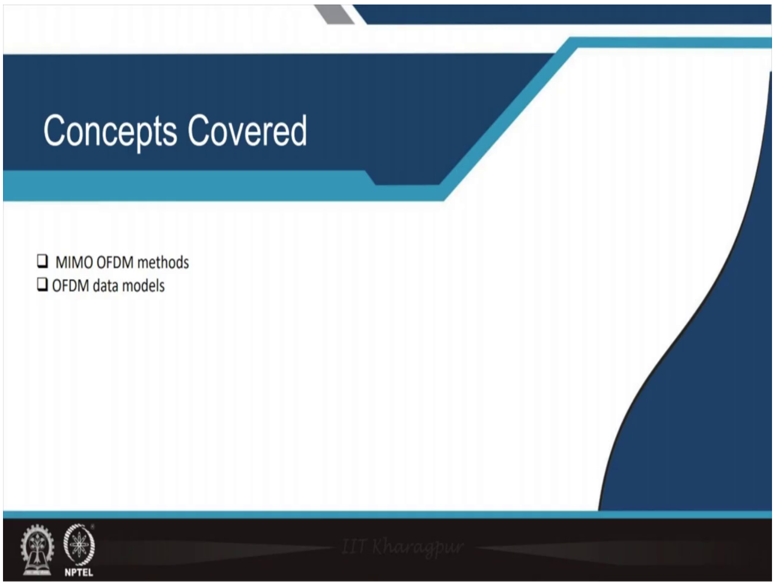
Signal Processing for mmWave Communication for 5G and Beyond

Prof. Amit Kumar Dutta
G.S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 10
MIMO-OFDM beamforming
Lecture - 49
OFDM Data Model

Welcome back. Welcome back to the milli metre Wave Communication for 6G and Beyond. So, today we will be covering the lecture number.

(Refer Slide Time: 00:34)

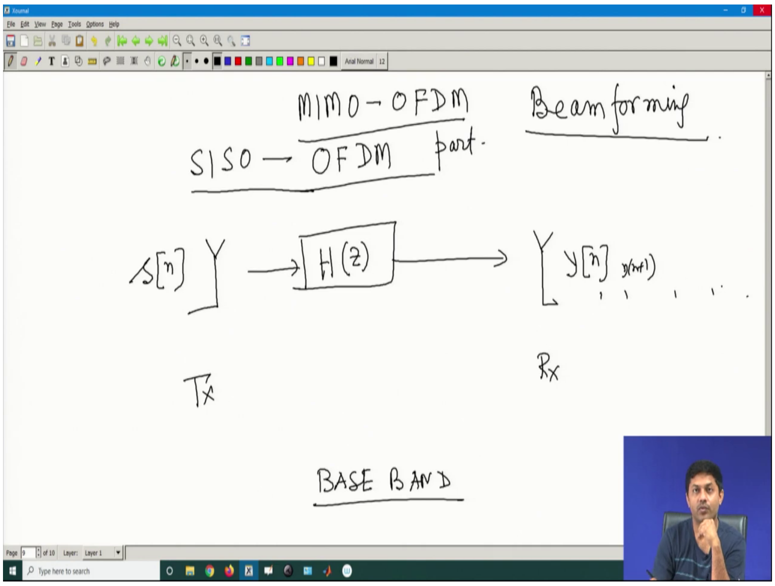


Things that will be covering are the following. So, now, what we will be talking about today is the MIMO OFDM parts. In the last class we have tentatively completed the MIMO beam forming part and in that context we have also covered how do how what is the; what is the first step of finding out the different parameters like equalizers and pre coders.

But we have not gone to the methods, method is just separate modules for that we will be discussing after that. But we just stopped at that point why we stopped at that point? Because we wanted to only show you how exactly the complete T x to R x data models are present here. Now in that context as you are still in the data model phase. So, now, we have learnt how the MIMO system works there and how the in the context of beam forming how MIMO things works.

Now, we will be; we will be extending this concept to the OFDM part MIMO OFDM part. So, you already know what is MIMO and then you are extending it to MIMO OFDM ok.

(Refer Slide Time: 01:48)



So, this is the topics and we will be taking few lectures on the MIMO beam forming MIMO OFDM beam forming what are the structural change? Probably it may be a very simple structural change, but overall you should know that. But before I get into the you know MIMO OFDM part you have to first know what is OFDM and I am pretty sure some of you know it and obviously, some of you may be knowing it MIMO OFDM things, but let us revise that because OFDM has its own flavor and MIMO OFDM has its own flavor.

