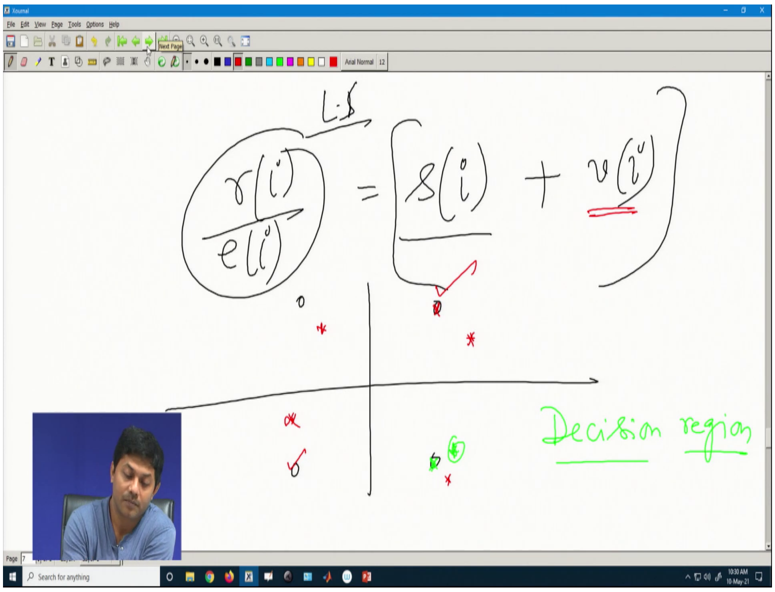

You just divide this by e i, what you get s i plus v i right. So, least what is least square? You take this because this is what is known to you. So, this is the quantities known to you. Total quantity is known to you, ok. Now, you know what are the constellation that has been sent say for example, I have sent q p s k. These are the all four constellation of s i, but here what you observe is r i by e i right. So, and you do not know the noise right. You do not have to know.

So, s i plus v i where it will be supposed actually this has been sent. So, s i plus v i will be somewhere here. If this is sent somewhere here, somewhere here depends wherever depends on how the noise is right, then how do you do a least square. Very simple. Whichever coordinate this particular fellow falls, suppose this one falls here you say ok this is the this is the constellation point that has been sent. If this gentleman falls in the third quadrant, you said ok this particular constellation has been sent.

This is simple least square. Now, whether it is correct or wrong that depends ok because suppose that noise is so large and because of which the data puts back here or the comes back here somewhere. Suppose you have actually sent this particular constellation, but noise is so large that your you know that your r i by e i comes somewhere here. It may because you do not know the noise, right.

So, you say ok I may have sent this constellation because that is the it falls into this decision region though this is some something to do with the detection estimation theory. So, you have to create a decision region that mean if the data falls into that region, I will vote for that particular constellation ok. So, this is so if the noise is so large and if it comes to the you know fourth quadrant, you can say I have already sent this.

I may have sent this, but that may be a wrong decision, but that is ok. That depends on how the v i is. You do not have any control on that, but what I am trying to say is that it is very very easy. It is extremely easy to decode a single data stream. If it is free from ISI because why? Because I have to just individually decode it. I have to individually decode r i. I do not have to worry about it.

So, I have to repeat this operation N times, but that is ok. How large the N that is ok, but I do not have to deal with any vector. I do not have to deal with any you know matrix, I do not have to deal with any inversion that is one of the worst thing right and that is why that is one of the reason why we wanted to avoid this equalizer based kind equalizer based detection.

Since all things are you know all things are removed from my side, now this has its now OFDM has its own other advantage. For example, it has a spectral benefit and I will explain little bit about that in the context of beam forming, but this is the key reason or key benefit that OFDMs always gives you, ok. I hope this part is making sense to you ok. So, now let us move on that ok. So, this you can do it. So, that is one of the simplistic things.

So, what is the key advantage? I am currently pushing it here is that this is completely a inversion free system and one of the advantage I have done a precoding. You remember you just see this I have done a precoding, but because of the precoding I really do not have to know my channel back to the transmitter side, ok.

So, it is completely independent of my channel knowledge because what is the precoding in OFDM? The precoder in OFDM is the FFT matrix IFFT matrix inverse Fourier transformation matrix and the equalizer at the receiver is also a FFT matrix. So, I really do not have to go for channel estimation because FFT matrix and inverse Fourier transformation matrix are all standard matrix.

If given a length N you know what is that matrix. You really do not have to know the channel. Now, the question is that does this OFDM system give you an advantage over not estimating a channel? What I am trying to say does it mean that OFDM system does not require any channel estimation, ok. That is the question you ask right because equalizer and precoder, they do not require a channel. Then why do I need, why do I need to even estimate my channel?

If you notice if you notice the equation here let us take this equation here itself or here. What about this r i e I; e i is the one which is the eigen value of my circulant channel matrix. Now, that is channel dependent. You cannot get rid of channel knowledge for e i. So, to know e i, I am repeatedly saying that e i if you know then only you can do the decoding.

