Signal Processing for mmWave Communication for 5G and Beyond
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Module - 09
Hybrid beamforming concept and Beamforming in MIMO
Lecture - 48
Parameter to be designed in MIMO Beamforming

Welcome back, welcome back to millimeter wave communication for 5G and 6G. So, today we will be covering the. So, last time we have discussed very details about, what are the parameter that needs to be configured or rather needs to be designed? But we have not put much effort on how they can be designed ok.

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So, we will continue the discussion things that will be covering are the following in the context of MIMO a beamforming cases. So, now let us go back to the mathematical models because from the mathematical model only you can create a lot of things. So, let me just write down the mathematical model for the MIMO beamforming cases.

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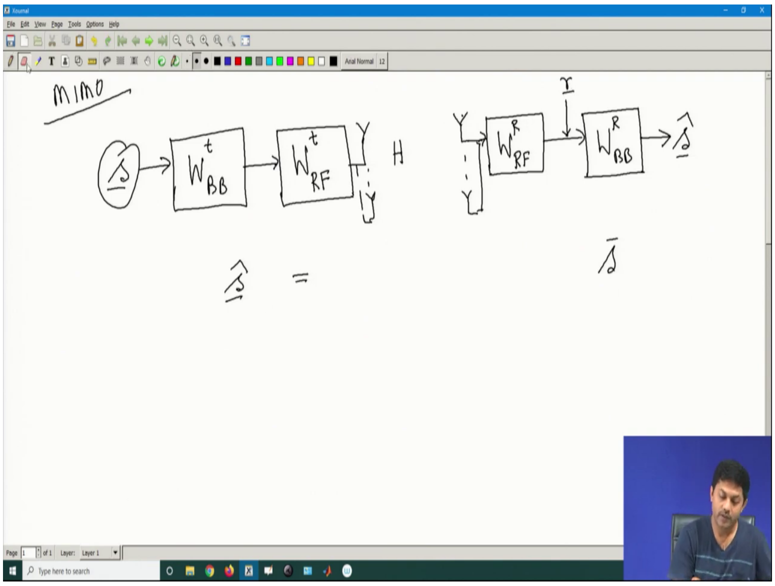

So, you have the s vector that is your data vector ok. So, this is a digital data vector. So, that first goes through W BB t transmitter side precoder and then you have the beam steering vectors beam steering not vectors matrix if it is a MIMO. So, I am still in a MIMO case.

So, that is why this is a vector had it been a SISO case this particular vector would be just a single element, I think we have discussed in details this should be the case. Now, this goes through the antenna, now this is the baseband model. So, let us not get into the RF and all.

Now, you may have multiple antenna here physically that is ok. So, now, physically this will be a channel. So, now, again the whole model will be more of a baseband context.

But I think you as long as you understand where you come from that should be fine because you are more interested in the data model part and then this goes to that is the equalizer right and probably W R or whatever I think you can put it W BB also the base band equalizer and from here the output would be s cap right. So, these are the things.

So, finally, see if I say at every level what my data model is. So, your s cap s cap s cap is what your estimated value of s, so that is an estimated value ok. So, at what level you want to data I mean as per that thing should be. So, it will be s bar or if you say this is my y vector or rather I would say this is r vector received r vector after the baseband. So, probably I will put it this is more fruitful way of writing it.

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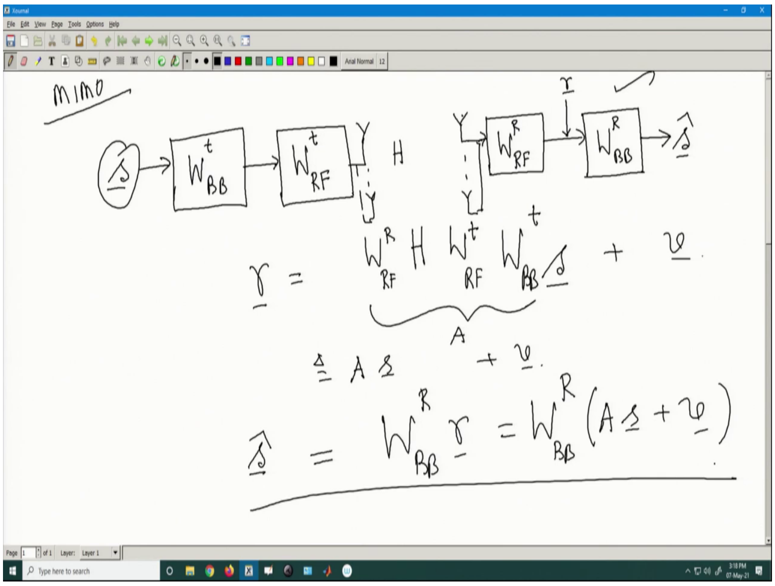