Probably, if you know MIMO OFDM beam forming would be just maybe just a simple extension to it. Now let us spend some time on understanding what exactly the OFDM part is, then what exactly the MIMO OFDM part is and subsequently the beam forming. So, we will be first talking about the OFDM part today. Now as you say there is a MIMO OFDM so; obviously, there is a single antenna OFDM also should be let us start with that.

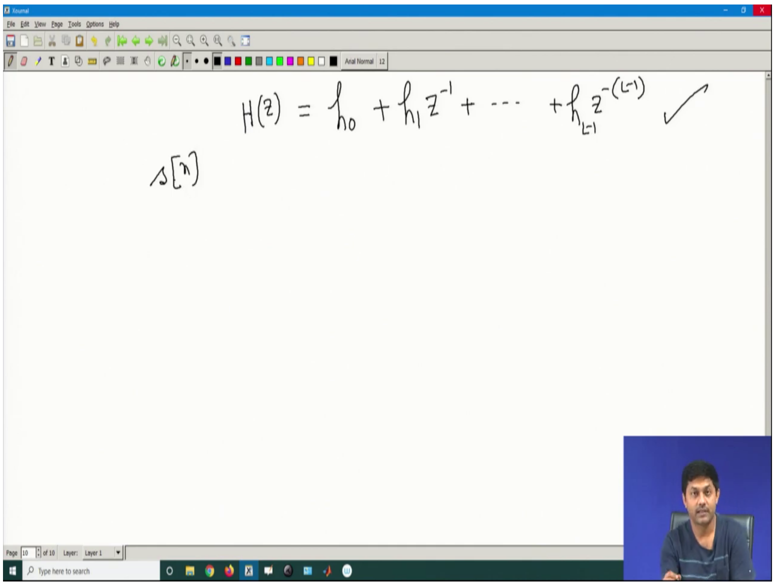
So, to understand MIMO OFDM beam forming, we have to first understand SISO OFDM beam forming ok. So, that is the step by step approach and once you finish the SISO OFDM beam forming then it will be just a you know cake work to go for MIMO OFDM beam forming ok. So, let us for the time being just keep aside the beam forming aspect because first understand SISO OFDM and then we can plug in the wave beam forming.

So, let us assume that I have just a single antenna physically single antenna in the transmitter side and I have a physically one antenna at the receiver side and do not bring the r e f and analog chains in this case we are completely in the baseband model ok. So, we are in baseband model we are completely in baseband now let us say. So, what was the channel model from one antenna to another antenna does not matter.

So, its a millimeter wave or anything the channel is a FIR filter. So, let us have a FIR filter let us say H of z z transformation of in a FIR filter right. So, you can always have that and this is how it is ok and let us say I have the s vector now its a single antenna. So, I cannot have s vector let us say s s n will be going there right and here I will be taking r n r n or y n you can say right y n also because consistently we were using the notation y n r n was more of a after the post equalization.

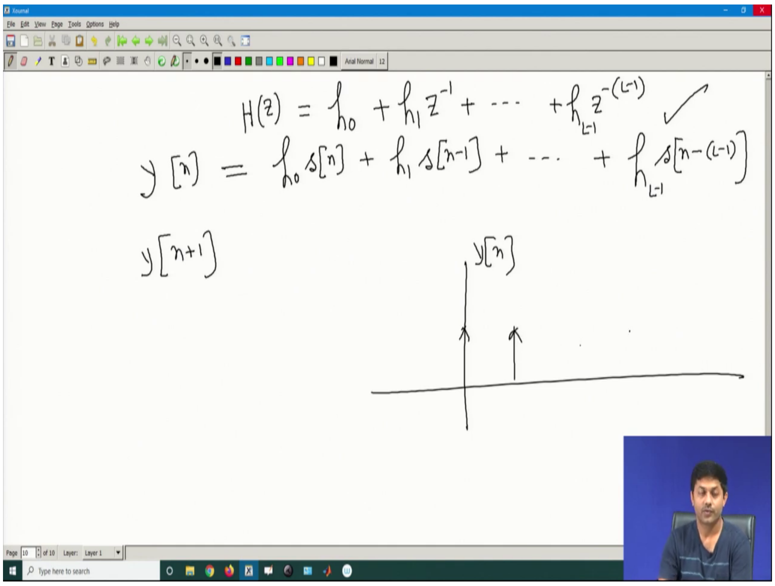
So, this is what the data is right. So, what is the data model that we want to create it here? Let us assume that first ok.

