Signal Processing for mmWave Communication for 5G and Beyond
Prof. Amit Kumar Dutta
G. S. Sanyal School of Telecommunication
Indian Institute of Technology, Kharagpur

Module - 09
Hybrid beamforming concept and Beamforming in MIMO
Lecture - 46
Mathematical description of M I M O Beamforming (cont'd)

Welcome, welcome to the millimeter Wave for 5G and beyond. So, in the last class what we have discussed is that we have to do a data detection.

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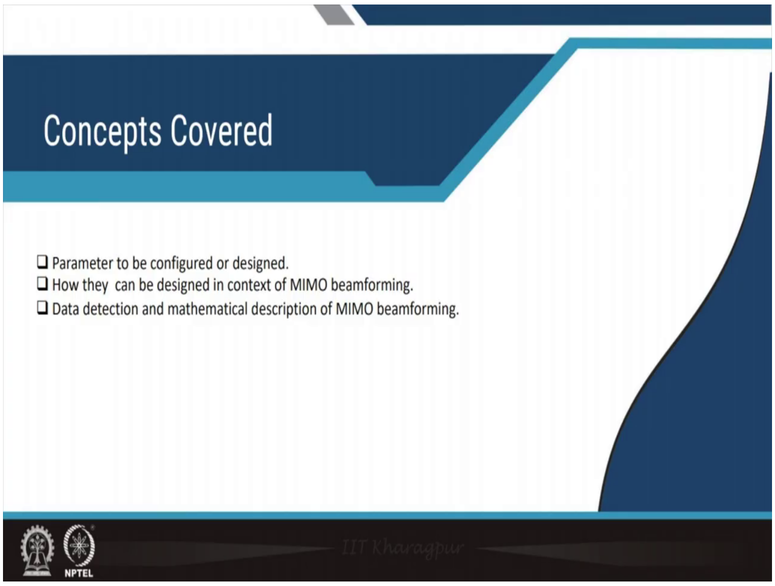

So, today we will be covering the like things that we will be covering are the following. So, the first step of the data detection was more of a R F side.

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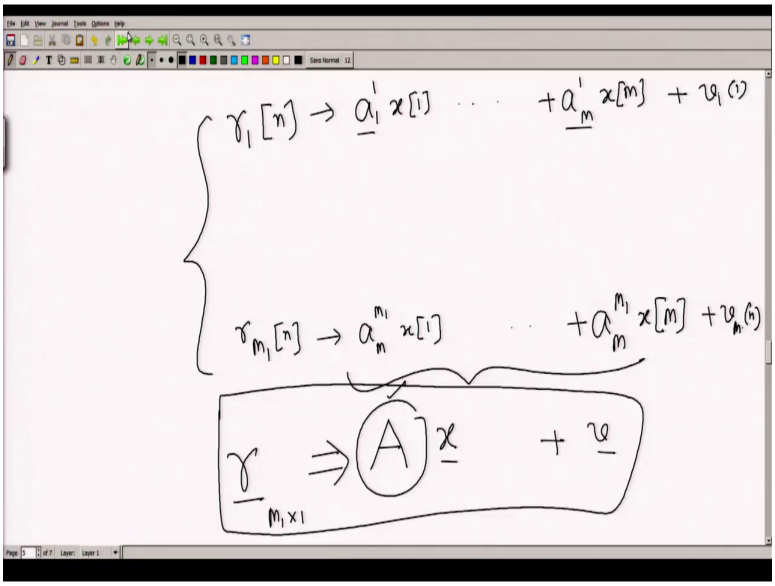