So, the r levels the r vector s this one then there is a channel, then your all receiver level aspect will be there. So, this is your vector v right this is what we have learnt its last time and this is what your A vector is. So, it will be something like here A s bar plus v and there is a final equalizer. So, by which you can get back the estimated value of your s.

So, that would be ok. So, this would be the case here right. So, you can say A of s vector plus v vector. So, this is what we have learnt it last time right. So, now, what we want to stress on the fact that, how would you know how do you in fact estimate all these parameters all these equalizers? How do you exactly get these equalizers?

Then we said that ok, to get back this equalizer you have to first now to get W BB you have to first know A then you said ok to get A, the only way to get a is that if you have to first know h or the channel if you know the channel you can you know you can get rest of the thing.

So, in this case we are splitting, I think last time also discussed we are splitting the whole problem in two domains right. One is knowing the channel first and then you know then finding out rest of the matrices, so four matrices you need to find out. So, today we will be explaining how what is the; what is the framework by which you really get those 4 matrices given H. So, there will be subsequent classes where I will be talking more on how exactly the optimization problem can be solved.

So, there is a separate you know. So, there is a separate module for that. So, today and subsequently another two, three classes we will be explaining, how exactly my cost function will be fabricated because once you know how a cost function is fabricated you can come up with your own optimization techniques.

So, that is the main goal in our case today. So, today we will be seeing the optimization case. So, now, when I want to develop some sort of equal any design parameters or you want to estimate any of these you know any of these parameters you have to create an optimization framework.

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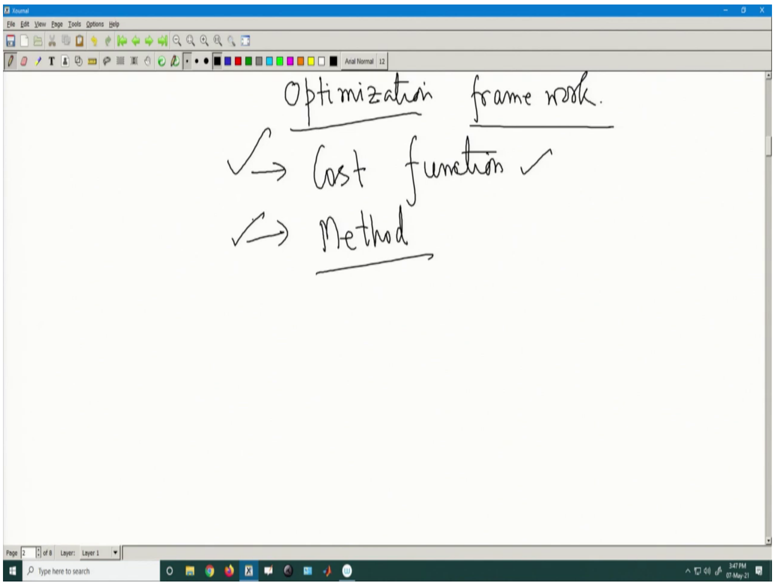

So, moment you say optimization framework, so the first criteria. So, now, you have to develop an optimization framework. So, now, this optimization framework is a one which determines all these parameters ok. Now, to do an optimization framework the first thing that you have to understand is.

So, what is the, what is the first criterion optimization framework, is to develop the cost function. So, first thing of an optimization framework is to develop the cost function and then once you develop the cost function based on that you can create your own optimization method. So, next comes the method.