(Refer Slide Time: 05:23)



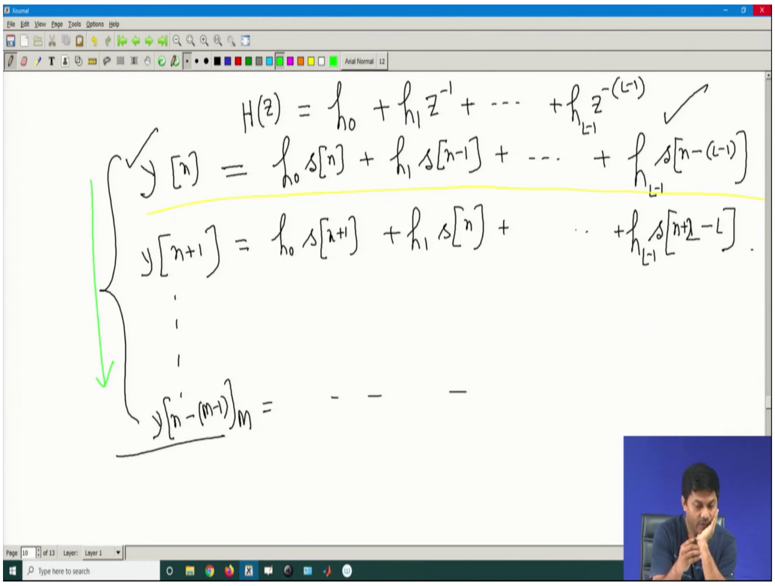
So, let us say this H z its an FIR filter let us say it has let us say this is what it is let us ok now this is the channel. Now we have explained extensively how a channel can be represented as an FIR filter. Now if I give s n as a data right to the system this system this system what is the y n there right. So, you can always create y n.

(Refer Slide Time: 06:21)



Let us say this is my y n how do I find the y n? If this is the channel it will be h 0 s n plus h 1 s n minus 1 plus h l minus 1 s of n minus l minus 1 right. This what it is as simple as that right is this perfectly fine right ok. Let us say I have y n plus 1 because this is the y n is the first sample. So, I was same it was as if like this is my sampling point somewhere I was sampling my y n the first one then y n plus 1 y n plus 2 and so on and so forth. So I am just sequentially observing all my data. So, what is y n plus 1?

(Refer Slide Time: 07:29)



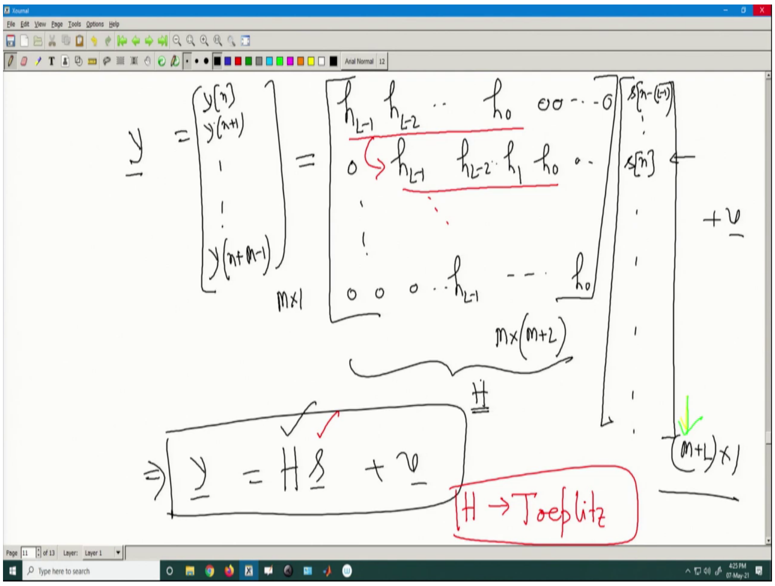
I am only doing few of them; obviously, this will be n plus 1 minus of L plus 1. So, probably this will be plus 2, I mean you know how it can be fit into it. So, that is not similarly I can have as many as data you want right straight forward. So, if I stack them together that mean what does it mean? It means that if I look at my transmission system I am sending the data 1 by 1 unlike your MIMO unlike your MIMO.

Why because in MIMO I am sending the data at the same time all the bunch of data I am sending, but as I am using a SISO single antenna system I cannot send s n, s n minus 1 or s n plus 1 whatever at the same time right because I do not have the antenna. So, at a time I am sending only just one, but I am observing one by one. So, I am putting them in memory ok.