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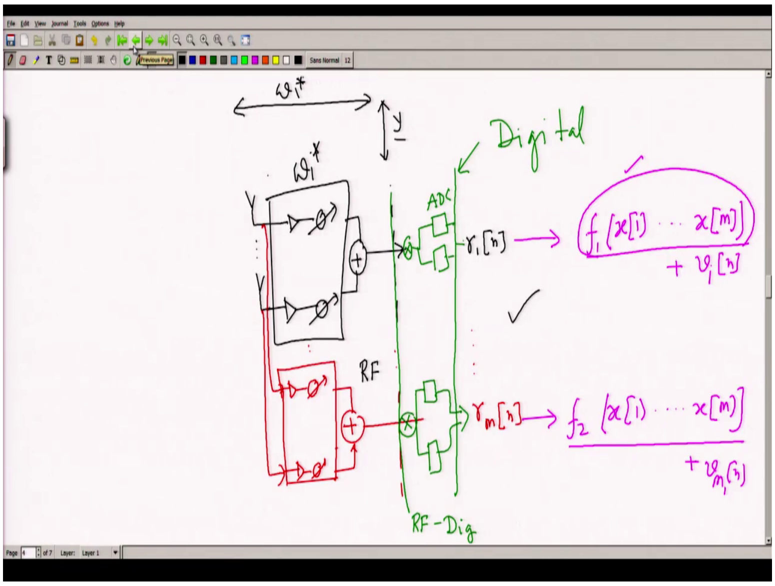

So, if I look go back to my diagram here again. So, this is my received antenna and from the received antenna, I do not directly get into the digital side. So, what I do? I first do a first level of extraction first level of in equalizer design and that is nothing but a phase shifter ok keeping the amplitude parts to the l n n. So, this is more of a phase shifter part.

Then we said once I do a reverse phase shifting, I get back my digital data here and that digital data gives me a data model like that r bar equal to A s plus v bar ok.

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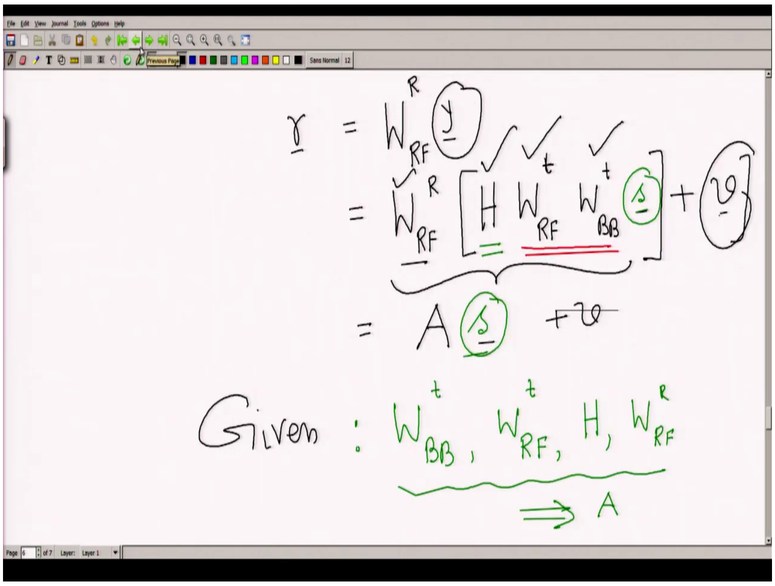

So, which means that now with all my R F extraction my R F level equalizer development, I get a digital data stream where still also some residual effect of channel and beam forming those parts are there. What is my intention? My intentions to get back my original data that I have transmitted. So, that is my key part of the story. So, which means that I need to know I have to do another level of processing, because I still have not got back my s.

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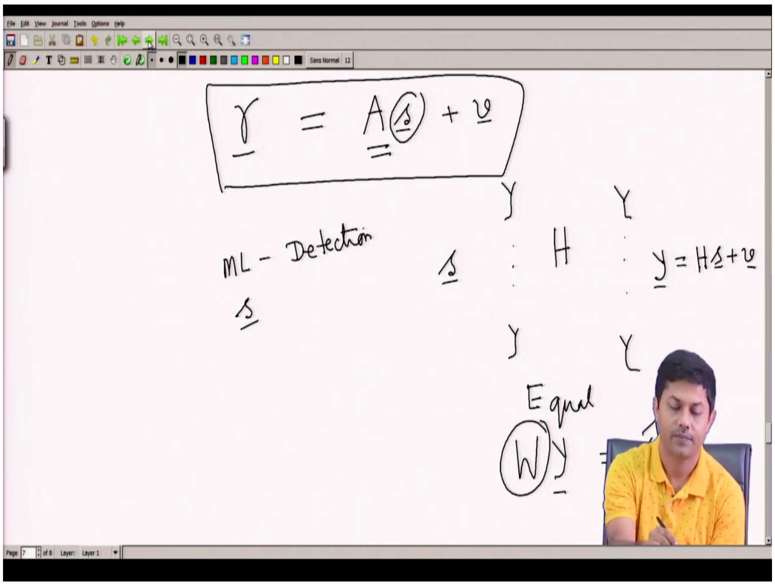