How exactly you actually solve it? Now, today I will be mostly talking about the cost function development. So, what are the different way you can develop that? This method part there is a separate modules and there will be four, five lectures on the methods, but how exactly different methods can be applied in the context of MIMO being forming to do the optimization. So, let us talk more on the cost function part.

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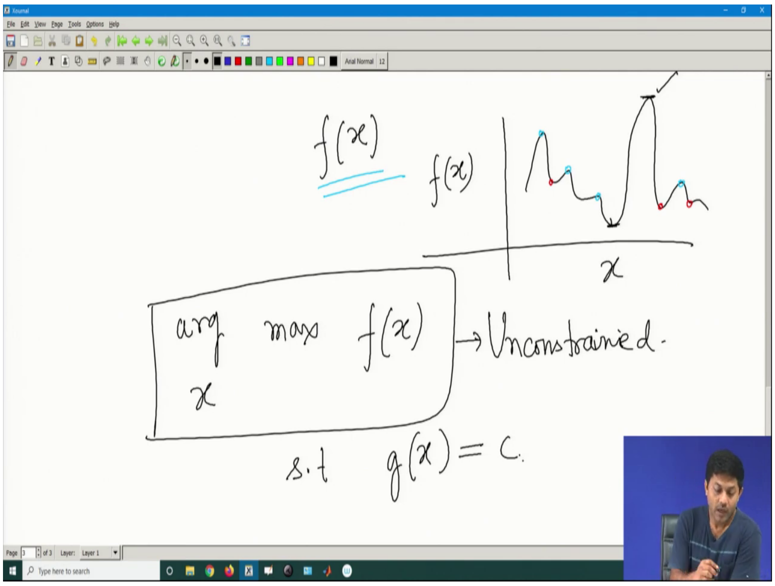

So, how exactly you want to do a cost function development. So, cost function will be based on some criteria ok. So, say f x is a function and I would like to optimize it optimize meaning, I want to find for what value of x, I will have my x minimum or maximum? Say for example, my f x can be something like that, now it is a single variable by the way or my explanation I just take the single variable.

So, this could be the function ok. So, it can be some random function. So, either I want to find out what is the minimum point or I want to find out what is the maximum point. Now, this is some sort of a global maximum I think if you have some notion of optimization probably you may know. This is the one which is called the global of maximum point, this is the one this point you call it global minimum point.

These are the points which are mainly the local minima probably this one and these are the points what you can call them as local maxima right. Now, your optimization framework there is no guarantee that it will always be converging to a local minima or sorry global minima or global maxima. It depends on which type of method you are utilizing it or what type of cost function you are using.

But this is the mainly seen main type of scenarios here. Now, in our context, who is that f x? Let us understand that. That mean, what is that cost function which can generate our parameters. So, in the context of normal optimization, so what I do here. So, basically if f x is a cost function I want to either maximize it or minimize it depends on what exactly want and this is what is called an optimization framework.

Now, this is unconstrained (Refer Time: 10:04) this is unconstrained you can also put constrained. Say for example, I can say optimize this function such that some other function of x should satisfy this criteria ok, so this is constant optimization. Now, let us see in our case whether I would like to have a constrained optimization or I would like to have a unconstrained optimizations ok. It depends on how you. Now in our case who are the cost functions.