And then I am stacking them together so; that means, if I look at here it is not just one observation rather I am observing the data in a time in time I am observing not in space I am observing that. So, this is my y n, this is y n plus 1, this is y n plus 2 and so on and so forth. So, this is how I am observing it now this is what I am writing it right. Now if I if I you know stack my data that mean I observe say certain length of data I am observing it, it can be m length data or whatever.

That mean y n minus m minus 1 it will have an equation I am not writing some equation with there. So, what will I get if I stack them together.

(Refer Slide Time: 09:32)



So, say I am calling it y bar, this will be what it would be. If you pay little attention you will see this will be some channel matrix will be coming right and some data will be coming as simple as that ok. So, who are they? So, here you can just look at the complete data here ok. So, how do I get it? You can say h l minus 1 h l minus 2 dot dot h 0 it will be all 0 0 right then this will be shifted h l minus 1 h l minus 2 and then here h 1 then h 0 some 0.

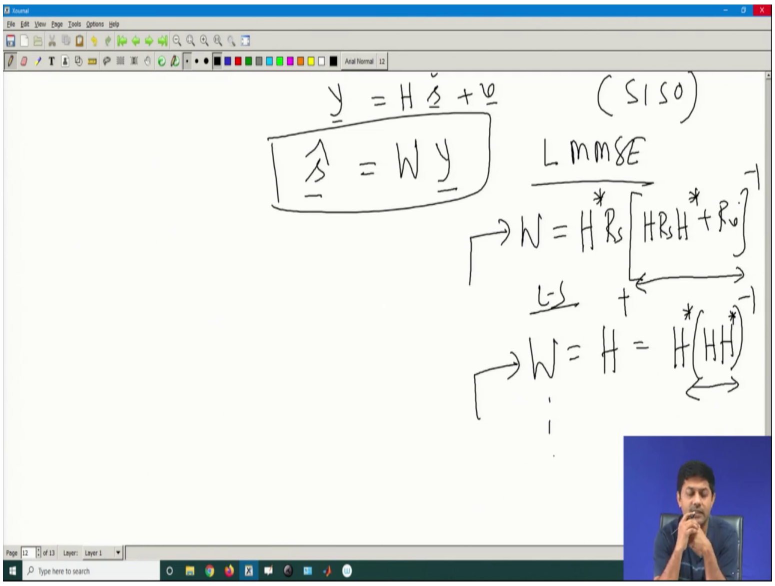
So, at the end it will be h 0 h l minus 1 in between and it will start from 0 this how the channel matrix is right and then take some little effort you can. So, at some point of time at this point you may have s n and before that you may have s n minus l minus 1 or something like that. And here you can have the data that is how it is right ok. So, what is the dimension? This should be m cross 1 because that is the number of observation I am having it definitely this will be m cross how many? It will be as many as data I want to observe it. So, it should be l plus m kind of thing m plus and this will be m plus L cross L.

So, that is the number of kind of data you will observe it. So, in a very simple matter plus v I am not writing the v part because that is anyway. So, now, in a very simple matrix notation this will be y is equal to now this whole thing I call it some equivalent H, this will be some vector plus v vector. So, this is how again the data model comes though it is a single antenna it is as if like I am creating a multiple antenna no, it is not a multiple antenna its still a single antenna, but its just that I am observing in time and I am stacking all my observation together ok.

Now, what is the nature of my H ok? You see that the nature of the H is something like a you know you are shifting the row to the right side if you look at if you look at this is the row right and this is one level shifted right side one level shifted and so and so forth and finally, it touches h 0 everything is like shifted. So, what type of matrix this is? This to a Toeplitz matrix.

So, my H comment is H is a Toeplitz matrix Toeplitz matrix that is ok. Now may be the nature this is how the nature would be, but I am not worried about that now you know how to get back your s ok. So, how do you get back your s you can create some equalizer.

(Refer Slide Time: 13:15)



Because you know your equation is Y is equal to H of s bar plus v bar [FL] it is a SISO model mind it as you like its a MIMO no its not a MIMO just because you are observing at different instance of time. In a MIMO you are observing a different antenna point. So, that is a; that means, your observation is at the spatially distributed, but here it is time wise distributed that mean I am observing in different time, but in a MIMO case I am observing in multiple antennas the space wise observation which is still as issue.