So, you have to know e i. This is the reason see if you look at the least square decoding, what I have done r i, I have received and then I divide it by e i. So, that mean I did not need to know what is e i, unless you know it you cannot do it. So, how do you know e i and that is something related to the channel estimation. So, which means this eigen value is a channel dependent function, ok.

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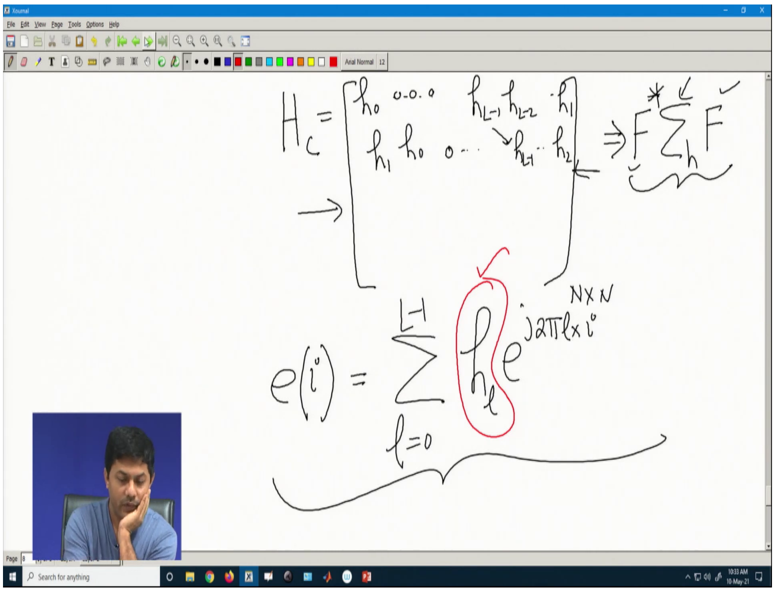

Now, if the channel is structured. So, this is a circular channel right. H c after CP constellation. So, this is your N cross N circular channel. How does it look like? If you notice this is h 0 L minus 1, this will be all zeros, then h L minus 2 dot dot h 1, ok and then this will be h 0 h 1 and then this will be pushed back to h 2 and this will be coming here h L minus 1, these are all 0. So, this is how the circulant channel matrix, right.

So, the eigen value e i the ith eigen value ok. So, there is I am just writing the formula or e i the ith point assume because if I do a EVD, this is this was F star sigma h and F and this I can decompose, right. So, these are standard, this is standard. So, this is the one I am talking. That is the that is the that is a square matrix, but it is a diagonal matrix which contains all the eigen values.

So, this is a very standard you know what this will be all your you know h l. That means the lth taps l equal to 0 to say I have L minus 1 number of taps are there into to the power j 2 pi l multiplied by this i. So, this is the standard formula. This will give you the eigen values appear. So, that means it boils down to the fact you have to know the individual channel tap ok. So, if you know individual channel tap and how do you know? That you have to do channel estimation.

So, that is the reason where channel estimation will come. So, you have to know channel, but only for knowing the eigen values, not the equalizer, not the precoder. So, that is one of the advantages. So, you are not really getting rid of channel estimation part. You have to do channel estimation. At least you have to know this and how do you do that? We will if the time permits, we will see that because intention of this course is more of a beam forming ok.

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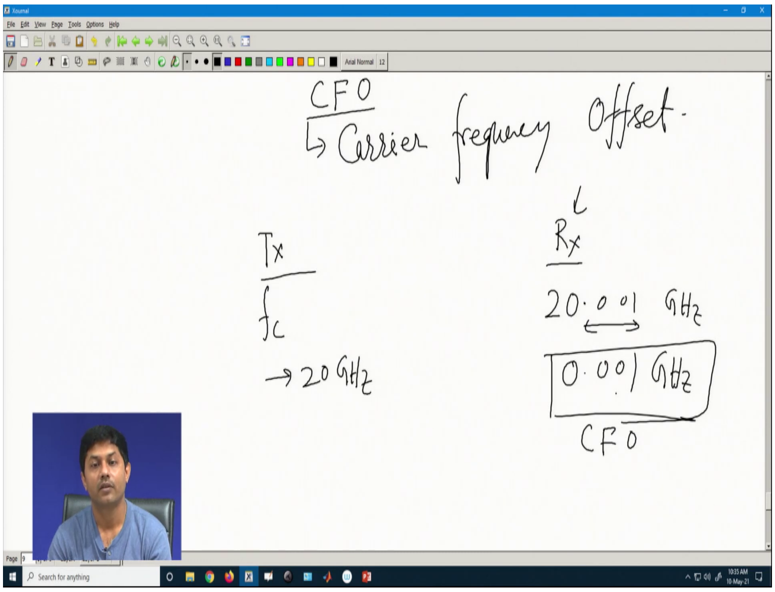

So, this is one part of the things. Next comes I will be just touching upon a problem called CFO. So, point here is that is the OFDM is like a rosy kind of things. Does it mean that it is the it is the gift of all problems all solutions? I will say no. It solves some of the problem, but it comes with some of the problems ok. Probably it solves a bigger problem, but it comes with some of the problem. So, one such problem is called carrier frequency offset, ok.