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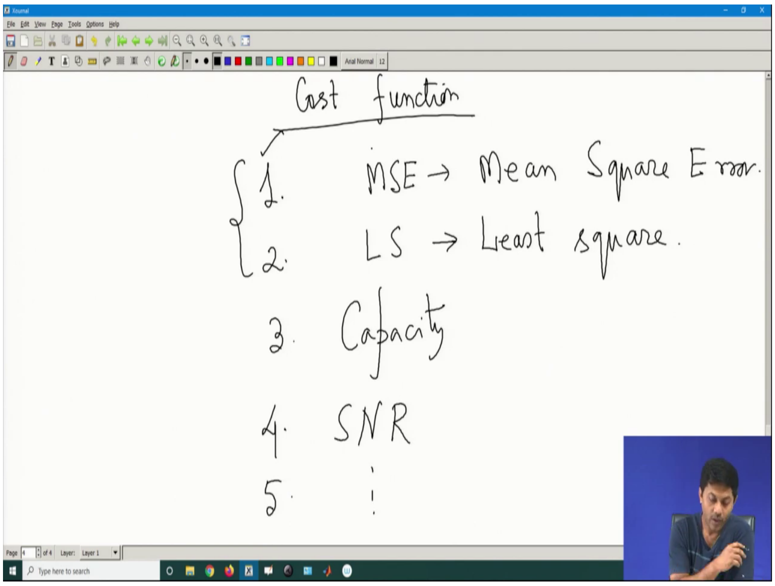

So, let us understand that ok. So, the main focus of this talk will be mainly to develop the cost functions ok. So, what are the different type of cost function that I will be most interested in? The most prominent and widely accepted cost function in the context of signal processing and communication world is called MSE ok, this is called mean square error ok.

This is very widely accepted cost function, then you can have LS or is called least square this could be two very prominent apart from that as your communication is, now these two are you know these two are very general cost function and this is widely accepted in the context of communication.

But there are some specific communication related things one such cost function is called capacity optimization. That mean, I would like to obviously, I would like to maximize my capacity and that would become a cost function. If I have some more some more are there you can have SNR as your cost function that mean I would like to have those parameters in such a way that it maximize my you know SNR ok, anything else? That could be mean right. So, the list can be more, so it depends on how would like to have it here.

Now, in this context I will be talking some of the MSE least current capacity. Now, probably least square I may not talk much because this is a non Bayesian framework and this one is more of a Bayesian framework. So, that is more widely accepted, but has a different pros and cons.

Whereas, least square is very simple and kind of low cost, the capacity is also I mean it is complexity wise it is pretty high and sometimes you may or may not get a closed form expression ok. And all this cost function whatever you develop there is no guarantee that you will get a closed form expression ok closed form in the sense that you do not have to iterate on the solution.

For example if you look at the lmms equalizer right whatever we developed in one of the classes that is a closed form expression, but its not that every time you may get a close form expression here ok, so it can be iterative ok. So, let us see, what is the MSE? In what context my MSE comes into picture? Ok.

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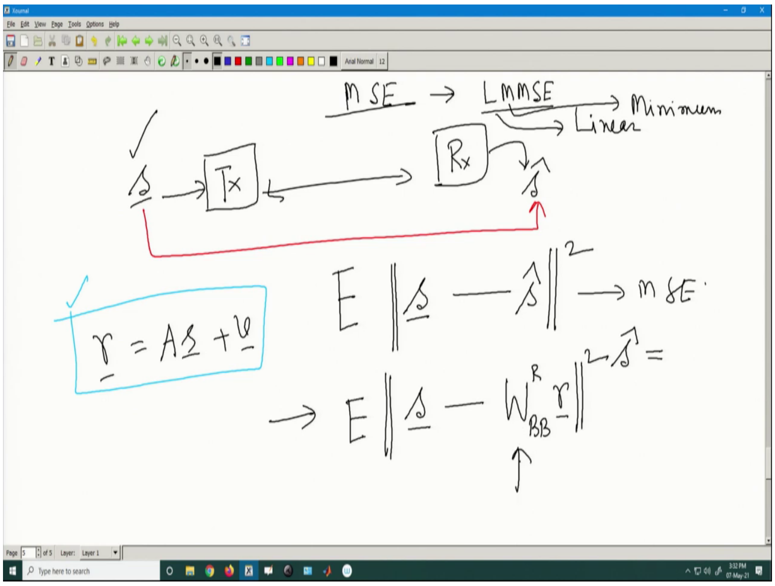

Now, MSE of what? It is a mean square error so ok. So, in a normal communication system if you have a Tx suppose this is my general text, I am just writing a you know general kind of things this is my Tx my Rx, now from the Tx what is my data? Because that is my data finally.