Now how do I get back my s? Now you know how to get back my s you can always develop some sort of equalizer and then you multiply this y and you know how to develop the equalizer right. So, let us say if it is an LMMS equalizer. If it is LMMS equalizer what will happen how do you develop W? This W would be something like your channel then R of s meaning the covariance matrix of the data vector then it will be H R s H star plus covariance matrix of your noise, this is the simple this is a simple equation.

If it is least square then your W would be H pseudo inverse which is nothing but H into whole inverse something like that ok yeah sorry yeah. So, many ways you can have; you can have some other. So, point is you can develop your equalizer and get back. So, one part of the story now the point is that suppose I would like to do some sort of a pre coding ok or rather I would like to have this question to you is this equalizer development simpler task actually if you notice its not a very simpler task what is the problem with this equality equalizer development.

The problem of equalizer development is that, this part inversion creation this part inverse creation why it is a problem? What if the dimension of y is large ok let us say I take a dimension of 30 or 40 what is the dimension of this inverse? 40 cross 40 matrix. So, it is 40 cross 40 matrix inversion is not as easy task it will take huge computation ok.

So, that mean I or we not I we never want such kind of equalizer development for a large system for a small system fine that is why if it is just a 2 cross 2 MIMO or a 4 cross 4 MIMO or even a 8 cross 8 MIMO probably this works fine, but even if it is SISO case I am observing now if I start making more and more observation why I need to make more and more observation I think that should be first answer.

Why it is not ok to just have one1 observation and detect my signal. If you will notice go back here if you notice here. If I just make one observation and try to extract my data which 1 I will extract just take this take this data now you have one observation and you have one number of unknown s n, s n minus 1 till s n minus l minus 1 one unknown one observation can I do it? Can I do a good detection of my s n? Can I do a good estimation of my s n?

Probably no. So, I have to increase my observation as well now another problem is that even if you increase your observation, this length will start increasing this length will start increasing. So, you will always have a rank deficient system where you have more and more unknown system compared to your observation, but if you make at least more and more observation more and more observation it is better than having just one observation and extracting so many information that may be a tougher job.

So, it is inevitable that you have to make more and more observation from a single antenna case whenever your channel is an FIR filter otherwise its performance will be very poor right think about it. Suppose l l itself is ten its a 10 length tap and you are making only one observation. So, you have one observation 10 unknown data.

So, how do you make a decision virtually impossible to do that virtually impossible to detect. 10 you know unknown from just one observation its a map does not support mathematics does not support right because this will be a rank diffusion system. So, you have to make at least some 100 observation let there be 110 unknown, but at least I can detect 100 better right out of 110.

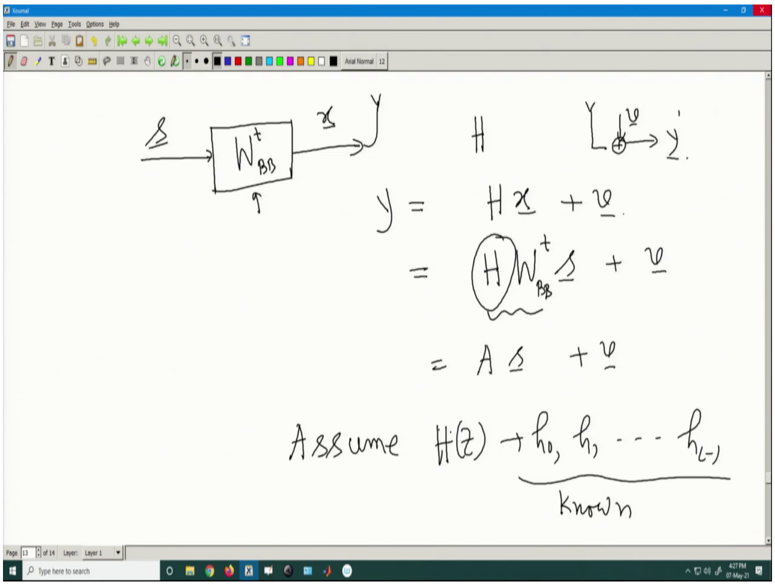
So, that is the whole idea why I need more and more observation here. Now the problem is that if I want to make more and more observation now I am stuck in the inversion problem of my equalizer. So, can I do some other thing by which I do not have to face all this equalizer business? This equalizer business I do not have to face it ok.

So, one such simple technique is that, I can have some simply pre coder not motivating you why how a OFDM comes into picture. Now there is a; there is a spectrum way of thinking OFDM, but I am not getting into that I am thinking from a very simple engineering point of view ok. Now this is one of the issue that I have an inversion problem. So, can I avoid the inversion in my detector and have a very simple kind of detector I do not want to get into this inversion problem.