So, now I got r cap r vector which is equal to A which is known to me A is known to me; obviously, because if you look at I make a strong comment here. If this four matrices W BB two or this W RF H W RF t W BB t; all these four matrixes are known to us I can at least I know what is the a part here, because all these four matrix multiplication are nothing but this A ok.

So, now this A this is what is coming right. Now, the issue is different. Now, I have not ended my story. Next is that how do you get back my s. Now, that is a simpler task, because I am in a digital domain ok. So, if you remember what we did in the MIMO case similar thing we did right. See if you have a similar case in the MIMO we did the similar case. So, for example, this is my H. I send let me put a let me not mess up the notation and this is my y. So, what was my the data? y is equal to H of s plus v.

So, how did you what did you do if it is a detection. I can do many things. I can do a m l detection I can do equalizer development whatever its sky is the limit for me. See if it is a equalizer based detection, what I do. I develop some sort of equalizer base matrix W which I multiplied with y and I get back my s cap estimation. This is what I did and how do you develop this W.

There are lot of such cost function. One was you know what was that l m m s e it can be least square it can be any other cost function you take it and you can develop your equalizer. So, now, essentially you probably may be smelling that now again, I am back to another equalizer development. So, that see there are so many equalizers now.

So, now this is what exactly happens when you go for a beam forming. So, the first matrix is this one. This is a first level of equalizer. What it does. It is mainly the phase shifting kind of thing ok, because your l n a is anyway coming. So, it is a very simple phase shifter level of kind of W RF, but amplitude I can push it back to the. So, that was the actual structure this how the structures are developed here ok.

Now, but this is in the RF domain. So, W RF I am repeating W RF is the RF domain equalizer. So, this W RF R is more of a RF domain equalizer, but what is not true what is not convenient for us is that we still have the A. So, now, our next level of activity will be to develop the A I mean to equalize the A. Now, if I am done here if somebody gives me that. Can I extract my s given A. Yes, I can do ML I can simply do ML detection and get my s ok. So, it is my choice. So, how do I do a ML detection for such kind of things.

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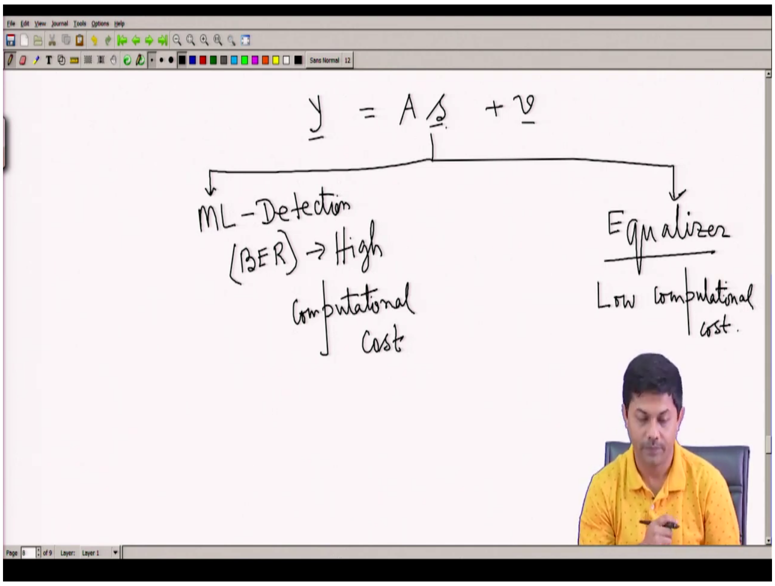

Now, you may be finding lot of literature that after this s, I can directly go for another level equalizer, but I prefer that it is your choice ok. If you really do not want to get into equalizer level of detection it is ok. If you want to do ML detection that is also ok. So, from here to extract s, I have two paths two predominant path. One is a very high cost that is kind of an M L and that is the best; that is the best in what sense? Best in BER sense that will be the best BER. If you do not, but cost is very high. So, its very high cost I can say high computational cost.