So, no matter what you do here whether you do a precoder or whether you do a ofdm whether you do you know beam formation beam steering does not really matter, at the end of the day what you are transmitting is your information symbol and what you are getting back is your information symbol that is all I know I am interested in ok. Now, in between whatever you do its your choice. So, ultimately. So, when I say MSE it is obviously, what I sent and what I received its the error between them right.

So, that mean I would like to have some sort of a parameter design by which I can minimize the MSE between these two parameter s and s cap. So, s cap is more of a what you perceive as your transmitted symbol ok. Now, it is perceived that mean you think that is what that would have been transmitted. So, that is something like that. So, it is the MSE it is the mean square error between s and s cap. So, I would like to have the MSE cost function to be build something like that ok minus s cap ok.

Now, this is my cost function, now still I am not done because this is more of a square of course, this is the mean square error. So, when you say mean square error obviously, there has to be mean meaning expectation. Now, so this is my fundamental cost function point, now usually we cannot use directly this kind of cost function. So, there would be some sort of a linearized cost function because the direct MSE formation is very tough to have it.

So, instead of MSE we always have LMMSE because you may not be able to get closed form expression for a MSE all the time though there is no guarantee that LMMSE can give you that, but its a simplistic approach. So, it is a linear mean square error linear minimum mean square error. So, this term is for linear and this term is for minimum and MSE you know mean square error. So, linear minimum mean square error is what we want.

So, if this is your MSE cost function moment it becomes LMMSE, the cost function is slightly changed that mean, this is my data vector original vector and I want to develop some sort of a linear equalizer. So, that mean, my s cap whatever my estimated data and that would be based on some sort of a linear equalizer model. So, what is the original data model? The original data model in our case was r is equal to A s plus v this was the original data model for MIMO beam forming right. So, this is what we are getting if you look at the function here.

So, this is what we are getting originally this is what I was getting. Now that would be something like that. So, now that means, this is my data received data r, I want to develop W BB along with other four matrices, such that the overall MSE between s and s cap is minimized. So, that is the main objective of this function ok. So, this is my original data model. So, whenever you know, whenever you want to develop a cost function you have to always know the received data model ok, otherwise you cannot develop cost function.

So, cost function always depends on your, what you observe. So, your observation vector is very important at what point you take the observation vector matters ok. Now, here this is what we were. So, this would be r vector right this is what we are getting it correct. Now, this is still not done I am still not because I am not only I am not only developing this W BB R I am actually developing everything for other 4 matrices as well right.

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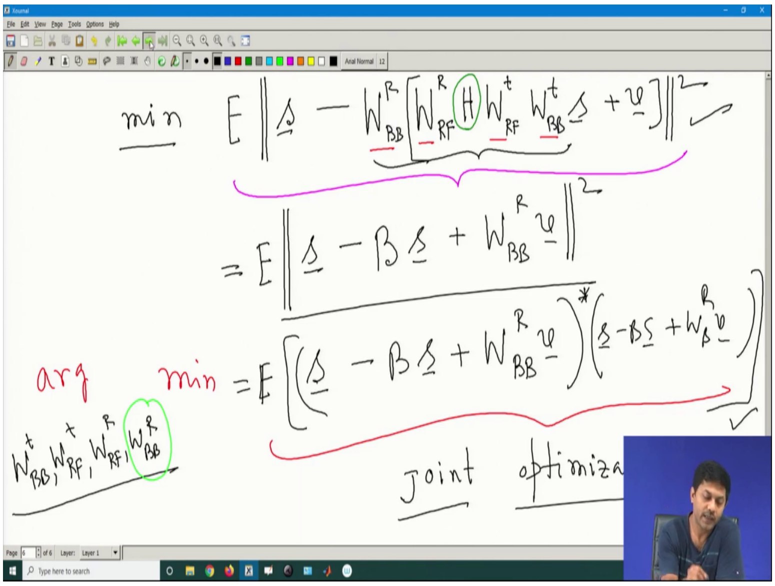

So, now I am just breaking it down. So, it is an expectation s minus though I am not solving it today because as I said there is a separate module for solving these equations. So, today I am only building the cost function part because that is important for us ok. So, now, this should be W BB (Refer Time: 00:00), then you have this W you know RF R. I am just breaking it down ok, then you have H, you have W just put it back everything W RF transmitter side, W BB transmitter side, then what then s was there right, this whole thing was within v cap.