But looks like it is not possible if the given system is like that. So, what I do is that I develop some sort of a pre coder ok. So, this is one such technique where you can have a pre coder and you can reduce the you can actually avoid the inversion in your receiver let us see how it can be done. Now pre coder itself is a big topics let me not get into that and you can have pre coder based on various criteria.

Again let me not get into that, because my intention is not to develop a pre coder based on various criteria rather my intention is to develop a particular pre coder a particular pre coder I am repeating that such that my decoder is of simple enough where there is no inversion that is my goal. So, let us say I have a peak I want to have such kind of pre coder.

(Refer Slide Time: 20:17)



So, which means that which means instead of sending my data vector directly what I do is that, I send it through some sort of a some sort of a pre coder. So, let us call it W baseband because that is the notation we have used it, let us call it x vector and that is the one1 which is serially being transmitted serially being transmitted through my transmitter. So, that is what it is let us assume that one.

So, this is the one which I have developed ok now how do I develop that itself is a very big topics ok. I am not getting into that by here intention is that I want to develop a particular one which will make my de coder life simpler ok. So, how do I do that? So, this is my H and this is where I receive my noise v vector and that is my y vector again this is a single antenna system which means this y vector is more of a time sample data ok.

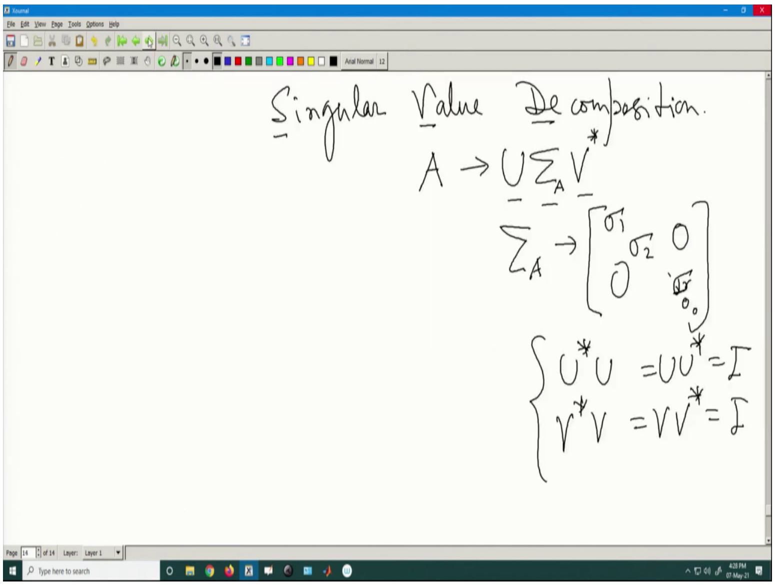
So, what is my data model? y is equal to H into x because I am now sending x instead of s because s will be you know modified this one it is ok let us put it back the x bar. So, what it would be? W BB t into s bar because my intention is to get back my s, but still I am not done it does not solve any problem because from one problem it just transfer the problem to another because W BB is also be some sort of a matrix.

If it is a matrix what will happen W h into W BB t is also some matrix right let us call it A matrix A bar plus v does it solve my problem of de coder? No because it is still have to do the inversion level of equalizer right because I want to again avoid that. So, I make some trick ok what is that trick? Trick is the following. Now let us say my channel is known to me because channel is its a FIR filter, let us say I have done a channel estimation and the channel is known to the receiver as well as to the transmitter ok.

So, does not it matter let us not worry about transmitter side, let us say I know the channel; that means, this matrix is known to me. Instead of matrix I would say the channel coefficient h 0 to h l minus are known to me. So, I know this H. So, I can develop this whole h this whole H can develop because I know all these coefficients its a Toeplitz matrix I can find out what the H is that is not a big issue ok.