So, I will just touch up on some of them, because this topic is more on the detection side high computational will cost ok and this is a low computational cost equalizer based. Advantage is that it is a low computational cost. So, these are the two main you know main thread of detection mechanism. So, it is your choice how do you want to do that this is ML detection this equalizes low computational cost ok.

So, we will discuss both of them, because I do not want to get into another level of sudden equalizer and there should be a logic behind it right. Though it is it has nothing to do with beam forming. So, the aspect of beam forming is nullified now. The moment I multiplied with W RF R, beam forming is over.

Now, what all I have now what I am left with is a simple digital data stream ok having a matrix multiplied by another data stream plus noise the simple linear model and from that I have two mechanism of extracting direction that is all. So, what is beam forming? Beam forming is over ok. So, this was the case now.

Now, let me just describe little bit on the ML detection part also, because that can be its your choice right, because if the number of data streams are less say two or three digital data stream like s. If it is just three or four, I can do ML if the computational cost permits me ok.

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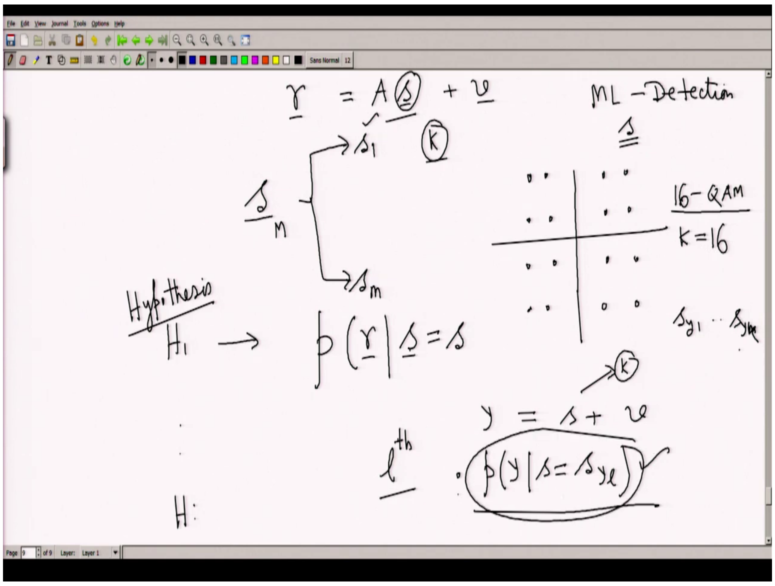

So, let us go back here. So, let me write again ok. Now, I want to do a ML detection. ML detection for s bar ok. What should I do or how do I do that ok. So, ML also is a cost function right. So, you have to first develop the cost function. So, first you have to develop what is your maximum likelihood function. So, what is the you know cost function here? The cost function here is the maximum likelihood function ok.

So, first of all what is the cost function in this case. So, if s has a dimension say M that is the number of data stream that we have sent it. So, which means that s can be s 1 I will I should not put I can put it this way, but it may be confusing I put it this way s 1 to say, s M. So, that many number of individual lines are there and this is a vector right ok and. So, that mean it has m number of elements and.

Let us say or let us say it takes a constellation which is K constellation K number of constellation it has a say, for example, QAM ok say 64 QAM or 16 QAM or that kind of things. So, let us say I have. So, that is the constellation. So, let us say it has chosen individual s 1 to s M has chosen a data from K number of constellation points. So, it can be something like that if it is just as a 64 QAM individual 60 sorry 16 square it will be something like that.