So, I am just breaking it down. So, that is my gigantic cost function, but what is important here is that, who are the variable? On which I want to minimize? So, that mean what I want finally, I want to minimize this particular cost function this cost function how to minimize that.

Now, who are the variable of interest on which I need to optimize this function. So, if you put an effort you can understand this is what I want, this is what I want, this is what I want and this is what this 4 matrix I would like to have it, but pay attention that this is known to me currently ok because I am not including the channel estimation part writer.

So, it means I am assuming that channel is known to me ok, that also you can include it ok that also you can include it in your argument, but for the timing I am going step by step for the timing you assume channel is somehow known to you. Now, we will see next modules that how exactly the channel itself is estimated ok. So, now this is more of a you know. So, you can just you can do some surgery here just for your simplicity you can say this whole thing you can say this is some B matrix ok.

Now, how do you even further break it? So, I think I may have done some techniques I may have shown you some of the techniques where I was showing it like how exactly in LMMSE context, how do you break down this? So, it will be something like expectation of s bar minus B matrix s bar plus and the same thing. Similarly ok you can do that, if you are not interested to break it down I mean I am pretty sure you must have known the other techniques as well that if you say this is slightly clumsy.

You can just from here you do not have to break it completely you can this whole thing you can assume it to be A and then you can you know you can break it down, but this breaking down is always easy because there you can actually see what are the parameter that is coming into picture, now this you have to estimate. Now, I am not going to do further processing of this particular equation because that is a part of our next module. So, I am just only putting the what are the different cost function that is coming into picture.

Now, this particular cost function you would like to minimize it, this whole cost function you would like to minimize it. So, I would like to minimize this cost function over who are the argument meaning what are the variables on which I want to optimize this function; obviously, the other 4. So, this will be all laundry list of your matrices. Now, this is the stock difference between what I said today compared to the earlier one. So, earlier one there also I was you know touching upon some of the optimization things.

But there I was more interested to find out this particular one the last one ok. So, there I was assuming that the other first three are known to you, but now today what I talked about is that given the H, how do I figure how do I find all the four matrices jointly together? Ok. Now, this sort of things are called jointly optimized. So, these are all joint optimization. Now, whether it will produce a closed form or do not produce a closed form that really that matters there is no such guarantee.

So, but so this is called a joint optimization why it is joint because I am jointly you know figure out figuring out who are the matrices. So, all you know can be figured out all can be found out by this optimization. So, exact methods obviously, explain later. So, this is only the cost function development this is the one such MSE cost function development ok. So, next go to I am not being the least square because least square is more of A you have to just remove the expectation part and that becomes your least square.

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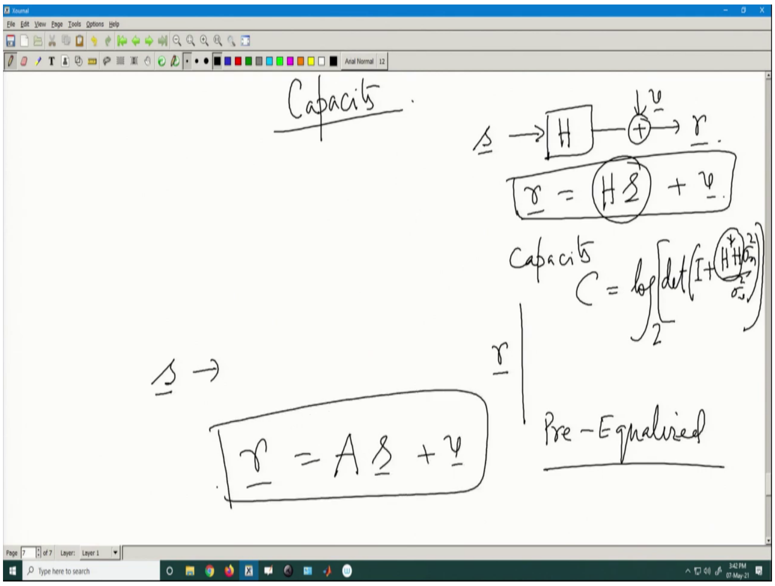