Carrier frequency offset. So, how exactly what is the carrier frequency offset? Forget about OFDM. So, carrier frequency offset is that if you have a transmitter and if you have a receiver ok, so you are sending or you are modulating with the frequency say f c ok. So, it can be say 20 gigahertz at the Rx, you may not be able to exactly demodulate at 20 gigahertz probably the you know the p l l that you are taking it exactly won't produce 20 megahertz. It may be something like that.

So, there is still some mismatch between the two, ok. So, this 0.001 gigahertz mismatch does it create any problem in my system? Yeah. So, this is actually called this terminology, this is your CFO, this is the carrier frequency offset. Now, we will see that how this particular one destroys the benefit that OFDM carries. So, the lesser the CFO; bigger the benefit, bigger the CFO; lesser the benefit.

So, we will explain in the next class how exactly this CFO can be modeled into our OFDM system and how that carries forwards to the beam forming case and how the beam forming also get disturbed by this. So, these are something called an impairment, but we should know it because this is one of the killer factors of the OFDM and subsequently for the beam forming. So, with this I end the session. In the next class, I will be talking more on the OFDM with CFO and that part.

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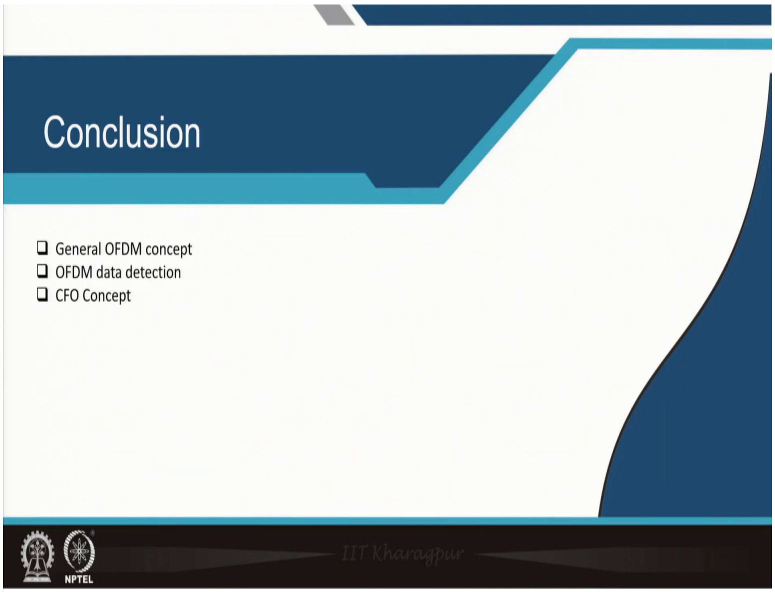

What are the points that we have covered? So, we have covered the OFDM. I think we have completed some of the part. One is the spectrum part is left and we just touched upon how OFDM data detection happens and the basics of you know CFO concept.

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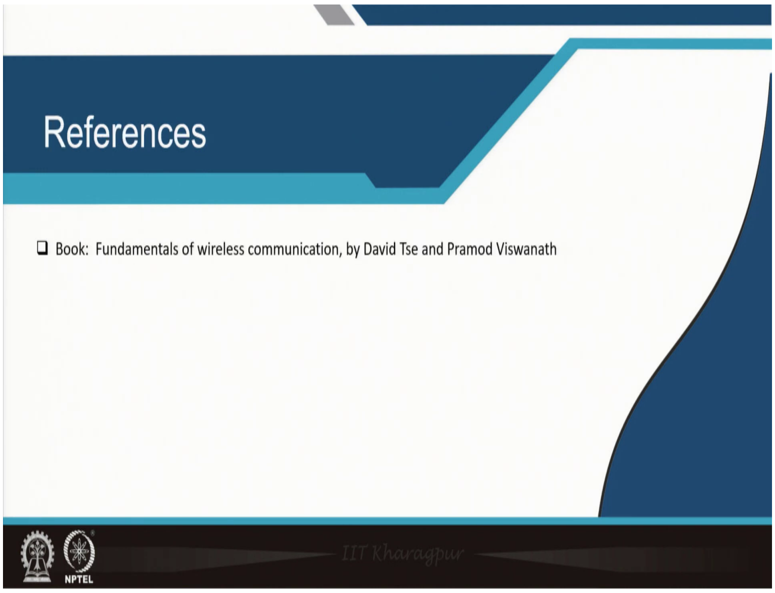

And the references are many. I just put one of the reference which I also gave it last time. So, you read chapter 3, chapter 4 where OFDM is given, but there is no one point reference for OFDM. There are millions of contents available in I triple explorer where you can find OFDM concepts.

Thank you.