So, it is a 16 QAM case ok. So, 16 QAM case. So, in this case K is equal to 16 so; that means, every individual s 1 to s M can pick up any of these points. So, if it is an ML as you know when you go for a ML detection it requires a hypothesis right. So, what is the hypothesis here in this case? So, how many hypotheses will be there? So, I can say the number of hypotheses depends on how many number of possibilities that s can have. s is a m length vector right and each point can come from K number of constellation right.

And what does a cost function for ML? Its a basically a maximum likelihood function. So, I need to choose, I have to first create maximum likelihood function for each and every s and then I have to see which is the best maximum likelihood that is the fundamental point. Now; that means, that it comes to a point where I have need to find out how many such hypothesis points, I can have it.

So, how many those hypothesis points that I have? See if I have if I start from H 1 to another H. So, these are all called hypothesis, and I have to choose the best hypothesis corresponding to particular s. So, what is that? So, how do you create a generalized hypothesis? Basically, its a probability of my r vector given my s vector is some. I mean it is basically for a particular point right ok.

So, for example, how did you do for a single case, meaning single antenna case say y is equal to s plus w or w [FL] v when this was the case it is just. So, m is just 1, but it still have K number of constellation point. So, what was your hypothesis? It was a probability of y given s is equal to one of this; one of this constellation points.

So, let us say these constellations points are say, s 1 to s K; these are my constellation points. I let me put a different notation, because this is a symbol. So, let us say, s y 1 to s y m or K ok. So, this s could be one of the constellation point. Say let us say I pick up the lth constellation point. So, this is my hypothesis function. It is my lth hypothesis function; lth hypothesis functions; it is my lth hypothesis or lth maximum likelihood function is this.

Now, this I will pick up and I will create K number of such hypothesis function, I choose the best out of it; that is basically my M L right. So, which means that I need to construct this hypothesis function. Now, when it is a vector. So, what does it mean how many such hypothesis point can come? It depends on what is the total number of possibilities of my input vector or the input signal.

Now, for a scalar it is easy to do, because it depends on number of constellation point, but when it is a vector, like s is a vector in our case. So, how many such hypothesis point I will get? That mean, how many such possibilities of my s vector would be that is the matter.

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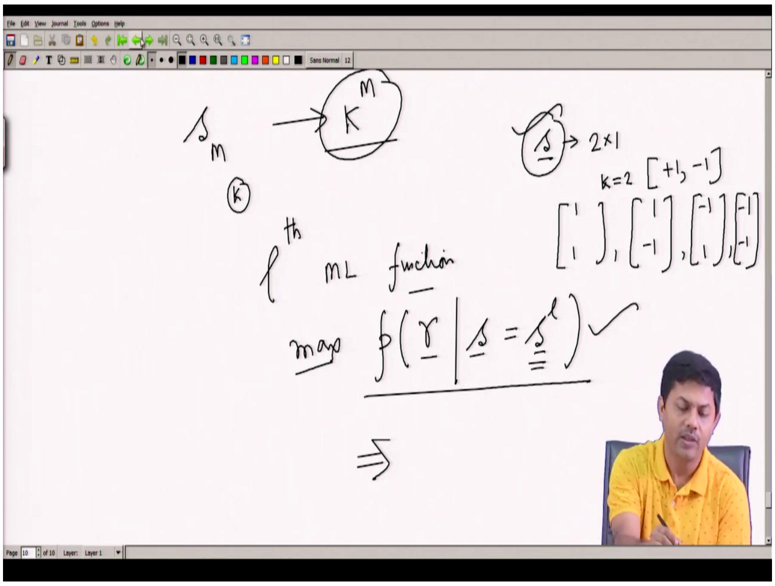

So, that is nothing but so if s has a length M and each one comes from K number of you know hypothesis. So, how many such points I can have? I have K to the power M right. So, that many number of hypothesis I can created. So, I can just take a generalized hypothesis. So, what is my lth cost function lth maximum likelihood function? What is my lth?

So, this is K to the power M number of hypothesis there and what is my lth M L function? It is nothing but probability of my r vector. Given that my s vector is nothing but something like that. So, this is one of the realization of this vector. Say for example, I give a very simple example, say my s vector has length 2. So, it is a 2 cross 1 vector.