So, I am now going to the capacity part. So, what if I optimize it based on capacity ok. So, what is the capacity formula. So, if you look at the way we have proceeded is that ah, whatever cost function you want to develop we already know the formula for the cost function that is it ok. So, we know MSE. So, from MSE we know what is called MSE and then we just proceeded. Now, when I say I am picking up some other cost function. So, for that I have to know the formula see if I say my capacity is my goal, that mean I want to increase my capacity.

So, I have to first develop what is the capacity of this system. So, in this in a very simplistic sense, how do I you know how do I think about the capacity formula for such systems? That you have to first figure out ok. So, this is my mathematical model where I am here. So, this is my complete mathematical model right, I would like to have all these 4 based on my capacity that mean my W BB W RF t and R all this force should be such that the whole system gives you the; gives you the maximum of my capacity that is as simple as that right.

So, how to develop it? Ok let us see. So, now, suppose I have a MIMO system let us start from there suppose I have a s as a system and this is my channel it goes through the channel and there is a noise addition and I am receiving say r vector. So, what is my data model? So, that is my data model. So, what is the capacity of this system?

The capacity of this system is if I do not have an equalizer, but if I have an equalizer, what is the capacity of this system? Suppose if I have no capacity if I do not consider any equalizer in that case, what is the capacity? Capacity will be C if I say C is my capacity it is a log base 2 determinant of I plus SNR ok.

So, SNR of this matrix system, SNR of this vector system say. Now, this SNR will be it will come to me some sort of a matrix format it I should not write an SMR because that may be confusing term for you. So, this will be H star H into sigma square x by sigma square v provided the noise is iid, so that is the case.

So, its basically it depends on H, now I just extend the similar thing to the other part of my story. So, what is my; what is my data model here, in this case I would say my data model is s cap right and it goes through all the data and finally, I am getting r cap, it is a pre equalized data; pre equalized data. So, what is that? It is r was A into s bar plus v this was the case right, this is what I have seen for the MIMO beam forming case I am not doing equalization yet ok.

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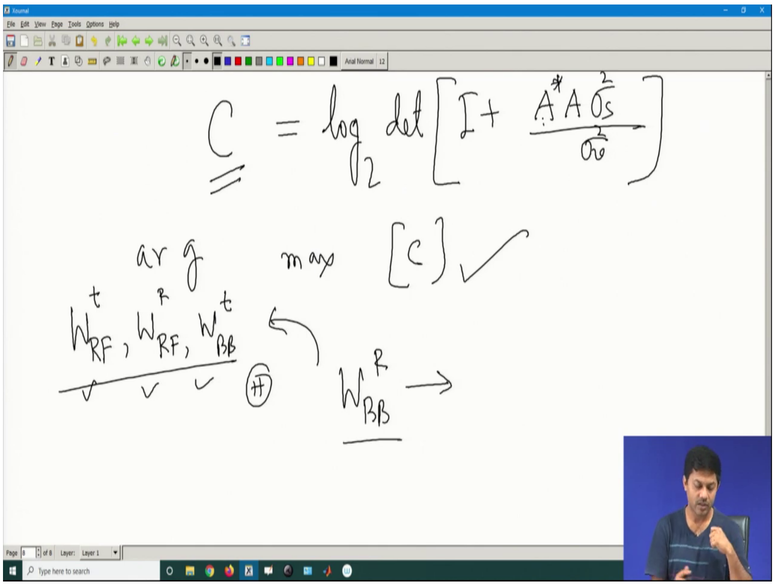

So, how can I develop the cost function for such system? Well, the capacity of that system would be log base 2 determinant of I plus A star A, let us assume my data is a iid data sigma v. So, there is a n and all these things will be there. So, I should you can just adjust it in the end. So, there is a N term comes N r terms come, that is adjusted in a square a I am assuming that part. So, I am not drawing that.