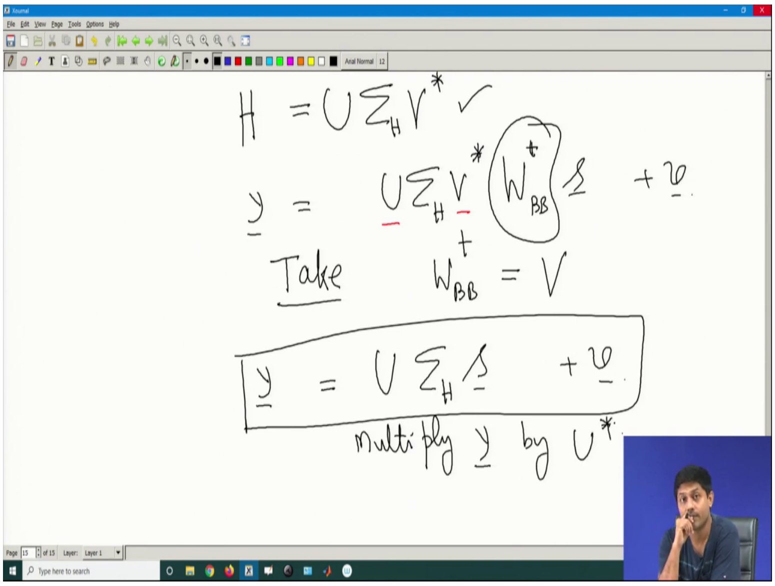
So, channel is known to me. So, let us assume channel H z; that means, this all this h 0, h 1, h l minus 1 they are all known to me they are all known to me ok. So, what is the advantage. So, that mean that whole h matrix I know. So, let us do a SVD.

(Refer Slide Time: 23:33)



So, let us do singular value decomposition SVD you know what is SVD right. So, A is a matrix, I can always break such matrix into three component U sigma V ok. Now, U and V are unitary matrix and sigma A is diagonal matrix is the diagonal matrix depending on the rank of this matrix you will have say let us say my rank is r. So, you will have sigma 1 to sigma r component and there will be 0, the other elements and rest of them will be 0. So, its the diagonal element and U and V are completely unitary matrix. So, which means U U star U is equal to U U star is equal to I similarly V star V is equal to V V star is equal to I. So, that is the property we have followed it here ok. So, similarly here any matrix and this SVD exists for any matrix no matter what the dimension whether its a square matrix or non square matrix it will exist right. So, let us assume that in my data model whatever H equal to H matrix.

(Refer Slide Time: 25:09)

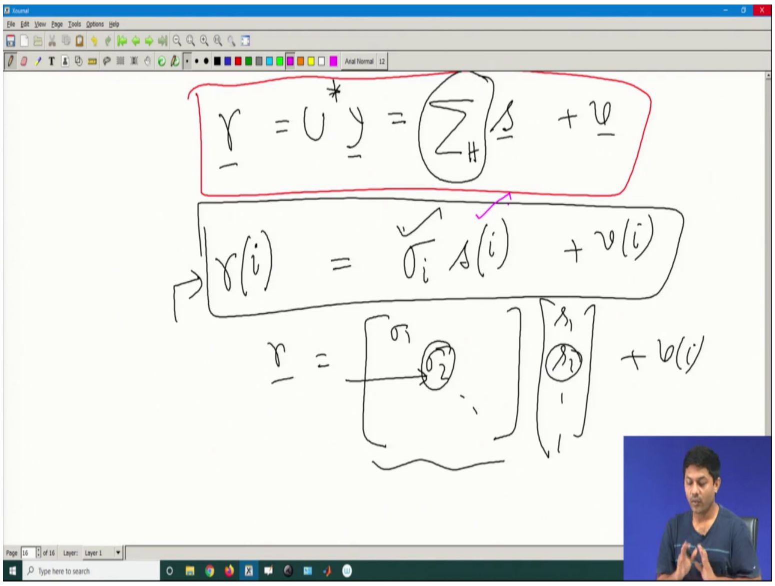


So, let us assume it has a SVD. So, let us assume that this H also has an SVD U sigma H and V star because if the H is known knowing SVD is also not impossible ok. I am only saying not impossible does not mean that its easy let us forget about that part how easy or how difficult it is let us assume that I somehow managed to get that ok. So, let us assume that what is done U sigma H V star ok.

Now you come back to the data model. So, your y cap is equal to U sigma H V star and this will be your BB r BB t ok. Now then you can guess what should be my W BB t now again I am saying W BB t can be found 100 ways or more than 100 ways let us not get into that. I am saying I am interested only 1 particular type of W pre coder design ok. So, let us assume I choose W BB t because it is in your hand you are the 1 which is transmitting rights you are the developer and you know U V all these matrixes you know because you know the channel right.