And each and every s coming from a v p s k data. So, the constellation is K is equal to 2 and this is nothing but plus 1 and minus 1. So, how many such s vector possible, its 2 to the power 2 4 who are they? This will be 1 1 1 minus 1 minus 1 1 minus 1 minus 1. So, these are the four possibilities that s vector can have right. So, I am talking of that. So, s l meaning lth realization. So, if there are four it could be one of the four realizations.

So, what does it mean? So, what it means? So, this is the function that I need to find it and then I maximize, maximize this function basic same. I need to find out K to the power M number of such function, I need to find out K to the power M number function and then, I need to find out the maximum of that function that is basically my M L. So, the fundamental point is to find out that function.

So, what does it means? This comes. Now, I am stuck here. So, I cannot move any more forward, unless I know the distribution of this noise and all this and you know I think if somebody has taken detection estimation courses.

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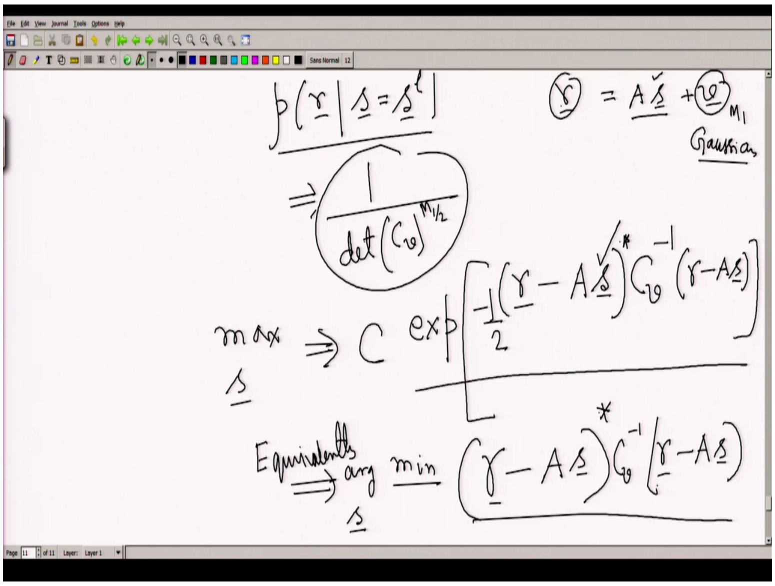

So, from there this probability of r given s vector is one of the realization, will be some sort of a if it is a. If r vector is A plus s vector plus noise. So, what does it mean? Given s this r is nothing but a Gaussian as simple as that right. And given s what is that? That means, this whole thing is known. What is left is a Gaussian vector if this is Gaussian.

See if this is Gaussian, this r will also be Gaussian if s is known. So; that means, this whole thing will also be a Gaussian provided v is known. So, if that is the if that is the case, then what will be the distribution of it, because that is what I am interested right.

Determinant of, if v has a covariance matrix you know C v this to the power if there are you know if the length is say M or M 1 is M 1 by 2. So, I am not interested really how this can be really constructed here. So, I do not get any kind of advantage here by considering this whole point. So, this could be some sort of a constant, so let us call it C constant exponential of r vector minus ok. If this C v I mean covariance matrix has that this will be r, this may be a star.

So, this is my cost function comes. Who is the variable? Yes, so, what is my job? I need to maximize this cost function over my s bar, that mean; I need to choose that particular s vector which for which this will be. Sorry, this will be minimum yeah this will be maximum yeah. So, my cost function is the maximum one.

So, now there are standard methods by which, so; that means, maximizing mean I take a log and all these things. So, this will be equivalently this comes around r minus A s vector. So, finally, instead of maximizing this becomes a minimizing function, because there is a negative here. And this is the case minimizing over the argument. This is my cost function.