So, some similar kind of thing, so this is my cost function ok, what do I want? I want to maximize this maximize this C, on what? Over all my argument, but be careful here I am taking only a right, but a does not contain the final baseband equalizer right that is ok. So, you can have as many as you want. So, this W RF t this W RF R and then W BB t, but you notice W BB R is not there. So, at least this can be a cost function development. So, once you get all these three based on that what? Based on that you can get the W BB R also.

So, W BB R you can find out because if this 3 are known as well as the channel then you can find W BB R using some other criteria say for example, LMMSE criteria that is I think we have explained it. Because if I know rest of the three I can also know in the first case in this case; in this case I am not discriminating W BB R. So, all four are you know put together, but when I am doing our second case I can split it.

I can you know I can first find out these three based on this capacity formula because capacity is between that chunk after that if I want to develop another equalizer, but also this W BB can also be plugged into this equation based on how exactly you want to build up the post processing cost capacity, but this is a pre you know pre equalized data based on that I developed one cost function. Now, this a contains all these three. So, this could be; this could be a cost function model. So, like that you can develop your own cost function and you can have your own optimization method.

So, at the end of the day using this optimization method you can find out all these four matrices, but again I am telling you it is a strategy that is there is no; there is no limit how exactly you create your own strategy ok, data model is given that is generic that is how the that is what the explanation is, but optimizations how do you find out this its completely up to you.

How you develop your cost functions I have shown only few of them sky is the limit you can have your own cost function ok. Another cost function could be a b e r or s e r bit error rate or symbol error rate and finding out such kind of things is even tougher, but I am saying that theoretically you can also get that.

So, my point here is that you build up your cost function and create a optimization framework based on that cost function and that can you know that can give you how things will be there. Now, how to solve this we have developed the cost function fine we have developed cost function we have you know we have created the framework, but this is nothing this is first step first step is the cost function now how do you solve it the method I was talking right this one this method second part the method what should be the method to solve it.

So, that I will be explaining in another module ok. So, with this I conclude this particular session. So, next class I will be talking or moving to the MIMO ofdm beam forming. Once that part is finished I will come back to the optimization frameworks method part ok because MIMO optimization is another interesting part where several aspect will come into picture there and I feel that would should be the next topics there ok. So, with this I conclude the session here.

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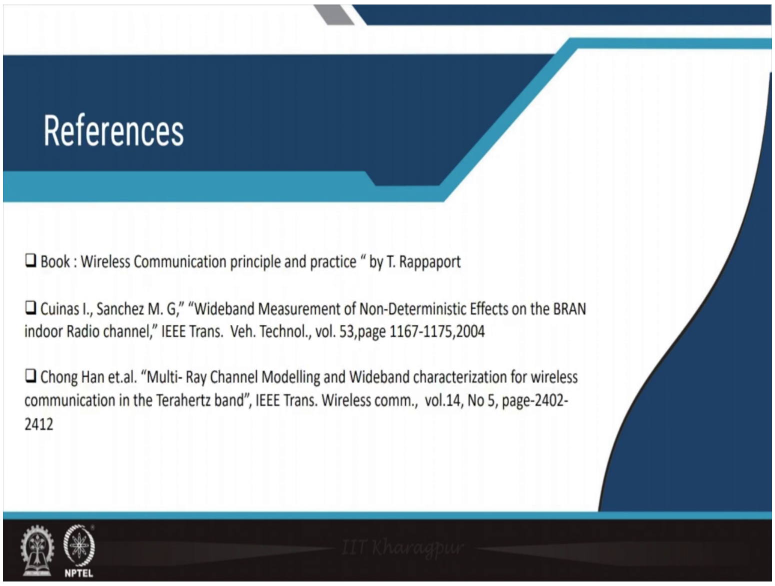

So, in conclusion we have kind of completed the yeah and the reference will be same as what we have.