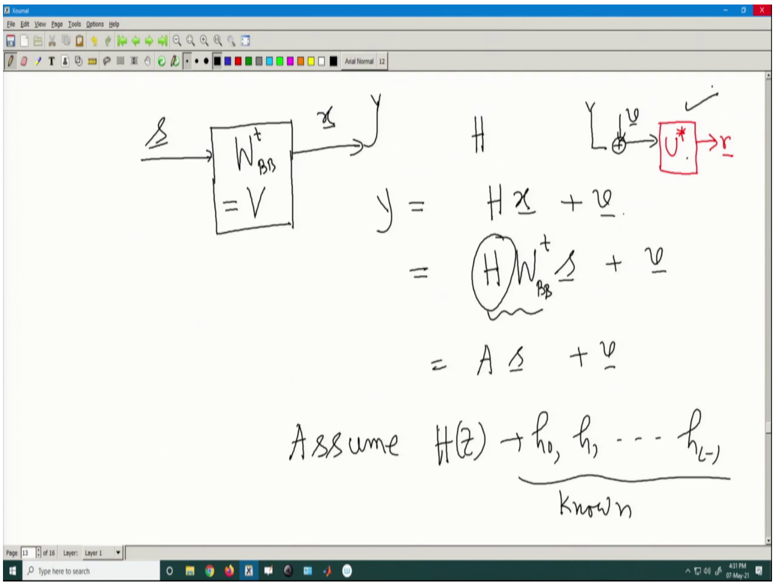
So, you assume that you take W BB is equal to V matrix note it is only V matrix ok. So, you replace it here so; that means, assume I should not say assume I should say take W BB t equal to V U pick up then what will be the data model? It will be U sigma H and V star into V it becomes I. So, that part is over ok now what you do here? Now you get Y cap right this is your y cap now what I say? After that you just multiply it with U. So, you multiply Y bar by U star let us see what happens ok.

(Refer Slide Time: 27:42)



So, I say let us call it r cap r bar which is equal to U star into Y bar what will happen? This U will be vanish it will vanish right. So, this will be sigma H into s bar plus v bar. Let us say that is my new data model. So, what does it mean? It means here whatever I am getting this antenna model instead of these what I do here is that.

(Refer Slide Time: 28:21)



I multiply this with U star and get my r bar and here what I do? I pick W BB t which is equal to V matrix that is the only change I do. I want this type of pre coder let us see what happens ok that mean I pick a coder which is equal to b pre coder equal to U star. So, what will happen? So, finally, I get this one r bar is equal to sigma H s plus v now let us think about it what is the de coder strategy? So, now, this is the diagonal matrix right. So, diagonal matrix. So, what do I do? If it is a diagonal matrix is it a difficult thing to do. So, it means that.

If it is a diagonal matrix, I always get individual element of r as whatever some sigma i plus s of this vectors ith element right plus this v i you can say diagonal matrix what does it mean? It means that you have sigma 1, sigma 2 dot dot dot and there is some s 1, s 2 dot dot dot and this is my r vector plus v vector. So, if I say the ith elements ith element; that means, only multiplication of this only multiplication of this and the ith element of this that is all will come.

So, I really do not have to worry about this matrix inversion and all because I though it is a matrix, but I am not now dealing with a matrix because I can individually pick up and individually the data model will be like this. Can I decode this? Can I decode s i? Very simple why because this is just like a scalar you scalar, this is scalar map this is just a scalar value; that means, r is equal to sigma into s in plus v.

If you know sigma you can if you know sigma because you know the channel and you can get back your s normal estimation what it takes sin theta I do not have to involve any matrix there because just like a. So, this is the beautiful part of a particular pre coder now why am I talking all this? Because OFDM when I talk about that this is the ramp up or this is more of a motivation part of why OFDMs come into picture in my life ok.

Because this simplicity in my decoder is something that we need to worry about that mean if I can create that particular pre coder followed by U star which is my I can think of an equalizer, the final equation is just like a diagonal matrix and I can pick up individual element of my data and I can you know I can estimate my s i or I can detect my s i very easily no matrix in machine in this whole process I have not involved any matrix inversion that is one of the easiest thing ok.

So, in the next class we will be now moving more towards the what are the issues with this part ok and that gives birth I would say to a new concept of OFDM called OFDM. So, this whole thing we just keep it in mind and we will see that how OFDM can solve some of the issues that even this particular solution gives.

Now, this particular solution whatever I have talked about pre coder based solution, it only solves the receiver part because it completely take away the complexity of my receiver design receiver is very simple there is no matrix inversion, but it has its own problem ok. So, with this I stop this session and in the next class I will be talking more about its problem and subsequent you know the birth of OFDM concept.

(Refer Slide Time: 32:17)

So, in conclusion we have kind of completed the yeah.

(Refer Slide Time: 32:21)

And the reference will be same as what we have ok.

Thank you.