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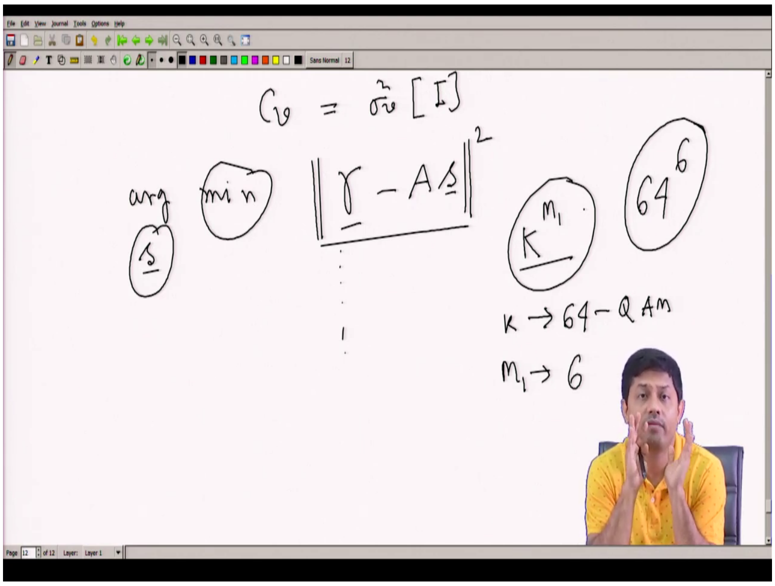

So, what does it mean? It means that if C v is an identity matrix. What when it happens? When all the noises are independent noise. In that case what is my cost function? It will be r vector minus A s vector mod square, the C v does not have an impact. So, minimize this particular cost function, two norm of this cost function with argument of all s; that is my M L detection. So, what does it mean? Now, I have to go back. So, this means that if I have K number of constellation and M 1 number of data length. So, K to the power M 1 number of realization of s vector will be happening.

So, I will create that many number of points and take the minimum of them, it is a standard M L method ok. Now, we can see this whole thing is very costly why it is a costly thing? Because the K can be large M 1 can be large and you cannot create so many such search points. Suppose K is a I say 64 its a 64 Q A M and M 1 that particular how many number of so data stream say I have created say not less than say 6.

So, you can imagine 64 to the power 6 that is a huge number. That many number of such points will be created and you have to choose the list of it, which what value of s bar will be that is a very huge search mechanism. So, if somebody has done M l detection, he probably he or she may be aware of that such a things may or may not be possible depending on how complex. But I have why I have talked about all this, because you may be given a chance for M 1 as well.

So, I do not want to get into an equalizer-based solution all the time ok. I should also give a fair chance to the M L detection. Now, it is up to us to decide whether I really want to go for a M L detection part or I want to go for other low-cost part. Now you can see this is a very high-cost detection method, but here the number of equalizers stops. That mean I just need only one equalizer at the receiver that is nothing but my W R F R. So, that is the only equalizer I have it. This is the only equalizer I have it ok.

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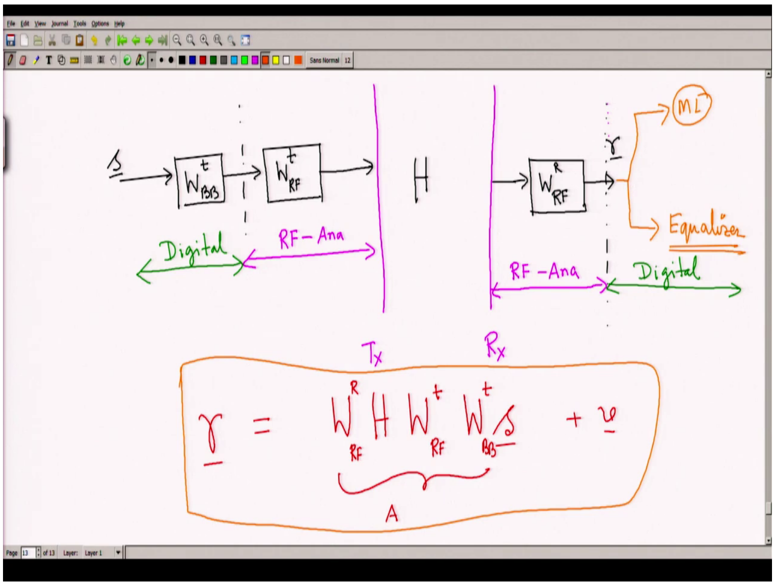

Now, to be frank what is my digital way of data model creation? Digital way of data model creation would be something like that. You have transmitter side; you have W BB baseband decoder and I put t just to ensure this is a transmitter side. Then, I do R F level. This is at the RF level W RF t side.

So, if I send s vector from here I. So, this is a; this is a mathematical model rather than a physical model. So, whatever diagram I have shown earlier they are all physically how they are connected and how the structure is. Now, this is a mathematical way of understanding. Now, this is my transmitter side ok. This part RF analog side, this part digital side ok.

Now, when it go to the receiver end and there is a after that what will happen? There is a channel here. Again, digital side starts, again the antenna side starts then here I have. So, this is my antenna side, this is my R X side; t X side this is R X side and at the receiver side and I stop at that moment, because I did not go anything beyond this. Now, from here, I get r vector.

Then you say this part again is my RF and analog ok, but here this r vector I am saying I am getting a digital side, because just after this RF I digitize it and get my. So, this is my digital side. From here, the digital side starts from my digital sides. Now, now from here, I can have two way of detection; one is an ML, another is equalizer-based. So, this is the complete pictorial diagram that I have here.

So, nut means; if I stand which means if I stand at the digital side, I would see r vector, it is a digital vector, how do I get it? So, this is my transmitted s vector which will be modified by W BB t, which will be further modified by W RF t, then it will be multiplied by channel part, then it will be multiplied by W RF r part that is it. So, this whole four part was my air layer and then I have this one.

So, this is a very nice pictorial view of my you know data models, where my digital received data I got it. So, this is my digital received data which I have obtained by calculating the different equalizer. Now, you may ask the questions how do I determine all these parameters and how do I you know how do I exactly estimate all these parameters? So, that is something that definitely we will be covering it.

But given this four matrixes, I think you tentatively know how it can be. The whole data model is like that. Now, this do and everything has a purpose ok. W BB has a purpose, because the purpose I think we have explained it. Then W RF t has a purpose that was more of a beam forming part.

And we have said that W BB t is very much essential and we have told you the reasons why W BB t or precoder digital precoder is essential in the beam forming case. Then H part channel part; obviously, that will be the physics of the air and W RF R. So, all four components together will form my digital received data and I have just explained the M L detection part.

So, in the next class I will be talking more on the equalizer side or the low-cost equalizer part in the next class. So, with this I conclude this particular session. And we will next move to the equalizer-based detector and there will then we will see one more matrix will appear because of that.

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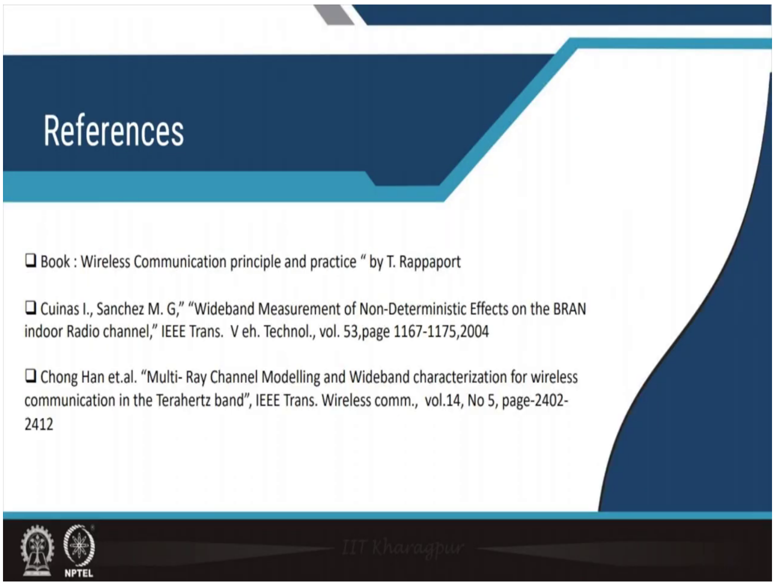

I conclude. So, in conclusion we have kind of completed the yeah and the reference will be same as what we have.